



Social Network Analysis: Unit 5

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AUEB, Master in Data Science, November 8, 2023

Outline of SNA

Properties	Models	Algorithms
Small diameter Edge clustering	Small-world model Erdős-Rényi model	Decentralized search
Scale-free	Preferential attachment Copying model	PageRank Hubs and authorities
Weak ties' strength Core-periphery	Kronecker graphs	Community detection
Densification power-law Shrinking diameters	Microscopic model of evolving networks	Link prediction Supervised random walks
Information virality	Independent cascade model Game-theoretic model	Influence maximiza- tion Outbreak detection

Outline:

- Network resilience
- Link prediction and network inference
- Social learning on networks

In real world, networks can be vulnerable!
How resilient are they to node/edge losses?

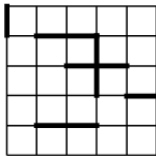
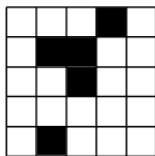
- How node or edge percolation affects the network functionality?
- Resilience of randomly vs. preferentially grown networks
- Resilience to random failures vs. targeted attacks

Network Resilience

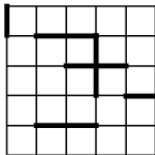
Assume that a fraction of nodes or edges are removed.
In the remaining graph,

- how large are the connected components?
- what is the average distance between nodes in the components?

This problem is related to percolation.



Edge removal and edge percolation



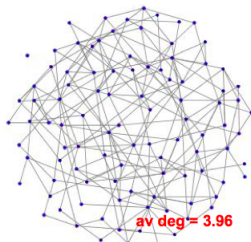
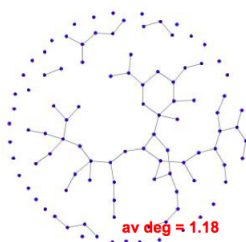
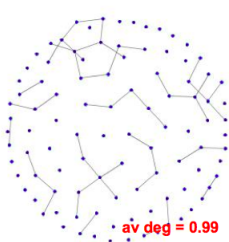
- bond percolation: each edge is removed with probability $(1 - p)$
It corresponds to random failure of links.
- targeted attack: causing the most damage to the network with the removal of the fewest edges
 - strategies: remove edges that are most likely to break apart the network or lengthen the average shortest path
e.g. edges with high betweenness

percolation in ER graphs

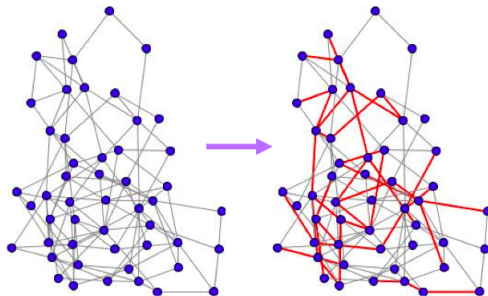
Attacks: reverse process of adding edges

As the average degree increases to 1, a giant component suddenly appears.

Edge removal is the opposite process – at some point the average degree drops below 1 and the network becomes disconnected.



Quiz

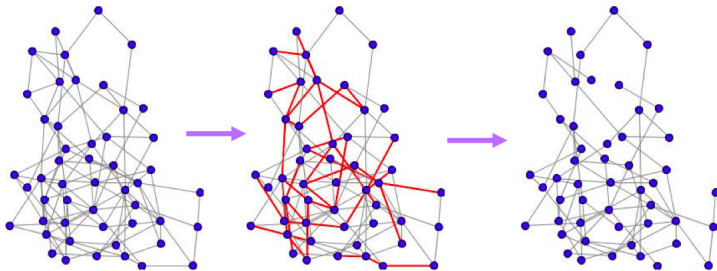


Quiz

In this network each node has average degree 4.64.

If you removed 25% of the edges, by how much would you reduce the giant component?

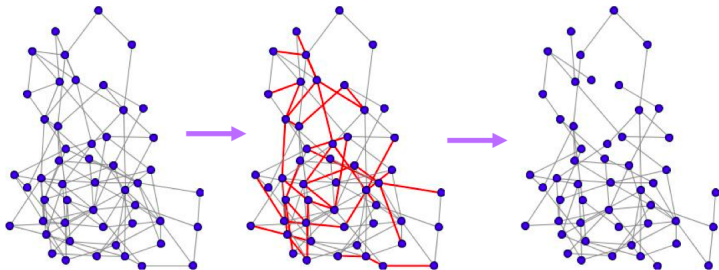
Quiz (cont.)



Answer

- 50 nodes, 116 edges, average degree 4.64
- After the 25% edge removal:
76 edges, average degree 3.04 – still well above percolation threshold

Quiz (cont.)

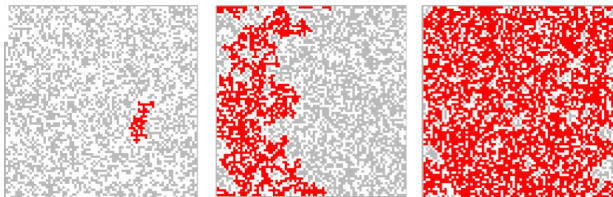


Answer

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Ordinary Site Percolation on Lattices:

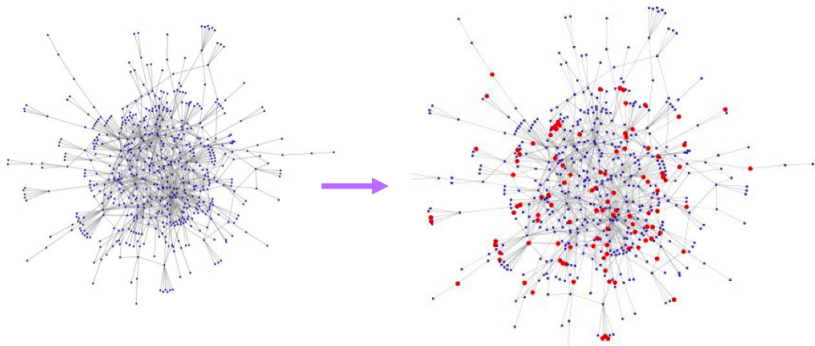
Fill in each site (site percolation) with probability p



- low p : small islands
- p critical: giant component forms, occupying finite fraction of infinite lattice
- p above critical value: giant component occupies an increasingly larger portion of the graph

<http://www.ladamic.com/netlearn/NetLogo501/LatticePercolation.html>

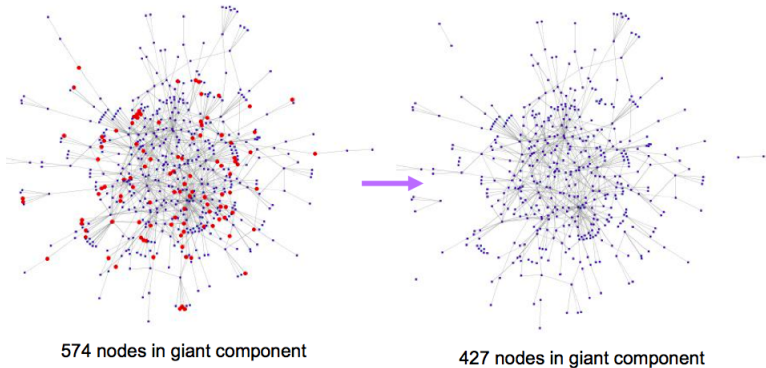
Percolation on networks



- Percolation can be extended to networks of arbitrary topology.
- We say the network percolates when a giant component forms.

Random attack on scale-free networks

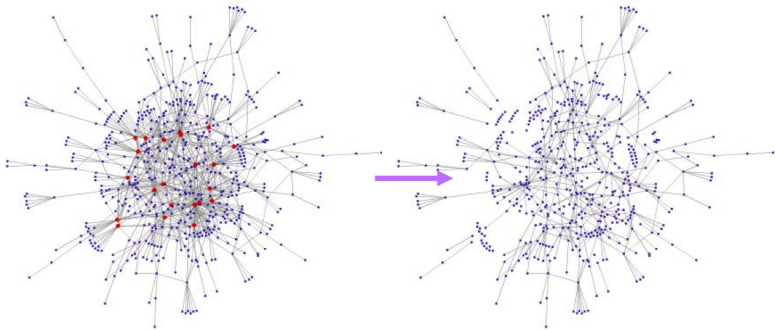
Example: gnutella filesharing network, 20% of nodes removed at random



Targeted attacks on power-law networks

Power-law networks are vulnerable to targeted attack

Example: same gnutella network, 22 most connected nodes removed (2.8% of the nodes)



574 nodes in giant component

301 nodes in giant component

Quiz

Why is removing high-degree nodes more effective?

- it removes more nodes
- it removes more edges
- it targets the periphery of the network

Answer

Removing high-degree nodes causes the removal of more edges.

Quiz

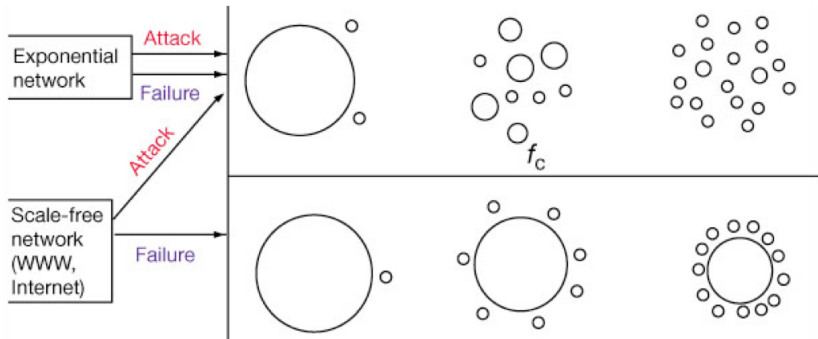
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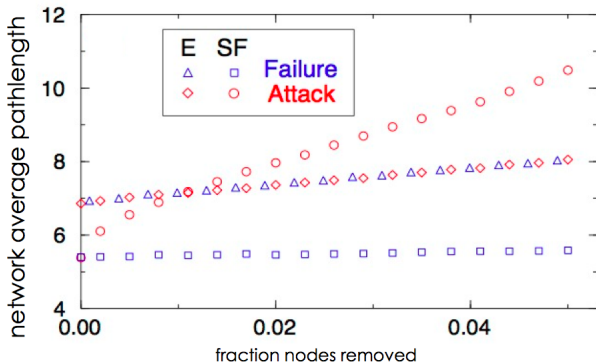
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Random failures vs. Attacks



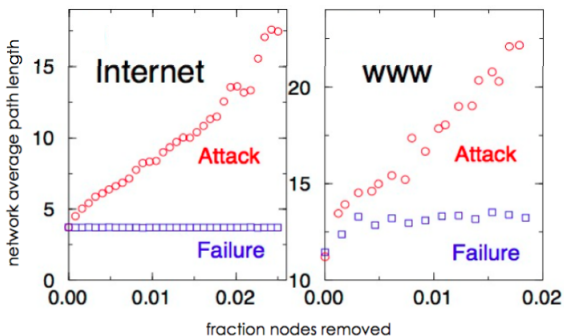
Source: Error and attack tolerance of complex networks. Reka Albert, Hawoong Jeong and Albert-Laszlo Barabasi. Nature 406, 378-382(27 July 2000);
<http://www.nature.com/nature/journal/v406/n6794/abs/406378A0.html>

effect on path length



Source: Error and attack tolerance of complex networks. Reka Albert, Hawoong Jeong and Albert-Laszlo Barabasi. Nature 406, 378-382(27 July 2000); <http://www.nature.com/nature/journal/v406/n6794/abs/406378A0.html>

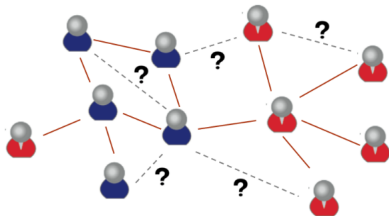
effect on path length of real networks



Source: Error and attack tolerance of complex networks. Reka Albert, Hawoong Jeong and Albert-Laszlo Barabasi. Nature 406, 378-382(27 July 2000); <http://www.nature.com/nature/journal/v406/n6794/abs/406378A0.html>

Link prediction

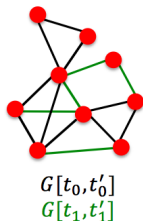
Given a network, which links are possible to be established next?



Link prediction and Network Inference

The link prediction problem

Given $G[t_0, t'_0]$ a graph on edges up to time t'_0 , output a ranked list L of links (not in $G[t_0, t'_0]$) that are predicted to appear in $G[t_1, t'_1]$.



Evaluation of a solution: If n is the number of new edges that appear during the test period $[t_1, t'_1]$, take the top n elements of L and count the correct edges.

Link prediction via Proximity

Since network data are very sparse, we usually consider a core, e.g., we take into account only nodes with degree of at least 3 (we don't know enough about nodes with less than 3 edges to make good inferences).

Algorithm:

- For each pair of nodes (x, y) compute score $c(x, y)$
For example, $c(x, y)$ could be the number of common neighbors of x and y
- Sort pairs (x, y) by the decreasing score $c(x, y)$
Note: Only consider / predict edges where both endpoints are in the core (degree ≥ 3)
- Predict top n pairs as new links
- See which of these links actually appear in $G[t_1, t'_1]$

Social learning

The process according to which which agents in a network learn about an underlying state by observing their neighbors' choices.

Models:

- Bayesian social learning
- Myopic (Non-Bayesian) social learning
- Bayesian observational social learning
Users observe past actions – Most relevant for markets
- Bayesian communication social learning
Communication of beliefs or estimates – Most relevant for friendship networks

Myopic Learning in Social Networks

- It was first introduced by DeGroot (1974) and more recently analyzed by Golub and Jackson (2007).
- Beliefs are updated by taking weighted averages of the neighbors' beliefs
- A finite set $\{1, \dots, n\}$ of agents
- Interactions captured by an $n \times n$ nonnegative interaction matrix W
 $w_{ij} > 0$ indicates the trust or weight that i puts on j
 W is a stochastic matrix (row sum=1)
- There is an underlying state of the world $\theta \in R$
- Each agent has initial belief $z_i(0)$; we assume $\theta = \frac{1}{n} \sum_{i=1}^n z_i(0)$
- Each agent at time k updates his belief $z_i(k)$

Some practical applications of SNA:

- Identifying important people (those linked by strong social ties) within an individual's network neighborhood, e.g., romantic partners
- Identifying influential people in a network
- Detecting outbreaks of contagious diseases

- Market trends identification
- Identification of opinion trends
- Biological networks
- etc

We discussed the following aspects:

- Network resilience
- Link prediction and network inference
- Social learning on networks

Concluding Remarks

Thank you for attending the course!

What I hope you got out of it:

- What are the key properties of empirically observed networks.
- What are the processes that shape them and help us understand the empirical observations.
- What implications of the processes on the functions the network is supposed to carry out.
- Research trends (also biological networks!).

Textbooks:

- David Easley and Jon Kleinberg, *Networks, crowds, and markets*, Cambridge University Press, 2010.
- Jure Leskovec, Anand Rajaraman and Jeffrey David Ullman, *Mining of massive datasets*, Cambridge University Press, 2014.
- M. E. J. Newman, *Networks – An Introduction*, Oxford University Press, 2010 (deriving knowledge using math and analytics, community detection, etc).
- Matthew O. Jackson, *Social and Economic Networks*, Princeton University Press, 2008 (how is a network created by self-interested users?).
- Albert-Làszlò Barabási, *Network Science*, Cambridge University Press, 2015 (for beginners).