Πρόβλημα

$$x_{n+1} = x_n + y_n$$
$$y_{n+1} = -x_n$$
$$x_0 = 0$$
$$y_1 = 1$$

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$$

Χαρακτηριστική εξίσωση. $\begin{vmatrix} 1-\lambda & 1 \\ -1 & -\lambda \end{vmatrix} = \lambda^2 - \lambda + 1 = 0$

$$\lambda_1 = \frac{1+i\sqrt{3}}{2} = e^{i\frac{\pi}{3}}$$
$$\lambda_2 = \frac{1-i\sqrt{3}}{2} = e^{-i\frac{\pi}{3}}$$

Τύπος του Sylvester

$$\boxed{A^n = \lambda_1^n \frac{A - \lambda_2 I}{\lambda_1 - \lambda_2} + \lambda_2^n \frac{A - \lambda_1 I}{\lambda_2 - \lambda_1}}$$

$$A^n = \frac{e^{in\frac{\pi}{3}}}{i\sqrt{3}} \begin{bmatrix} 1 - \frac{1-i\sqrt{3}}{2} & 1 \\ -1 & -\frac{1-i\sqrt{3}}{2} \end{bmatrix} - \frac{e^{-\frac{in\pi}{3}}}{i\sqrt{3}} \begin{bmatrix} 1 - \frac{1+i\sqrt{3}}{2} & 1 \\ -1 & -\frac{1+i\sqrt{3}}{2} \end{bmatrix}$$

$$A^n = \frac{e^{\frac{in\pi}{3}}}{i\sqrt{3}} \begin{bmatrix} \frac{1+i\sqrt{3}}{2} & 1 \\ -1 & \frac{-1+i\sqrt{3}}{2} \end{bmatrix} - \frac{e^{-\frac{in\pi}{3}}}{i\sqrt{3}} \begin{bmatrix} \frac{1-i\sqrt{3}}{2} & 1 \\ -1 & \frac{-1-i\sqrt{3}}{2} \end{bmatrix}$$

$$A^n = \frac{e^{in\frac{\pi}{3}}}{i\sqrt{3}} \begin{bmatrix} e^{i\pi/3} & 1 \\ -1 & e^{i2\pi/3} \end{bmatrix} - \frac{e^{-\frac{in\pi}{3}}}{i\sqrt{3}} \begin{bmatrix} e^{-i\pi/3} & 1 \\ -1 & e^{-i2\pi/3} \end{bmatrix}$$

$$A^n = \frac{2}{\sqrt{3}} \begin{bmatrix} \frac{e^{i(n+1)\frac{\pi}{3}} - e^{-i(n+1)\frac{\pi}{3}}}{2i} & \frac{e^{in\frac{\pi}{3}} - e^{-in\frac{\pi}{3}}}{2i} \\ -\frac{e^{in\frac{\pi}{3}} - e^{-in\frac{\pi}{3}}}{2i} & \frac{e^{i(n+2)\frac{\pi}{3}} - e^{-i(n+2)\frac{\pi}{3}}}{2i} \end{bmatrix}$$

$$A^n = \frac{2}{\sqrt{3}} \begin{bmatrix} \sin (n+1)\frac{\pi}{3} & \sin \frac{n\pi}{3} \\ -\sin n\frac{\pi}{3} & \sin (n+2)\frac{\pi}{3} \end{bmatrix}$$

$\sin \frac{\pi}{3}$	$\sin \frac{2\pi}{3}$	$\sin \frac{3\pi}{3}$	$\sin \frac{4\pi}{3}$	$\sin \frac{5\pi}{3}$	$\sin \frac{6\pi}{3}$
$\sqrt{3}/2$	$\sqrt{3}/2$	0	$-\sqrt{3}/2$	$-\sqrt{3}/2$	0

$$A^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A^1 = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$$