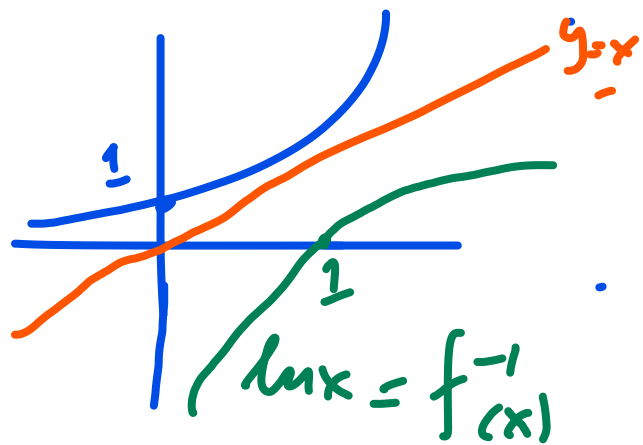


1D Μαθ.

$$f(x) = e^{kx}$$



$$f(x) \rightarrow e^{Ax}$$

$$A = n \times n$$

= nivaras

McLaurin

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

σερα!:

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

e^{At} = πίνακας | ΕΚΘΕΤΙΚΟΣ ΠΙΝΑΚΑΣ

συνήθως σε Mc Laurin.

$$(e^{At}, \sin A, \cos A \rightarrow A^{n \times n})$$

Ⓐ ⓪ A διαγωνιστοποιείται:

$$\lambda_i, \vec{u}_i \rightarrow \Delta = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \dots \\ & & & \lambda_n \end{bmatrix}$$

$$A = P \Delta P^{-1}$$

$$e^{At} = P e^{\Delta t} P^{-1} = P \begin{bmatrix} e^{\lambda_1 t} & & 0 \\ & e^{\lambda_2 t} & \\ 0 & & \dots \\ & & & e^{\lambda_n t} \end{bmatrix} P^{-1}$$

$n \times 1$: $A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} -2-\lambda & 1 \\ 1 & -2-\lambda \end{vmatrix} = 0 \Rightarrow$$

$$(2+\lambda)^2 - 1 = 0 \Rightarrow \begin{cases} \lambda_1 = -1 \\ \lambda_2 = -3 \end{cases}$$

Ιδιοδιανύσματα $\vec{u}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\vec{u}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$$P = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad \left(Q = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \right)$$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix} \left(-\frac{1}{2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \right)$$

Άρα $e^{At} = P e^{\Delta t} P^{-1} =$

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-3t} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}}$$

= ...

$$\begin{bmatrix} \frac{e^{-t} + e^{-3t}}{2} & \frac{e^{-t} - e^{-3t}}{2} \\ \frac{e^{-t} - e^{-3t}}{2} & \frac{e^{-t} + e^{-3t}}{2} \end{bmatrix}$$

$$= e^{At}$$

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$$

B) A ή 0 A δει διαγωνοποιείται

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\boxed{x \rightarrow At}$$

δίνεται:

$$e^{At} = I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \dots$$

(συγκρίνει πάντα)

Ιδιότητες: $e^{At} \cdot e^{As} = e^{A(t+s)}$

⊙ $e^0 = I$

$e^{At} \cdot e^{-At} = I$

$\downarrow \quad \downarrow$
 $B \quad B^{-1}$

θεzw:	$e^{At} = B$
-------	--------------

Αρα e^{At} ΠΑΝΤΑ

αντιστρέφεται.

$n \times 2$

$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$e^{At} ?$

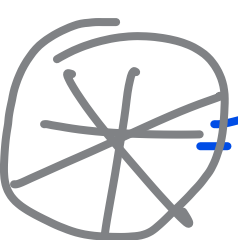
↓ ΠΙΝΑΚΑΣ

Ιδιοτιμες? $\begin{vmatrix} \lambda & -1 \\ 1 & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 + 1 = 0$
 $\Rightarrow \lambda^2 = -1$

$\lambda \notin \mathbb{R} \Rightarrow \lambda = \pm i$

McLaurin:

$$e^{At} = I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \dots$$

 $= I + At + \frac{A^2 t^2}{2} + \frac{A^3 t^3}{6} + \frac{A^4 t^4}{24}$

??

u:

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 + I = 0$$

$$\Rightarrow A^2 = -I$$

$$A^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= -I$$

$$A^2 = -I$$

MOTIVO

$$A^3 = A \cdot A^2 = -A$$



$$A^4 = A^2 \cdot A^2 = (-I) \cdot (-I) = I$$

$$A^5 = A^4 \cdot A = A \dots \dots$$

$$e^{At} = I + At + \frac{A^2 t^2}{2} + \frac{A^3 t^3}{6} + \frac{A^4 t^4}{24} + \dots =$$

$$I + At - \frac{I t^2}{2} - \frac{A t^3}{6} + \frac{I t^4}{24} + \dots =$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} t - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{t^2}{2} - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \frac{t^3}{6} +$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \frac{t^4}{24}$$

$$= \begin{bmatrix} 1 + 0 - \frac{t^2}{2} + \frac{t^4}{24} & -t + \frac{t^3}{3} - \dots \\ t - \frac{t^3}{6} + \dots & 1 - \frac{t^2}{2} + \frac{t^4}{24} - \dots \end{bmatrix}$$

(McLaurin)

$$= \begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix} = e^{At}$$

KATI

ΣΑΝ: $(e^{i\theta} = \cos\theta + i\sin\theta)$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \Rightarrow$$

$x \rightarrow A$
 $n \times n$

$$\sin A = A - \frac{1}{3!} A^3 + \frac{1}{5!} A^5 - \dots$$

ΕΑΝ.
π* Διαμορφώσεις:

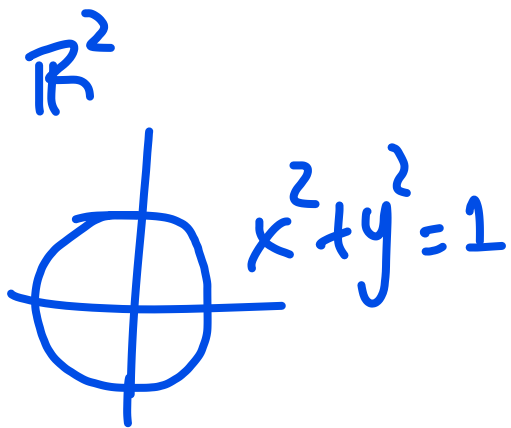
$$\sin A = P \sin(\Delta) P^{-1} =$$

$$P \begin{bmatrix} \sin \lambda_1 & & & 0 \\ & \sin \lambda_2 & & \\ & & \ddots & \\ & & & \sin \lambda_n \end{bmatrix} P^{-1} \dots$$

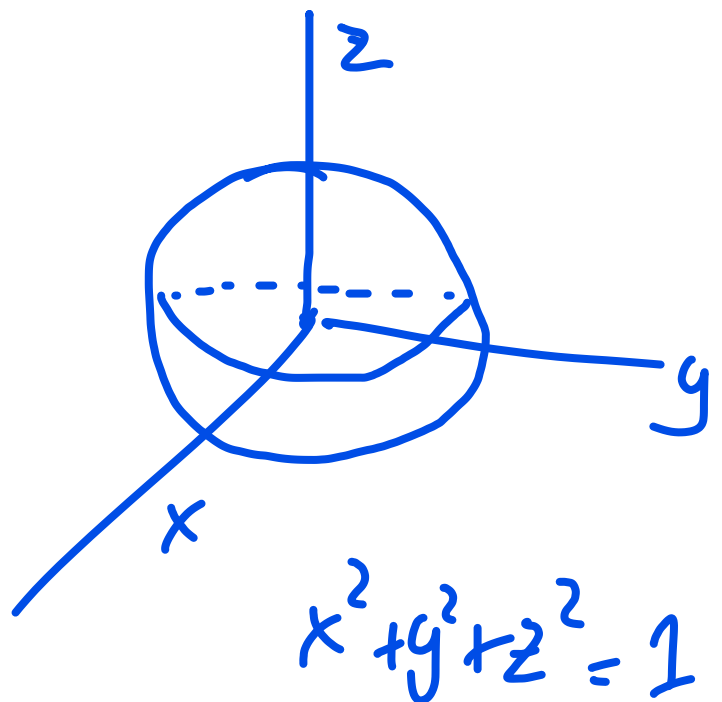
ΑΚΡΟΤΑΤΑ ΤΕΤΡΑΓΩΝΙΚΩΝ

ΜΟΡΦΩΝ ΣΤΗΝ ΜΟΝΑΔΙΑΙΑ

ΣΦΑΙΡΑ.



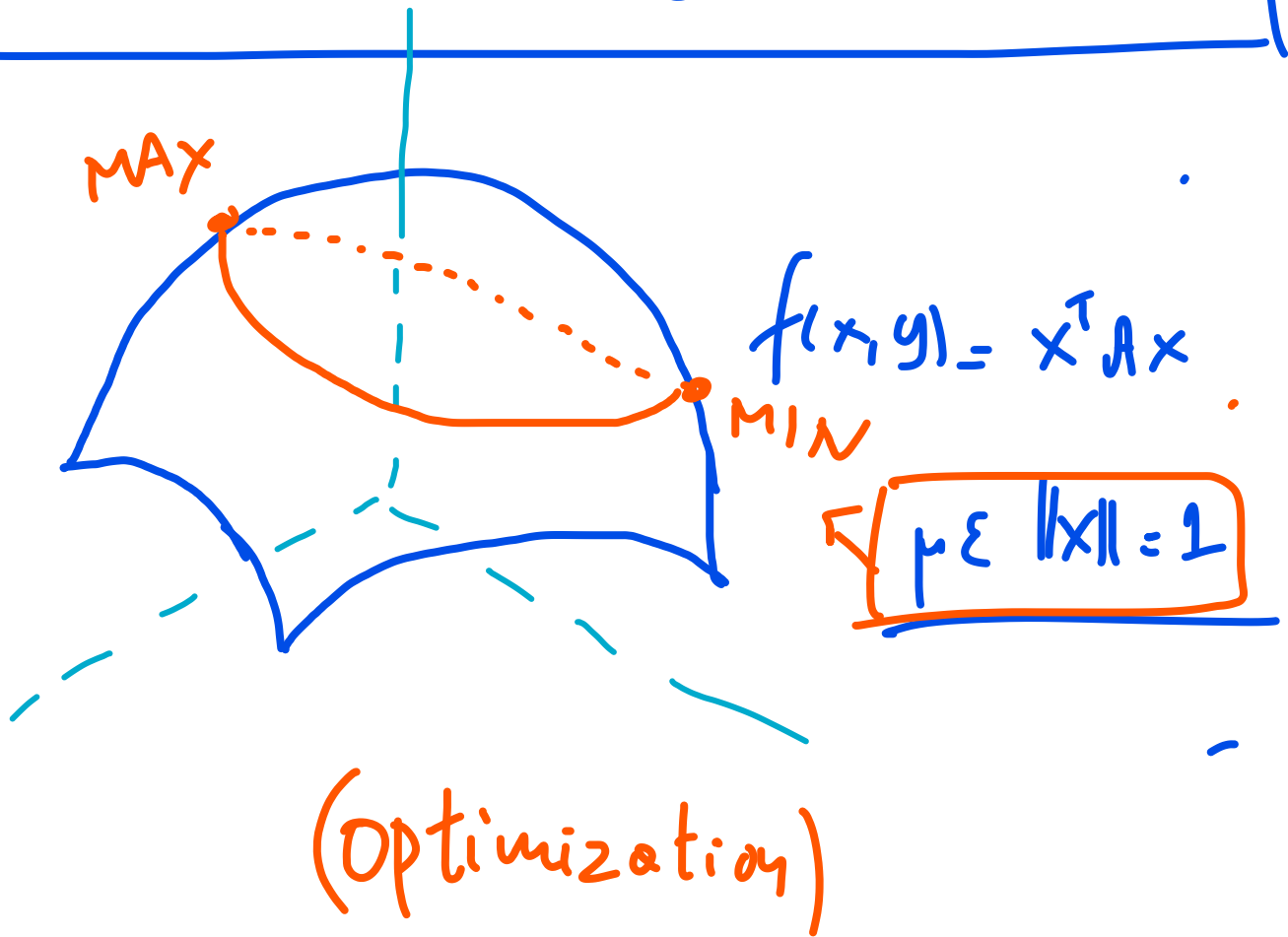
Μοναδ. κύκλος



$f(x) = \text{MIN?}$
 $x \cdot y$, με $x + y = 8$

ΥΠΟ ΠΕΡΙΟΡΙΣΜΟΥΣ

πχ πολ/οζει Lagrange : λογ. 2



$f(\vec{x}) = x^T A x$, (A συμμετρικός)
 $\|x\| = 1$.

$$\left\{ \begin{array}{l} f_{\text{MAX}} \rightarrow \lambda_{\text{MAX}} \quad \vec{X} = \delta_1 \delta_2 \delta_3 \\ f_{\text{MIN}} \rightarrow \lambda_{\text{MIN}} \quad \text{for } \vec{x}: \\ \delta_1 \delta_2 \delta_3 \end{array} \right.$$

ex 1 $f(x, y, z): \mathbb{R}^3 \rightarrow \mathbb{R}$

$$= 2xy + 2xz.$$

$$\boxed{1 = \sqrt{x^2 + y^2 + z^2}}$$

$f_{\text{MAX}}, f_{\text{MIN}}?$

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Diagonals: $|A - \lambda I| = 0 \Rightarrow$

$$\begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 0 \\ 1 & 0 & -\lambda \end{vmatrix} = 0 \Rightarrow \begin{cases} \lambda_1 = \sqrt{2} \\ \lambda_2 = -\sqrt{2} \\ \lambda_3 = 0 \end{cases}$$

$$f_{MAX} = \sqrt{2} \rightarrow \text{διοδικαιώματα } \vec{x} \rightarrow \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$f_{MIN} = -\sqrt{2} \rightarrow \text{διοδικαιώματα } \vec{x} \rightarrow \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$A\vec{u} = \lambda\vec{u}$$

$$A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \sqrt{2} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow$$

$$\underline{\lambda = +\sqrt{2}}$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sqrt{2}x \\ \sqrt{2}y \\ \sqrt{2}z \end{pmatrix} \Rightarrow$$

$$\begin{cases} y + z = \sqrt{2}x \\ x = \sqrt{2}y \\ x = \sqrt{2}z \end{cases} \quad (y = z)$$

Αν θέλω $\|\vec{x}\|=a$?

$$f_{\text{MAX}} = \lambda_{\text{MAX}} \cdot a \rightarrow \text{ιδιοδιαν.}$$

$$f_{\text{MIN}} = \lambda_{\text{MIN}} \cdot a \rightarrow \text{ιδιοδιαν.}$$

$n \times 2$: $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix}$

$$f(x) = x^T A x \rightarrow f_{\text{MAX}}? \quad \left(\|x\| = 1 \right)$$

~~f_{MAX}~~
 λ_{MAX}

Apa:
$$\begin{vmatrix} 3-\lambda & 2 & 1 \\ 2 & 3-\lambda & 1 \\ 1 & 1 & 4-\lambda \end{vmatrix} = 0 \Rightarrow$$

$$-\lambda^3 + 10\lambda^2 - 27\lambda + 18 = 0$$

$$\lambda = 6, \lambda = 3, \lambda = 1$$

$$f_{\max} = 6 = f(\vec{u})$$

$$A\vec{u} = 6\vec{u} \Rightarrow \dots \quad \vec{u} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \hat{u} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$$
