15 Μαρ. (α)

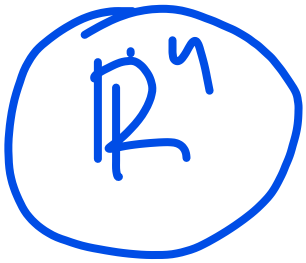
A $n\times n$ → A^{-1} υπάρχει

→ A^{-1} δεν υπάρχει

$A^{n\times n} \longrightarrow A^{-1}$ ΟΧΙ

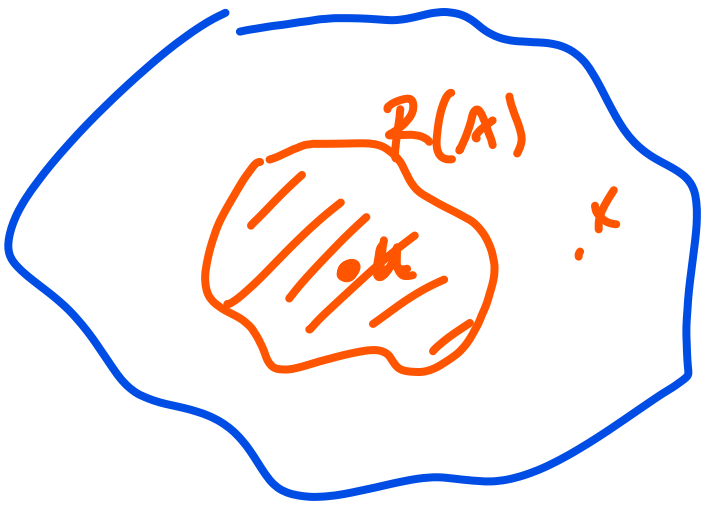
A_L $A^{n\times n}$, αντίστροφίσιμος

P_A ?



$R(A) = \mathbb{R}^n$

$R(A) = \mathbb{R}^n \Rightarrow P_A = I$



$\mathbb{R}^n$

$P_A \cdot \vec{x} = \vec{u}$

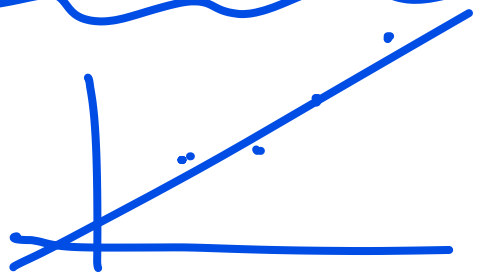
A μη αντιστ.

Απόδ. A = αντιστ. (A^{-1} υπάρχει)

$$P_A = A(A^T A)^{-1} A^T =$$

$$(AB)^{-1} = B^{-1} \cdot A^{-1}$$

$$\underbrace{A \cdot A^{-1}}_{I} \cdot \underbrace{A^{T^{-1}} \cdot A^T}_{I} = I$$



Λ.Ε.Τ. "Βέλτιστη λύση"

$\|\varepsilon\|$ = minimum

ε = error

$A\vec{x} = b \Rightarrow$

Λ.Ε.Τ. $\hat{x}$: $(A\hat{x} \neq b)$

$$\|\varepsilon\| = \|A\hat{x} - b\| = \min.$$

Πχ

$$\left.\begin{aligned} 4x &= 2 \\ 2y &= 0 \\ x+y &= 11 \end{aligned}\right\}$$

Αδύνατο.

Λ.Ε.Τ. ?

$$\begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$$

$$A\vec{u} = \vec{b}$$

$$\Rightarrow A^T A u = A^T b \longrightarrow \hat{u} = (A^T A)^{-1} \cdot A^T b$$

$$\hat{u} = A^+ b$$

$$\begin{bmatrix} 4 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 17 & 1 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ 11 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\hat{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{cases} 4x = 2 \\ 2y = 0 \\ x + y = 11 \end{cases}$$

Αντιστοιχεί $= \begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix} = b_1$

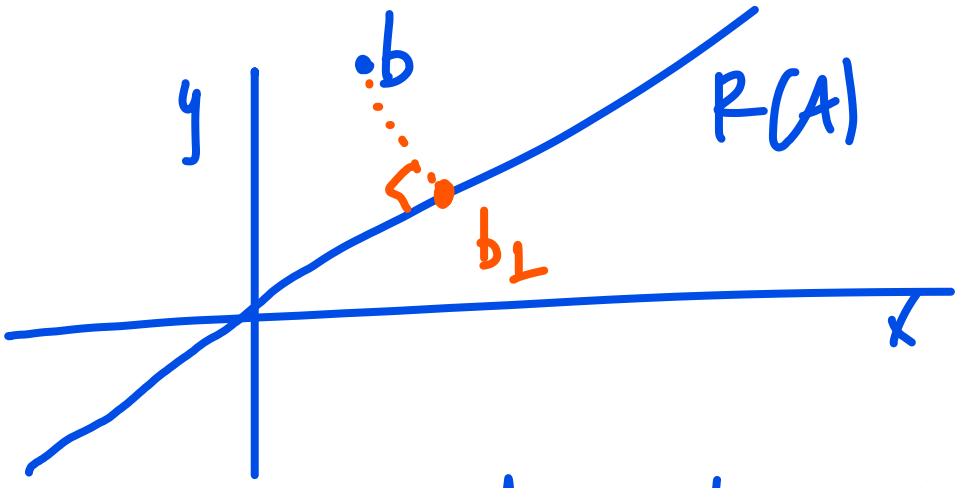
$$\vec{b} = \begin{pmatrix} 2 \\ 0 \\ 11 \end{pmatrix}$$

$\|b - b_1\| = \min$

(Error $= \|b - b_1\| = \left\| \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix} \right\| = \sqrt{4 + 16 + 64}$

$= \sqrt{84} = 9,16$)

A^+ Ψευδοαντίστροφος



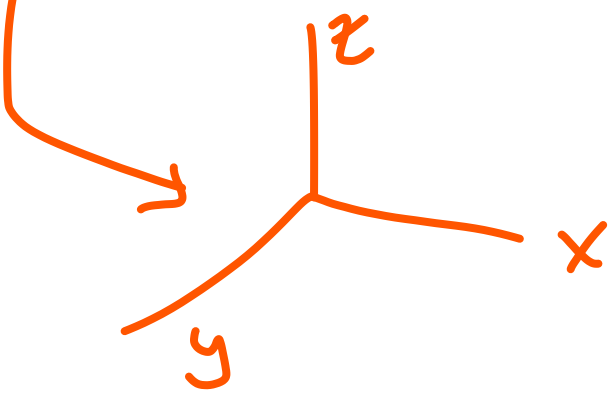
$Ax = b$, Αδύνατο

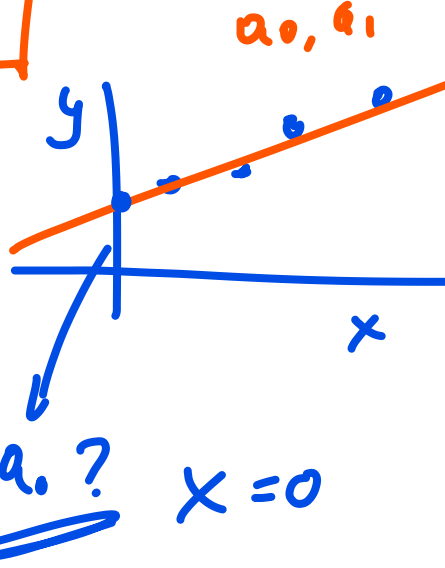
$\longrightarrow$ $Ax_1 = b_1$, λύνεται.

$(y = a_0 + a_1 x)$ ΛΕΤ: $\begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$

ΕΠΕΚΤΑΣΗ ΤΗΣ ΙΔΕΑΣ (πολλές μεταβλητές)

$$y = a_0 + a_1 x + a_2 z + \dots$$





πχ Παγωτό: $\begin{cases} t = \text{μέρες ψυγείου} \\ c = {}^{\circ}C \\ g = \text{gr. χάνει το παγωτό} \end{cases}$

$$G = a_0 + a_1 t + a_2 c$$

Λύση: Αντικαθιστώ: $\begin{cases} 0,15 = a_0 + a_1 \cdot 1 - 10 a_2 \\ 0,18 = a_0 + 1 \cdot a_1 - 5 a_2 \\ \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \ddots \end{cases}$

$$\underbrace{\begin{bmatrix} 1 & 1 & -10 \\ 1 & 1 & -5 \\ 1 & 1 & 0 \\ \vdots & 2 & -10 \\ \vdots & 2 & \vdots \\ \vdots & \vdots & \vdots \\ 1 & 3 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}}_{\vec{a}} = \underbrace{\begin{bmatrix} 0,15 \\ 0,18 \\ 0,20 \\ \vdots \\ 0,25 \end{bmatrix}}_{b}$$

$$A\vec{a} = b \Rightarrow A^T A \vec{a} = A^T b$$

$$\hat{u} = (A^T A)^{-1} A^T b \Longrightarrow \dots$$

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 0,174 \\ 0,025 \\ 0,005 \end{pmatrix} \Longrightarrow$$

$$G = 0,174 + 0,025t + 0,05c$$

gr χαΐδικε μέρες °C

π.χ. $t = 9$ μέρες, $c = -35°$ $\Rightarrow$ $G = 0,224$ g

Ελάχιστα τετράγωνα σε καμπύλη

Βάσεις: $P(t) = a_0 + a_1 t + a_2 t^2 + \dots$

$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \end{pmatrix}$?

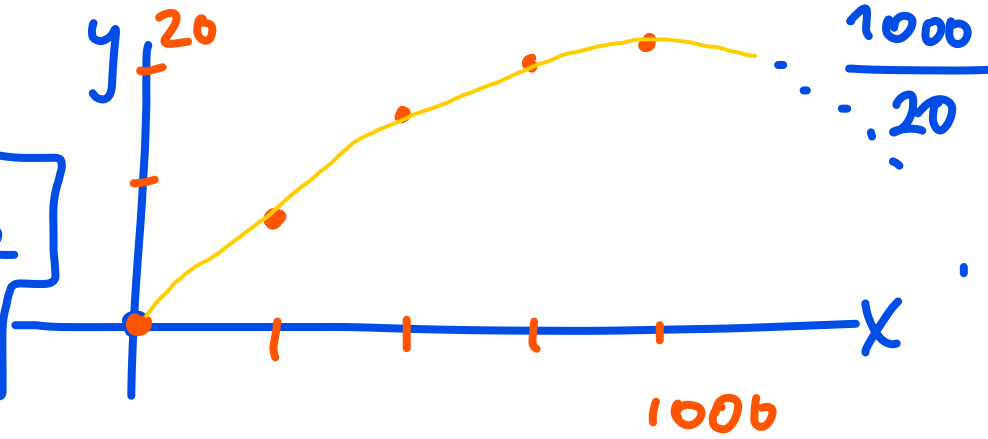
Βολή πυραύλου

(m) x	0	250	500	750
(m) y	0	8	15	19

RADAR:

$y = f(x)$

$y = a_0 + a_1 x + a_2 x^2$



$A\vec{a} = b \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0{,}25 & 0.25^2 \\ 1 & 0.5 & 0.5^2 \\ 1 & 0.75 & 0.75^2 \\ 1 & 1 & 1^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$

$= \begin{bmatrix} 0{,}008 \\ 0{,}015 \\ 0{,}019 \\ 0{,}02 \end{bmatrix}$

x : 1000

y : 1000

για πρακτικούς

$$Au = b \Rightarrow A^T A u = A^T b \Rightarrow$$

$$\hat{u} = (A^T A)^{-1} \cdot A^T b \qquad (\hat{u} = A^+ b)$$

πείτε $\hat{u} = \begin{bmatrix} -0,00023 \\ 0,03983 \\ -0,0194 \end{bmatrix} = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}$

$$\boxed{y = -0,00023 + 0,03983x - 0,0194x^2}$$

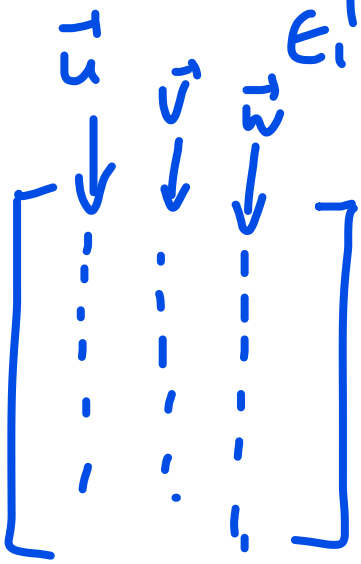
Πού θα πέσει το εδαφος?

$y = 0 \Longrightarrow x = 2,044 \rightarrow$

σε 2044 m

ΟΡΘΟΓΩΝΙΟΙ ΠΙΝΑΚΕΣ Q

$Q \in \mathbb{R}^{n \times n}$: οι στήλες είναι ορθοκανονικές.



$\langle u, v \rangle = 0$, $\langle u, w \rangle = 0$

$\langle v, w \rangle = 0$

$\|\vec{u}\| = \|\vec{v}\| = \|\vec{w}\| = 1$

$$Q \cdot Q^T = I = Q^T \cdot Q$$

$$Q^T = Q^{-1}$$