15 Απρ: ΤΕΤΡΑΓΩΝΙΚΕΣ ΜΟΡΦΕΣ.

παρ 1: $f(x,y) = x^2 + 10xy + y^2$

i) $\vec{X} = \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow X^T A X$, $A = \begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}$

Διαγωνοποίηση: (A) Ιδιοτιμές:

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 5 \\ 5 & 1-\lambda \end{vmatrix} = 0 \Rightarrow$$

$$(1-\lambda)^2 - 25 = 0 \Rightarrow \lambda = 6, \ \lambda = -4$$

$$\Delta = \begin{bmatrix} 6 & 0 \\ 0 & -4 \end{bmatrix} \quad Q \rightarrow \text{ιδιοδιανυσματικός}$$

$$A = Q \Delta Q^T$$

$Q^{-1} = Q^T$ ορθογώνιος

(B) Ιδιοδιανύσματα:

για $A = A^T$

$\lambda = 6$: $A\vec{u} = 6\vec{u} \Rightarrow$

$\begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 6\begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \dots \quad \vec{u} = \begin{pmatrix} k \\ k \end{pmatrix} \eta\chi \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda = -4$: $A\vec{v} = -4\vec{v} \Rightarrow \begin{cases} x + 5y = -4x \\ 5x + y = -4y \end{cases} \Rightarrow$

$\vec{v} = \begin{pmatrix} -k \\ k \end{pmatrix} \quad \eta\chi \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$Q = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

(ΟΧΙ ΕΞΙΣΟΥ ακόμα)

ΚΑΝΟΝΙΚΟΠΟΙΗΣΗ: $\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} : Q$

Κανονική μορφή | $\vec{X} = Q\vec{Y}$ θέτω

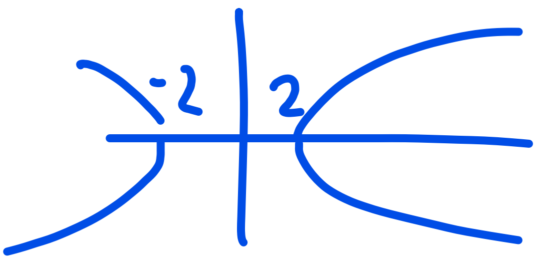
$F(x,y) \to F(x',y')$ για $\begin{pmatrix} x \\ y \end{pmatrix} = Q \begin{pmatrix} x' \\ y' \end{pmatrix}$

$= 6x'^2 - 4y'^2$

ii) Αν $F(x,y) = 24$, ΚΩΝΙΚΗ ΤΟΜΗ? σχήμα?

$$6x'^2 - 4y'^2 = 24 \Rightarrow$$

$$\frac{x'^2}{4} - \frac{y'^2}{6} = 1$$ ΥΠΕΡΒΟΛΗ



Q στροφή

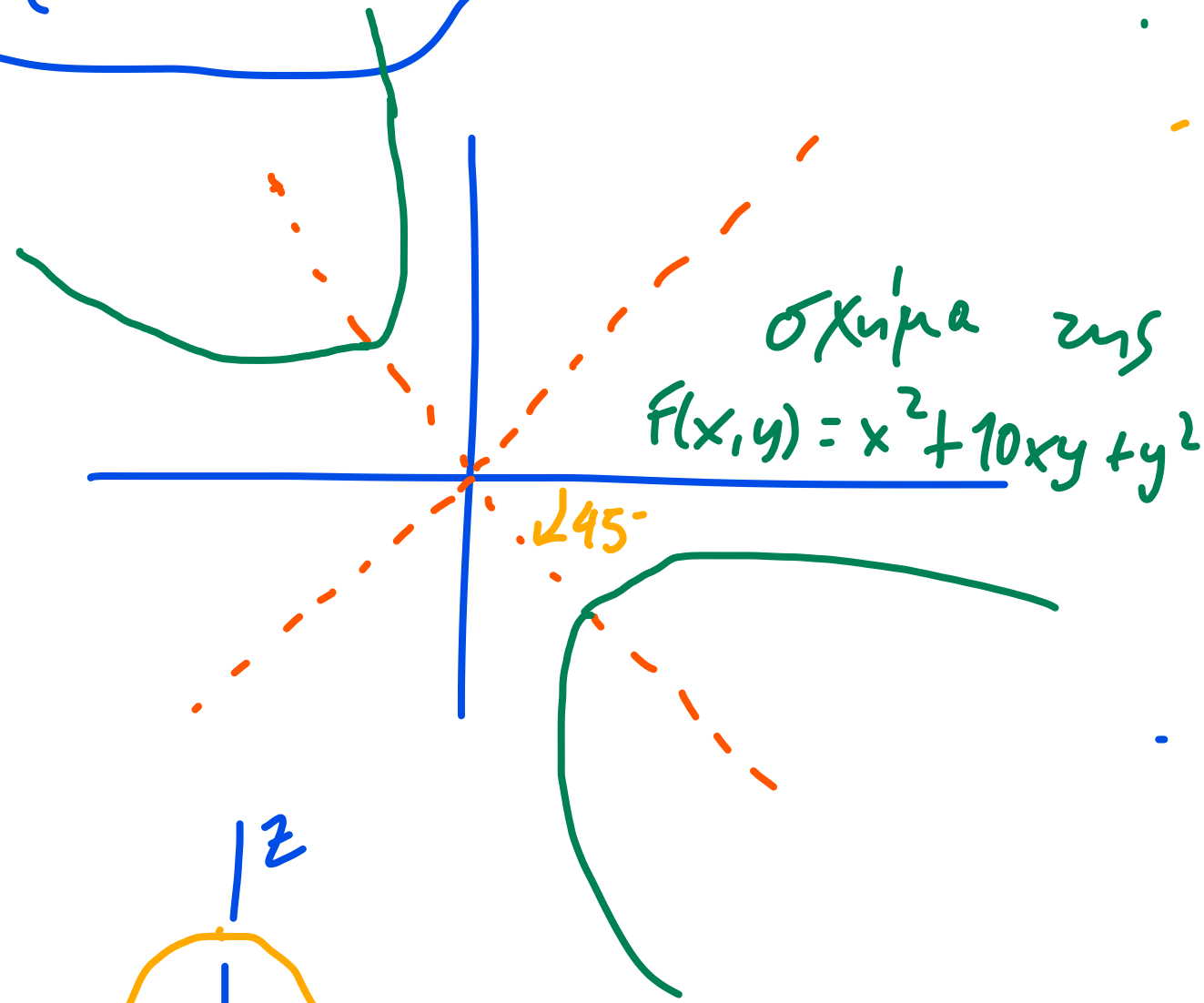
$$Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Πίνακας στροφής

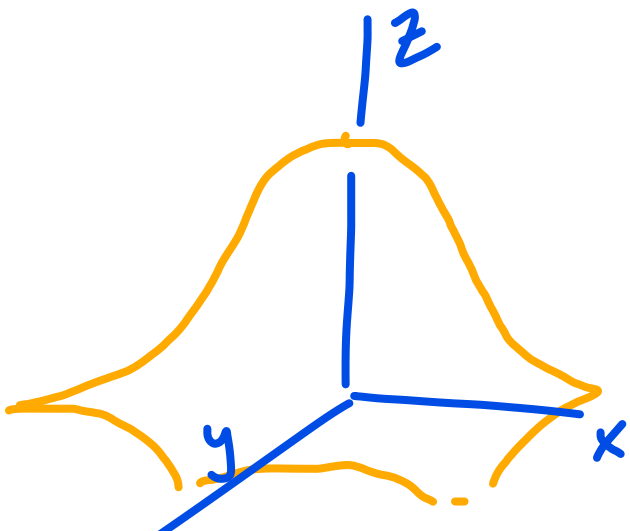
$$P = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}$$

φ

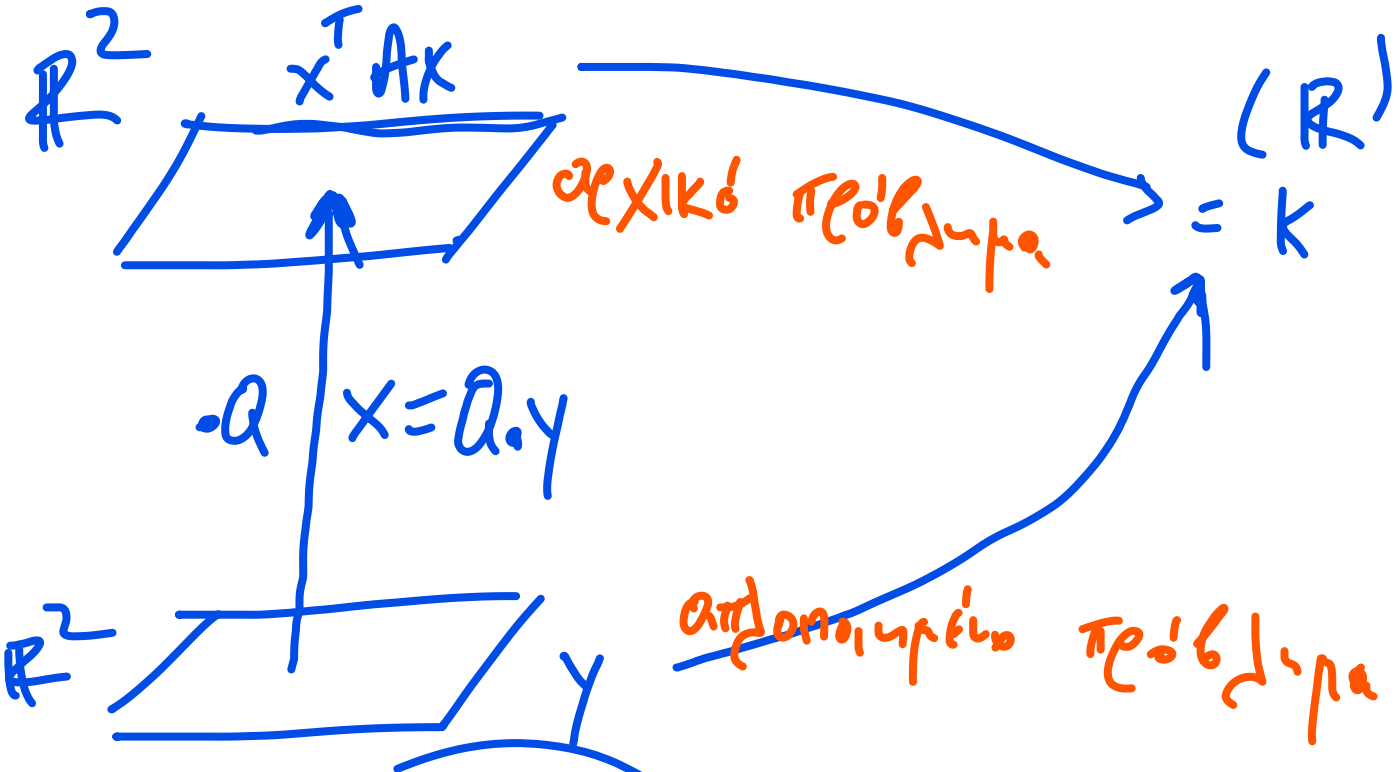
$$\varphi = -45^\circ$$



σχήμα της $f(x,y) = x^2 + 10xy + y^2$



Σχηματική η αλλαγή μεταβλητής:



ΥΤΔ y πιο εύκολο διάγωνιος Δ.

ΓΕΝΙΚΗ ΜΟΡΦΗ

$$f(x,y) = \underbrace{ax^2 + 2bxy + \gamma y^2}_{} + \delta x + \varepsilon y + k = 0$$

ΣΕ ΜΟΡΦΗ ΠΙΝΑΚΩΝ $\boxed{\mathbb{R}^2 \to \mathbb{R}}$

$$\vec{X} = \begin{pmatrix} x \\ y \end{pmatrix} \quad A = \begin{bmatrix} a & b \\ b & \gamma \end{bmatrix}, \; E = \begin{bmatrix} \delta \\ \varepsilon \end{bmatrix}$$

$$\boxed{f(x,y): \vec{X}^T A \vec{X} + E\vec{X} + K = 0}$$

διαγωνοποίηση κανονικά...

Θέτω $X = QZ \rightsquigarrow$

$$f(z_1, z_2) = \lambda_1 z_1^2 + \lambda_2 z_2^2 + b_1 z_1 + b_2 z_2 + K = 0$$

(λ_1, λ_2 = ιδιοτιμές του A)

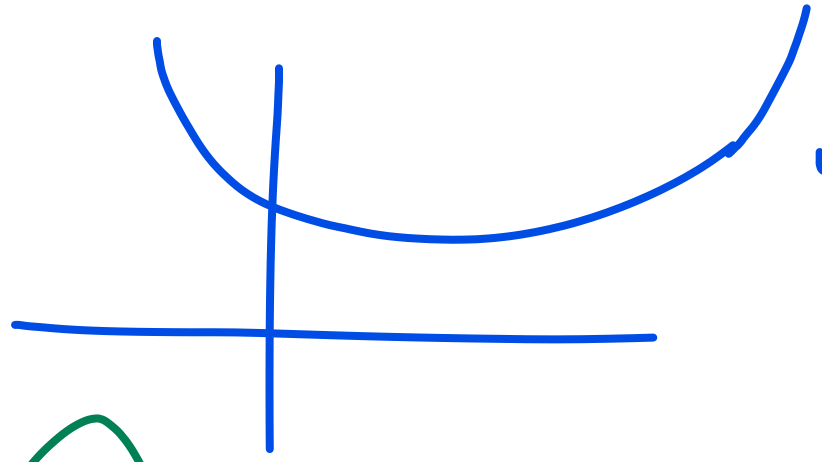
Διακρίνουσα: $\Delta = \frac{b_1^2}{4\lambda_1} + \frac{b_2^2}{4\lambda_2} - K$

- $\Delta = 0 \Rightarrow$ σημείο ή ζεύγος ευθειών

- $\Delta \neq 0 \Rightarrow$ ΕΛΛΕΙΨΗ ή ΥΠΕΡΒΟΛΗ
- Αν $\lambda_i = 0 \Rightarrow$ ΠΑΡΑΒΟΛΗ
- Αν $\lambda_1 = \lambda_2 = 0 \Rightarrow$ ΕΥΘΕΙΑ

ΘΕΤΙΚΑ (Αρνητικα..)

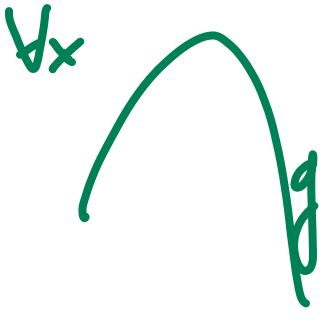
ΟΡΙΣΜΕΝΗ Τ.Μ.



$y = x^2 + x + 1 > 0$

$\forall x$

$\forall x$

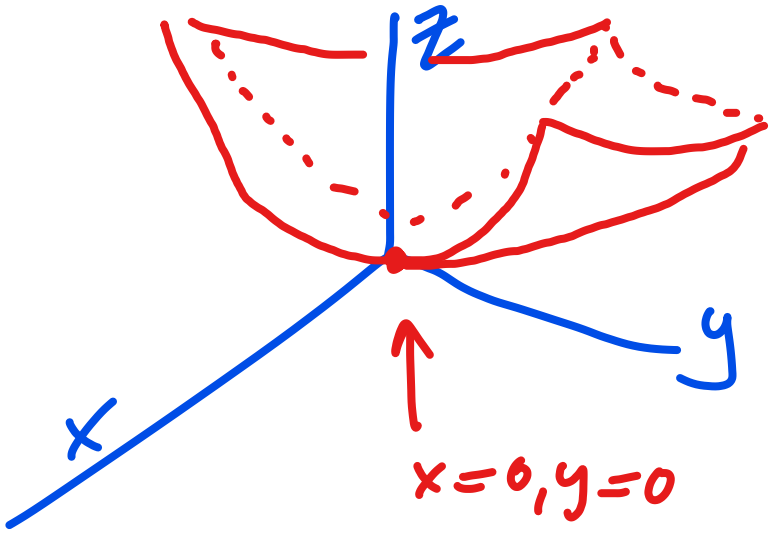


$g_H < 0$

Αρνητικά ορισμένη.

$\Rightarrow$ ΘΕΤΙΚΑ ΟΡΙΣΜΕΝΗ

πχ $\mathbb{R}^3$

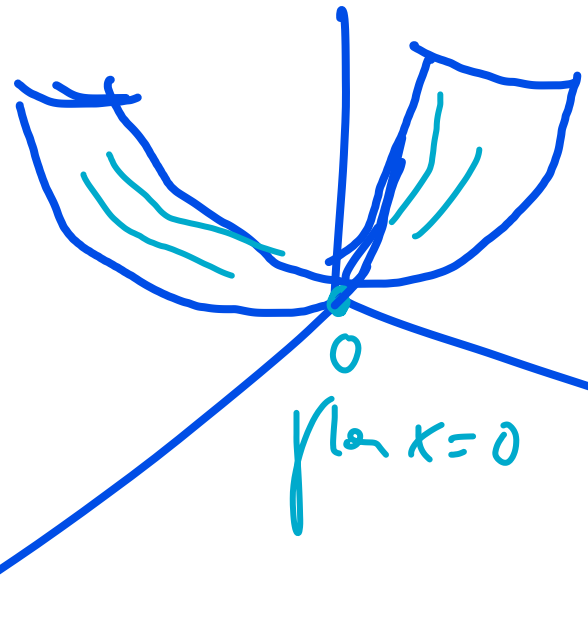


$z = 3x^2 + 7y^2$

ΘΕΤΙΚΑ ΗΜΙΟΡΙΣΜΕΝΗ

$z \geq 0$

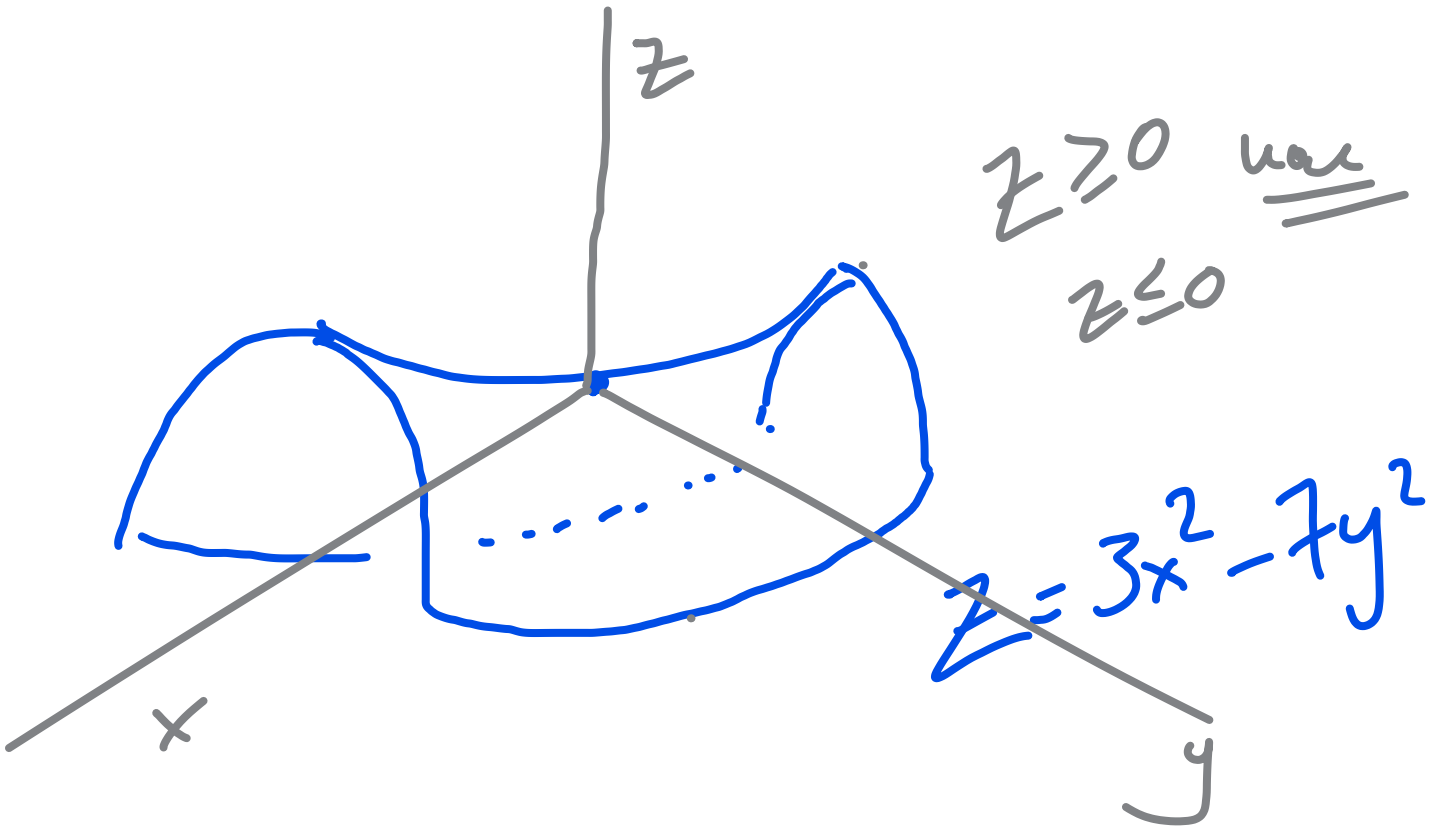
πχ2



$z = 3x^2 \geq 0$

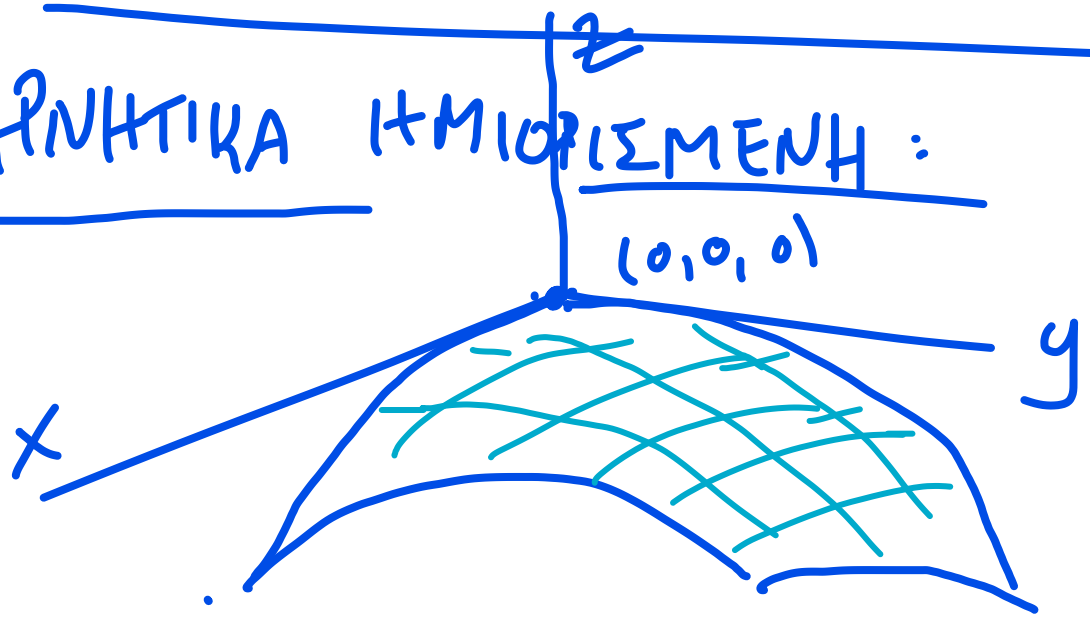
ΘΕΤΙΚΑ ΗΜΙΟΡΙΣΜΕΝΗ

πχ3: ΑΟΡΙΣΤΗ Τ.Μ.



$z \geq 0$ και $z \leq 0$

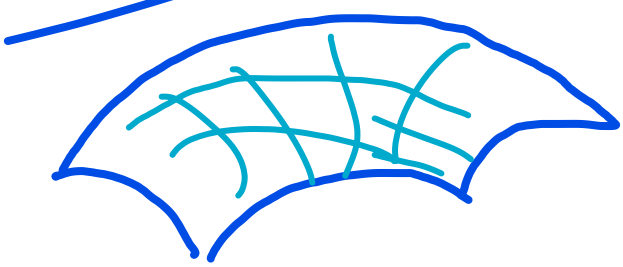
ΑΡΝΗΤΙΚΑ ΗΜΙΟΡΙΣΜΕΝΗ:



$$z = -3x^2 - 7y^2$$

ΑΡΝΗΤ.

ΟΡΙΣΜΕΝΗ:



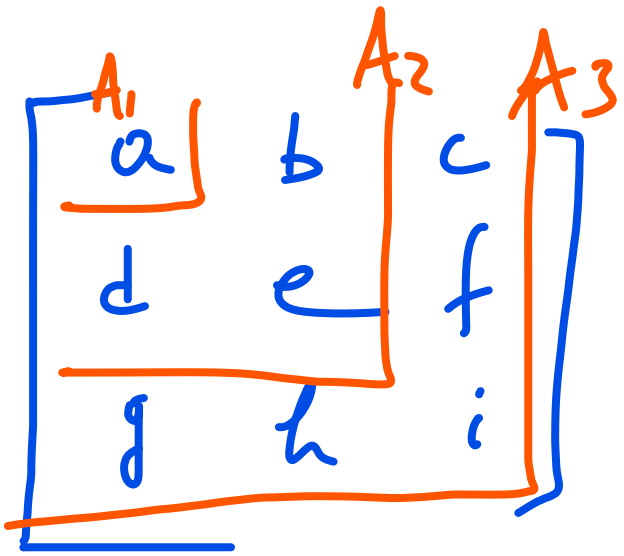
$z = -3x^2 - 7y^2 - 1$

(A) Ιδιοτιμές

(B) ΜΕ ΟΡΙΖΟΥΣΕΣ

$f(x_1, x_2, x_3, x_4 \dots)$ $\quad \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \end{pmatrix}$

πχ (3×3)



πχ $A = \begin{bmatrix} -2 & -1 \\ 10 & 3 \end{bmatrix}$?

$A_1 = -2 < 0$.

$A_2 = \begin{vmatrix} -2 & -1 \\ 10 & 3 \end{vmatrix} = -6 + 10 = 4 > 0$. ✓

Αρνητικά ορισμένος .

πχ²

$$B = \begin{bmatrix} 10 & 2 \\ 1 & 3 \end{bmatrix}$$

$10 = B_1 > 0$

$B_2 = \begin{vmatrix} 10 & 2 \\ 1 & 3 \end{vmatrix} = 28 > 0$

Θετικά ορισμένος

πχ

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 4 & x & 6 \\ 5 & 6 & 1 \end{bmatrix} > 0$$

x ?

A = θετικά ορισμένος > 0

$A_1 = 1 > 0$ πάντα

$$A_2 = \begin{vmatrix} 1 & 4 \\ 4 & x \end{vmatrix} = x - 16 > 0 \Rightarrow \boxed{x > 16}$$

$$A_3 = \begin{vmatrix} 1 & 4 & 5 \\ 4 & x & 6 \\ 5 & 6 & 1 \end{vmatrix} = \begin{matrix} 1 & 4 & 5 & 1 & 4 \\ 4 & x & 6 & 4 & x \\ 5 & 6 & 1 & 5 & 6 \end{matrix}$$

$\Leftrightarrow \cdots \quad -24x + 188 > 0$

$x < \frac{47}{6} \simeq 7,8$ $\boxed{x < 7,8}$

ΑΔΥΝΑΤΟΝ

$\rightarrow$ ΠΟΤΕ θ.0ρ.