

5 ΜΑΘΟΥ

# ΣΥΝΑΡΤΗΣΕΙΣ

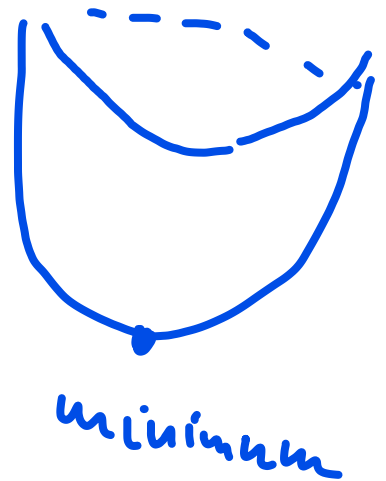
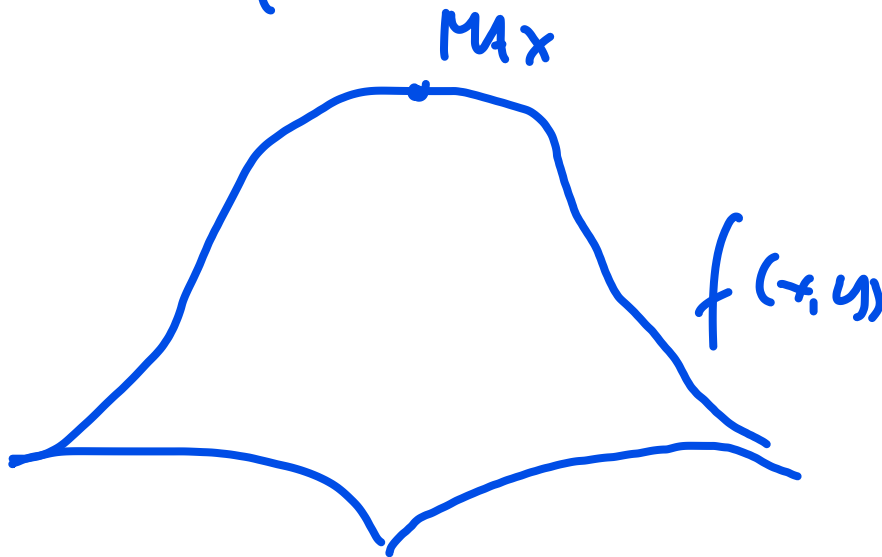
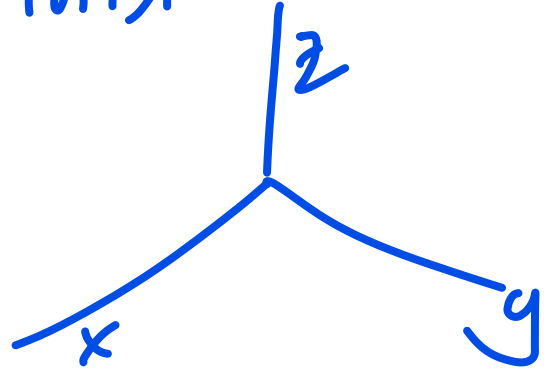
ΠΟΛΛΩΝ ΜΕΤΑΒΛΗΤΩΝ

ΑΚΡΟΤΑΤΑ

$\eta\tau$

$$f(x) = x^2 - y^2 = z$$

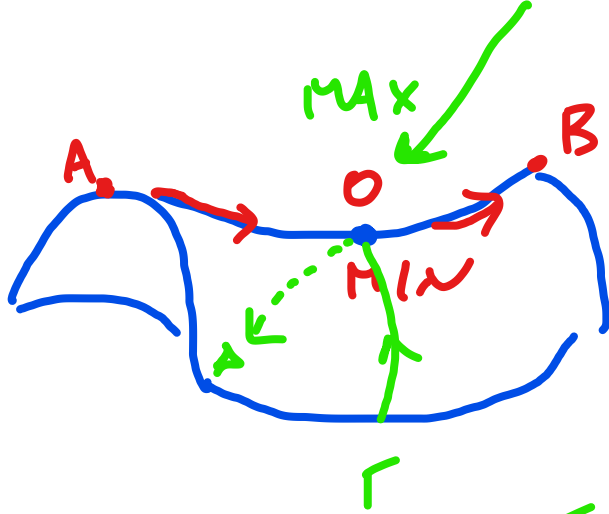
Επιφάνεια



$f(x) \rightarrow$  Σ.Κ. σημείο καμπύλης

ΕΠΙΦΑΝΕΙΑ

$\rightarrow$  Σ. Σέλιος



$f(x, y) = \dots$

ΤΑΥΤΟ ΧΡΟΝΑ

MAX, MIN  $\rightarrow$

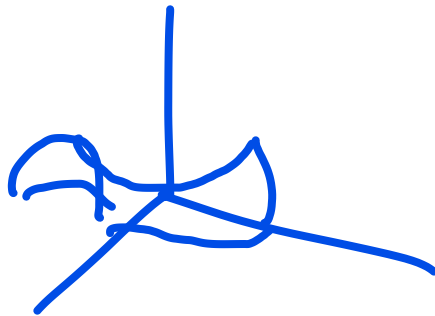
ΑΝΑΛΟΓΟΣ ΤΗΝ ΔΙΑΣΠΟΜΗ

MAX, MIN, Σ-Σέλας ?

$f: \mathbb{R}^{\textcircled{2}} \rightarrow \mathbb{R}$

πχ  $f(x, y) = x^2 - y^2 = z$

$\mathbb{R}^2 \rightarrow \mathbb{R}$



$f''|_A \rightarrow$

2<sup>ος</sup> ΠΑΡΑΓΩΓΟΥ  
ΠΙΝΑΚΑΣ  
( $n \times n$ )

# ΜΕΡΙΚΕΣ ΠΑΡΑΓΕΓΜΑΤΑ

πχ

$$f(x,y) = 3x^2 - 5xy + y^3$$

$$\left. \begin{array}{l} (f_x =) \frac{\partial f}{\partial x} = 6x - 5y = 0 \\ (f_y =) \frac{\partial f}{\partial y} = -5x + 3y^2 = 0 \end{array} \right\} \Rightarrow$$

$$\left\{ \begin{array}{l} x = \frac{5}{6}y \\ -\frac{25}{6}y + 3y^2 = 0 \Rightarrow y \left[ 3y - \frac{25}{6} \right] = 0 \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} y = 0 \Rightarrow x = 0 \\ y = \frac{25}{18} \Rightarrow x = \frac{125}{108} \end{array} \right.$$

$$A(0,0) \quad B\left(\frac{125}{108}, \frac{25}{18}\right)$$

ΠΩΑΝΑ ΑΚΡΟΤΑΤΑ

B: ΕΝΑΧΙΣΤΟ (Από σχήμα)

A: ~~ΜΕΓΙΣΤΟ~~ (= =)

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$$f(x, y) = \dots$$

1<sup>η</sup> ΠΑΡΑΓΩΓΟΣ

$$\begin{cases} \frac{\partial f}{\partial x} = \\ \frac{\partial f}{\partial y} = \end{cases}$$

$$2^{\text{η}} \text{ ΠΑΡΑΓΩΓΟΣ} = A = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

↑  
ΜΕΓΙΣΤΟ, ΕΝΑΧΙΣΤΟ,

ΣΕΛΑΣ?

Πχ1:  $f(x,y) = 2x^2 - 4xy + 5y^2$

ΤΕΤΡ. ΜΟΡΦΗ

$x = \begin{pmatrix} x \\ y \end{pmatrix} \quad x^T A x = f(x,y)$

$$A = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$$

Ⓐ  $\frac{\partial f}{\partial x} = 4x - 4y \quad \longrightarrow \quad \frac{\partial^2 f}{\partial y \partial x} = -4$

$$\frac{\partial f}{\partial y} = -4x + 10y$$

Ⓑ  $\begin{cases} 4x - 4y = 0 \Rightarrow x = y \\ -4x + 10y = 0 \end{cases} \quad \longleftarrow \quad y = 0$

$\circ (0,0)$

$x=0$

\*κροτακτο??

Γ) 2η παραγωγός?

$$\frac{\partial^2 f}{\partial x^2} = 4$$

$$\frac{\partial^2 f}{\partial y^2} = 10$$

$$\frac{\partial^2 f}{\partial x \partial y} = -4$$

$$H = \begin{bmatrix} 4 & -4 \\ -4 & 10 \end{bmatrix}$$

$$\underbrace{(\det H = 24)}_{\text{det } H}$$

$\Rightarrow$  Η τετραγωνική

ορισμένη  $\Rightarrow$

ΕΛΑΧΙΣΤΟ

$\omega (0,0)$

$\pi \times 2$   $f(x,y) = \sin x + \sin y + \sin(x+y)$

$D_f:$   $\begin{cases} 0 < x \leq \pi \\ 0 < y \leq \pi \end{cases}$

(A)  $\frac{\partial f}{\partial x} = \cos x + \cos(x+y)$

$\frac{\partial f}{\partial y} = \cos(x+y) + \cos y$

(B)  $\begin{cases} \cos x + \cos(x+y) = 0 \\ \cos y + \cos(x+y) = 0 \end{cases} \Rightarrow$

$\cos x - \cos y = 0 \Rightarrow \cos x = \cos y$



$0 < x < \pi$

$\begin{cases} x = y \\ x = -y \end{cases}$

$0 < x, y \leq \pi$

Αντίστροφο  $x=y$

$$\cos x + \cos 2x = 0 \Rightarrow \cos x + 2\cos^2 x - 1 = 0$$

Θέτω  $\cos x = w$ :  $2w^2 + w - 1 = 0$

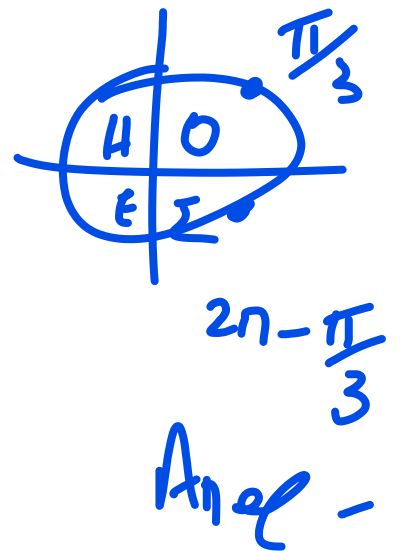
$$\Rightarrow \dots w = -1, w = \frac{1}{2}$$

Αρα

$$\begin{cases} \cos x = -1 \\ \cos x = \frac{1}{2} \end{cases}$$

$$0 < x \leq \pi$$

$$\rightarrow \begin{cases} x = \pi \\ x = \frac{\pi}{3} \end{cases}$$



$x=y$   $A(\frac{\pi}{3}, \frac{\pi}{3})$

$$B(\pi, \pi)$$

πίθασα  
αυτοζα



① Πίνακας 2<sup>ης</sup> παραγώγων:

$$\frac{\partial f}{\partial x} = \cos x + \cos(x+y) \Rightarrow \frac{\partial^2 f}{\partial x^2} = -\sin x - \sin(x+y)$$

$$\frac{\partial^2 f}{\partial y \partial x} = -\sin(x+y)$$

$$\frac{\partial f}{\partial y} = \cos y + \cos(x+y) \Rightarrow$$

$$\frac{\partial^2 f}{\partial y^2} = -\sin y - \sin(x+y)$$

Αρα

$$H = \begin{bmatrix} -\sin x - \sin(x+y) & -\sin(x+y) \\ -\sin(x+y) & -\sin y - \sin(x+y) \end{bmatrix}$$

ΠΙΝΑΚΑΣ 2<sup>ης</sup> ΠΑΡΑΓΩΓΟΥ

σημείο A  $(\frac{\pi}{3}, \frac{\pi}{3}) \rightarrow H = \begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\sqrt{3} \end{bmatrix}$

Αφαιρούμε ορισμούς.  $H_1: -\sqrt{3} < 0$

$H_2: \begin{bmatrix} -\sqrt{3} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\sqrt{3} \end{bmatrix} = 3 - \frac{3}{4}$

$\Rightarrow$  A: MAXIMUM

$\frac{9}{4} > 0$

σημείο B?  $(\pi, \pi)$ ? (συν H)

$H = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$  Αόριστος

$\det(H) = 0$

ΔΕΝ ΜΠΟΡΟΥΜΕ ΝΑ ΑΠΑΝΤΗΣΟΥΜΕ





















