10 ΙΟΥΝΙΟΥ

ΘΕΜΑΤΑ (ΙΟΥΝΙΟΣ 2018)

1

α) $\begin{bmatrix} -3 & -2 \\ 3 & 4 \end{bmatrix}$ λ ?

$$\begin{vmatrix} -3-\lambda & -2 \\ 3 & 4-\lambda \end{vmatrix} = 0 \Rightarrow$$

$$(-3-\lambda)(4-\lambda)+6=0 \Rightarrow$$

$$-12+3\lambda-4\lambda+\lambda^2+6=0 \Rightarrow \lambda^2-\lambda-6=0$$

$$\lambda=3, \quad \lambda=-2.$$

β) A^{-1} ? (A, I) $h_A(x) = x^2 - x - 6$

Θ. C-H : $A^2 - A - 6I = \mathbb{O} \Rightarrow$

$$A^2 - A = 6I \Rightarrow$$

$$\frac{1}{6}(A^2 - A) = I \Rightarrow A \underbrace{\left[\frac{1}{6}(A - I)\right]}_{A^{-1}} = I$$

γ) A^4 ? (A, I)

$(6) \rightarrow \boxed{A^2 = A + 6I} \rightarrow (\cdot A)$

$$A^3 = A^2 + 6A = A + 6I + 6A =$$

$$7A + 6I.$$

$(\cdot A)$ $A^4 = 7A^2 + 6A = 7(A + 6I) + 6A$

$$A^4 = 13A + 42I.$$

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(2) $B = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ $\lambda = 2, 1$

Ιδιοδιανύσματα: α) $B\vec{u} = 2\vec{u} \Longrightarrow$

$$\begin{cases} x - z = 2x \\ y = 2y \\ 2z = 2z \end{cases} \Longrightarrow \begin{cases} x = -z \\ y = 0 \\ 0 = 0 \end{cases}$$

$$\vec{u} = \begin{pmatrix} k \\ 0 \\ -k \end{pmatrix} \quad \text{πχ} \quad \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \vec{u}$$

β) $B\vec{v} = 1\vec{v}$

$$\begin{cases} x - z = x \\ y = y \\ 2z = z \end{cases} \Longrightarrow \begin{cases} z = 0 \\ 0 = 0 \\ z = 0 \end{cases}$$

$$\vec{v} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} = \begin{pmatrix} \kappa \\ \mu \\ 0 \end{pmatrix}$$

$$= \kappa \underline{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}} + \mu \underline{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}$$

$$\vec{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \vec{w} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Άρα έχω 3 ιδιοδιανύσματα:

$\vec{u}, \vec{v}, \vec{w} \Rightarrow$ ΔΙΑΓΩΝΟΠΟΙΕΙΤΑΙ

$$B = P \Delta P^{-1}$$

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}, \quad \Delta = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

b) $B^3 = P\Delta^3 P^{-1} \Rightarrow$

$$B^3 = P \cdot \begin{bmatrix} 8 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot P^{-1}.$$

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3) $m_A(x) = (x+3)^4 (x-1)^3 x$

$p_A(x) = (x+3)^2 (x-1)^2 x$

$$\left[\begin{array}{cc|cc|c} -3 & 1 & & & \\ & -3 & & & \\ \hline 0 & & \boxed{-3} & 0 & \\ & & & & 0 \end{array}\right]$$

$$
\begin{array}{c} -3 \\ \\ \end{array}
$$

$$
J = \begin{bmatrix}
0 & & \\
& \begin{array}{cc|c} 1 & 1 & \\ 0 & 1 & \\ \hline 0 & & 1 \end{array} & 0 \\
& & \boxed{0}
\end{bmatrix}
$$

n

$$
J = \begin{bmatrix}
\begin{matrix} -3 & 1 \\ 0 & -3 \end{matrix} & & & \\
& \begin{matrix} -3 & 1 \\ 0 & -3 \end{matrix} & 0 & \\
0 & & \begin{matrix} 1 & 1 \\ 0 & 1 \end{matrix} & 0 \\
& & 0 \quad 1 & \\
& & 0 & \boxed{0}
\end{bmatrix}
$$

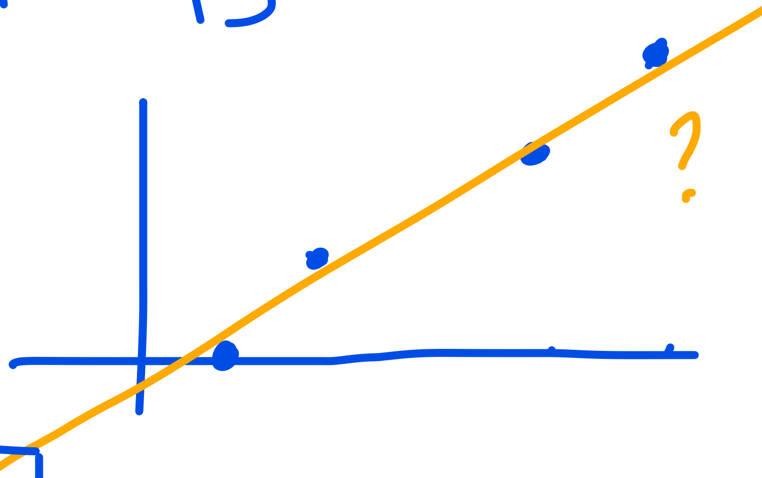
nx

$$\left(\begin{bmatrix} 4 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{bmatrix} \right)$$

4

x	1	2	4	5
y	0	1	2	3

$$\boxed{y = b_0 + b_1 x}$$



$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} \qquad y = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\boxed{Ax = y} \longrightarrow \left(\hat{x} = A^T y\right)$$

$$\underbrace{A^T A x = A^T y}_{} \Rightarrow \quad \hat{x} = \left(A^T A\right)^{-1} A^T y$$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 12 \\ 12 & 46 \end{bmatrix}$$

$$A^T y = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 25 \end{bmatrix}$$

<u>Aea</u>

$$\begin{bmatrix} 4 & 12 \\ 12 & 46 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} 6 \\ 25 \end{bmatrix}$$

$$(\det = -80)$$

$$\hat{x} = \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} 4 & 12 \\ 12 & 46 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 6 \\ 25 \end{bmatrix} =$$

$$\begin{bmatrix} -3/5 \\ \frac{7}{10} \end{bmatrix} \Rightarrow \boxed{y = -0,6 + 0,7x}$$

5

$$f(x) = x_1^2 - 8x_1x_2 - 5x_2^2$$

$$A = \begin{bmatrix} 1 & -4 \\ -4 & -5 \end{bmatrix}$$ <u>Ιδιοτιμές;</u>

$$\begin{vmatrix} 1-\lambda & -4 \\ -4 & -5-\lambda \end{vmatrix} = 0 \Rightarrow$$

$$(1-\lambda)(-5-\lambda)-16=0 \Rightarrow$$

$$\lambda^2+4\lambda-21=0$$

$$\begin{cases} \lambda=3 \\ \lambda=-7 \end{cases}$$

Θέτοντας $x=Qy$ (Q = ιδιοδιαν)

ΚΑΝΟΝΙΚΗ ΜΟΡΦΗ:

$$3y_1^2-7y_2^2=F(y)$$

Q?

Ιδιοδιανύσματα:

- $A\vec{u} = 3\vec{u} : \begin{cases} x - 4y = 3x \\ -4x - 5y = 3y \end{cases} \Rightarrow$

$$\boxed{x = -2y} \qquad \vec{u} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

- $A\vec{v} = -7\vec{v} \Rightarrow \left.\begin{matrix} x - 4y = -7x \\ -4x - 5y = -7y \end{matrix}\right\}$

$$\Rightarrow 8x = 4y \Rightarrow y = 2x$$

$$\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

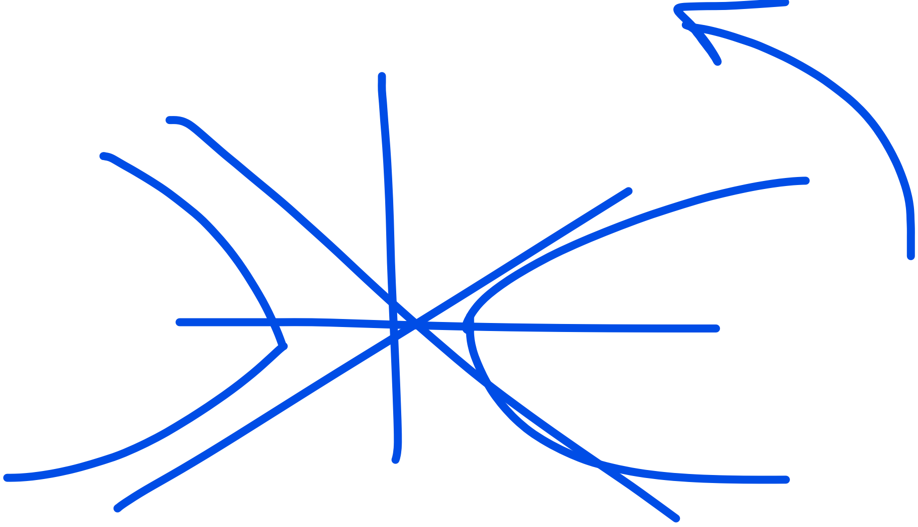
ΚΑΝΟΝΙΚΟΠΟΙΗΣΗ:

$$\vec{u} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \longrightarrow \begin{pmatrix} -2/\sqrt{5} \\ \frac{1}{\sqrt{5}} \end{pmatrix}$$

$$\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \longrightarrow \begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$$

$$Q = \begin{bmatrix} -2/\sqrt{5} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix}$$

$$\left(\cos\theta = -\frac{2}{\sqrt{5}} \rightarrow \theta = 153{,}4^\circ \text{ στροφή}\right)$$



150°