

11 Μαρτ.

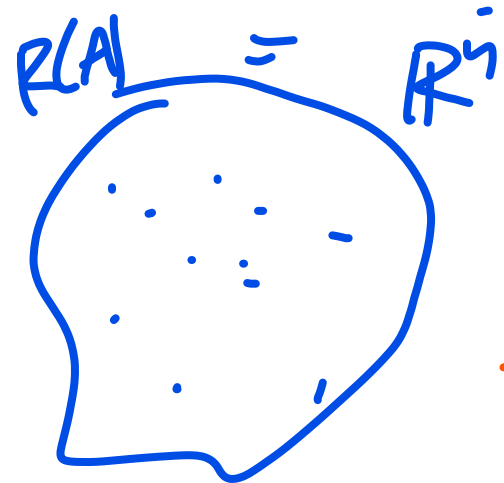
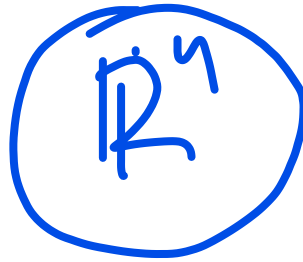
$A^{n \times n}$

$\rightarrow A^{-1}$  υπάρχει  
 $\rightarrow A^{-1}$  δεν υπάρχει

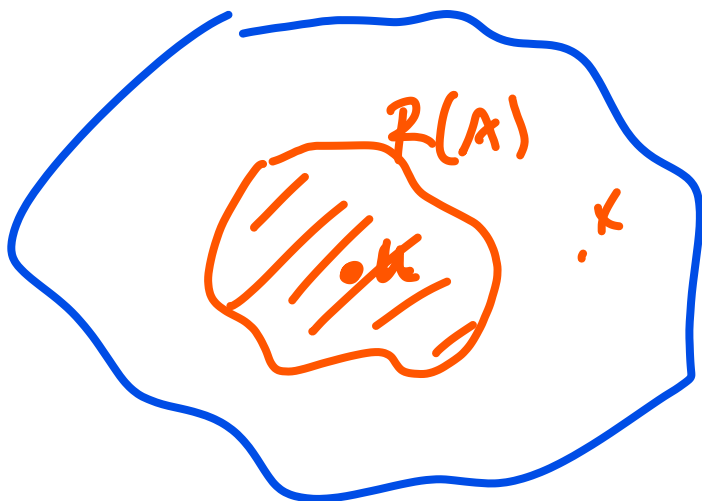
$A^{n \times n} \rightarrow A^{-1}$  OXI

$P_A?$

$A \in \mathbb{R}^{n \times n}$ , αντιστρέψιμος



$R(A) = \mathbb{R}^n \Rightarrow P_A = I$



$\mathbb{R}^n$

$P_A \vec{x} = \vec{u}$

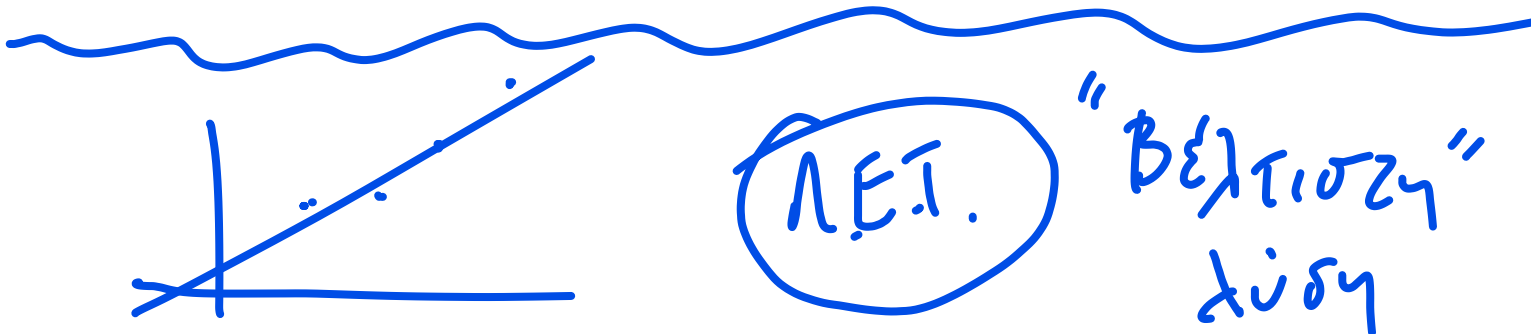
$A$  μη αντιστρεψιμή.

Απόδ  $A = \text{αντίστροφη}$ . ( $A^{-1}$  υπάρχει)

$$P_A = A(A^T A)^{-1} A^T =$$

$$(AB)^{-1} = B^{-1} \cdot A^{-1}$$

$$\underbrace{A \cdot A^{-1}}_I \cdot \underbrace{A^T \cdot A^{-T}}_I = I$$



$\| \varepsilon \| = \text{minimum}$

$\varepsilon = \text{error}$

$$A \hat{u} = b \Rightarrow$$

ΛΕΙ.  $\hat{x}$  :  $(A \hat{x} \neq b)$

$$\| \varepsilon \| = \| A \hat{x} - b \| = \text{min.}$$

$$\frac{n \times}{\quad} \left. \begin{array}{l} 4x = 2 \\ 2y = 0 \\ x + y = 11 \end{array} \right\}$$

A solução

A.E.T. ?

$$\begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$$

$$A \vec{u} = \vec{b}$$

$$\Rightarrow A^T A u = A^T b \Rightarrow \hat{u} = \underbrace{(A^T A)^{-1} \cdot A^T b}$$

$$\begin{bmatrix} 4 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix} \Rightarrow$$

$$\hat{u} = A^+ b$$

$$\begin{bmatrix} 17 & 1 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ 11 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\hat{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\left. \begin{array}{l} 4x = 2 \\ 2y = 6 \\ x + y = 11 \end{array} \right\}$$

$$\text{ΑΥΤΙΜΟΝΩΣΗ} = \begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix} = b_1$$

$$\vec{b} = \begin{pmatrix} 2 \\ 0 \\ 11 \end{pmatrix}$$

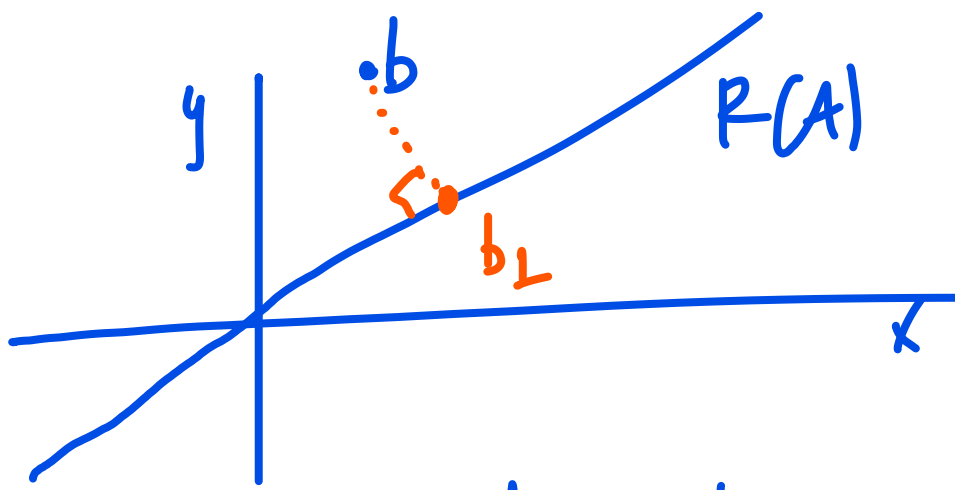
$$\|b - b_1\| = \min$$

$$\text{Error} = \|b - b_1\| = \left\| \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix} \right\| = \sqrt{4 + 16 + 64}$$

$$= \sqrt{84} = 9,16$$

$A^+$

ή ευδοκίσιμος



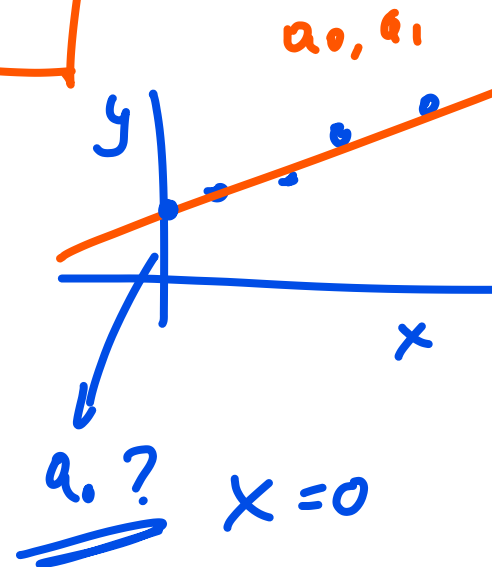
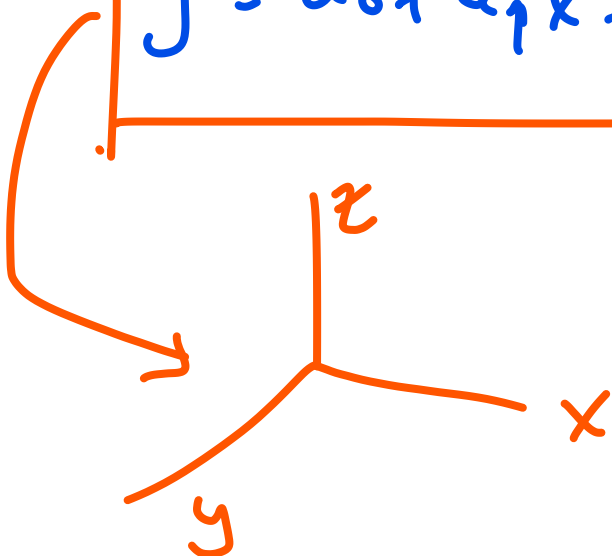
$$Ax = b, \text{ A δύνανται}$$

$$\rightarrow Ax_1 = b_1, \text{ λύνεται.}$$

$$(y = a_0 + a_1 x) \text{ ΛΕΤ: } \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

ΕΠΕΚΤΑΣΗ ΤΗΣ ΙΔΕΑΣ (ΠΟ) <sup>είναι</sup> μεταβλητές

$$y = a_0 + a_1 x + a_2 z + \dots$$



πλ

Παγωτό :

$t =$  μέρες ψυγείου  
 $c =$  °C

$g = g^i$  . χάνει το παγωτό

$$G = a_0 + a_1 t + a_2 c$$

Λύση : Αντιματρικό :

$$0,15 = a_0 + a_1 \cdot 1 - 10a_2$$

$$0,18 = a_0 + 1 \cdot a_1 - 5a_2$$

$\vdots$   
 $\vdots$     $\vdots$     $\vdots$   
 $\vdots$     $\vdots$     $\vdots$

$$A = \begin{bmatrix} 1 & 1 & -10 \\ 1 & 1 & -5 \\ 1 & 1 & 0 \\ \vdots & 2 & -10 \\ \vdots & 2 & \vdots \\ \underline{1} & \vdots & 0 \end{bmatrix}$$

$$u = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$$b = \begin{bmatrix} 0,15 \\ 0,18 \\ 0,20 \\ \vdots \\ 0,25 \end{bmatrix}$$

$$A u = b \Rightarrow A^T A u = A^T b$$

$$\hat{u} = (A^T A)^{-1} A^T b \Rightarrow \dots$$

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 0,174 \\ 0,025 \\ 0,005 \end{pmatrix} \Rightarrow$$

$$G = 0,174 + 0,025t + 0,005c$$

$\downarrow$   $\downarrow$   $\downarrow$   
gr χαίμακα      μέρες      %

~~dx~~  $t = 9$  μέρες }  $G = 0,224g$   
 $c = -35^\circ$

Ελάχιστα ζεγράφωνα σε καμνίτη

Πείρα:  $P(t) = a_0 + a_1 t + a_2 t^2 + \dots$

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \end{pmatrix} ?$$

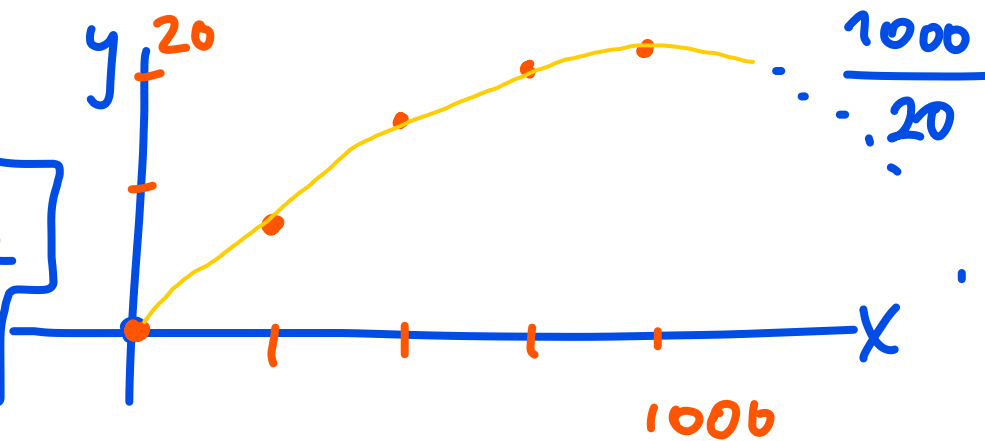
Βοήθη Πυραμίδου

(m)	X	0	250	500	750
(m)	Y	0	8	15	19

RADAR:

$$y = f(x)$$

$$y = a_0 + a_1 x + a_2 x^2$$



$$A \vec{u} = b \Rightarrow$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0,25 & 0,25^2 \\ 1 & 0,5 & 0,5^2 \\ 1 & 0,75 & 0,75^2 \\ 1 & 1 & 1^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$$x : 1000$$

$$y : 1000$$

για την τιμή

$$= \begin{bmatrix} 0,008 \\ 0,015 \\ 0,019 \\ 0,02 \end{bmatrix}$$



$$Au = b \Rightarrow A^T Au = A^T b \Rightarrow$$

$$\tilde{u} = (A^T A)^{-1} \cdot A^T b \quad (\tilde{u} = A^T b)$$

πρέπει

$$\tilde{u} = \begin{bmatrix} -0,00023 \\ 0,03983 \\ -0,0194 \end{bmatrix} = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}$$

$$y = -0,00023 + 0,03983x - 0,0194x^2$$

πού θα πέσει στο έδαφος?

$$\underline{y=0} \Rightarrow x = 2,044 \Rightarrow$$

σε 2044 m

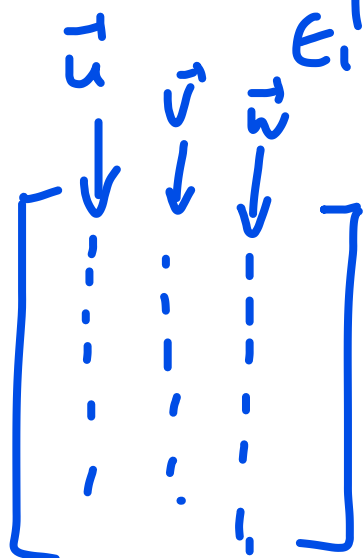
ΟΡΘΟΓΩΝΙΑ

ΠΙΝΑΚΕΣ  $Q$

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$Q \in \mathbb{R}^{n \times n}$  : οι στήλες

είναι ορθοκανονικές.



$$\langle u, v \rangle = 0, \langle u, w \rangle = 0$$

$$\langle v, w \rangle = 0$$

$$\|\vec{u}\| = \|\vec{v}\| = \|\vec{w}\| = 1$$

$$Q \cdot Q^T = I = Q^T \cdot Q$$

$$Q^T = Q^{-1}$$