Additive dummy variables

- $$Fe_i = \begin{cases} 1 \,, & \text{if female} \\ 0 \,, & \text{if male} \end{cases}$$

$$wage_i = \alpha + \boxed{\beta Fe_i} + e_i$$

$$\hat{\alpha} = 11.52 \,, \quad \hat{\beta} = -2.65$$

$$wage_i = \begin{cases} \alpha + e_i \,, & \text{if } Fe_i = 0 \ \text{(Male)} \\ \alpha + \beta + e_i \,, & \text{if } Fe_i = 1 \ \text{(Female)} \end{cases}$$

$$E(wage_i \mid Fe_i) = \begin{cases} \alpha \,, & \text{if } Fe_i = 0 \ \text{(Male)} \\ \alpha + \beta \,, & \text{if } Fe_i = 1 \ \text{(female)} \end{cases}$$

$$\hat{E}(wage_i \mid Fe_i) = \begin{cases} \hat{\alpha} = 11.52 \,, & \text{if } Fe_i = 0 \\ \hat{\alpha} + \hat{\beta} = 11.52 + (-2.65) = 8.86 \,, & \text{if } Fe_i = 1 \end{cases}$$

$$\hat{\beta} = (\hat{\alpha} + \hat{\beta}) - \hat{\alpha} = -2.65$$

$$Male_i = \begin{cases} 1, & \text{if male} \\ 0, & \text{if female} \end{cases}$$

- $wage_i = \alpha + \beta Male_i + e_i$

$$\boxed{\hat{\alpha} = 8.86, \quad \hat{\beta} = 2.65}$$

$$E(wage_i \mid Male_i) = \begin{cases} \alpha + \beta, & \text{if } Male_i = 1 \text{ (Male)} \\ \alpha, & \text{if } Male_i = 0 \text{ (Female)} \end{cases}$$

$$\hat{E}(wage_i \mid Male_i) = \begin{cases} \hat{\alpha} + \hat{\beta} = 8.86 + 2.65 = 11.52, & \text{if } Male_i = 1 \\ \hat{\alpha} = 8.86, & \text{if } Male_i = 0 \end{cases}$$

$$\hat{\beta} = (\hat{\alpha} + \hat{\beta}) - \hat{\alpha} = 2.65$$

$$Male_i = \begin{cases} 1, & \text{if Male} \\ 0, & \text{if Female} \end{cases} , \quad Fe_i = \begin{cases} 1, & \text{if Female} \\ 0, & \text{if Male} \end{cases}$$

$$wage_i = \beta \, Male_i + \gamma \, Fe_i + e_i$$

$$\hat{\beta} = 11.52, \quad \hat{\gamma} = 8.86$$

$$E(wage_i \mid Male_i, fe_i) = \begin{cases} \beta, & \text{if male} \\ \gamma & \text{if Female} \end{cases}$$

$$\hat{E}(wage_i \mid male_i, fe_i) = \begin{cases} \hat{\beta}, & \text{if male} \\ \hat{\gamma}, & \text{if Female} \end{cases}$$

$$\hat{\beta} - \hat{\gamma} = 11.52 - 8.86 = 2.64$$

Τελεια πολυσυγγραμμικοτητα (Dummy variable trap)

$$wage_i = \alpha + \beta \, Male_i + \gamma \, Fe_i + e_i$$

$$wage_1 = 1\cdot\alpha + \beta\cdot 1 + \gamma\cdot 0 + e_1$$

$$wage_2 = 1\cdot\alpha + \beta\cdot 0 + \gamma\cdot 1 + e_2$$

$$wage_3 = 1\cdot\alpha + \beta\cdot 1 + \gamma\cdot 0 + e_3$$

$$\begin{bmatrix} wage_1 \\ wage_2 \\ wage_3 \end{bmatrix} = \alpha \overset{\overset{x_1}{\uparrow}}{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}} + \beta \overset{\overset{x_2}{\uparrow}}{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}} + \gamma \overset{\overset{x_3}{\uparrow}}{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

$$\Rightarrow \underset{3\times 1}{wage} = \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}}_{X} \underset{\Delta}{\underset{\downarrow}{\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}}} + e = X \cdot \Delta + e$$

Αρα $r(X) = 2 = r(X'X) \rightarrow$

Αρα X και $X'X$ ειναι ιδιαζουσες και μη αντιστρεψιμες. $\Rightarrow$ $\boxed{\hat{\Delta} = (X'X)^{-1} X' wage}$

- $wage_i = \alpha + \beta Fe_i + e_i$

$$wage_i = \alpha + \boxed{\beta\, Fe_i} + \gamma\, Ed_i + e_i$$

$\hat{\alpha} = -3.50$, $\hat{\beta} = -2.52$, $\hat{\gamma} = 1.12$

$$E(wage_i \mid Fe_i, Ed_i) = \begin{cases} \alpha + \gamma Ed_i, & \text{if } Fe_i = 0 \text{ (Male)} \\ (\alpha+\beta) + \gamma Ed_i, & \text{if } Fe_i = 1 \text{ (female)} \end{cases}$$

$$\frac{\partial E(wage_i \mid Ed_i, Fe_i)}{\partial Ed_i} = \begin{cases} \gamma, & \text{if } Fe_i = 0 \\ \gamma, & \text{if } Fe_i = 1 \end{cases}$$

$$\beta = E(wage_i \mid Fe_i = 1, Edu_i) - E(wage_i \mid Fe_i = 0, Edu_i)$$

$$= \Big[\underbrace{(\alpha+\beta) + \gamma Ed_i^F}_{\text{female}}\Big] - \Big[\underbrace{\alpha + \gamma Ed_i^M}_{\text{male}}\Big]$$

$$= \beta + \gamma(\underbrace{Ed_i^F - Ed_i^M}_{=0 \text{ για ισο}})$$

επιπεδο εκπαιδευση

Both additive and multiplicative dummies

$$wage_i = \alpha + \overset{\text{additive}}{\boxed{\beta Fe_i}} + \gamma Ed_i + \overset{\text{multiplicative}}{\boxed{\delta \cdot Ed_i \cdot Fe_i}} + e_i$$

$\hat{\alpha} = -3.56$, $\hat{\beta} = -2.40$, $\hat{\gamma} = 1.13$, $\hat{\delta} = -0.0095$

$$E(wage_i \mid Fe_i, Ed_i) = \begin{cases} \alpha + \gamma \cdot Ed_i, & \text{if } Fe_i = 0 \text{ (male)} \\ (\alpha+\beta) + (\gamma+\delta) Ed_i, & \text{if } Fe_i = 1 \text{ (female)} \end{cases}$$

$$\frac{\partial E(wage_i \mid Fe_i, Ed)}{\partial Ed_i} = \begin{cases} \gamma, & \text{if } Fe_i = 0 \text{ (male)} \\ \gamma + \delta, & \text{if } Fe_i = 1 \text{ (female)} \end{cases}$$

$$\delta = (\gamma + \delta) - \gamma$$

$$\hat{\gamma} + \hat{\delta} = 1.13 - 0.0095 = 1.12$$

$$\beta = \left[(\alpha+\beta) + (\gamma+\delta) Ed_i^F\right] - \left[\alpha + \gamma Ed_i^M\right]$$

$$= \beta + \gamma (Ed_i^F - Ed_i^M) + \delta Ed_i^F$$

$$= \beta, \quad \text{as} \quad Ed_i^F = Ed_i^M = 0$$

$$black_i = \begin{cases} 1, & \text{if black} \\ 0, & \text{if white} \end{cases}, \quad fe_i = \begin{cases} 1, & \text{if female} \\ 0, & \text{if male} \end{cases}$$

- $wage_i = \alpha + \beta Ed_i + \boxed{\gamma fe_i} + \boxed{\delta\, black_i} + e_i$

$\hat{\alpha} = -3.27$, $\hat{\beta} = 1.11$, $\hat{\gamma} = -2.50$, $\hat{\delta} = -1.51$

$$E(wage_i \mid Ed_i, fe_i, black_i) = \begin{cases} \alpha + \beta Ed_i, & \text{if } fe_i = black_i = 0 \text{ (white male)} \\ (\alpha + \gamma) + \beta Ed_i, & \text{if } fe_i = 1, black_i = 0 \text{ (white female)} \\ (\alpha + \delta) + \beta Ed_i, & \text{if } fe_i = 0, black_i = 1 \text{ (black male)} \\ (\alpha + \gamma + \delta) + \beta Ed_i, & \text{if } fe_i = black_i = 1 \text{ (black female)} \end{cases}$$

$$\frac{\partial E(wage_i)}{\partial Ed_i} = \beta$$

$$\gamma = \left[\alpha + \gamma + \beta Ed_i^{w,F}\right] - \left[\alpha + \beta Ed_i^{w,M}\right]$$

$$= \gamma + \beta \underbrace{\left(Ed_i^{w,F} - Ed_i^{w,M}\right)}_{=0}$$

interaction dummy

$$\bullet\ wage_i = \alpha + \beta Ed_i + \boxed{\gamma Fe_i} + \boxed{\delta\, black_i} + \boxed{\theta \cdot Fe_i \cdot black_i} + e_i$$

$$\hat{\beta} = 1.16,\quad \hat{\gamma} = -2.55,\quad \hat{\delta} = -1.83,\quad \hat{\theta} = 0.58$$

$$E(wage_i \mid Ed_i, black_i, fe_i)$$

$$\begin{cases} (1)\ \alpha + \beta Ed_i, & \text{if } Fe_i = black_i = 0 \text{ (white males)} \\ (2)\ (\alpha + \gamma) + \beta Ed_i, & \text{if } Fe_i = 1,\ black_i = 0 \text{ (white females)} \\ (3)\ (\alpha + \delta) + \beta Ed_i, & \text{if } Fe_i = 0,\ black_i = 1 \text{ (black males)} \\ (4)\ (\alpha + \gamma + \delta + \theta) + \beta Ed_i, & \text{if } Fe_i = black_i = 1 \text{ (black females)} \end{cases}$$

$$\theta = \left[\overset{F,b}{(4)} - \overset{M,b}{(3)}\right] - \left[\overset{F,w}{(2)} - \overset{M,w}{(1)}\right]$$

$$= [\gamma + \theta] - [\gamma]$$