**Econometrics - TA session 1**

**QUESTION 1**

Assume a regression model with intercept. Data are as follows:

(i) What is the sample size?

(ii) Compute the regression equation with using LS.

(iii) Estimate the standard error of and test the hypothesis that is zero. (Write down the null and the alternative. Derive the test statistic and explain how you will decide for the null or for the alternative hypothesis.)

(iv) Test the hypothesis that . (Write down the null and the alternative. Derive the test statistic and explain how you will decide for the null or for the alternative hypothesis.)

(v) Compute .

**Solution**

(i) Sample size: The sample size is . To see this, notice that since we have an intercept in the model

(ii) Compute the regression equation for LS: We know that

Now to find the inverse , we use the rule for inverse of block diagonal matrices

So,

So, the estimated regression is

(iii) Standard error of and test hypothesis that is zero.

We know that

and that

To obtain we replace with , where

The normal equations imply the orthogonal decomposition

We are told that

We know and so we need to find . We know

Therefore,

Going back

Now need to find , where .

Consequently,

Finally,

Hypothesis testing: We have the following two hypotheses:

To test this hypothesis we employ the test statistic

which under the assumption and the null hypothesis . At a size of test , we are going to reject the null if , where .

So, in our case

So, the null is rejected.

(iv) Test the hypothesis that

Two approaches:

t-test: The hypotheses is

the test statistic would be

which again under normality assumption and the null, we have that . We reject the null at size of test if , where .

In our example,

So, we reject the null. F-test: The hypotheses are

where

The test statistic would be

which again under normality assumption and the null, we have that . We reject the null at size of test if , where .

In our example,

So, we reject the null.

Remark: Since one restriction we have the result that . In the case of one linear restriction the t-test should be preferred to F-test.

(v) Compute

Centered :

Uncentered :

**QUESTION 2**

Consider the linear regression model

where . You know that (true) and that

In a sample of 32 observations, the LS coefficients are .

1. Test at the significance level the joint null hypothesis that . State analytically the alternative hypothesis against which you are testing.
2. Assume that you do not know but you are given that its estimate is . In what sense, can the statistic you used to test the joint hypothesis be valid?

It is suggested that you find critical values using proper tables and -values with the software of your preference using proper commands.

**Solution**

1. The null hypothesis

can be written as with

For the Normal Classical Linear Regression model, we have that (conditional on ) . It follows that, under ,

Since is known, can be computed for a given sample and its (exact) sampling distribution can be used to conduct hypothesis testing.

Given the particular form of in this case, the statistic takes the form

which is convenient since it does not require inversion of . With our data, and

From the table of we get that the and the null hypothesis is not rejected at level of statistical significance .

1. The test statistic is distributed as chi-sq with 2 degrees of freedom for any sample size . When the is replaced by the unbiased estimator the statistic, say , is validated only asymptotically:

Since the sample size is relatively small, but we maintain the normality assumption, a natural choice is to use the statistic for testing this null hypothesis:

with critical value .

**Question 3**

Suppose that x follows the Weibull distribution

1. Obtain the log-likelihood function for a random sample of n observations.
2. Obtain the likelihood equations for maximum likelihood estimation of α and β. Note that the first provides an explicit solution for α in terms of the data and β. But, after inserting this in the second, we obtain only an implicit solution for β. How would you obtain the maximum likelihood estimators?
3. Obtain the second derivatives matrix of the log-likelihood with respect to α and β. The exact expectations of the elements involving β involve the derivatives of the gamma function and are quite messy analytically. Of course, your exact result provides an empirical estimator. How would you estimate the asymptotic covariance matrix for your estimators in part b?
4. Prove that (Hint: The expected first derivatives of the log-likelihood function are zero.)

**Solution**

1. We form the likelihood function:

It is usually simpler to work with the log of the likelihood function:

1. The F.O.C are given by:

 Since the first likelihood equation implies that at the maximum, one approach to obtain the ml estimators would be to scan over the range of β and compute the implied value of α. Two practical complications are the allowable range of β and the starting values to use for the search.

1. The second derivatives are:

If we had estimates in hand, the simplest way to estimate the expected values of the Hessian would be to evaluate the expressions above at the maximum likelihood estimates, then compute the negative inverse.

1. First, the expected value of the first FOC is zero which implies the following:

Now, the expected value of the second FOC is also zero:

Divide by n, use the fact that every term in the sum has the same expectation and substitute to obtain:

 Now multiply by to obtain

 Since .