Lecture 1. : Public Goods (A refresher)

* Reason of State Intervention → Public Goods
(Benefits). ↳ Externalities

→ Normative Analysis : Social Planner. would decide the optimal provision of Public Goods

→ Positive Analysis : Discuss the political economy incentives that distort the decisions of the policy makers.

Definitions :

Pure Public Goods : Goods that are perfectly non-rival in consumption and are also non-excludable

* Non-rival : One individual's consumption does not affect another's opportunity to consume the good.
* Non excludable: Individuals cannot deny each other the opportunity to consume a good.

	Rival Goods	Non-rival Goods
Excludable	Private Goods (food, clothing)	Impure Public Goods (Parks, cinemas)
Non-Excludable	Impure Public goods (city park)	Pure Public Goods (national defense)

A Simple (2 agents) model of Public Goods.

→ An economy of agents (agent i / agent J)
→ Utility function of agent i is the following

$$U_i = \log(c_i) + \log(G)$$

where c_i : Private Consumption of i
and G : Public Good

→ Private Consumption : $c_i = y_i - g_i$
→ Public Good : $G = g_i + g_J$

$$\rightarrow \max_{g_i} U_i = \log(y_i - g_i) + \log(g_i + g_J)$$

The question for agent i is : What is the optimal g_i^*

$$U_J = \log(c_J) + \log(G)$$

$$U_J = \log(y_J - g_J) + \log(g_i + g_J)$$

$$\max_{g_i} U_i = \underbrace{\log(Y_i - g_i)} + \underbrace{\log(g_i + g_J)}$$

$\boxed{\text{FOC}}$

$$\frac{\partial U_i}{\partial g_i} = 0 \Leftrightarrow \frac{1}{Y_i - g_i}(-1) + \frac{1}{g_i + g_J} = 0 \Leftrightarrow 2g_i = Y_i - g_J$$

$$\Leftrightarrow \boxed{g_i^* = \frac{1}{2}\left[Y_i - g_J\right]}$$ Reaction Function i

$\uparrow g_J \Rightarrow \downarrow g_i^*$ Agent i has an incentive to reduce the provision of public good g_i when the other guy increases his/her contribution g_J [Incentive to free-ride]

$$\max_{g_J} U_J = \log(Y_J - g_J) + \log(g_i + g_J)$$

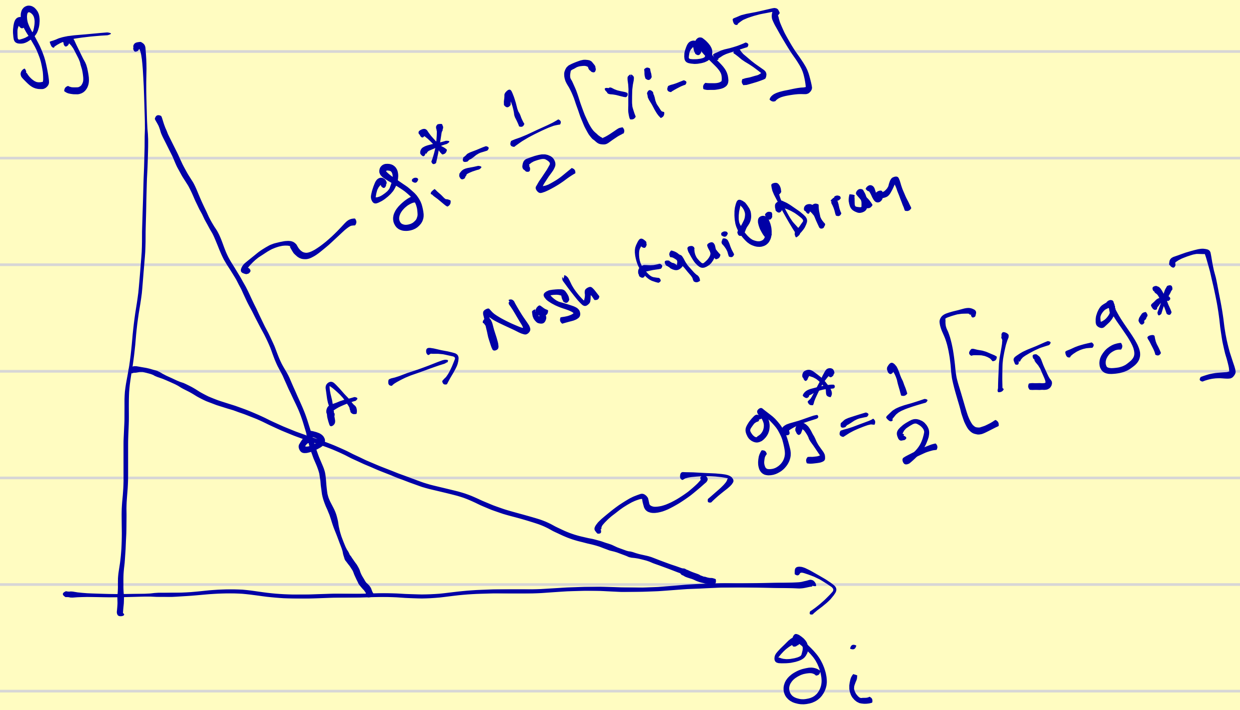
$$\partial U_J / \partial g_J = 0 \Leftrightarrow \frac{1}{Y_J - g_J}(-1) + \frac{1}{g_i + g_J} = 0 \Leftrightarrow$$

$$\Leftrightarrow \boxed{g_J^* = \frac{1}{2}\left[Y_J - g_i\right]}$$ Reaction Function J

Agent J has also an incentive to free-ride

$\uparrow g_i \Leftrightarrow \downarrow g_J^*$ (Free-ride incentive)

What is the Equilibrium? (Non cooperation)



Nash Equilibrium

For simplicity we assume symmetric agents $y_i = y_J = \bar{y}$

System
$$\begin{cases} g_i^* = \frac{1}{2}[\bar{y} - g_J] \\ g_J^* = \frac{1}{2}[\bar{y} - g_i] \end{cases}$$

$$\boxed{g_i = g_J = \frac{\bar{y}}{3}}$$

Non-cooperative Provision of Public good.

Cooperative Solution (Social Optimum)

→ We assume that there exists a "social planner" that chooses g_i and g_J as to maximize the Social Welfare Function $W = U_i + U_J$

$$W = \underbrace{\log(y_i - g_i) + \log(g_i + g_J)}_{U_i} + \underbrace{\log(y_J - g_J) + \log(g_i + g_J)}_{U_J}$$

Social Planner (Benevolent) $\max_{g_i, g_J} W = U_i + U_J$

$$W = \boxed{\log(Y_i - g_i)} + \underline{\log(Y_J - g_J)} + \boxed{2 \log(g_i + g_J)}$$

$$\frac{\partial W}{\partial g_i} = \boxed{\frac{1}{Y_i - g_i}(-1)} + \boxed{2\frac{1}{g_i + g_J}} = 0 \Leftarrow$$

PMC as before

SMB = PMB$_1$ + PMB$_2$

$$\Rightarrow 3 g_i = 2Y_i - g_J \Leftarrow \boxed{g_i^* = \frac{1}{3}\left[2Y_i - g_J\right]}$$

$$\partial W / \partial g_J = 0 \Leftarrow \ldots\ldots \boxed{g_J^* = \frac{1}{3}\left[2Y_J - g_i\right]}$$

By assuming (as before) symmetric agents for simplicity $Y_i = Y_J = \bar{Y}$

SOCIAL OPTIMUM

We conclude that $\boxed{g_i = g_J = \frac{1}{2}\bar{Y}}$

$$g_i^{NASH} < g_i^{COOP.} \quad / \quad g_J^{NASH} < g_J^{COOP}$$

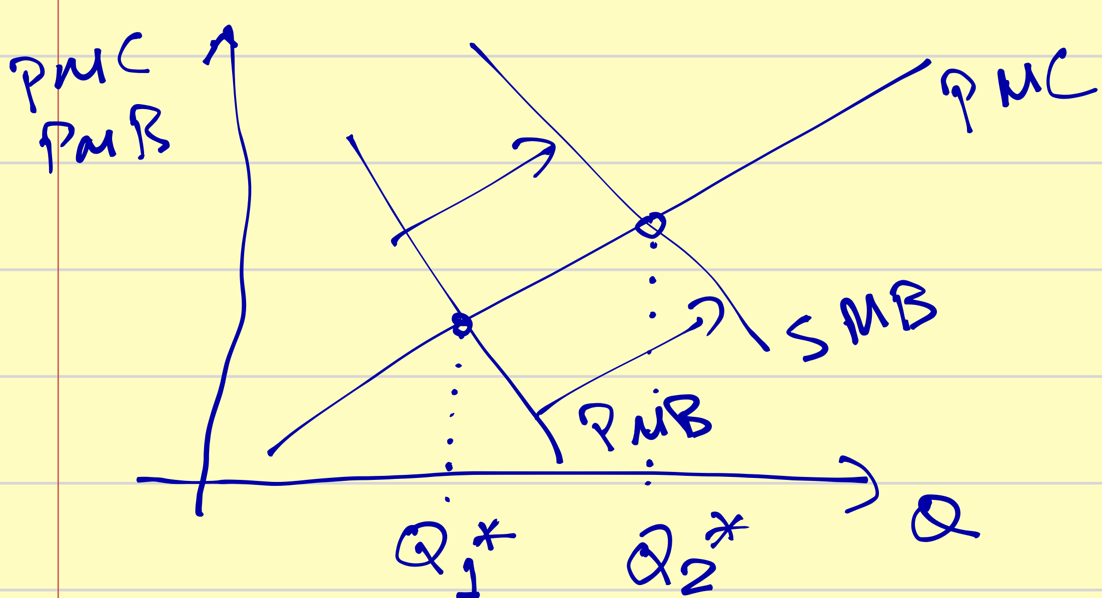
⇒ Back to the FOCs

$$\frac{\partial U_i}{\partial g_i} = \overbrace{\boxed{\frac{1}{Y_i - g_i}(-1)}}^{PMC_i} + \overbrace{\boxed{\frac{1}{g_i + g_J}}}^{PMB_i} = 0$$

FOC Non Coop

$$\frac{\partial W}{\partial g_i} = \underbrace{\boxed{\frac{1}{Y_i - g_i}(-1)}}_{PMC_i} + \underbrace{\boxed{2\frac{1}{g_i + g_J}}}_{PMB_i + PMB_J = SMB} = 0$$

FOC Cooperative



Positive Externality Consumption.

Q is underprovided

PMB = PMC

⇒ PMC = SMB ⇒ ↑ Q (Social optimum)

[GPUBER]

⇒ Question : If $U_i(g_i^{*COP}) > U_i(g_i^{*NON\ COP})$

If $U_J(g_J^{COP}) > U_J(g_J^{*NON\ COP})$

$Y = 1200$ € Solve the Problem

Hint: What happens when I plays NC and J plays cooperatively and vice versa