Homework Set 3

1.a)

$$\begin{bmatrix} z_{t+1} \\ x_{t+1} \\ y_{t+1} \end{bmatrix} = \begin{bmatrix} -5 & 4 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} z_t \\ x_t \\ y_t \end{bmatrix} + \begin{bmatrix} t \\ 0 \\ 0 \end{bmatrix}$$

1.b)

$$\begin{bmatrix} y_t \\ x_t \\ z_t \\ u_t \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & -0.25 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \\ z_{t-1} \\ u_{t-1} \end{bmatrix}$$

1.c)

$$\begin{bmatrix} \varphi_{t+1} \\ z_{t+1} \\ x_{t+1} \\ y_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & -5 & 0 & -4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \varphi_t \\ z_t \\ x_t \\ y_t \end{bmatrix}$$

2.a)

$$\begin{bmatrix} x_{t+1} = -x_t + y_t - 8 \\ y_{t+1} = -0.3x_t + 0.9y_t + 4 \end{bmatrix}$$

General Solution:

$$\begin{aligned} x_t &= \mathrm{A}_1(0.73)^t + \mathrm{A}_2(-0.83)^t + 6.37 \\ y_t &= \mathrm{A}_1 1.73(0.73)^t + \mathrm{A}_2 0.17(-0.83)^t + 20.8 \end{aligned}$$

2.b)

$$\begin{bmatrix} x_{t+1} = x_t - y_t \\ y_{t+1} = x_t + 3y_t \end{bmatrix}$$

General Solution:

$$\begin{aligned} x_t &= \mathrm{A}_1 2^t + \mathrm{A}_2 t 2^{t-1} + \mathrm{A}_2 2^t \\ y_t &= -\mathrm{A}_1 2^t - \mathrm{A}_2 t 2^{t-1} - 2\mathrm{A}_2 2^t \end{aligned}$$

Homework Set 4

1. a)

General Solution: $y_t = \mathrm{A}_1 e^t + \mathrm{A}_2 e^{0.5t} + (\frac{t^3}{3} - 2t^2 + 9t - 18)e^t$

1. b)

General Solution: $y_t = \mathrm{A}_1 e^{-t} + \mathrm{A}_2 t e^{-t} + t^2 - 4t + 6$

2. a)

General Solution:

$$\begin{aligned} x_t &= \mathrm{A}_1 e^{3t} + \mathrm{A}_2 e^{-t} \\ y_t &= \mathrm{A}_1 2e^{3t} + \mathrm{A}_2(-2)e^{-t} \end{aligned}$$

2.b)

General Solution:

$x_t = A_1 2e^{2t} + A_2 0.5e^{-t}$

$y_t = A_1 2e^{2t} + A_2 e^{-t}$

3)

The system of the two differential equations:

$$\dot{x} = -0.1x + 5\lambda$$
$$\dot{\lambda} = -\frac{20}{x} + 0.1\lambda$$

The fixed point of the system:

$$x^* = 100$$
$$\lambda^* = 2$$

The linearization of the system around the fixed point is:

$$\dot{x} = -0.1(x - x^*) + 5(\lambda - \lambda^*)$$
$$\dot{\lambda} = 0.002(x - x^*) + 0.1(\lambda - \lambda^*)$$

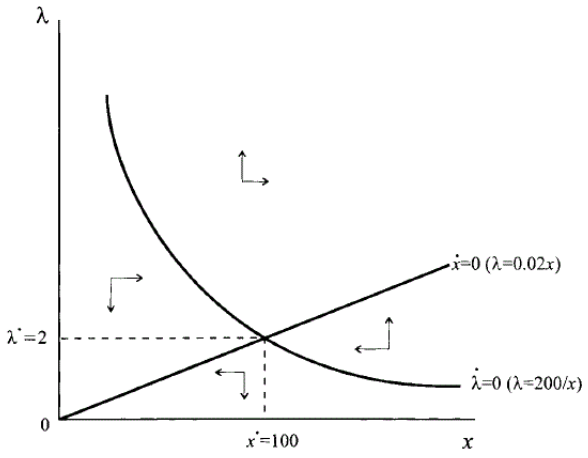

Figure 1: Phase Diagram