

Part I

Homework Set 1

0.1 Numerical Application

The system of differential equations that describe the solution:

$$\begin{aligned}\dot{\lambda} &= -\lambda \\ \dot{y} &= y + \frac{a+\lambda}{2}\end{aligned}$$

0.2 Exhaustible Resources

$$\begin{aligned}\dot{\lambda} &= \rho\lambda + 2x + \delta\lambda \\ \dot{x} &= \frac{1+\lambda}{2} - \delta x\end{aligned}$$

0.3 The Saving Problem

Euler equation: $u'(c_t) = u'(c_{t+1})\beta R_{t+1}$

Part II

Homework Set 2

0.1. a) $2y_{t+1} + 3y_t + 2 = 0, y_0 = 1$

$$y_t = \frac{7}{5} \left(-\frac{3}{2}\right)^t - \frac{2}{5}$$

The term of the homogeneous equation dominates the equilibrium value as time goes to infinity. y_t movement is unstable-explosive and alternates in signs.

b) $y_{t+1} - \frac{1}{2}y_t = t + 3, y_0 = 3$

$$y_t = \left(\frac{1}{2}\right)^t + 2t + 2$$

y_t converges to its equilibrium value, but this equilibrium value is not stable, so y_t goes to infinity as time goes to infinity.

0.2.a)

$$y_t = -3^t + \frac{3}{2}2^t + \frac{1}{2}4^t$$

Since both distinct roots are in absolute value bigger than one, then y_t goes to infinity as t goes to infinity.

b)

$$y_t = 3^t - \frac{1}{6}t^3 - \frac{1}{2}t^2 - \frac{1}{3}t$$

y_t goes to infinity as t goes to infinity

0.3.

$$y_t = [-c_1 + c_2(t - 0.5)](-1)^t - \frac{2a}{(a+1)^2}a^t$$

$$x_t = [-c_1 - c_2(t - 1)](-1)^t + \frac{a-1}{(a+1)^2}a^t$$

This is the same application as in Tutorial 3, case $\Delta=0$