Differential Equations & Phase Diagrams

Mathematics for Economists, Fall 2024-25

Homework Exercises Set 4

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Due: 30/12/2024

1 Find the solution of the following differential equations, determine the arbitrary constants and characterize the movements:

a) $2y''(t) - 3y'(t) + y = (t^2 + 1)e^t,\ y(0) = 5,\ y'(0) = 14$

b) $y''(t) + 2y'(t) + y = t^2,\ y(0) = 0,\ y'(0) = 1$

2 Solve the following linear autonomous differential equations systems and construct their phase diagrams:

a) $\dot{x} = x + y,\ \dot{y} = 4x + y$

b) $\dot{x} = 3x - 2y,\ \dot{y} = 2x - 2y$

3 Solve the equilibrium for the following control problem. Linearise the system around its equilibrium and establish its stability properties:

$$\underset{\{u\}}{max} J = \int_0^\infty (20lnx - 0.1u^2)dt$$

$$s.t. \quad \dot{x} = u - 0.1x$$