Optimal Control & Dynamic Programming Applications

Mathematics for Economists, Fall 2024-25

Homework Exercises Set 1

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Application - Numerical (analytical solution - no discounting)

Solve the following problem using optimal control:

$$max \int_0^T -(\alpha u + \beta u^2)dt$$

$$s.t. \quad \dot{y} = y - u, \;\; y(0) = y_0, \;\; y(t) - free$$

Application - Exhaustible Resources example

In this problem, production of a good y, yields economic benefits but also contributes to the stock of pollution, x, which is an economic bad. Instantaneous net benefits are $y - y^2 - x^2$. If the stock of pollution depreciates (is broken naturally in the environment) at the rate δ, find the path of consumption that solves the following:

$$max \int_0^T e^{-\rho t}(y - y^2 - x^2)dt$$

$$s.t. \quad \dot{x} = y - \delta x, \;\; x(0) = x_0, \;\; x(T) = x_T$$

Application - The Saving Problem

A consumer wants to maximize

$$\max_{\{c_t, A_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$s.t. \quad A_{t+1} = R_{t+1}(A_t + y_t - c_t)$$

where $\beta \epsilon (0,1)$, u is a twice continuously differentiable, increasing, strictly concave utility function, A_{t+1} is the consumer's holdings of an asset at the beginning of period $t+1$, y_t an endowment sequence, c_t the consumption of a single good, R_{t+1} is the gross rate of return on the asset between t and $t+1$. Find the Euler equation using
1) Lagrange multipliers
2) Dynamic programming