**Optimal Control & Dynamic Programming Applications**

Mathematics for Economists, Fall 2023-24

Homework Exercises Set 1

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**Application** - Numerical (analytical solution - no discounting)

Solve the following problem using optimal control:

$$max \int_0^T -(\alpha u + \beta u^2)dt$$

$$s.t. \quad \dot{y} = y - u, \ \ y(0) = y_0, \ \ y(t) - free$$

**Application** - Exhaustible Resources example

In this problem, production of a good $y$, yields economic benefits but also contributes to the stock of pollution, $x$, which is an economic bad. Instantaneous net benefits are $y - y^2 - x^2$. If the stock of pollution depreciates (is broken naturally in the environment) at the rate $\delta$, find the path of consumption that solves the following:

$$max \int_0^T e^{-\rho t}(y - y^2 - x^2)dt$$

$$s.t. \quad \dot{x} = y - \delta x, \ \ x(0) = x_0, \ \ x(T) = x_T$$

**Application** - The Saving Problem

A consumer wants to maximize

$$\max_{\{c_t, A_{t+1}\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t u(c_t)$$

$$s.t. \quad A_{t+1} = R_{t+1}(A_t + y_t - c_t)$$

where $\beta \epsilon (0,1)$, $u$ is a twice continuously differentiable, increasing, strictly concave utility function, $A_{t+1}$ is the consumer's holdings of an asset at the beginning of period $t+1$, $y_t$ an endowment sequence, $c_t$ the consumption of a single good, $R_{t+1}$ is the gross rate of return on the asset between $t$ and $t+1$. Find the Euler equation using
  1) Lagrange multipliers
  2) Dynamic programming