Problem Set
LECTURE VI: Public Finance in the Canonical RBC

Exercise 1. Lump Sum Taxes:

(a) Show how a higher government share of GDP (ω) along the deterministic steady state affects the level of steady state capital (K).

(b) Does this imply a higher level of steady state output (Y)? Explain why.

(c) What happens to the steady state level of aggregate private output, $Y^p = C + I$, as the government share of GDP (ω) increases? Explain.

Exercise 2. Distortionary Taxation, No Lump Sum Taxes: A representative household gets utility from consumption and government spending and disutility from labor. It gets to choose how much labor to supply and how much to consume; it takes government spending as given. It can save by accumulating bonds or capital. It leases capital to a representative firm on a period-by-period basis. It owns the firm and receives profit from the firm via a lump sum payment. It faces a common distortionary tax rate on its labor and capital income. Its problem can be written:

$$\max_{\{C_t, N_t, K_{t+1}, B_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - \theta \frac{N_t^{1+\chi}}{1+\chi} + \psi \ln G_t \right\}$$

subject to

$$C_t + K_{t+1} - (1-\delta)K_t + B_{t+1} \leq (1-\tau_t)(w_t N_t + R_t K_t) + \Pi_t + (1 + r_{t-1})B_t$$

(a) Derive the first order conditions for the household problem.

(b) A representative firm produces output according to $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$. Each period, it solves the static problem of picking K_t and N_t to maximize static profit. Derive the first order conditions for the firm.

The government's budget constraint is given by:

$$G_t + r_{t-1}D_t = \tau_t \left(w_t N_t + R_t K_t\right) + D_{t+1} - D_t$$

D_t is government debt (so that positive values of D_t mean that the government faces an interest expense). Government spending follows an exogenous AR(1) process with non-stochastic mean of G:

$$\ln G_t = (1-\rho_G)\ln G + \rho_G \ln G_{t-1} + \epsilon_{G,t}$$

The tax rate follows an AR(1) process with non-stochastic mean of τ, but reacts to deviation of the debt-gdp ratio from an exogenous steady state target, D/Y:

$$\tau_t = (1-\rho_\tau)\tau + \rho_\tau \tau_{t-1} + (1-\rho_\tau)\gamma_\tau \left(\frac{D_t}{Y_t} - \frac{D}{Y}\right)$$

The parameter γ_τ is such that the there is a determinate, non-explosive equilibrium. Exogenous productivity follows an AR(1) in the log with non-stochastic mean of unity:

$$\ln A_t = \rho_A \ln A_{t-1} + \epsilon_{A,t}$$

(c) Assume that $B_t = D_t$ initially. What must be true about household bond-holdings and government debt going forward? Write down the definition of a competitive equilibrium. Derive an expression for the aggregate resource constraint (i.e. the expenditure identity expression for output).

(d) Assume that in steady state the government consumes an exogenous fraction of output, i.e. $\frac{G}{Y} = \omega > 0$. Assume further that the steady state debt-gdp ratio is exogenous, $\frac{D}{Y}$. Solve for the steady state value of τ consistent with the government's budget constraint holding in steady state.

(e) Given exogenous values of ω and $\frac{D}{Y}$, as well as the steady state value of τ you found above, solve for expressions for the steady state values of all other endogenous variables in terms of these parameters and the other deep parameters of the model.