LECTURE II cont.

FIRMS, PRODUCTION, AND SUPPLY

Technology and the Production Function

- Goods are produced in firms. Firms use capital, labor, land, energy, materials and other inputs to produce output. In economics, we characterize the technology used to produce outputs from inputs by a mathematical relationship, called the production function, of the form:

$$Q = f(x_1, x_2, \dots.)$$

where: Q stands for the quantity of output produced and $x_1, x_2, \dots.$are the quantities of inputs $X_1, X_2, \dots.$ For the most part in these notes we will consider the case, where output of given product is produced by capital and labor inputs:

$$Q = f(K, L)$$

- The marginal product of a factor is the extra amount of output that can be produced by an additional unit of that factor's input, keeping all other inputs constant.
- The marginal product of capital and labor associated with the preceding production are given by:

$$MP_K \equiv \frac{\Delta \mathrm{Q}}{\Delta K}, MP_{\mathrm{L}} \equiv \frac{\Delta \mathrm{Q}}{\Delta L}, \text{ respectively}$$

Typically, we assume that technology is characterized by positive and diminishing marginal products.

That is, $MP_K, MP_{\mathrm{L}} > 0$ and $\frac{\Delta MP_K}{\Delta K}, \frac{\Delta MP_L}{\Delta L}$<0

Note that marginal product of a factor is the slope of the graph of the production function, with respect to that factors input, keeping all other inputs constant.

Isoquants

- A production function can be represented by isoquants. An isoquant is the locus of points representing alternative combinations of inputs that produce a given level of output.

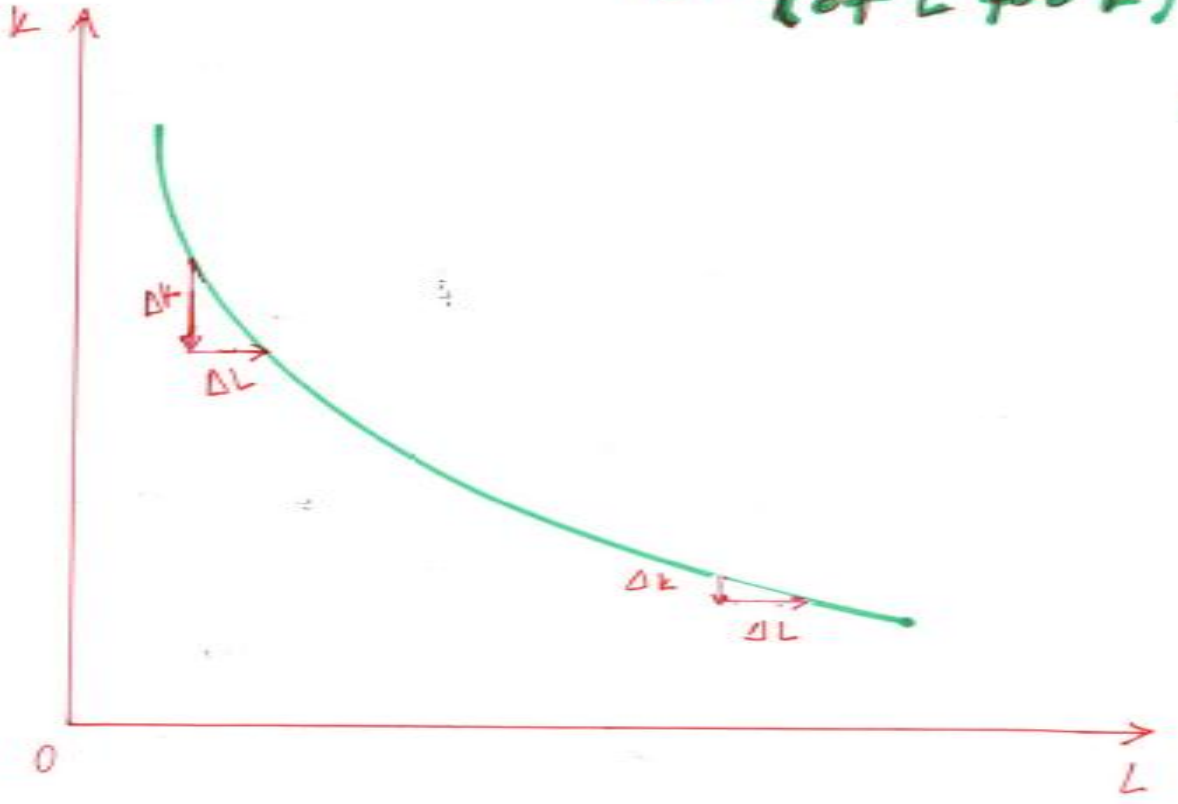
- Typical properties of isoquants are that:
- Higher isoquants are associated with a greater level of output
- Isoquants have a negative slope
- Isoquants are convex to the origin

Diminishing Marginal Rate of Technical Substitution

- The negative of the slope of an isoquant is called the marginal rate of technical substitution and indicates the rate at which one input can be reduced, while holding output constant, by increasing the amount of another input by one unit. The last property of isoquants means that this rate is diminishing.

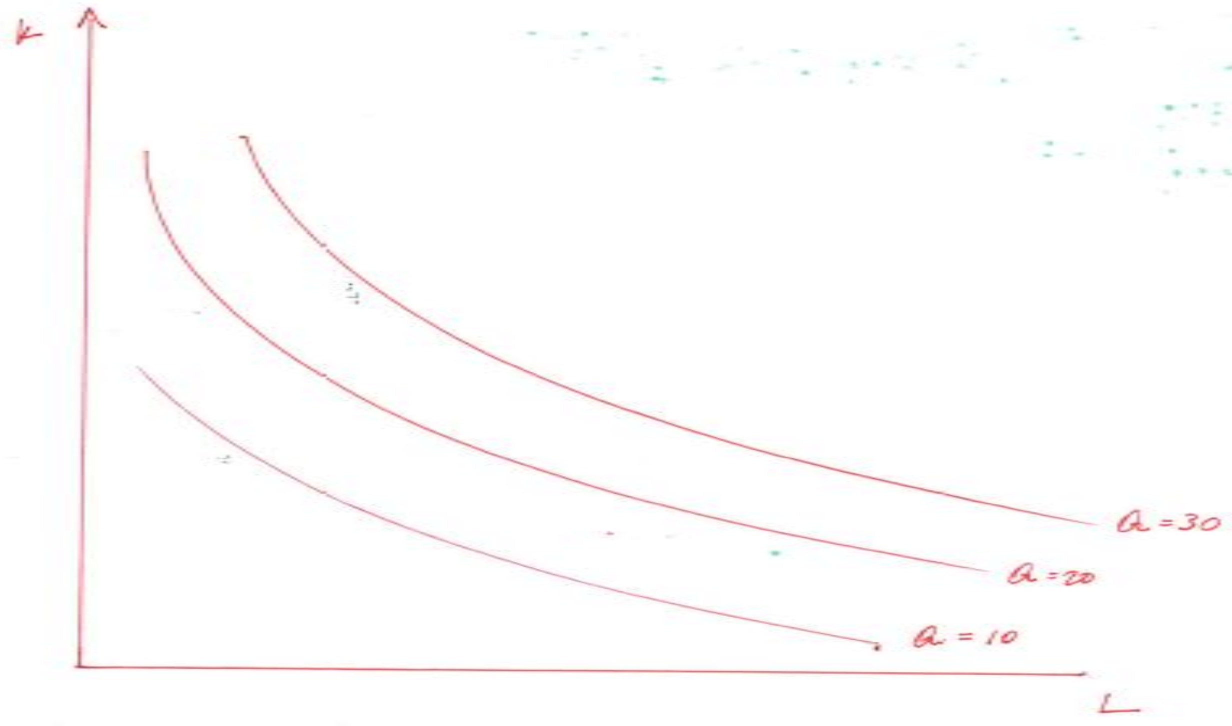
$$MRTS_{(\text{of } L \text{ for } K)} = -\frac{\Delta K}{\Delta L}\bigg|_{Q\ \text{const}}$$

$$= \frac{MP_L}{MP_K}$$



Returns to Scale

- The Rate at which output increases in response to a proportional increase in inputs. When inputs double, output less than doubles, exactly doubles, or more than doubles, we say that technology is characterized by decreasing, constant, or increasing returns to scale, respectively.
- Economies of scale vs coordination problems and physical constraints

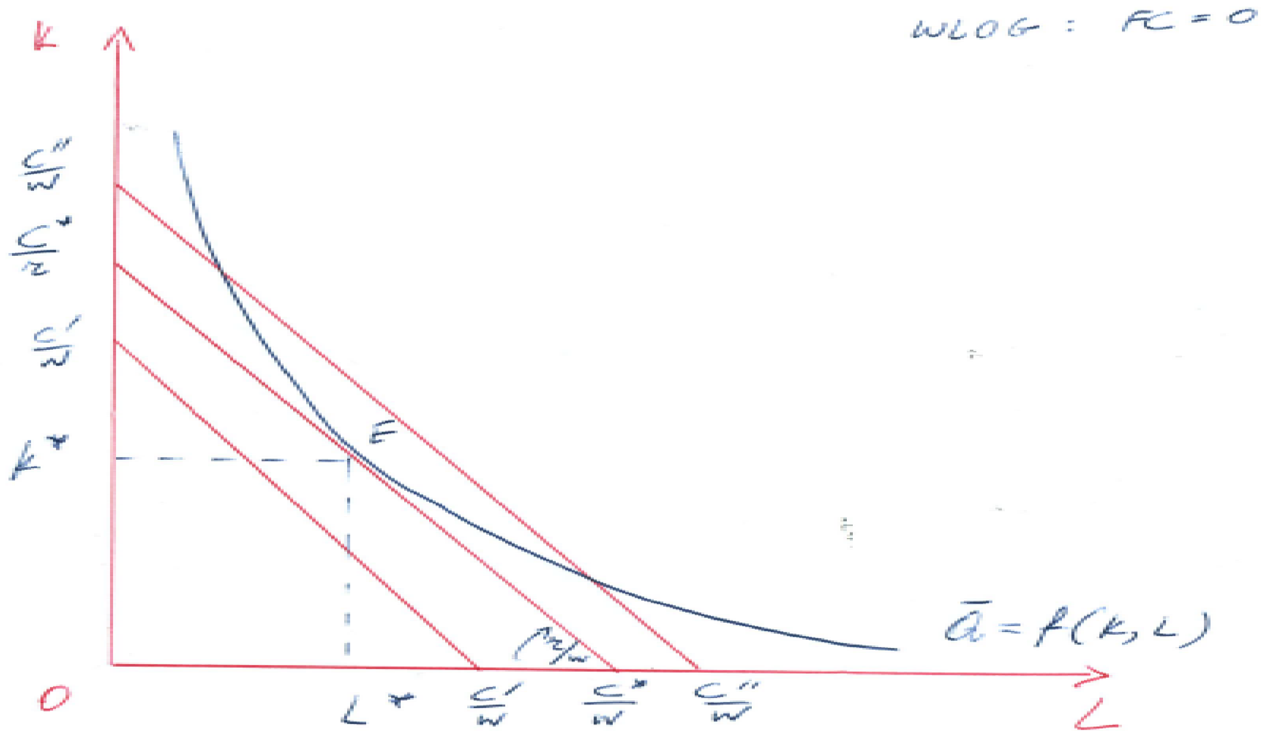

K
L
Q = 30
Q = 20
Q = 10

Profit Maximization subject to Technology constraint

- Firm’s profits:

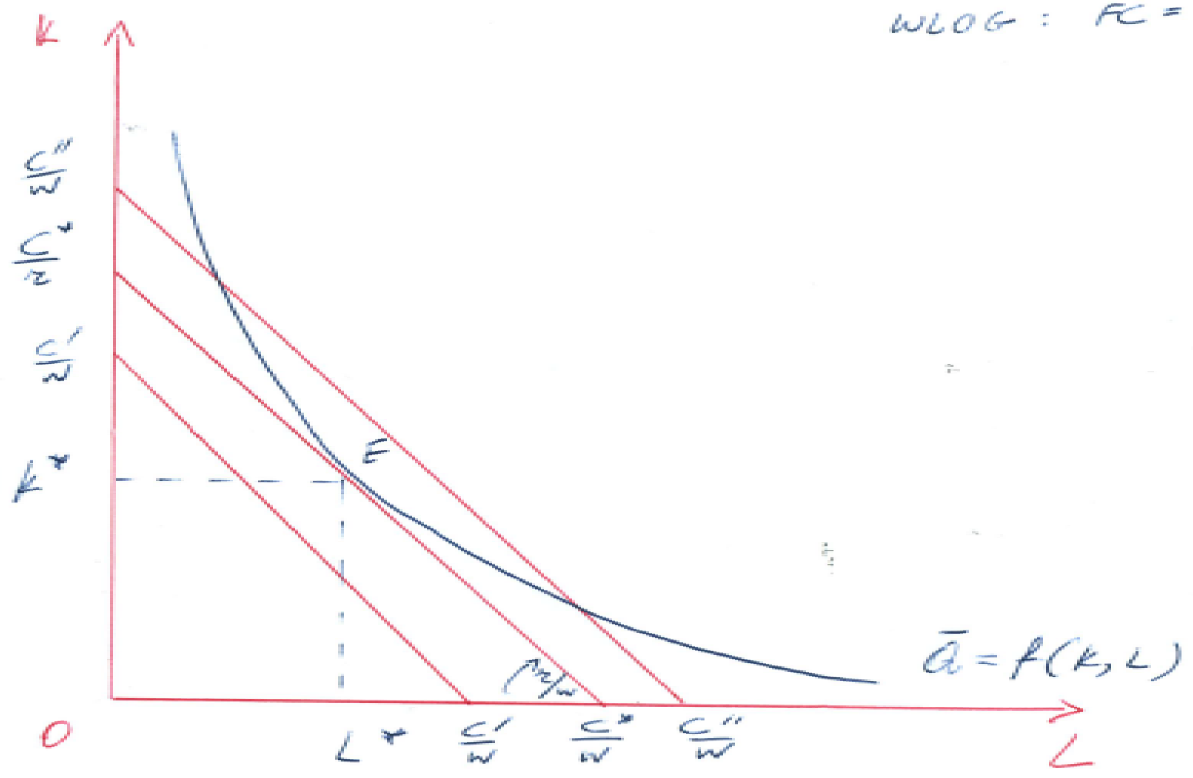
- $\pi = pQ - rK - wL - FC$
- p price of output
- $r$cost of capital
- w wage rate
- FC fixed costs

- Firms decide inputs and output to maximize profits subject to the production technology constraint.
- Clearly, for any given level of output they decide to produce they must minimize costs:
- $C = rK + wL + F\mathrm{C}$
- associated with this level of output:
- $Q = f(K, L)$

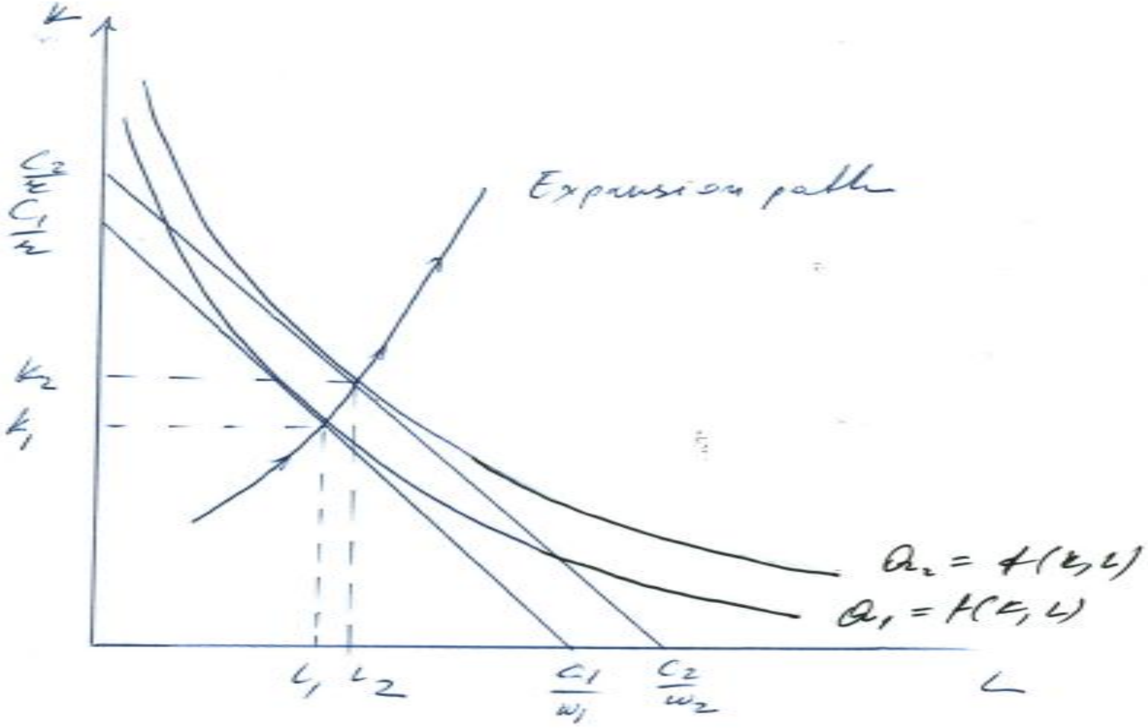


Constrained cost minimization

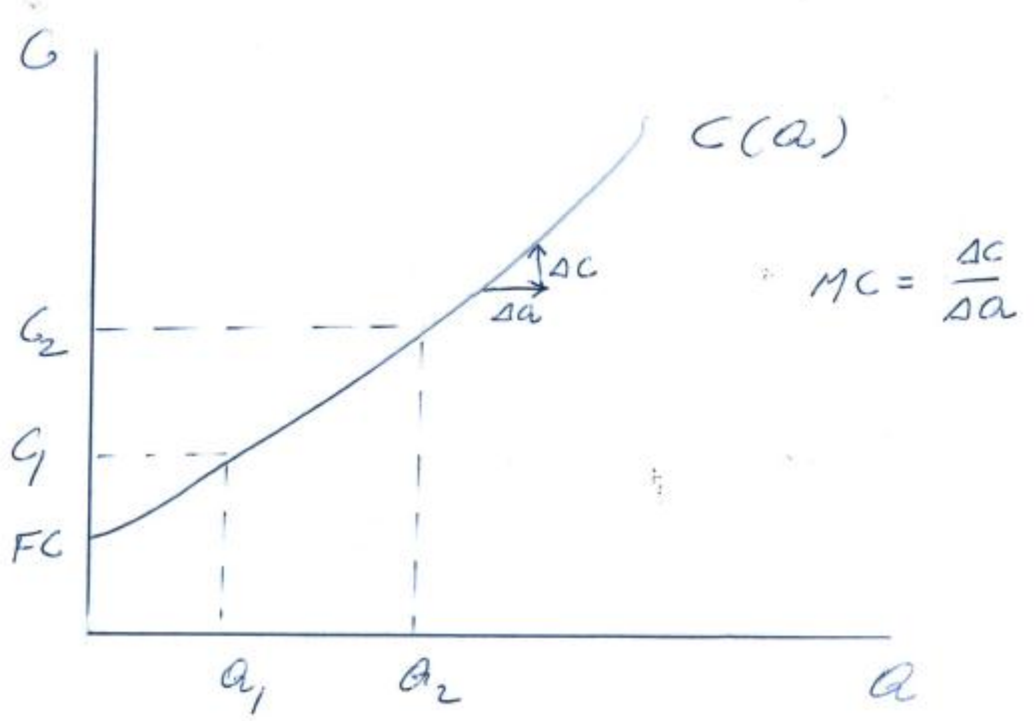
WLOG : FC = 0



Constrained cost minimization



Output expansion and costs

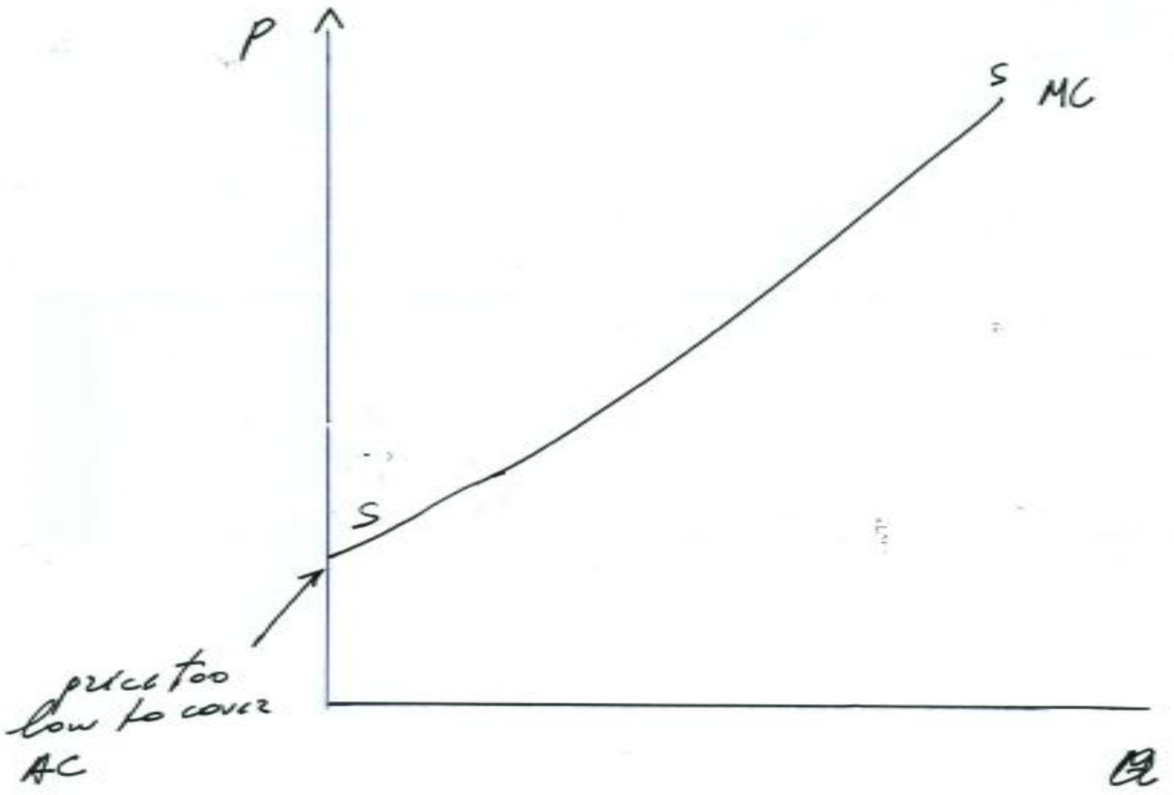


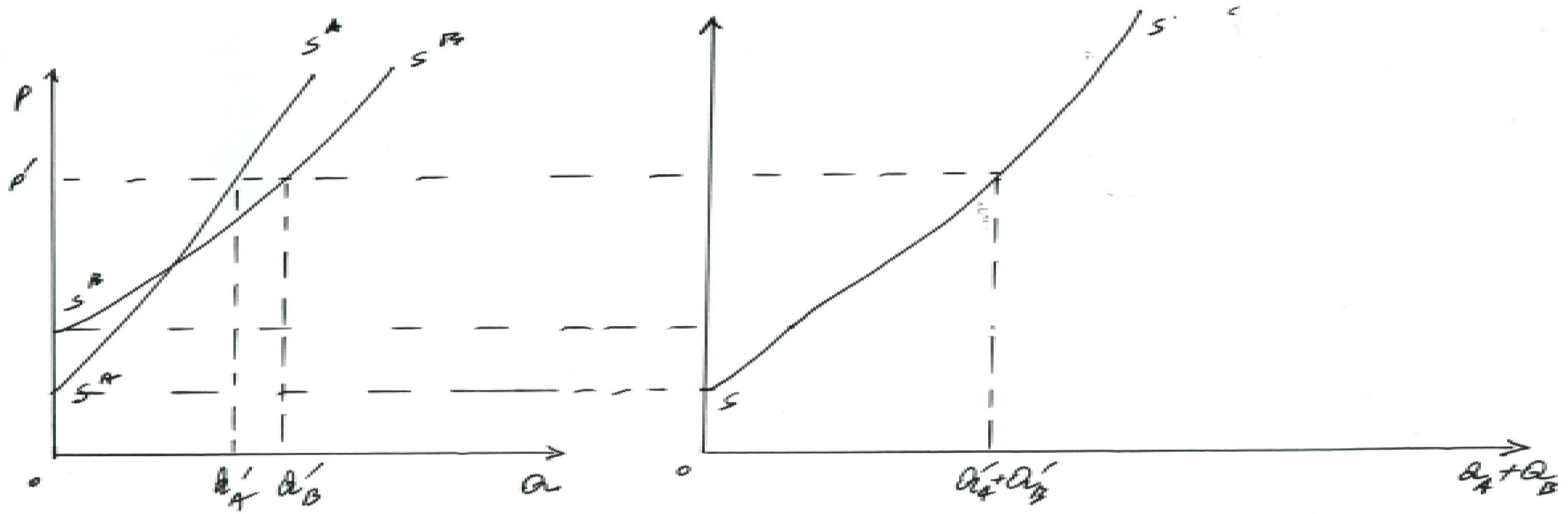
The expansion path translates to a cost function

Marginal Cost, MC, is the slope of the (total) cost curve

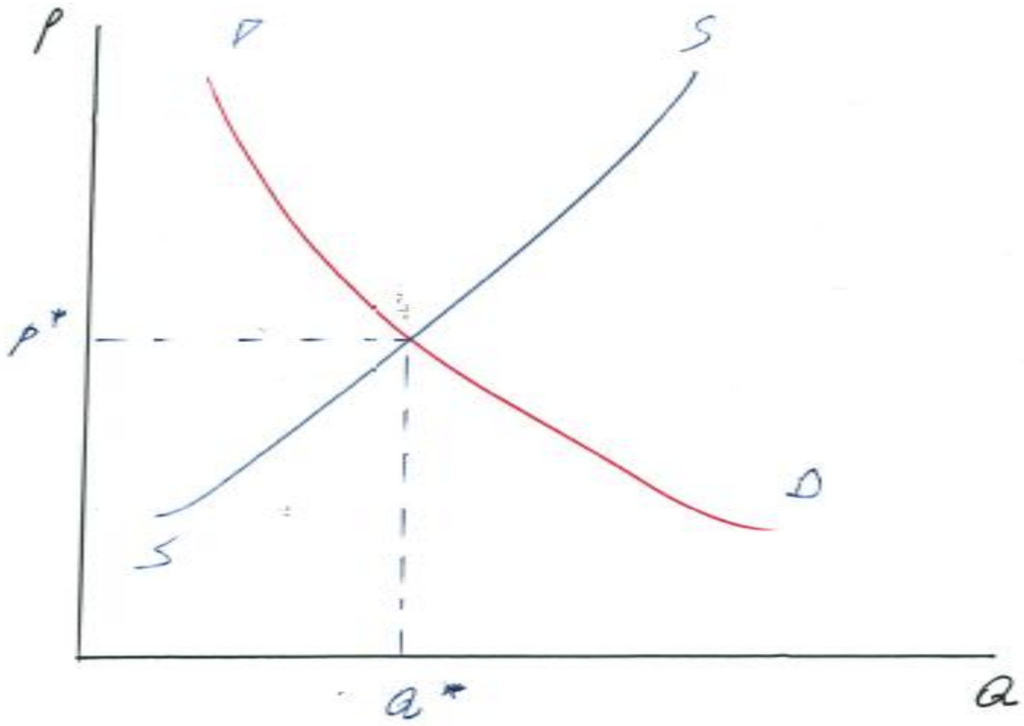
Profit Maximization

- Note, profits can be re-expressed as:
- $\pi = pQ - C(Q)$
- A firm that takes price of output as given will maximize profits, provided that revenues cover fixed costs, producing up to the point were :
- $p = MC$

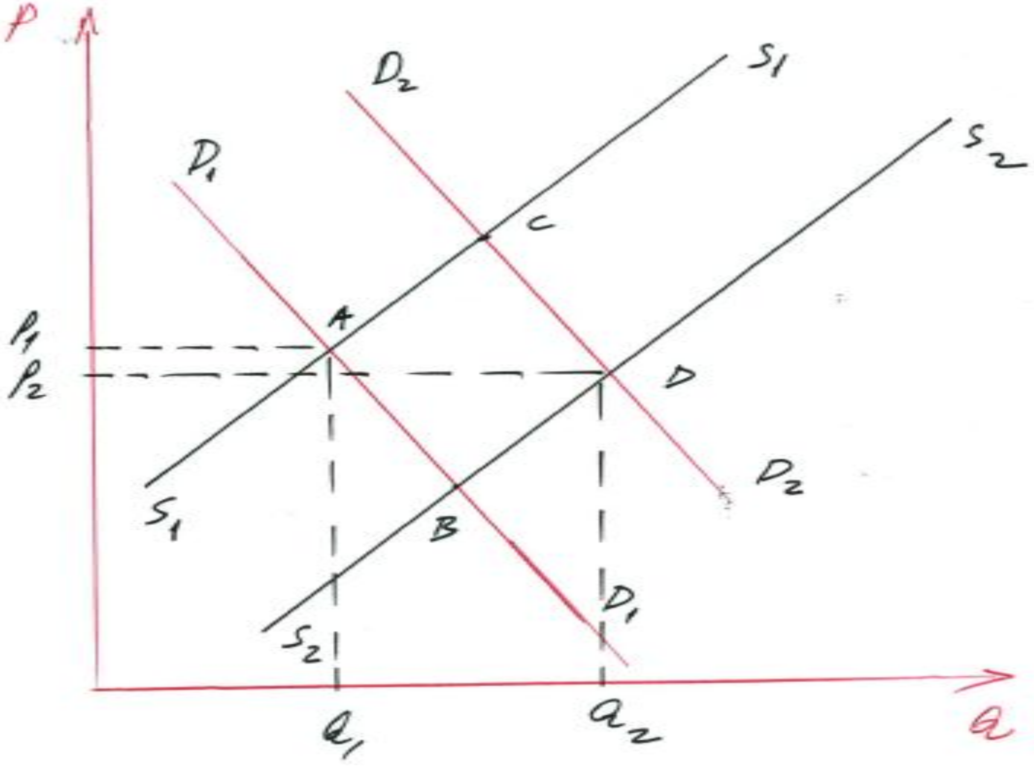

P
S
MC
S
price too
low to cover
AC
Q



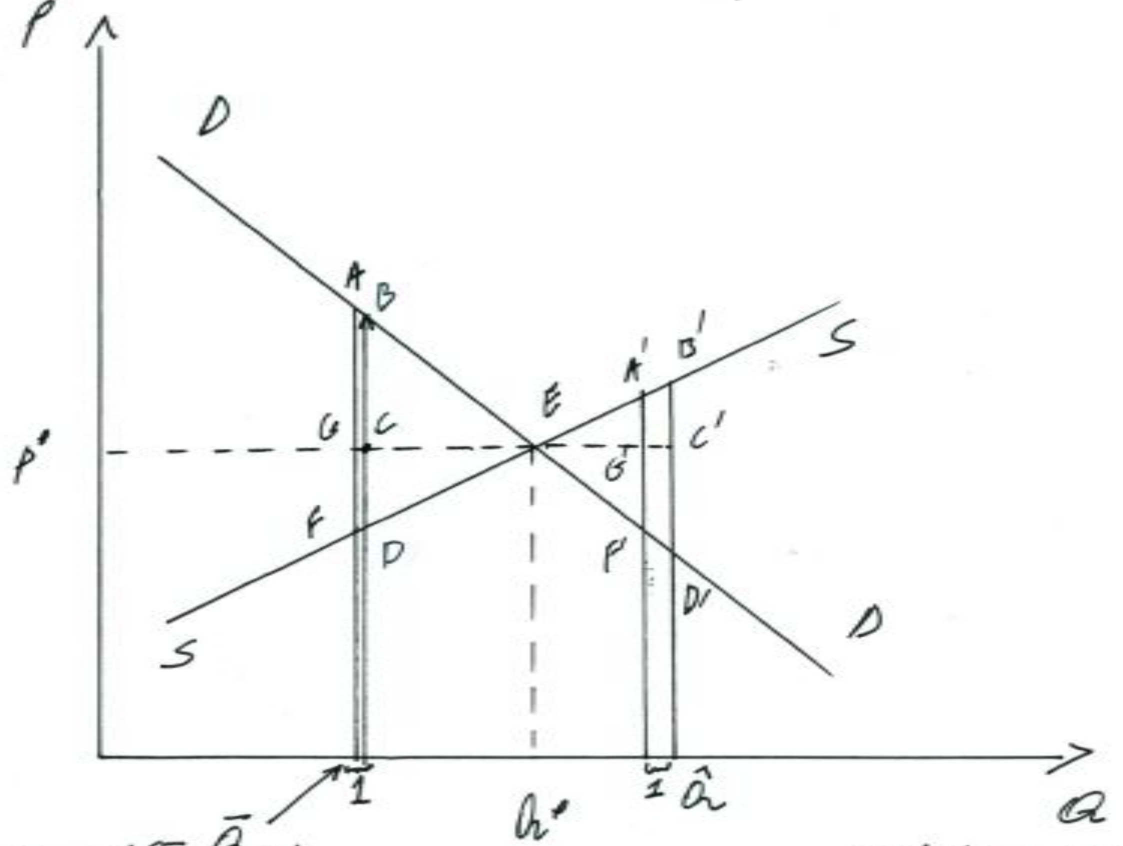
P
P'
S^A
S^B
S
Q
Q'_A
Q'_B
Q'_A+Q'_B
Q_A+Q_B
0



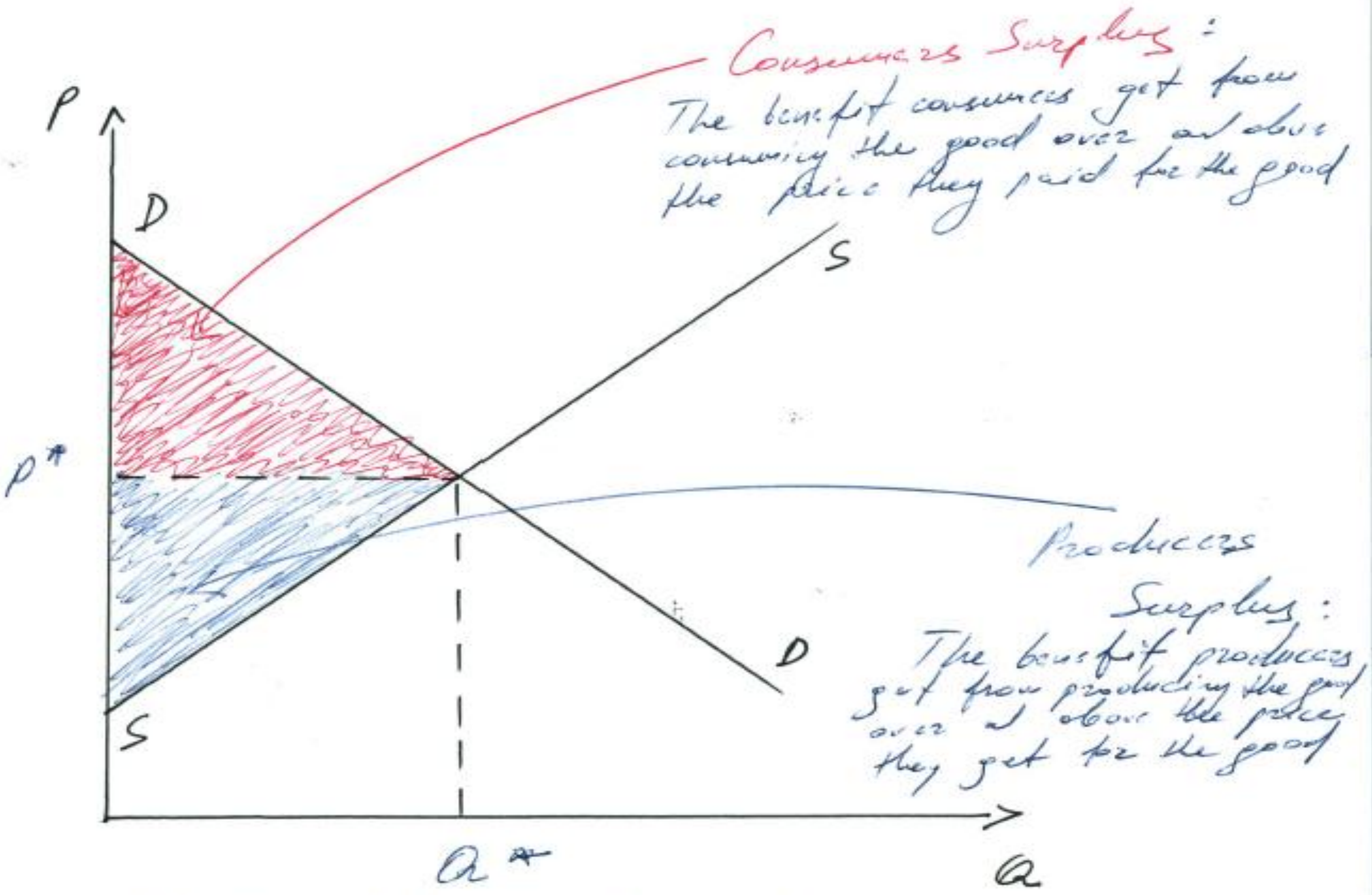
Market Equilibrium



Shifts in Aggregate Demand and Aggregate Supply



Suppose we are at $(\bar{Q}, \bar{Q}_{p^*})$.
Producing one more unit of Q increases the benefit to consumers by ABCG and increase the benefit (profits) to producers by FDCG. Clearly, the sum of these benefits is maximized at E.



In Market Equilibrium Consumers + Producers Surplus is maximized

Another Way to Look at Efficiency

$$\frac{P_2}{P_1} = \frac{MU_{A1}}{MU_{A2}} = \frac{MU_{B1}}{MU_{B2}} \quad (1)$$

$$P_1 = MC_1 \quad (2)$$

$$P_2 = MC_2 \quad (3)$$

$$\frac{r}{w} = \frac{MP_{L1}}{MP_{K1}} = \frac{MP_{L2}}{MP_{K2}} \quad (4)$$

(1) => Every individual in society has the same $MRS_{2\to 1}$ (allocative efficiency)

(4) => Every firm in society has the same $MRTS_{L\to K}$

(1)-(3) => $\frac{P_2}{P_1} = MRS_{2\to 1} = \frac{MC_2}{MC_1} = MRT_{2\to 1}$

$$MRS_{2\to 1} = MRT_{2\to 1}$$

The rate at which individuals in society is willing to exchange good 2 for good 1 is the same at which producers are willing to transform good 2 to good 1.