A Simple Example of a Gaussian Process

Let $\emptyset \neq \Theta \subseteq \mathbb{R}^d$ and $Y \sim N(0_{d\times 1}, V)$, where V is a symmetric positive semi-definite matrix.

Define $X = \{X_\theta := \theta' Y,\ \theta \in \Theta\}$. X is Gaussian since

$\forall \theta, \theta_1, \theta_2 \in \Theta$,

$$E X_\theta = \theta' E Y = \theta' 0_{d\times 1} = 0,$$

$$Cov(X_{\theta_1}, X_{\theta_2}) = E(X_{\theta_1} X_{\theta_2}) = E(\theta_1' Y \theta_2' Y) =$$

$$= E(\theta_1' Y Y' \theta_2) = \theta_1' E(Y Y')\theta_2 = \theta_1' V \theta_2.$$

Since Y is a Gaussian vector, the previous imply that if Θ^* is an ordered finite subset of Θ, i.e. $\Theta^* = \{\theta_1, \theta_2, \dots, \theta_n\}$ (assume that it is accordingly ordered) then

$$\begin{pmatrix} X_{\theta_1} \\ X_{\theta_2} \\ \vdots \\ X_{\theta_n} \end{pmatrix} \sim N\left(0_{n\times 1}, \Gamma_{n\times n}\right), \text{ where}$$

$$\Gamma = \left(Cov(X_{\theta_i}, X_{\theta_j})\right)_{i,j=1,\dots,n} = \left(\theta_i' V \theta_j\right)_{i,j=1,\dots,n}. \ \square$$