Addendum: ARMA Models with non-zero Mean

In the context of the UDC we can extend the definition of the ARMA Models so as to include the case of non-zero Mean. In this respect, and if $\mu \in \mathbb{R}$ (using the already established notation) define:

$$\Phi(L)y_t = \mu + \Theta(L)\varepsilon_t, \ \forall t \in \mathbb{Z} \iff$$
$$y_t = \Phi^{-1}(L)(\mu + \Theta(L)\varepsilon_t), \ \forall t \in \mathbb{Z} \iff$$
$$y_t = \Phi^{-1}(L)\mu + \Phi^{-1}(L)\Theta(L) \ \forall t \in \mathbb{Z} \iff$$
$$\Phi(L)(y_t - \Phi^{-1}(1)\mu) = \Theta(L)\varepsilon_t, \ \forall t \in \mathbb{Z} \iff$$
$$\Phi(L)\, y_t^* = \Theta(L)\varepsilon_t, \ \forall t \in \mathbb{Z}, \text{ where } y_t^* = y_t - \Phi^{-1}(1)\mu, \ t \in \mathbb{Z}.$$

and the latter form is essentially the typical form of the ARMA(p,q) recursion for the transformed process $(y_t^*)_{t\in\mathbb{Z}}$ which is essentially a pointwise translation by $-\Phi^{-1}(1)\mu$ of the $(y_t)_{t\in\mathbb{Z}}$ process.
(Notice that for the translation constant Φ^{-1} is evaluated at 1 due to the fact that L acts as identity on constant sequences).

Thereby in our framework $(y_t^*)_{t\in\mathbb{Z}}$ has all the relevant properties we have already established (under the appropriate conditions)* which implies that:

1. $E(y_t) = E(y_t^* + \Phi^{-1}(1)\mu) = \Phi^{-1}(1)\mu$,
2. $Var(y_t) = Var(y_t^* + \Phi^{-1}(1)\mu) = Var(y_t^*) = Var(y_0^*)$,
3. $k>0$, $\gamma_k = E(y_t - \Phi^{-1}(1)\mu)(y_{t-k} - \Phi^{-1}(1)\mu) = E(y_t^* + \Phi^{-1}(1)\mu - \Phi^{-1}(1)\mu)(y_{t-k}^* + \Phi^{-1}(1)\mu - \Phi^{-1}(1)\mu) = E(y_t^* y_{t-k}^*) = \gamma_k^*$, where γ_k^* is the relevant autocovariance of $(y_t^*)_{t\in\mathbb{Z}}$.

Example. When $p=1$, then $\Phi^{-1}(1)\mu = \sum_{i=0}^{\infty} B_1^i 1^i \mu = \frac{\mu}{1-B_1}$. $\square$

It is easy to derive the modification for the Gaussian Quasi-Likelihood function, if the actual value of μ (say μ_0) is unknown and the analogous extension for the definition of GQMLE. $\square$

* e.g. when $(\varepsilon_t)_{t\in\mathbb{Z}}$ is strictly stationary and ergodic, then $(y_t)_{t\in\mathbb{Z}}$ is also strictly stationary and ergodic (why?)

[The notes are in a state of perpetual correction. They do not substitute the lectures. Please r eport any typos to stelios@aueb.gr or the course's e-class.]