0.1 Natural monopolies

An industry is a natural monopoly if one firm is viable but not two or more, i.e., $\Pi(1) \geq 0 > \Pi(2)$. Below, we present three market structures that give rise to a natural monopoly.

0.1.1 Contestable markets

A market structure that describes the behavior of incumbent firms constantly faced by threats of entry. Entry does not require any sunk cost. Firms are subject to a hit-and-run entry. Therefore, if incumbent firms have no cost advantage over entrants, a contestable market equilibrium will result in having an incumbent firm making zero profits. Homogeneous product industry. All firms have the same costs, $C_i = F + cq_i$ and the inverse demand function is $p = a - Q$.

Definition (see also figure **??**).

1. An industry configuration is the incumbent's pair $\left(p^I, q^I\right)$.
2. An industry configuration is said to be feasible if,
   (a) At the incumbent's price p^I, the quantity demanded equals the incumbent's quantity supplied, $p^I = a - q^I$.
   (b) The incumbent makes nonnegative profit, $p^I q^I \geq F + cq^I$.
3. An industry configuration is said to be sustainable if no potential entrant can make a profit by undercutting the incumbent's price.
4. A feasible industry configuration is said to be a contestable markets equilibrium if it is sustainable.

The price p^I in a contestable market is above marginal cost, because the firm has to cover its fixed costs.

0.1.2 War of attrition

Another popular approach to natural monopoly is the war of attrition. Time is continuous from 0 to $+\infty$. The rate of interest is r. Two firms with identical cost structures $C(q) = f + cq$, if $q > 0$ and $C(0) = 0$, per unit of time. Price adjustments are instantaneous. If the two firms are in the market at time t, price equals marginal cost c and each firm looses f per-unit of time. If only one firm is in the market, the price is equal to the monopoly price p^m and the firm makes instantaneous profit $\pi^m - f > 0$. The other firm makes zero profit. Both firms are in the market at date 0. At each time each firm decides whether to exit (conditional on the other firm still being in the market). Exit is costless and once a firm exits it never returns.

We will construct a symmetric stationary equilibrium in which, at any instant, each firm is indifferent between dropping out and staying.[1] The equilibrium will be in mixed strategies.

Let $G(t)$ denote the probability that one firm drops at or before t. The marginal benefit of waiting is that the rival will drop out, in which case the firm remains a monopoly forever. The probability that the rival will drop out in the next instant of time conditional that it has not dropped up until t is $dG/(1-G)$. The benefit is $(\pi^m - f)/r$.[2] Hence, the expected marginal benefit is $(\pi^m - f)\, dG/r\,(1-G)$. The marginal cost of waiting is f. The marginal cost should equal the marginal benefit

$$\frac{dG}{1-G} = x \equiv \frac{rf}{\pi^m - f}.$$

Note that the probability a firms drops out in $t + dt$, given that it has not dropped out till time t (hazard function) is given by $dG/(1-G)$ and it does not depend on t (stationarity). This suggests that $G(t)$ is the exponential distribution $G(t) = 1 - e^{-xt}$.[3]

To check that it is indeed an equilibrium, note that if $G(t)$ denotes player 2's probability that drops at or before t, then firm 1's expected payoff is zero at any t (hence, no incentive to deviate). Suppose we are at time t and both firms are in the market. If player 1 drops out its payoff is zero. If it does not drop, then its expected payoff is calculated as follows. The benefit for player 1 is that player 2 will drop out in the next instant of time with probablity $rf/(\pi^m - f)$ in which case firm 1's discounted profit is $(\pi^m - f)/r$, but the cost is f, which yields zero expected profits.

The industry outcome is stochastic. Each firm drops out according to a Poisson process with parameter x.[4]

Figure **??** illustrates the difference in price dynamics in the contestability and war of attrition theories. In a contestable market, the price p^c (same as p^I in figure **??**) is above marginal cost. The monopoly profit in a contestable market is zero. No expenditure is made to obtain it. In contrast, in the war of attrition, the monopoly profit is the regular

[1] Stationary means that the strategy does not depend on time t.

[2] The present value of monopoly profits is,

$$\int_0^\infty e^{-rt}\pi^m dt = \pi^m \left[-\left. \frac{e^{-rt}}{r} \right|_0^\infty \right] = \frac{\pi^m}{r}.$$

[3] See also Fudenberg and Tirole (1991, pp.119-121).

[4] For a Poisson process the waiting time until the first arrival has an exponential distribution $e^{-\lambda t}$. Hence, in our context, e^{-xt} is the probability that a firm has not dropped out by time t conditional on the other firm has not dropped out.

one. The expenditure corresponds to the duopoly losses incurred prior to giving up or getting the monopoly situation.[5]

0.1.3 Short-term capital accumulation

There is room for only one firm in the market, and in equilibrium there will be only one firm. The firm makes a profit and deters entry through capital accumulation. Capital is sunk only in the short-run and must be renewed periodically. The length of time over which capital is sunk determines the period of commitment.

Eaton and Lipsey (Bell, 1980).

Time is continuous and the horizon is infinite. One unit of capital is necessary for production and gives access to constant marginal cost c. A second unit of capital is useless. One unit of capital costs f per unit of time and has deterministic duration H. The fixed cost of production,

$$F \equiv \int_0^H f e^{-rt} dt$$

is paid up-front when the unit is installed. Therefore, with equipment of age $\tau < H$ the firm never has an incentive to leave the market even if another firm enters. The monopoly profit is π^m. An entrant can enter at any point between $[0, H]$.

How can a firm secure the market for itself? When the incumbent faces the threat of entry he renews the plant at $H - \Delta$. If there is no threat, then $\Delta = 0$.

Assumption: $f < \pi^m < 2f$ (natural monopoly).

The firms' sole decision is when to build units of capital. One firm invests at time 0. The strategies are otherwise symmetric. They are also Markovian in that they depend on the current payoff-relevant state (firms' capital structures). The incumbent firm always purchases a second unit of capital Δ years before its current unit depreciates. The other firm invests in a unit of capital if the incumbent has only one unit and this unit is more than $H - \Delta$ years old. In equilibrium, the length Δ is chosen such that when the incumbent's unit of capital is $H - \Delta$ old, the entrant is indifferent between entering and not entering. If he does not enter, the incumbent remains a monopoly forever. If he enters he makes a profit of $-f$ for Δ years and enjoys monopoly profit forever after. Along the equilibrium path the incumbent always renews his capital before it depreciates and the entrant never enters.

[5]There are war of attrition games that are non-stationary. For example, two firms are engaged in a patent race and until discovery both firms are losing money. Only the firm that makes the discovery first claims the prize. However, the R&D productivity increases over time (see Fudenberg and Tirole (1991, p.123)), making the equilibrium strategies non-stationary.

Let's compute Δ. In equilibrium, the incumbent's present discounted profit from date 0 on is,

$$V = \int_0^\infty \pi^m e^{-rt} dt - \left(\int_0^H f e^{-rt} dt \right) \left(1 + e^{-r(H-\Delta)} + e^{-r2(H-\Delta)} + \cdots \right).$$

The first term represents the flow of monopoly profits forever. The second term is the cost of one unit of capital repeated at dates 0, $H-\Delta$, $2(H-\Delta)$ and so on. Then,

$$V = \frac{\pi^m}{r} - \frac{f}{r} \left(\frac{1-e^{-rH}}{1-e^{-r(H-\Delta)}} \right).$$

If the entrant enters, he will do so just before the incumbent renews his capital, i.e., at $H-\Delta$. If the entrant enters the incumbent sticks around for Δ units of time before exiting the market. The entrant's profit from the entry date on is thus equal to,

$$V - \int_0^\Delta \pi^m e^{-rt} dt = V - \pi^m \frac{1-e^{-r\Delta}}{r}.$$

This is because the only difference between the incumbent's profit and the entrant's profit is the duopoly situation for Δ units of time and the foregone monopoly profits during that time. The incumbent will choose Δ so that entry is deterred,

$$V - \pi^m \frac{1-e^{-r\Delta}}{r} = 0, \tag{*}$$

or substituting V,

$$\frac{\pi^m}{f} = \frac{1-e^{-rH}}{e^{-r\Delta} - e^{-rH}}.$$

So, $\Delta > 0$. Moreover, $\Delta < H/2$, which implies that the incumbent never has more than two units of capital.[6]

[6] This can be seen as follows. Under our assumptions, the highest value of π^m/f is 2. Also,

$$\frac{1-e^{-rH}}{e^{-r\Delta} - e^{-rH}}$$

is increasing in Δ which means that the highest Δ is when π^m/f is the highest. So, the Δ that solves

$$2 = \frac{1-e^{-rH}}{e^{-r\Delta} - e^{-rH}}$$

is,

$$\Delta = -\frac{\ln\left(\frac{e^{-rH}}{2} + \frac{1}{2}\right)}{r}$$

What happens when commitments are very short? Let H (and thus Δ) go to zero. From (*) we see that $V \to 0$. Thus, even though there is only one firm in the market, this firm makes no profits. The monopoly rent is entirely dissipated by the accumulation of the second unit of capital (when commitment is short).

0.2 Dynamic competition with incomplete information. Applications of signaling

Overview: *a) Can limit price be rationalized as an equilibrium behavior (signal) to deter entry? Is it socially inefficient? b) Investment in misinformation. Signal jamming*

0.2.1 Limit pricing

Milgrom and Roberts (Econometrica, 1982).

An incumbent can charge a low price to signal to a potential entrant that its cost is low and if the entrant enters its profits will be low. For this to work we need information to be incomplete. With complete information a low price, charged by the incumbent, will have no real effects in the subgame that will be played if the entrant enters.

The incumbent moves first in period 1 and makes profit. Firm 2 observes the price and in period 2 decides whether to enter or not. The marginal cost of the incumbent, which is private information, is,

$$c_1 = \begin{cases} c_1^L, & \text{with probability } x \\ c_1^H, & \text{with probability } 1-x. \end{cases}$$

Let p_m^t be the monopoly price of firm 1, $t = L, H$, with $p_m^L < p_m^H$. Let $M_1^t(p_1)$ be the incumbent's profit function and let M_1^L and M_1^H denote the incumbent's maximum short-run profit depending on its type. We assume that the profit function is concave in price. Firm 2 learns the type of the incumbent immediately upon entry. The duopoly profits, net of fixed cost of entry, are, D_1^t, D_2^t. We assume that,

$$D_2^H > 0 > D_2^L.$$

In other words, the entrant wants to enter only when the incumbent's cost is high. The discount factor is δ. We will look for a separating perfect Bayesian equilibrium, where the two types choose different prices.

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which is less than $H/2$. This can be seen as follows. When $H = 0$, the above Δ is zero. The derivative of Δ with respect to H is

$$\frac{e^{-rH}}{e^{-rH}+1}$$

which is less than $1/2$, the derivative of $H/2$.

Since the high type will face entry his optimal strategy is to set the monopoly price. Now let's look at p_1^L. The incentive compatibility constraint for the high type is,

$$\begin{aligned} M_1^H + \delta D_1^H &\geq M_1^H\left(p_1^L\right) + \delta M_1^H \Rightarrow \\ \underbrace{M_1^H - M_1^H\left(p_1^L\right)}_{\text{SR loss}} &\geq \underbrace{\delta\left(M_1^H - D_1^H\right)}_{\text{Discounted LR gain}}. \end{aligned}$$

The IC constraint for the high type is satisfied when p_1^L is in $[A, B] \cup [C, \infty)$, see figure **??**. Similarly the IC constraint for the low type can be written as follows,

$$\underbrace{\delta\left(M_1^L - D_1^L\right)}_{\text{Discounted LR gain}} \geq \underbrace{M_1^L - M_1^L\left(p_1^L\right)}_{\text{SR loss}}.$$

When $p_1^L \in [D, E]$ the IC constraint for the low type is satisfied, see figure **??**.

Assumptions

1. Single-crossing. (It is more costly for the high type to charge a low price).[7]

2. $M_1^L - D_1^L > M_1^H - D_1^H$. (Lower cost firm has a bigger advantage of being a monopolist).

Both of the above conditions must be satisfied in order for the intersection $[A, B] \cap [D, E]$ to be non-empty. There is a continuum of separating equilibria, but the reasonable one is the least-costly one, i.e., the one that involves a price p_1^L that is closest to p_m^L. If $p_1^L < p_m^L$, then we have a limit price. The incumbent, by setting a price that is lower than the monopoly price, credibly signals to the entrant that he is of a low type and therefore entry will not be profitable. Some profits are sacrificed.

[7] In other words,

$$\frac{\partial\left[M_1^H\left(p_1^L\right) - M_1^L\left(p_1^L\right)\right]}{\partial p_1^L} > 0.$$

This condition is satisfied because,

$$\frac{\partial^2\left[\left(p_1 - c_1\right) D_1^m\left(p_1\right)\right]}{\partial p_1 \partial c_1} = -\frac{dD_1^m}{dp_1} > 0.$$

0.2.2 Signal jamming

Two firms 1 and 2 and two periods A and B. Costs are zero and products are differentiated. Each firm only observes its own price and quantity, so rivals can secretly cut prices. The demand is,

$$q_i = a - p_i + p_j,$$

where a is a random variable with mean a^e. The realization of demand stays the same in both periods. Let's assume first that there is no second period. The expected profit is,

$$\pi_i = p_i \left(a^e - p_i + p_j\right).$$

The static equilibrium is,

$$p_1 = p_2 = a^e.$$

Now let's assume that there is a second period. If p in period A is α then firm i learns a perfectly by observing its first period demand, i.e., $q^A = a$. Now firm 1 deviates from α in period A. Firm 1 knows p_1^A and $p_2^A = \alpha$. So, firm 1 knows the intercept, but firm 2 does not. Firm 2 thinks that $q_2^A = a - \alpha + p_1^A$ and in the second period $p_2^B = a - \alpha + p_1^A$. The profit function of firm 1 in period B is,

$$\begin{aligned} \pi_1^B &= p_1^B \left(a - p_1^B + p_2^B\right) \\ &= p_1^B \left(a - p_1^B + a - \alpha + p_1^A\right). \end{aligned}$$

Then,

$$\frac{\partial \pi_1^B}{\partial p_1^A} = p_1^B.$$

Also, the optimal price for firm 1 in period B, using $\pi_1^B = p_1^B \left(a - p_1^B + a - \alpha + p_1^A\right)$ from above, is,

$$p_1^B = \frac{2a - \alpha + p_1^A}{2}.$$

The expected profit function of firm 1 in period A is,

$$\pi_1^A = p_1^A \left(a^e - p_1^A + p_2^A\right) = p_1^A \left(a^e - p_1^A + \alpha\right).$$

$$\frac{\partial \pi_1^A}{\partial p_1^A} = a^e - 2p_1^A + \alpha.$$

$$\begin{aligned} \frac{\partial \pi_1^A}{\partial p_1^A} + \delta \left.\frac{\partial \pi_1^B}{\partial p_1^A}\right|_{p_1^A = \alpha} &= 0 \Rightarrow \\ a^e + \alpha - 2p_1^A + \delta \left.\frac{2a^e - \alpha + p_1^A}{2}\right|_{p_1^A = \alpha} &= 0 \Rightarrow \\ \alpha &= a^e \left(1 + \delta\right) > a^e. \end{aligned}$$

Each firm charges a higher price to make its rival think that demand is high.

1 COLLUSION

Overview: *a) Ability to sustain higher profit than under static competition, using punishment strategies. b) Deriving conditions on primitive (e.g., frequency of interaction) under which this can be done. c) Illustrating what can be achieved under different modes of competition–Bertrand versus Cournot. d) Price wars during booms/recessions.*

1.1 Bertrand model

Consider the Bertrand game of price competition, where firms interact in each period t, with $t = 0, 1, 2, ..., \infty$. There is a discount factor $\delta < 1$ and each firm i attempts to maximize the discounted value of its profits, $\sum_{t=0}^{\infty} \delta^t \pi_{it}$, where π_{it} is firm i's profit in period t. In this repeated Bertrand game, firm i's strategy specifies what price p_{it} it will charge in each period t as a function of the history of all past price choices by the two firms, $H_{t-1} = \{p_{1\tau}, p_{2\tau}\}_{\tau=0}^{t-1}$. These strategies allow firms to tacitly collude, i.e., to sustain a price that is higher than the Nash-price, without entering into an "explicit" agreement. To see this denote by p^m the monopoly price and consider the following strategies for firms $i = 1, 2$,

$$p_{it}(H_{t-1}) = \begin{cases} p^m, & \text{if all elements of } H_{t-1} \text{ equal } (p^m, p^m) \text{ or } t = 0 \\ c, & \text{otherwise.} \end{cases} \tag{*}$$

This strategy calls for each firm to start the game by charging the monopoly price. Then in each period firms should continue to charge the monopoly price, if in every previous period both firms have charged the monopoly price and otherwise to charge the static Nash equilibrium price, which is equal to marginal cost c. This is called a *grim trigger* strategy. Both firms cooperate until one firm deviates. This deviation triggers a permanent retaliation in which firms revert to Nash equilibrium.

Proposition. *The strategies described in (*) constitute a SPNE of the infinitely repeated Bertrand duopoly game if and only if* $\delta \geq \frac{1}{2}$.

Proof. Recall that a set of strategies is a SPNE of an infinite horizon game if and only if it specifies Nash equilibrium play in every subgame. First, note that all subgames have an identical structure: each is an infinitely repeated Bertrand duopoly game, exactly like the game as a whole. Thus, we need to show that after any previous history of play, the strategies specified for the remainder of the game constitute a Nash equilibrium of an infinitely repeated Bertrand game. Moreover, given the form of strategies specified by (*) we need to be concerned with only two types of previous histories: those in which there has been a previous deviation and those in which there has been no deviation.

Consider first a subgame arising after a deviation has occurred. The strategies call for each firm to play c. This is a Nash equilibrium since no firm can earn more than zero profits by deviating.

Now consider a subgame starting in period t where no deviation had occurred in a previous period. Each firm knows that its rival strategy calls for it to charge p^m until a deviation takes place. Is it in firm i's interest to unilaterally deviate from this strategy? Suppose that firm i deviates in period $\tau \geq t$. From period t through period τ, firm i earns $\frac{(p^m - c)x(p^m)}{2}$ in each period, exactly as it does if it never deviates. In period τ firm i optimally deviates by charging $p^m - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small. Firm i's profit in period τ is, $(p^m - c - \varepsilon)\, x(p^m)$. Since ε is negligible, firm i's profit after deviation in period τ is arbitrarily close to $(p^m - c)\, x(p^m)$. From period $\tau + 1$ onward firm i will earn in each period a profit equal to 0. If firm i does not deviate in period τ its discounted sum of profits is,

$$\sum_{t=0}^{\infty} \delta^t \frac{(p^m - c)\, x(p^m)}{2} = \frac{(p^m - c)\, x(p^m)}{2(1 - \delta)}.$$

Hence, deviation is profitable if and only if,

$$(p^m - c)\, x(p^m) > \frac{(p^m - c)\, x(p^m)}{2(1 - \delta)} \Rightarrow \delta < \frac{1}{2}.$$

Thus, the strategies in (*) constitute a SPNE if and only if, $\delta \geq \frac{1}{2}$.

■

If firms meet frequently, then δ is close to one, if not, then it is close to zero. Another way to interpret δ is to write it as $\delta = e^{-r\tau}$, where τ is continuous time.

More generally, if there are n firms collusion is sustainable if and only if $\delta \geq 1 - 1/n$.

The monopoly price is sustainable if and only if the present value of losses that a firm incurs if it deviates is large enough relative to the short-run gains from deviation. This happens when the discount factor is high.

1.2 Imperfect monitoring

With perfect monitoring, "price wars" or punishment phases are never observed in equilibrium. In practice, however, firms cannot monitor perfectly the actions of rivals and price wars may be observed.

Demand and prices are not publicly observed. The probability of low demand is α. Prices are conditioned on quantities. If in each period both firms sell positive quantities, then they continue to charge monopoly prices. If at least one firm did not sell anything, then firms enter into a punishment phase for T periods. Thus, there are two states:

- Good state: Last period both firms sold positive quantities. The value function is V^+.
- Bad state: Last period at least one firm sold zero for the first time. The value function is V^-.

If there is no deviation the expected discounted profits are,

$$(1-\alpha)\left[\frac{\pi^m}{2}+\delta V^+\right]+\alpha\delta V^-.$$

If a firm deviates its expected discounted profits are,

$$(1-\alpha)\left[\pi^m+\delta V^-\right]+\alpha\left[0+\delta V^-\right].$$

Collusion is sustainable if and only if,

$$\delta\left(V^+-V^-\right)\geq\frac{\pi^m}{2}.$$

Also, the following must hold,

$$V^-=\delta^T V^+$$

$$(1-\alpha)\left[\frac{\pi^m}{2}+\delta V^+\right]+\alpha\delta V^-=V^+.$$

Solve the last two equalities for V^+ and V^- and then plug them into the IC constraint. It turns out that,

$$V^+=\frac{(1-\alpha)\frac{\pi^m}{2}}{1-\alpha\delta^{T+1}-(1-\alpha)\delta}\text{ and }V^-=\delta^T V^+.$$

Collusion is sustainable if and only if,

$$\alpha\leq\frac{1-2\delta+\delta^{T+1}}{2\delta^{T+1}-2\delta}.$$

For collusion to be sustainable we need low α and high δ. For example, if recession is imminent (i.e., high α) then it is more difficult to sustain collusion.

1.3 Collusion under Cournot competition

Two firms, zero costs. Inverse demand is $p = 1 - X$. Grim-trigger strategy. The monopoly quantity is,

$$x_1 = x_2 = \frac{1}{4}.$$

The present value of profits along the collusive path is,

$$V = \frac{1}{1-\delta}\frac{\pi^m}{2}.$$

The one-time deviation profits are,

$$\pi^d = \max_{x_2} x_2 \left[1 - \left(\frac{1}{4} + x_2\right)\right] = \frac{9}{64}.$$

The Cournot profits are,

$$\pi^C = \frac{1}{9}.$$

The incentive compatibility constraint is,

$$\frac{1}{1-\delta}\frac{1}{8} \geq \frac{9}{64} + \frac{\delta}{1-\delta}\frac{1}{9} \Leftrightarrow \delta \geq \frac{9}{17}.$$

Now suppose that $\delta < 9/17$. The firms can sustain $x > 1/4$.

$$\pi^* = 2x^*\left(1 - 2x^*\right).$$

$$\frac{1}{1-\delta}\frac{\pi^*}{2} \geq \pi^d + \frac{\delta}{1-\delta}\frac{1}{9},$$

where,

$$\pi^d = \max_{x_2} x_2 \left[1 - \left(x^* + x_2\right)\right].$$

1.3.1 More severe punishments

Abreu (JET, 1986).

If there is no deviation, each firm produces,

$$\frac{x^m}{2}.$$

In the period after deviation they produce (x, x) for 1 period. Then they go back to $\pi^m/2$. The net present value from collusion starting at x is,

$$V(x) = x(1-2x) + \frac{\delta}{1-\delta}\frac{\pi^m}{2}.$$

$$\frac{1}{1-\delta}\frac{\pi^m}{2} \geq \pi^d + \delta V(x), \text{ (IC Cooperation).}$$

Before, we had only one IC constraint because the punishment was repetition of Nash, which is self-enforcing (static equilibrium). Now we need one more constraint,

$$V(x) \geq \pi_{d_p} + \delta V(x) \Rightarrow V(x) \geq \frac{\pi_{d_p}}{1-\delta}, \text{ (IC Punishment),}$$

where $\pi_{d_p} = \pi_2(x, y)$ with $y = BR_2(x)$. It turns out that,

$$\pi_{d_p} = \frac{(1-x)^2}{4}.$$

If firms do not play (x, x) in one period, then they play again (x, x) in the next period. Solve the ICC and the ICP with respect to x.

For example, it turns out that when $x \in \left(\frac{3}{8}, \frac{1}{2}\right)$ monopoly profits are sustainable with $\delta = 1/2$. Thus, even when $\delta < 9/17$ monopoly profits can be sustained with punishments that are more severe than Nash.

1.3.2 Collusion with fluctuating demand

Rotemberg and Saloner (AER, 1986).

There are two states of the world: a low demand state 1 and a high demand state 2. Each state occurs with probability $\frac{1}{2}$. There are two firms that produce homogeneous products and compete in prices (Bertrand). Hence, the demand functions are given by,

$$Q = \begin{cases} D_1(p), \text{ with probability } \frac{1}{2} \\ \\ D_2(p), \text{with probability } \frac{1}{2}. \end{cases}$$

Both firms have the same marginal cost $c > 0$. The firms interact in an infinitely repeated setting where in each stage they choose prices. Collusion is sustained via the *grim-trigger* strategy. Firms begin by charging the collusive prices. If in a previous period no firm deviated from the collusive path, then they continue charging the collusive prices, otherwise they revert to the one-shot Nash equilibrium forever (punishment phase). The firms have a common discount factor $\delta \in (0, 1)$.

Case 1: *Firms sustain monopoly profits.* First, we assume that firms try to sustain monopoly profits. Denote the monopoly prices and profits in each state of the world by, p_1^m, p_2^m, π_1^m and π_2^m (recall 1 and 2 denote the states not the firms). We assume that firms divide equally the collusive profits. The ex-ante present value of the profits from collusion for each firm is given by,

$$V = \frac{1}{1-\delta}\left[\frac{\pi_1^m}{4} + \frac{\pi_2^m}{4}\right].$$

In each period a firm observes the realized state and then chooses whether to deviate from the implicit agreement or not. If it deviates its profits are π_1^m or π_2^m, depending on the state of the world. This is because competition is à la Bertrand and a very small undercut of the rival's price is enough to steal all the demand. Collusion is sustainable as a subgame perfect equilibrium in the infinitely repeated game if and only if the following two incentive compatibility constraints are satisfied,

$$\underbrace{\delta V}_{\text{Long-run loss}} \geq \underbrace{\frac{\pi_1^m}{2}}_{\text{Short-run gain}} \quad \text{and} \quad \underbrace{\delta V}_{\text{Long-run loss}} \geq \underbrace{\frac{\pi_2^m}{2}}_{\text{Short-run gain}}.$$

The punishment profits are zero (due to the Bertrand assumption). That is why the long-run loss is δV. For a firm not to deviate it must be that the short-run gain from deviation is less than the long-run loss due to the break-down of collusion. Since $\pi_2^m \geq \pi_1^m$, only the second constraint is binding. In other words, collusion is more difficult in the high demand state. Therefore, for collusion to be sustainable in both states the following must be satisfied,

$$\delta V \geq \frac{\pi_2^m}{2} \Leftrightarrow \delta \geq \delta_o \equiv \frac{2\pi_2^m}{3\pi_2^m + \pi_1^m} > \frac{1}{2}.$$

When demand is not fluctuating (i.e., $\pi_2^m = \pi_1^m$), then collusion is sustainable if and only if $\delta \geq \frac{1}{2}$. With fluctuating demand collusion is more difficult, i.e., a higher discount factor is needed.

To summarize, if $\delta \geq \delta_o$, then monopoly profits are sustainable in both states. The price in the high demand state is not lower than the price in the low demand state. If $\delta \in \left(\underline{\delta} \equiv \frac{2\pi_1^m}{3\pi_1^m + \pi_2^m}, \delta_o\right)$, then collusion can be sustained in the low demand state, but we have a price war (collusion breaks down) in the high demand state. In the price war case prices fall. If $\delta < \underline{\delta} < \frac{1}{2}$, then no collusion can be sustained.

Case 2: *Firms sustain less monopoly profits.* Now let's assume that $\delta \in (\underline{\delta}, \delta_o)$, but firms try to sustain less than the monopoly profits. We denote the collusive prices by, p_1 and p_2. The incentive compatibility (IC) constraints now become,

$$IC_1 : \underbrace{\delta V(p_1, p_2)}_{\text{Long-run loss}} \geq \underbrace{\frac{\pi_1(p_1)}{2}}_{\text{Short-run gain}} \quad \text{and } IC_2 : \underbrace{\delta V(p_1, p_2)}_{\text{Long-run loss}} \geq \underbrace{\frac{\pi_2(p_2)}{2}}_{\text{Short-run gain}},$$

where $V(p_1, p_2) = \frac{1}{1-\delta}\left[\frac{\pi_1(p_1)}{4} + \frac{\pi_2(p_2)}{4}\right]$. The firms now solve the following maximization problem,

$$\max_{(p_1, p_2)} V(p_1, p_2)$$
$$\text{s.t. } IC_1 \text{ and } IC_2.$$

Ignore for the moment IC_1. We will solve the relaxed problem and then check whether the ignored constraint is satisfied. If it is then we are done. IC_2 is equivalent to (using $V(p_1, p_2) = \frac{1}{1-\delta}\left[\frac{\pi_1(p_1)}{4} + \frac{\pi_2(p_2)}{4}\right]$),

$$\pi_2(p_2) \leq \frac{\delta}{2 - 3\delta}\pi_1(p_1).$$

Increase $\pi_1(p_1)$ till π_1^m. This increases the maximum value of the objective function because of the following two reasons. First, the constraint is relaxed and second the objective function itself increases. Then it follows that collusive profits are maximized if the inequality in IC_2 becomes equality,

$$\pi_2(p_2) = \frac{\delta}{2 - 3\delta}\pi_1^m.$$

Now we check whether IC_1 is satisfied.

$$IC_1 \Leftrightarrow \pi_1(p_1) \leq \frac{\delta}{2 - 3\delta}\pi_2(p_2) \Leftrightarrow \pi_1^m \leq \frac{\delta^2}{(2 - 3\delta)^2}\pi_1^m \Leftrightarrow \delta \geq \frac{1}{2}.$$

IC_1 is satisfied if and only if $\delta \geq \frac{1}{2}$. Some collusion can be sustained in both states even when $\delta < \delta_o$, provided that $\delta \geq \frac{1}{2}$.

The profits in the high demand state are less than the monopoly profits, i.e., $\pi_2(p_2) < \pi_2^m$. In the low demand state firms sustain monopoly profits.

How about the prices? Assume that $D_2 = \lambda D_1$, where $\lambda > 1$ (i.e., inverse demand rotates). In this case, $p_1^m = p_2^m$. Thus, $p_2 < p_1^m = p_2^m$. In the good state prices fall. Margins also fall.[8]

Therefore, prices may fall during a period of high demand either because a price war breaks out (e.g. case 1), or because firms have to accept lower than monopoly profits in order to sustain some collusion (e.g. case 2).

[8] However, if the demand shifts out (instead of a rotation), then prices may increase during high demand.