Problem set # 3
Econ 950

1. (Modified Hotelling model). There are two firms, $i = 1, 2$, located at the two endpoints of the $[0, 1]$ interval. Consumers are uniformly distributed on the interval. Firms are selling homogeneous products and each consumer buys multiple units. The consumer who is located at point $z \in [0, 1]$ has the following utility function,

$$U = ax - \frac{bx^2}{2} + y - td,$$

where x is the good the two firms are selling, y is the numeraire good (with price normalized to one), t is the per-unit transportation cost and d is the distance from the consumer's location to a firm. The per-unit price of good x is p_i and each consumer's income is m. The two firms compete on prices simultaneously.

a) Show that a unique symmetric equilibrium exists.

2. (Vertical differentiation). Consider the Shaked and Sutton vertical differentiation model. The parameter θ is uniformly distributed in $[a, b]$, with $2a < b < 4a$. Assume that there are only two firms to begin with. Assume also that these firms are already "in" (have sunk F) and that they have already chosen their qualities to be m and M. So it only remains for them to play a price game. Assume however that the cost of producing one unit of M is c_M and the cost of producing one unit of m is c_m, with $c_M > c_m$.

a) Determine the equilibrium in this game (and in particular whose profits are higher) as a function of parameters (M, m, c_M, c_m, a, b).

b) Do a similar exercise when the density of consumer types is skewed towards a, e.g., when the density is described by a triangle decreasing from a to b.