0.1 Cournot's pricing game with complementary products

Overview: *Modeling competition in hardware-software industries. The "collusive" incentive of firms is to lower price, which benefits consumers as well.*

Firm 1 produces a hardware and charges p_1 and firm 2 produces a software and charges p_2. Both products are needed for the system to be functional. The price of the system is $p_1 + p_2 = p_S$. The system demand is $D(p_S) = 1 - p_S$. The non-cooperative equilibrium prices are,

$$p_1 = p_2 = \frac{1}{3} \text{ and } p_S = \frac{2}{3}.$$

Now suppose that 1 and 2 merge. The monopoly price is 1/2, less than 2/3. A monopolist chooses lower prices than the duopoly price. The intuition is as follows. Since the two products are perfect complements a price increase by one firm reduces the demand of both products. This negative externality on the rival's demand is not internalized by the firms and as a result each firm, when firms act independently, overprices. This externality is internalized by the monopolist and that is why prices are lower.

Sonnenschein (JPE, 1968) has shown that the Nash equilibrium where firms compete in price and sell perfect complements is isomorphic to the case where firms compete in quantities and sell perfect substitutes. In the latter case, a merger to monopoly would reduce aggregate quantity and increase price.

0.2 A quadratic utility, representative consumer model

Overview: *Integrate price/quantity and complements/substitutes models. Compare prices and quantities. Strategic complements and substitutes. Reinterpret the model in the context of a heterogeneous population of individuals, as opposed to a representative individual.*

Singh and Vives (RAND, 1984).

Two firms 1 and 2, each producing one product. The utility function of the representative consumer is,

$$U(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - \frac{1}{2}\left(\beta_1 q_1^2 + 2\gamma q_1 q_2 + \beta_2 q_2^2\right).$$

The UMP is,

$$\max_{q_1, q_2} U(q_1, q_2) - p_1 q_1 - p_2 q_2.$$

There is a third good (many goods) that the consumer buys competitively. We impose the following restrictions,

1. $\beta_1 \beta_2 - \gamma^2 > 0$
2. $\alpha_i \beta_j - \alpha_j \gamma > 0$, $i \neq j$
3. $\alpha_i > 0$, $\beta_i > 0$ and $\gamma \lesseqgtr 0$

The demand system is,

$$\begin{aligned} p_1 &= \alpha_1 - \beta_1 q_1 - \gamma q_2 \\ p_2 &= \alpha_2 - \beta_2 q_2 - \gamma q_1. \end{aligned}$$

Cournot game
Firms compete in quantities. The profit function of firm 1 is,

$$\pi_1 (q_1, q_2) = p_1 q_1.$$

The Nash equilibrium is denoted by q_1^C, q_2^C.
Bertrand game
We solve the demand system with respect to quantities.

$$\begin{aligned} \beta_1 q_1 + \gamma q_2 &= \alpha - p_1 \\ \gamma q_1 + \beta_2 q_2 &= \alpha - p_2. \end{aligned}$$

Using Cramer's rule we solve for q_1,

$$q_1 = \frac{\begin{vmatrix} \alpha - p_1 & \gamma \\ \alpha - p_2 & \beta_2 \end{vmatrix}}{(\beta_1 \beta_2 - \gamma^2)} = \frac{\alpha_1 \beta_2 - \alpha_2 \gamma - \beta_2 p_1 + \gamma p_2}{\delta} = a_1 - b_1 p_1 + c p_2.$$

Similarly, we can solve for q_2. The profit function of firm 1 is,

$$\pi_1 (p_1, p_2) = p_1 q_1.$$

The Nash equilibrium is denoted by p_1^B, p_2^B.

It turns out that $p_i^C > p_i^B$ and $q_i^B > q_i^C$ for any γ. Thus, Bertrand competition is more efficient.

When the goods are substitutes $\pi^C > \pi^B$. In this case firms prefer higher prices.

When the goods are complements $\pi^C < \pi^B$. In this case firms prefer lower prices.

0.3 Horizontal differentiation and spatial competition

Overview: *Firms may have incentives to differentiate their products so as to soften price competition, but this depends on "transportation costs." Welfare analysis of location/product offering under linear and quadratic models. Welfare analysis of the number of products (variety).*

Consumers are uniformly distributed on the $[0, L]$ interval. Firm 1 is located at a and firm 2 is located b distance from L. The per-unit transportation cost is $t > 0$.

The mill prices are p_1 and p_2. The effective prices are $p_1 + tx$ if the consumer buys from firm 1 and $p_2 + ty$ if the consumer buys from firm 2. The marginal consumer satisfies (see figure **??**),

$$p_1 + tx = p_2 + ty \Rightarrow x - y = \frac{p_2 - p_1}{t}.$$

We must also have $x+y=L-(a+b)$. Solving the system of two equations we get,

$$\begin{aligned} x &= \frac{1}{2}\left[L-(a+b)+\frac{p_2-p_1}{t}\right] \\ y &= \frac{1}{2}\left[L-(a+b)+\frac{p_1-p_2}{t}\right]. \end{aligned}$$

The demand functions are,

$$\begin{aligned} q_1 &= a+x=\frac{1}{2}\left[L+a-b+\frac{p_2-p_1}{t}\right] \\ q_2 &= b+y=\frac{1}{2}\left[L-a+b+\frac{p_1-p_2}{t}\right]. \end{aligned}$$

Note that this model is a special case of the previous (Singh and Vives) model.

The first order conditions yield,

$$\begin{aligned} L+a-b+\frac{p_2}{t}-\frac{2p_1}{t} &= 0 \\ L-a+b+\frac{p_1}{t}-\frac{2p_2}{t} &= 0. \end{aligned}$$

For both firms to have positive sales, it must be the case that,[1]

$$(L-a-b)\,t>|p_1-p_2|\,.$$

Fix p_2. When $p_1\in(p_2-(L-a-b)\,t,\ p_2+(L-a-b)\,t)$, the profit function of firm 1 is concave and both firms have strictly positive sales. In this case, the solution to the system of first order conditions yields,

$$p_1=t\left(L+\frac{a-b}{3}\right)\text{ and }p_2=t\left(L+\frac{b-a}{3}\right).$$

The equilibrium profits are,

$$\pi_1=\frac{t}{2}\left(L+\frac{a-b}{3}\right)^2\text{ and }\pi_2=\frac{t}{2}\left(L+\frac{b-a}{3}\right)^2.$$

Note that π_1 is increasing in a and π_2 is increasing in b. This suggests that $a=b=L/2$ (principle of minimum differentiation). However, this is not true. Equilibrium prices must satisfy $(L-a-b)\,t>|p_1-p_2|$. This holds as long as $3L>5b+a$. This condition is violated when $a=b=L/2$. Hence, the principle of minimum differentiation does not hold.

The problem is that when the inequality $(L-a-b)\,t>|p_1-p_2|$ is violated the profit function is discontinuous. When p_1 is slightly below $p_2-(L-a-b)\,t$,

[1] This comes from the conditions that say that x and y (from above) must be positive.

firm 1 grabs the 'captive' market of firm 2, which has size b and consequently its profit function jumps up. When p_1 is slightly above $p_2 + (L-a-b)\,t$, firm 1 looses its 'captive' market, which has size a and consequently its profit function jumps down. When firms are very close to each other, i.e., a and/or b are large, the discontinuities are more pronounced and this causes a non-existence problem. When firms are very close to each other the firm who charges a higher price makes zero profits and has incentive to undercut the rival and grab the whole market. This will drive the prices down to zero, but this is not an equilibrium either, unless $a+b=L$.

0.3.1 Quadratic transportation costs

D'Aspermont et. al., (Econometrica, 1979).

Instead of linear, the transportation cost is quadratic. The marginal consumer satisfies,

$$\begin{aligned} p_1 + tx^2 &= p_2 + ty^2 \Rightarrow t\left(x^2 - y^2\right) = p_2 - p_1 \\ &\Rightarrow (x+y)(x-y) = \frac{p_2 - p_1}{t} \text{ using } x+y = L-a-b \\ (L-a-b)(x-y) &= \frac{p_2 - p_1}{t} \Rightarrow (x-y) = \frac{p_2 - p_1}{t\,(L-a-b)}. \end{aligned}$$

Solving with respect to x and y we obtain,

$$x = \frac{1}{2}\left[L-a-b+\frac{p_2-p_1}{t\,(L-a-b)}\right] \text{ and } y = \frac{1}{2}\left[L-a-b+\frac{p_1-p_2}{t\,(L-a-b)}\right].$$

The demand function of firm 1 is,

$$D_1 = a + x = \frac{1}{2}\left[L+a-b+\frac{p_2-p_1}{t\,(L-a-b)}\right].$$

Similarly, we can derive D_2. The equilibrium prices are,

$$p_1 = t\,(L-a-b)\left(L+\frac{a-b}{3}\right) \text{ and } p_2 = t\,(L-a-b)\left(L+\frac{b-a}{3}\right).$$

These prices are valid for any a and b. Once we substitute the prices back into the profit functions, we can compute that $\partial\pi_1\left(p_1(a,b), p_2(a,b); a, b\right)/\partial a$ and $\partial\pi_2\left(p_1(a,b), p_2(a,b); a, b\right)/\partial b$ are negative. Hence, firms will choose maximum differentiation.

0.3.2 Social planner's problem

The social planner will choose the two locations a and b to minimize the total transportation cost given by (see also figure **??**),

$$\begin{aligned} C(a,b) &= \int_0^a t\,(a-s)\,ds + \int_a^{\frac{L-b+a}{2}} t\,(s-a)\,ds + \int_{\frac{L-b+a}{2}}^{L-b} t\,(L-b-s)\,ds + \int_{L-b}^{L} t\,(s-L+b)\,ds \\ &= \frac{tL^2}{4} + \frac{3ta^2}{4} + \frac{3tb^2}{4} - \frac{tL}{2}(a+b) + \frac{tab}{2}. \end{aligned}$$

Minimization of C yields,

$$a = b = \frac{tL}{4}.$$

The social planner would locate one firm at 1/4 and the other at 3/4. Hence, the non-cooperative outcome generates "too much" differentiation from the social point of view.

0.3.3 Circular city model

Salop (Bell, 1979).

In stage 1 firms choose whether to enter the market. Each firm that enters pays a fixed cost F. For simplicity, we assume that the n firms that have entered locate equidistantly around the circle. In stage 2 firms compete in prices. We search for a symmetric equilibrium. Suppose each of the $n-1$ firms charges p' and firm i charges p. The marginal consumer satisfies,

$$p + tx = p' + t\left(\frac{1}{n} - x\right).$$

The demand of firm i is,

$$2x = \frac{p' - p + t/n}{t}.$$

The profit function of firm i is,

$$\pi = \frac{p\left(p' - p + t/n\right)}{t}.$$

After taking the first order conditions and imposing symmetry, $p = p'$, we derive the equilibrium prices,

$$p = \frac{t}{n}.$$

The firm profit is,

$$\pi = \frac{t}{n^2} - F.$$

In the free entry equilibrium the number of firms is,

$$n^* = \sqrt{\frac{t}{F}}.$$

Social planner's problem The social planner chooses the number of firms to minimize the sum of the total transportation cost and the cost of entry, given by,

$$C(n) = \frac{t}{4n} + nF.$$

The socially optimal number of firms is,

$$n^o = \frac{1}{2}\sqrt{\frac{t}{F}} < n^*.$$

Too much variety from the social perspective.