1 STATIC MODELS OF OLIGOPOLY

1.1 The Cournot model and its implications

Antoine Augustin Cournot earned his doctorate in science in 1821, with a main thesis in mechanics and astronomy. Cournot was the first who examined noncooperative oligopoly theory in his book *Researches into the Mathematical Principles of the Theory of Wealth* which published in France in 1838. He essentialy found the Nash equilibrium in a game where firms use their production levels as strategies. Cournot contributed in the development of modern mathematical economics in many other ways. He derived the marginal revenue equals the marginal cost condition in the study of the monopoly. He also introduced firms' cost functions and the system of first order conditions to be solved (Shy 6.1).

Overview: *The Cournot model can be used to shed light on empirical findings such as the positive relationship between a firm's market share and its profit, or between industry concentration and overall industry profits.*

Let the inverse demand function be,

$$p(X) = a - bX,$$

where X is the aggregate output. The intensity of consumer demand is given by the intercept a and the size of the market, i.e., the number of consumers, by the reciprocal of the slope, i.e., $1/b$. For simplicity, we fix the demand to $p = 1 - X$. There are two firms 1 and 2. The marginal costs are: c_1, $c_2 < 1$. Firms choose quantities $X = x_1 + x_2$. The profits of firm i are,

$$\pi_i(x_1, x_2) = x_i(1 - X - c_i).$$

The first order condition of firm 1 is,

$$\frac{\partial \pi_1}{\partial x_1} = 1 - 2x_1 - x_2 - c_1 = 0,$$

which give rise to the following best-response function for firm 1,

$$x_1 = \frac{1 - x_2 - c_1}{2}.$$

Similarly, the best-response of firm 2 is,

$$x_2 = \frac{1 - x_1 - c_2}{2}.$$

Solving the two best-response functions we obtain the equilibrium quantities,

$$x_1^* = \frac{1 + c_2 - 2c_1}{3} \text{ and } x_2^* = \frac{1 + c_1 - 2c_2}{3}.$$

Industry output is,

$$X = \frac{2 - c_1 - c_2}{3}.$$

The equilibrium market price given by,

$$p^* = \frac{1 + c_1 + c_2}{3}$$

is the weighted average of three things: the intensity of demand and the two costs. The equilibrium profits are,

$$\pi_i = \frac{(1 - 2c_i + c_j)^2}{9} = (x_i^*)^2 .$$

The mark-up for firm i given by $p^* - c_i$ is equal to x_i^*. One can then generate some correlations between the endogenous variables by changing the parameters of the model. For example, allow c_i to increase. Two things happen: i) the mark up of firm i decreases, because x_i^* decreases and ii) the share of firm i's output relative to the industry output, x_i/X decreases as well. Thus, there exists a positive relationship between the mark-up of firm i and its quantity share relative to industry output.

Concentration indices try to summarize the distribution of market shares among firms in a single index. One such index is the *Herhindahl index* given by,[1]

$$H = \sum_{i=1}^{n} s_i^2, \text{ where } s_i = \frac{x_i}{X}.$$

A higher value for H means that the industry is more concentrated. The question is: How is H related to profitability and consumer welfare?

First, let's assume that firms are symmetric, i.e., all have the same marginal costs. Then the share of each firm is $1/n$ and the Herfindahl index is $H = 1/n$. More firms implies a less concentrated market, lower profits and higher welfare.

Second, let's drop the symmetry assumption. Industry profits are,

$$\Pi = \sum_{i=1}^{n} \pi_i = \sum_{i=1}^{n} (x_i^*)^2 = X^2 \sum_{i=1}^{n} \left(\frac{x_i^*}{X} \right)^2 = HX^2.$$

A higher concentration index, holding total output X fixed, implies higher industry profits. One way to achieve this is to increase the cost dispersion, while total cost stays the same, i.e., $c_1 + c_2 = 2c$, with fixed c. When firms become more asymmetric in terms of costs, the market shares also become more asymmetric. The Herfindahl index in this case increases (the lowest value of H is attained in the symmetric case and the highest when there is only one seller, monopoly).

[1] See Tirole (1988, pp.221-223) for more concentration indices and for a discussion about the axioms that these indices should satisfy.

On the other hand, industry output is fixed and as cost dispersion increases, H increases and total profits increase.[2]

General model

The inverse demand and cost functions are,

$$p(X) \text{ and } c_i(x_i).$$

The profit function of firm i is,

$$\pi_i(x_1, ..., x_n) = x_i p(X) - c_i(x_i).$$

The first order condition is,

$$\frac{\partial \pi_i}{\partial x_i} = p(X) - c_i^{'}(x_i) + x_i p'(X) = 0.$$

Let's assume constant marginal cost, c_i. The percent mark-up is,

$$\mu_i = \frac{p - c_i}{p} = (\text{using the foc}) \;\; = \frac{x_i p'(X) X}{X p(X)} = \frac{s_i}{\varepsilon},$$

where ε is the elasticity of demand. Hence, as the market share of firm i, s_i, increases, the %mark-up also increases.

1.2 Free entry and its welfare analysis

Overview: a*) The number of firms increases less than proportionately with the size of the market. b) Business stealing effect. Too much or too little entry? Depends on homogeneity/heterogeneity of products.*

Bresnahan and Reiss (JPE, 1991) have shown empirically that there exists a concave relationship between the number of active firms in a given market and the size of the market.

1.2.1 Entry

Consider the following inverse demand function,

$$p(X) = 1 - \frac{1}{S} X,$$

where S captures the size of the market. The profit function of a representative firm is given by,

$$\pi = x\left(1 - \frac{1}{S} X\right)$$

[2] Social welfare increases, because total benefit remains the same (since aggregate output did not change), but total cost of production has gone down (since the lower cost firm is now producing disproportionately more output). Consumer welfare does not change (since price and output have not changed), implying that the increase in social welfare comes entirely from the increase in industry profits.

and the first order condition is given by,

$$1 - \frac{1}{S}X - \frac{1}{S}x = 0.$$

In a symmetric equilibrium,

$$\frac{(n+1)\,x}{S} = 1 \Rightarrow x = \frac{S}{n+1}.$$

The equilibrium profits are,

$$\pi = \frac{S}{(n+1)^2}.$$

Denote the cost of entry by F. Firms will enter until their net profits are driven down to zero. Then, the number of firms in the market is given by,

$$n^* = \sqrt{\frac{S}{F}} - 1.$$

Hence the Cournot model predicts that the relationship between the active number of firms n and the market size S is concave.

1.2.2 Welfare analysis

For simplicity, we assume that $S = 1$. Social welfare as a function of the number of firms $W(n)$ is equal to the difference between how much consumers value output and the cost of entry and is given by,

$$W(n) = \frac{1}{2}\frac{n(n+2)}{(n+1)^2} - nF.$$

A social planner will choose n to maximize $W(n)$. This yields,

$$n^{sb} = \sqrt[3]{\frac{1}{F}} - 1 < n^*.$$

Therefore, free entry is excessive from the social viewpoint. For example, if $F = 1/64$, then the social planner would want 3 firms in the market, while in the free entry equilibrium there will be 7 firms.

Mankiw and Whinston (Rand, 1986) studied the free entry problem using more general demand and cost functions,

$$p(X),\, p'(X) < 0,\ c(x),\ c(0) = 0 \text{ and } c',\ c'' \geq 0.$$

After entry, an equilibrium q_N emerges. The profit function is,

$$\pi_N = q_N p(Nq_N) - c(q_N) - K.$$

In the free entry equilibrium, $\pi_N = 0$. Social welfare is given by,

$$W(N) = \int_0^{Nq_N} p(s)\,ds - Nc(q_N) - NK.$$

Maximization of $W(N)$ yields the second best, *sb*, number of firms denoted by N^{sb}. We would like to show that free entry is excessive, i.e., $N^* > N^{sb}$. We make the following three assumptions,

A1. Nq_N increases with N and $\lim_{N\to\infty} Nq_N = M < \infty$.

A2. q_N decreases with N.

A3. $p(Nq_N) - c'(q_N) \geq 0$. For any N, the resulting equilibrium price is above marginal cost.

Differentiating $W(N)$ with respect to N we obtain,

$$\begin{aligned} W'(N) &= p(Nq_N)\left(q_N + N\frac{\partial q_N}{\partial N}\right) - c(q_N) - Nc'(q_N)\frac{\partial q_N}{\partial N} - K \\ &= \pi_N + N\underbrace{\frac{\partial q_N}{\partial N}}_{(-)}\left[\underbrace{p(Nq_N) - c'(q_N)}_{(+)}\right] \leq \pi_N. \end{aligned}$$

Since at the social optimum N, $W'(N) = 0$ it must be that $\pi_N \geq 0$, implying that there is still profit to be made by entry. If, on top of that, π_N decreases as N increases, then free entry must be excessive.

$$\begin{aligned} \frac{\partial \pi}{\partial N} &= \frac{\partial q_N}{\partial N}p(Nq_N) + q_N p'(Nq_N)\frac{\partial(Nq_N)}{\partial N} - c'(q_N)\frac{\partial q_N}{\partial N} \\ &= \underbrace{\frac{\partial q_N}{\partial N}}_{\substack{(-) \\ \text{Business stealing}}}\left[\underbrace{p(Nq_N) - c'(q_N)}_{(+)\text{ mark-up}}\right] + \underbrace{q_N p'(Nq_N)\frac{\partial(Nq_N)}{\partial N}}_{(-)} < 0. \end{aligned}$$

1.3 Using the Cournot model to study mergers

Overview: *The Cournot model cannot be used to shed light on mergers. Other factors must be brought to bear.*

Salant et. al. (QJE, 1983).

We maintain the Cournot assumptions: n firms, marginal cost is zero and $p(X) = 1 - X$. The equilibrium quantity and profits are given by,

$$x^* = \frac{1}{n+1} \text{ and } \pi = \frac{1}{(n+1)^2}.$$

Suppose that k firms merge. The total number of firms becomes $n - k + 1$. The profits of the merged firm are,

$$\pi = \frac{1}{(n-k+2)^2}.$$

The merger is profitable if and only if,

$$\Delta(k) = \frac{1}{(n-k+2)^2} - k\frac{1}{(n+1)^2} > 0.$$

We compute the derivative of $\Delta(k)$,

$$\Delta'(k) = \frac{2}{(n-k+2)^3} - \frac{1}{(n+1)^2}.$$

The second derivative of $\Delta(k)$, given by,

$$\Delta''(k) = \frac{6}{(n-k+2)^4} > 0$$

reveals that $\Delta(k)$ is convex. Moreover, and since $\Delta(1) = 0$ coupled with $\Delta'(1) < 0$, such a merger is unprofitable for low values of k. For example, if $n \geq 6$, then it takes 80% of the firms to merge for a merger to be profitable.