Άσκηση 5

δ) $Y_B = K_B^{1/2} \cdot L_B^{1/2} \Rightarrow \hat{y}_B = \frac{Y_B}{L_B} = (\hat{k}_B)^{1/2}$

$\hat{y}_B^{1950} = 3^{1/2} = 1,7$, $\hat{k}_B^{1950} = \frac{3}{1} = 3$.

$\hat{C}_B^{1950} = (1-s) \cdot \hat{y}_B^{1950} = (1-0,16) \cdot 1,73 = 1,49$

$i_B^{o\,1950} = s\, y_B^{1950} = 0,2768$

$y_{1950}^B < y_{1950}^A$ εφόσον $k_{1950}^B < k_{1950}^A$

η A είναι πλουσιότερη από B (το 1950)

Σύγκριση χώρα B/A

$\frac{y_{1950}^B}{y_{1950}^A} = \frac{1,73}{3,46} = 0,5$, $\frac{C_{1950}^B}{C_{1950}^A} = \frac{1,49}{2,77} = 0,52$

$$\frac{K^B_{1950}}{K^A_{1950}} = \frac{3}{12} = 0{,}25.$$

β) Steady State

- $\hat{k}^*_B = \left(\frac{s}{n+\delta}\right)^{\frac{1}{1-\alpha}} \Rightarrow \hat{k}^*_B = 10{,}24 < \hat{k}^*_A$

- $\hat{y}^*_B = (10{,}24)^{1/2} = 3{,}2 < \hat{y}^*_A$

- $\hat{c}^*_B = (1-s^B)\cdot \hat{y}^*_B \Rightarrow \hat{c}^*_B = 2{,}68 < \hat{c}^*_A$

⇒ χαμηλότερη S_B, → άλλο (χαμηλότερο σημείο ισορροπίας)

ρυθμοί μεταβολής (όπως και στη Α)

$$\frac{\Delta K_t}{K_t} = \frac{\Delta C_t}{C_t} = \frac{\Delta Y_t}{Y_t} = n.$$

Άσκηση 6

$$Y_t = E_t^{1-a} \cdot K_t^{a} \cdot L_t^{1-a} = K_t^{a} \cdot (E_t \cdot L_t)^{1-a}$$

$$n = \frac{\Delta L_t}{L_t}, \quad g = \frac{\Delta E_t}{E_t}.$$

Ξέρω ότι: $K_{t+1} - (1-\delta)K_t = I_t \Leftrightarrow$

$$K_{t+1} - K_t = sf(K_t, L_t) - \delta K_t \Leftrightarrow$$

$$\frac{\Delta K_t}{K_t} = \frac{sf(K_t, L_t)}{K_t} - \delta \quad (1)$$

Ανά αποδοτικό εργαζόμενο:

$$\frac{Y_t}{E_t \cdot L_t} = \frac{K_t^{a} \cdot (E_t \cdot L_t)^{1-a}}{E_t \cdot L_t} = \frac{K_t^{\alpha}}{(E_t \cdot L_t)^{\alpha}} = \left(\frac{K_t}{E_t \cdot L_t}\right)^{\alpha} = \hat{K}_t^{\alpha}$$

$$\Rightarrow \boxed{\hat{y} = \hat{K}_t^{\alpha}} \quad (2)$$

Επίσης $\hat{K}_t = \frac{K_t}{E_t \cdot L_t} \Rightarrow \frac{\Delta \hat{K}_t}{\hat{K}_t} = \frac{\Delta K_t}{K_t} - \frac{\Delta E_t \cdot L_t}{E_t \cdot L_t} \Leftrightarrow$

$$\Rightarrow \frac{\Delta \hat{K}_t}{\hat{K}_t} = \frac{\Delta K_t}{K_t} - \left[\frac{\Delta E_t}{E_t} + \frac{\Delta L_t}{L_t}\right] \Rightarrow \boxed{\frac{\Delta K_t}{K_t} = \frac{\Delta \hat{K}_t}{\hat{K}_t} + g + n} \quad (3)$$

$$(1) \xrightarrow[(3)]{(2)} \frac{\Delta \hat{K}_t}{\hat{K}_t} = \frac{s \cdot f(K_t, L_t)}{K_t} - (g + n + \delta) \Leftrightarrow$$

$$\frac{\Delta \hat{K}_t}{\hat{K}_t} = s f(\hat{K}_t) - (n + g + \delta).$$

Ανά εργαζόμενο

$$\hat{K}_t = \frac{K_t}{L_t} \Rightarrow \frac{\Delta \hat{K}_t}{\hat{K}_t} = \frac{\Delta K_t}{K_t} - \frac{\Delta L_t}{L_t}$$

$$\hat{K}_t = \frac{K_t}{E_t} \Rightarrow \underbrace{\frac{\Delta \hat{K}_t}{\hat{K}_t}}_{\to 0} = \frac{\Delta K_t}{K_t} - \underbrace{\frac{\Delta E_t}{L_t}}_{\to g}$$

Στην ισορροπία: $\frac{\Delta \hat{k}_t}{\hat{k}_t} = g$

$$\boxed{\text{Άρα } \frac{\Delta \hat{k}_t}{\hat{k}_t} = g}$$

Επίσης $\hat{\hat{y}}_t = \frac{\hat{y}_t}{E_t} \Rightarrow \overbrace{\frac{\Delta \hat{\hat{y}}_t}{\hat{\hat{y}}_t}}^{\Rightarrow 0} = \frac{\Delta \hat{y}_t}{\hat{y}_t} - \frac{\Delta E_t}{E_t}$

$$\boxed{\frac{\Delta \hat{y}_t}{\hat{y}_t} = g}$$

$$\hat{\hat{C}}_t = \frac{\hat{C}_t}{E_t} \Rightarrow \frac{\Delta \hat{\hat{C}}_t}{\hat{C}_t} = \frac{\Delta \hat{C}_t}{\hat{C}_t} - \frac{\Delta E_t}{E_t} \Rightarrow \dots$$

$\Rightarrow \ldots$ άλγεβρα δ. $\Rightarrow$

$$\frac{\Delta \hat{C}_t}{\hat{C}_t} = g.$$

$$\boxed{\frac{\Delta \hat{C}_t}{\hat{C}_t} = \frac{\Delta \hat{Y}_t}{\hat{Y}_t} = \frac{\Delta \hat{K}_t}{\hat{K}_t} = g}$$

Β. Συνολικά μεγέθη

Από (3)

$$\frac{\Delta K_t}{K_t} = \frac{\Delta Y_t}{Y_t} = \frac{\Delta C_t}{C_t} = n + g.$$

Άσκηση 7

$$Y_t = K_t^{a} \cdot (E_t \cdot L_t)^{1-a}$$

K_t, Y_t, I_t, C_t: συνολικά μεγέθ

$\hat{K}_t, \hat{y}_t, \hat{I}_t, \hat{C}_t$: κατά κεφαλήν μεγέθη

$\hat{\hat{k}}_t, \hat{\hat{y}}_t, \hat{\hat{i}}_t, \hat{\hat{c}}_t$: κατά κεφαλήν και αποτελεσματικά μεγέθη

$$\frac{K_t}{Y_t} = 2{,}5 \qquad n + g = 0{,}03$$

$\alpha = 0{,}3$

$\delta = 0{,}04$

α) $$\Delta \hat{\hat{k}}_{t+1} = s \cdot \hat{\hat{k}}_t^{\alpha} - (n+g+\delta)\hat{\hat{k}}_t \Rightarrow$$

$$0 = s\hat{\hat{k}}_t^{\alpha} - (n+g+\delta)\hat{\hat{k}}_t \Rightarrow$$

$$s \cdot \hat{\hat{k}}_t^{-0{,}7} = 0{,}07 \Rightarrow \boxed{s = 0{,}07 \cdot \hat{k}_t^{0{,}7}} \quad (1)$$

Προσοχή εδώ!

$$\frac{K_t}{Y_t} = 2{,}5 \Rightarrow \frac{K_t}{K_t^{0{,}3} \cdot (E_t \cdot L_t)^{0{,}7}} = 2{,}5 \Leftrightarrow$$

ορισμός κατά κεφαλήν k αποτελεσματικών εργαζομένων

$$\frac{K_t^{0,7}}{(E_t \cdot L_t)^{0,7}} = 2,5 \Leftrightarrow \boxed{\hat{k}_t = 2,5^{\frac{1}{0,7}}} \quad (2)$$

$$(1) \overset{(2)}{\Longrightarrow} s = 0,07 \cdot (2,5)^{\frac{1}{0,7}} = 0,175 \text{ ή } \boxed{s = 17,5\%}$$

β) MPK στο s.s (οριακό προϊόν εργασίας: αύξηση Y, σε οριακή αύξηση του k)

$$MPK = \alpha K_t^{\alpha-1} (E_t \cdot L_t)^{1-\alpha} = \alpha \cdot \frac{K_t^{\alpha-1}}{(E_t \cdot L_t)^{\alpha-1}} = \alpha \left(\hat{k}_t\right)^{\alpha-1}$$

$$= 0,3 \cdot 2,5^{\frac{1}{0,7}} = \ldots = 0,12$$

γ) $\hat{C}_t = (1-s) \cdot \hat{y}_t$

$$\hat{C}_t + \hat{i}_t = \hat{y}_t \Rightarrow \boxed{\hat{C}_t = \hat{y}_t - \hat{i}_t} \quad (3)$$