Session 6:
Basic Replenishment Policies

Agenda

- EOQ & Discounts Recap
- Supply Chain Coordination
- Replenishment Policies

Expected outcomes

- You will be able to:
  - **Select** the proper inventory model under given circumstances / model assumptions
  - **Calculate** the optimal order quantity and the related cost
  - **Use** different replenishment policies

Recap: EOQ formula

Total annual cost incorporates: i) inventory holding cost, ii) ordering (or setup) cost, and iii) the materials cost

$$C = \boxed{\frac{Q}{2}H + \frac{D}{Q}S} + PD$$

Operating cost

Where:

C = total cost per year

Q = order quantity (in units)

H = cost of holding per unit of inventory for a year

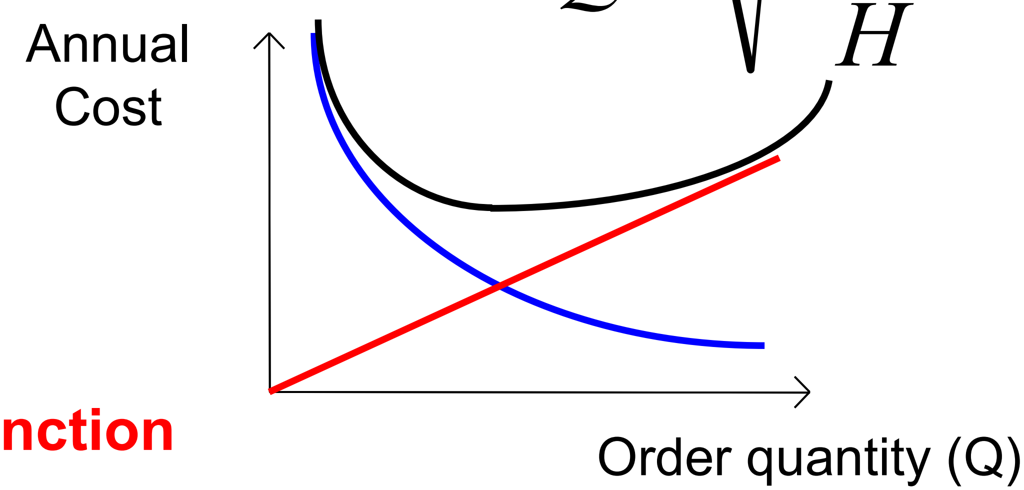
D = demand in units per year

S = fixed cost of ordering or setting up one lot, per lot

P = unit price

Price can be a function of order quantity

$$EOQ = \sqrt{\frac{2DS}{H}}$$



Calculating Order Quantity

- To determine the optimal order quantity in the case of discounts proportional to the quantity ordered, the following approach should be followed:
  1. Calculate EOQ, starting from the lowest price to the next higher price
  2. Check for feasibility (EOQ within the volume range from the respective price)
  3. If the first feasible EOQ is found for the lowest price, it is optimal
  4. Otherwise, calculate total cost for the first feasible EOQ and the larger price break quantity at each lower price level

Homework (1)

- A candy wholesaler faces steady daily demand of 2 tons
- The value of one ton of candies is £1,000
- The holding cost is 20% of the value of the candies
- The ordering cost is £400 per ordering
- The wholesaler operates 200 days per year
- Determine:
    1. the annual operating cost if the lot size is equal to 100 tons
    2. the lot size that minimises the total annual cost
    3. the total annual setup cost for the optimal lot size
    4. the time between orders (cycle length) if the wholesaler orders under the EOQ

Solution Homework (1)

- Annual demand D is 2*200 = 400 tons
- Holding cost H is 20%*1000 = £200
- Ordering cost S is £400

1. If Q = 100, then the annual operating cost is:

   400*(400/100) + ½* 200*100 = £11,600

2. $EOQ = \sqrt{\frac{2DS}{H}}$ = 40 tons

3. If Q = 40, the total annual setup cost is 400*(400/40) = £4,000
4. Since Q = 40 and the daily demand is 2 tons, this means the wholesaler places an order every 20 days

Homework (2)

- The value of one ton of candies is depends on the order quantity as follows:

Order Quantity	Price per ton
0-19	1000
20-39	900
40-59	800
60-	700

- Determine the lot size that minimises the total annual cost

Solution Homework (2)

- Calculate EOQ, starting from the lowest price to the next higher price

$$EOQ = \sqrt{\frac{2DS}{H}}$$

Order Quantity	Price per ton	Optimal order quantity
0-19	1000	No reason to calculate that price
20-39	900	No reason to calculate that price
40-59	800	**44.72 tons, feasible**
60-	700	**47.81 tons, not feasible**

The total annual cost when ordering 44.72 tons is:

$$\frac{44.72}{2} * 0.20 * 800 + \frac{400}{44.72} * 400 + 400 * 800 = 327{,}155$$

The total annual cost when ordering 60 tons is:

$$\frac{60}{2} * 0.20 * 700 + \frac{400}{60} * 400 + 400 * 700 = 286{,}867$$

Supply Chain Coordination

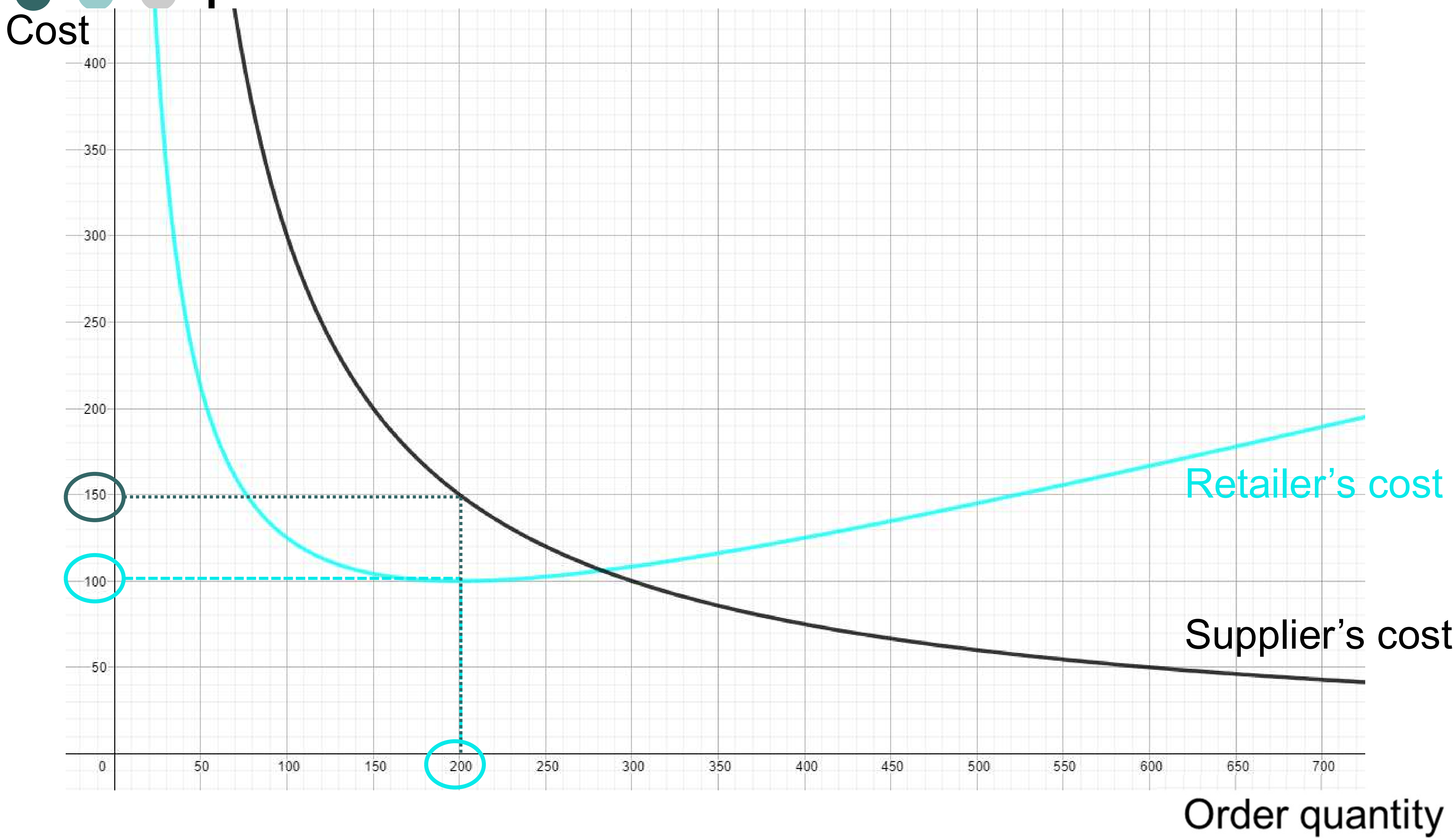
Supplier

- **Lot-for-lot** policy; i.e. a setup cost per order

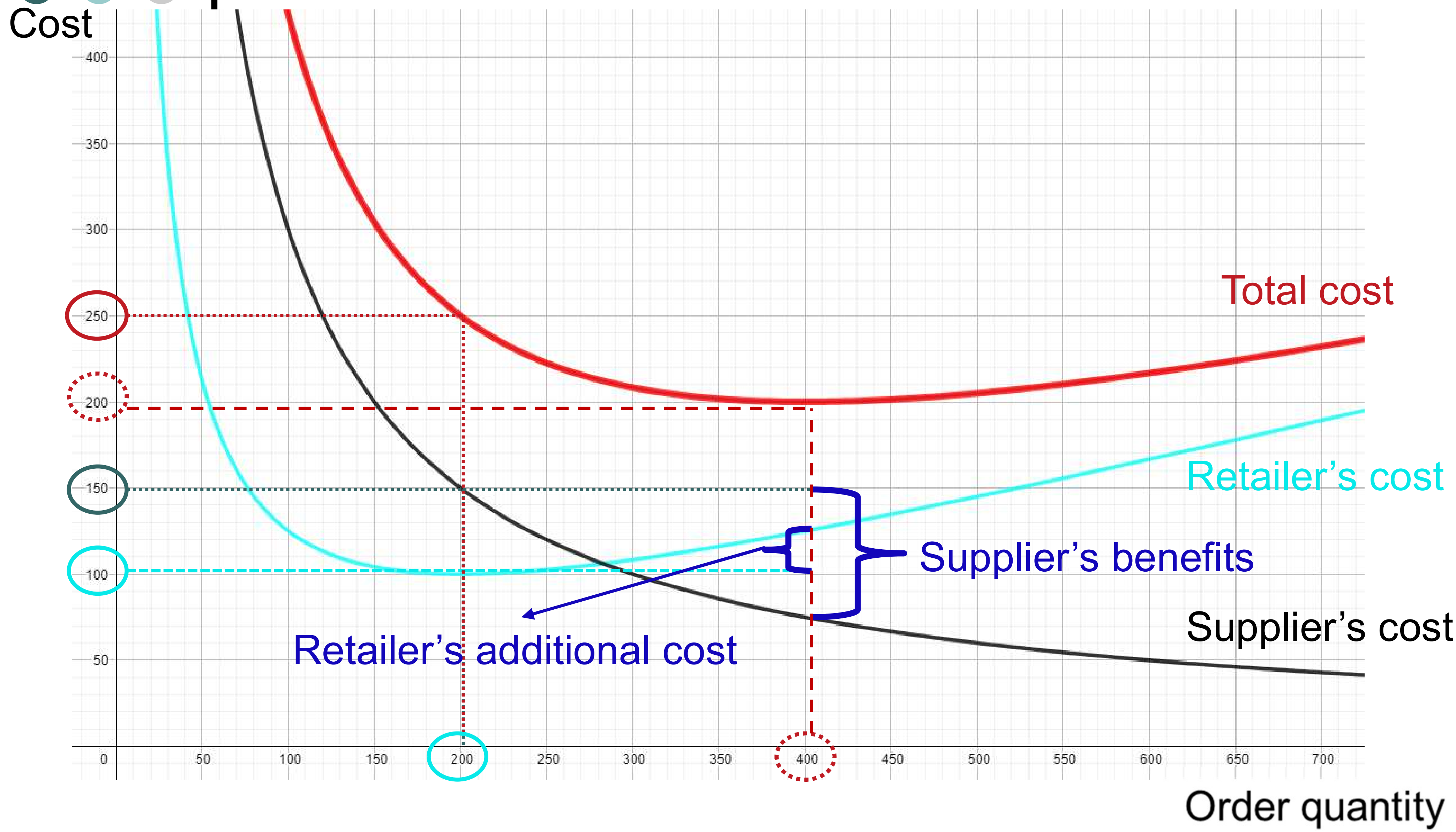
Retailer

- **EOQ** model; i.e. setup cost per order & holding cost per unit
- **Decision: Order Quantity**
- **Criterion: Individualistic**

Individual Costs



Individual & Total Costs



Solution

Induce retailer to change the order quantity by providing incentives (quantity discount)

A *quantity-price pair*
e.g. if you order 400 items you will have £30 discount

Replenishment Policies

Questions: **How much you will order?**

When?

Basic Replenishment Policies:

1. Lot for Lot,
2. Fixed Order Quantity,
3. Fixed Order Period,
4. Lot Size (order size equals to a multiple of a given number).

Moreover, it could be applied some constraints such as minimum and/or maximum levels

Notation

- Parameters
  - *t:* a **period** (e.g., day, week, month, etc.); we will consider t = 1,… , T, where T represents the **planning horizon**
  - D_t: **demand** in period t (in units)
  - c_t: **unit production** / purchasing cost (in pounds)
  - S_t: **setup/ordering cost** (in pounds) to place an order in period t
  - h_t : **holding cost** (in pounds) to carry a unit of inventory from period *t* to period *t+1*
  - I_t: **inventory** level at the end of period *t*
- Variables - Decisions
  - Q_t: **order quantity** in period *t*

1. Lot-for-Lot

Data:

t	1	2	3	4	5	6	7	8	9	10
D_t	20	50	10	50	50	10	20	40	20	30
S_t	100	100	100	100	100	100	100	100	100	100
h_t	1	1	1	1	1	1	1	1	1	1

Ordering policy: **Lot-for-Lot**

How much should we order? & When?

t	1	2	3	4	5	6	7	8	9	10	Total
D_t	20	50	10	50	50	10	20	40	20	30	300
Q_t	20	50	10	50	50	10	20	40	20	30	300
I_t	0	0	0	0	0	0	0	0	0	0	0
Setup cost	100	100	100	100	100	100	100	100	100	100	1000
Holding cost	0	0	0	0	0	0	0	0	0	0	0
Total cost	100	100	100	100	100	100	100	100	100	100	1000

1. Lot-for-Lot

- Could be the Lot-for-Lot policy optimal?
- What is the necessary and sufficient condition to be the Lot-for-Lot policy optimal?

t	1	2	3	4	5	6	7	8	9	10	Total
D_t	20	50	10	50	50	10	20	40	20	30	300
Q_t	20	50	10	50	50	10	20	40	20	30	300
I_t	0	0	0	0	0	0	0	0	0	0	0
Setup cost	~~100~~	~~100~~	~~100~~	~~100~~	~~100~~	~~100~~	~~100~~	~~100~~	~~100~~	~~100~~	~~1000~~
Holding cost	0	0	0	0	0	0	0	0	0	0	0
Total cost	~~100~~	~~100~~	~~100~~	~~100~~	~~100~~	~~100~~	~~100~~	~~100~~	~~100~~	~~100~~	~~1000~~

- If and only if **setup cost equals to zero**

2. Fixed Order Quantity

Ordering policy: Fixed Order Quantity of **100** units

t	1	2	3	4	5	6	7	8	9	10
D_t	20	50	10	50	50	10	20	40	20	30
S_t	100	100	100	100	100	100	100	100	100	100
h_t	1	1	1	1	1	1	1	1	1	1

How much?

When?

Q_t:	**100**	**0**	**0**	**100**	**0**	**0**	**100**	**0**	**0**	**0**
I_t:	**80**	**30**	**20**	**70**	**20**	**10**	**90**	**50**	**30**	**0**

Place an order (of 100 units) at periods 1, 4, and 7

2. Fixed Order Quantity

How much is the related cost?

Setup cost = £100, holding cost = £1 per unit per period

t	1	2	3	4	5	6	7	8	9	10	Total
D_t	20	50	10	50	50	10	20	40	20	30	300
Q_t	100	0	0	100	0	0	100	0	0	0	300
I_t	80	30	20	70	20	10	90	50	30	0	210
Setup cost	100	0	0	100	0	0	100	0	0	0	300
Holding cost	80	30	20	70	20	10	90	50	30	0	400
Total cost	180	30	20	170	20	10	190	50	30	0	700

Under Fixed Order Quantity (Q=100), the operating cost is **£700**, which is less than £1,000 (Lot-for-Lot)

Wagner-Whitin Property

- Can we do better than Fixed Order Quantity?
- Yes because of the **Wagner-Whitin Property** (1958):
  - *Under an optimal lot-sizing policy either the inventory carried to period t+1 from a previous period will be zero or the production quantity in period t+1 will be zero*

t	1	2	3	4	5	6	7	8	9	10	Total
D_t	20	50	10	50	50	10	20	40	20	30	300
Q_t	100	0	0	100	0	0	100	0	0	0	300
I_t	80	30	20	70	20	10	90	50	30	0	0
Setup cost	100	0	0	100	0	0	100	0	0	0	300
Holding cost	80	30	20	70	20	10	90	50	30	0	400
Total cost	180	30	20	170	20	10	190	50	30	0	700

3. Fixed Order Period

Ordering policy: Fixed Order Period of 2 periods

How much you will order? When?

t	1	2	3	4	5	6	7	8	9	10	Total
D_t	20	50	10	50	50	10	20	40	20	30	300
Q_t	70	0	60	0	60	0	60	0	50	0	300
I_t	50	0	50	0	10	0	40	0	30	0	180
Setup cost	100	0	100	0	100	0	100	0	100	0	500
Holding cost	50	0	50	0	10	0	40	0	30	0	180
Total cost	150	0	150	0	110	0	140	0	130	0	680

Under Fixed Order Period of 2 weeks, the operating cost is £680

Can we do better than Fixed Order Period?

Wagner-Whitin Algorithm

4. Lot Size

Ordering policy: Lot Size of 30 units

How much you will order? When?

Shortages or backorders are not allowed

Setup cost = £100, holding cost = £1 per unit per period

t	1	2	3	4	5	6	7	8	9	10	Total
D_t	20	50	10	50	50	10	20	40	20	30	300
Total cost											

4. Lot Size

Ordering policy: Lot Size of 30 units

How much you will order? When?

t	1	2	3	4	5	6	7	8	9	10	Total
D_t	20	50	10	50	50	10	20	40	20	30	300
Requirements	-20	-40		-40	-30	-10		-40		-30	
Q_t	30	60	0	60	30	30	0	60	0	30	300
I_t	10	20	10	20	0	20	0	20	0	0	100
Setup cost	100	100	0	100	100	100	0	100	0	100	700
Holding cost	10	20	10	20	0	20	0	20	0	0	100
Total cost	110	120	10	120	100	120	0	120	0	100	800

Under Lot Size of 30 units, the operating cost is £**800**.

Questions

Example to Practice

Data:

t	1	2	3	4	5	6
D_t	200	220	150	100	90	40
S_t	180	280	270	230	210	190
h_t	2	1	1	2	4	2

- Calculate the operating cost under
  1. Lot-for-lot policy
  2. Fixed order quantity of 300 units
  3. Fixed order period of 2 and 3 weeks
  4. Lot size of 100 units

1. Lot-for-lot

Data:

t	1	2	3	4	5	6
D_t	200	220	150	100	90	40
S_t	180	280	270	230	210	190
h_t	2	1	1	2	4	2
Q_t	200	220	150	100	90	40
I_t	-	-	-	-	-	-
Setup	180	280	270	230	210	190
Holding	0	0	0	0	0	0
Total	180	280	270	230	210	190

Total cost = Setup Cost

= 180 + 280 + 270 + 230 + 210 + 190 = £1,360

2. Fixed Order Quantity

Data:

t	1	2	3	4	5	6
D_t	200	220	150	100	90	40
S_t	180	280	270	230	210	190
h_t	2	1	1	2	4	2
Q_t	300	300	-	300	-	-
I_t	100	180	30	230	140	100
Setup	180	280	0	230	0	0
Holding	200	180	30	460	560	200
Total	380	460	30	690	560	200

Fixed Order Quantity of 300 units

690

1,630

Total cost = Setup + Holding = £2,320

3. Fixed Order Period (2 weeks)

Data:

t	1	2	3	4	5	6
D_t	200	220	150	100	90	40
S_t	180	280	270	230	210	190
h_t	2	1	1	2	4	2
Q_t	420	-	250	-	130	-
I_t	220	-	100	-	40	-
Setup	180	0	270	0	210	0
Holding	440	0	100	0	160	0
Total	620	0	370	0	370	0

Fixed Order Period of 2 weeks

660

700

Total cost = Setup + Holding = £1,360

3. Fixed Order Period (3 weeks)

Data:

t	1	2	3	4	5	6
D_t	200	220	150	100	90	40
S_t	180	280	270	230	210	190
h_t	2	1	1	2	4	2
Q_t	570	-	-	230	-	-
I_t	370	150	-	130	40	-
Setup	180	0	0	230	0	0
Holding	740	150	0	260	160	0
Total	920	150	0	490	160	0

Fixed Order Period of 3 weeks

410

1,310

Total cost = Setup + Holding = £1,720

4. Lot Size

Data:

Lot Size of 100 units

t	1	2	3	4	5	6
D_t	200	220	150	100	90	40
S_t	180	280	270	230	210	190
h_t	2	1	1	2	4	2
Q_t	200	300	100	100	100	-
I_t	-	80	30	30	40	-
Setup	180	280	270	230	210	0
Holding	0	80	30	60	160	0
Total	180	360	300	290	370	0

1,170

330

Total cost = Setup + Holding = £1,500