PROBLEM SET 5

THE ECONOMY

- N+1 consumers, N>4.
- Two goods: A and X, written in this order.
- One firm, with production function $\hat{A} = 2\sqrt{\hat{X}}, \hat{X} \geq 0$
- Consumer 0 is the sole owner of the firm.
  Consumption set $\mathbb{R}^2_+$
  Endowment vector $e_0 = [0,0]$
  utility function $U_0 = X_0$
- Consumers i=1...N
  Consumption set $\mathbb{R}^2_+$
  Endowment vector $e_i = [0,1]$.(the endowment of good A is zero).
  utility function $U_i = A_i X_i$

1.Compute all competitive equilibria of this economy or show that none exist.

2.Draw equilibrium utilities as a function of N.

- Normalize the price of X to 1. Let p be the price of A.
- The firm chooses $(\hat{A}, \hat{X})$ to maximize profit $\Pi = p\hat{A} - \hat{X}$ subject to the constraint $\hat{A} = 2\sqrt{\hat{X}}, \hat{X} \geq 0$ .The profit maximizing choices are

$$\begin{aligned} \hat{X} &= p^2 \\ \hat{A} &= 2p \\ \Pi &= p^2 \end{aligned} \tag{0.1}$$

- The utility maximizing choices of consumers are

$$X_0 = p^2 \tag{0.2}$$

$$A_i = \frac{1}{2p}, X_i = \frac{1}{2} \tag{0.3}$$

Equilibrium conditions

$$\sum_{i=1}^{N} A_i = \hat{A} \tag{0.4}$$
$$\sum_{i=0}^{N} X_i + \hat{X} = N$$

equilibrium prices

$$p^E = \frac{\sqrt{N}}{2} \tag{0.5}$$

equilibrium quantities

$$A_i = \frac{1}{\sqrt{N}}, X_i = \frac{1}{2}, X_0 = \Pi = \frac{N}{4} \tag{0.6}$$
$$\hat{A} = \sqrt{N}, \hat{X} = N/4$$

Equilibrium utilities

$$U^E{}_0 = \frac{N}{4}$$
$$U^E{}_i = -\frac{1}{2}\log N - \log 2$$