PS1,answers

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PS1

PROBLEM SET 1

Consider a firm whose technology is described by a production function $Q = F(K, L), Q \geq 0, K \geq 0, L \geq 0$ **.In other words, the only thing we know about the technology is that good Q is the (only) output of the firm and goods K, L are the (only) inputs. The ratio** K/L **is called the capital intensity of the firm.**

QUESTION 1

express the same technology with a production set.

QUESTION 2

suppose that we have two observations of this firm's behavior, namely

$$p^t = [q_t \quad r_t \quad w_t], y^t = [Q_t \quad -K_t \quad -L_t], t = 1,2 \qquad \textbf{(1)}$$

where q = price of good Q,r=price of good K,w=price of good L **,and**

$$\begin{aligned} p^1 &= [1 \quad 1 \quad 1], y^1 = [2 \quad -\theta \quad -1] \\ p^2 &= [1 \quad 1 \quad 2], y^2 = [\alpha \quad -1 \quad -1] \end{aligned} \qquad \textbf{(2)}$$

θ, α **are nonnegative parameters. Which values of** θ, α **are consistent with producer theory?**

Use your answer to question 2 to answer the following questions.

QUESTION 3

Is the following pattern consistent with producer theory?

$$\underbrace{\frac{w_1}{r_1} < \frac{w_2}{r_2} \text{ and } \frac{K_1}{L_1} > \frac{K_2}{L_2}}_{\text{as labor becomes relatively more expensive,capital intensity decreases}} \qquad \textbf{(3)}$$

If your answer is yes, describe a production set that rationalizes this pattern.

QUESTION 4

Is the following pattern consistent with producer theory?

as labor becomes relatively more expensive,capital intensity increases
$\frac{w_1}{r_1} < \frac{w_2}{r_2}$ and $\frac{K_1}{L_1} < \frac{K_2}{L_2}$

(4)

If your answer is yes, describe a production set that rationalizes this pattern.

ANSWER TO QUESTION 1

$$Y = \left\{ \begin{bmatrix} Q \\ -K \\ L \end{bmatrix} : 0 \leq Q \leq F(K, L), K \geq 0, L \geq 0 \right\}$$

ANSWER TO QUESTION 2

$$p^1 y^1 = [1 \ 1 \ 1]\begin{bmatrix} 2 \\ -\theta \\ -1 \end{bmatrix} = 1-\theta$$

$$p^1 y^2 = [1 \ 1 \ 1]\begin{bmatrix} \alpha \\ -1 \\ -1 \end{bmatrix} = \alpha - 2$$

$$p^2 y^2 = [1 \ 1 \ 2]\begin{bmatrix} \alpha \\ -1 \\ -1 \end{bmatrix} = \alpha - 3$$

$$p^2 y^1 = [1 \ 1 \ 2]\begin{bmatrix} 2 \\ -\theta \\ -1 \end{bmatrix} = -\theta$$

WAPM $\quad p^1 y^1 \geq p^1 y^2 \quad | \quad 1-\theta \geq \alpha - 2$

$\quad p^2 y^2 \geq p^2 y^1 \quad | \quad \alpha - 3 \geq -\theta$

PARAMETER VALUES CONSISTENT WITH PRODUCER THEORY

$$\boxed{\alpha + \theta = 3, \ \alpha \geq 0, \ \theta \geq 0} \qquad (1)$$

WE ARE GIVEN THAT

$$\frac{w_1}{r_1} = 1 < 2 = \frac{w_2}{r_2} \qquad (2)$$

$$\frac{K_1}{L_1} = \theta \qquad (3)$$

$$\frac{K_2}{\ } = 1 \qquad (4)$$

$$\frac{K_2}{L_2} = 1 \qquad (4)$$

HENCE $\frac{K_1}{L_1} > \frac{K_2}{L_2} \iff \theta > 1$

AND THEREFORE BOTH PATTERNS ARE COMPATIBLE WITH PRODUCTION THEORY

ANSWER TO QUESTION 3

ALL VALUES $1 < \theta \leq 3$, $\alpha = 3 - \theta$ ARE CONSISTENT WITH PRODUCTION THEORY AND THE PATTERN $\frac{K_1}{L_1} > \frac{K_2}{L_2}$.

IN ORDER TO FIND RATIONALIZING PRODUCTION SETS, WE SET $\theta = 3$, $\alpha = 0$

THE SMALLEST PRODUCTION SET RATIONALIZING THIS PATTERN IS

$$Y_{MIN} = \{y^1, y^2\} = \left\{ \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} \right\}$$

TO FIND THE LARGEST PRODUCTION SET THAT RATIONALIZES THIS PATTERN, LET

$$Q(p^1, y^1) = \{ y \in R^3 : p^1 y^1 \geq p^1 y \} =$$

$$= \{ y = [Q, -K, -L] : p^1 y^1 \geq p^1 y, Q \geq 0, K \geq 0, L \geq 0 \} =$$

$$= \{ y = [Q, -K, -L] : p^1 y^1 \geq p^1 y,\ Q \geq 0, K \geq 0, L \geq 0 \} =$$

$$= \left\{ \begin{bmatrix} Q \\ -K \\ -L \end{bmatrix} \in \mathbb{R}^3 : -2 \geq Q - K - L,\ Q \geq 0, K \geq 0, L \geq 0 \right\}$$

$$Q(p^2, y^2) = \left\{ \begin{bmatrix} Q \\ -K \\ -L \end{bmatrix} : p^2 y^2 \geq p^2 \begin{bmatrix} Q \\ -K \\ -L \end{bmatrix} \right\} =$$

$$= \left\{ \begin{bmatrix} Q \\ -K \\ -L \end{bmatrix} : -3 \geq Q - K - 2L,\ Q \geq 0, K \geq 0, L \geq 0 \right\}$$

HENCE

$$YO = Q(p^1, y^1) \cap Q(p^2, y^2) =$$

$$\left\{ \begin{bmatrix} Q \\ -K \\ -L \end{bmatrix} : \begin{array}{l} Q \leq K + L - 2 \\ Q \leq K + 2L - 3 \\ Q \geq 0,\ L \geq 0,\ K \geq 0 \end{array} \right\} \quad (5)$$

TO FIND THE PRODUCTION FUNCTION DESCRIBED BY THE SET YO IN (5), SOLVE THE PROBLEM

$$\max Q$$

SUBJECT TO $[Q, -K, -L] \in YO$

VARIABLES: Q

PARAMETERS: K, L

CONDITIONS ON PARAMETERS: $K \geq 0, L \geq 0$

NOTICING THAT $K+L-2 \leq K+2L-3$ iff $L \geq 1$

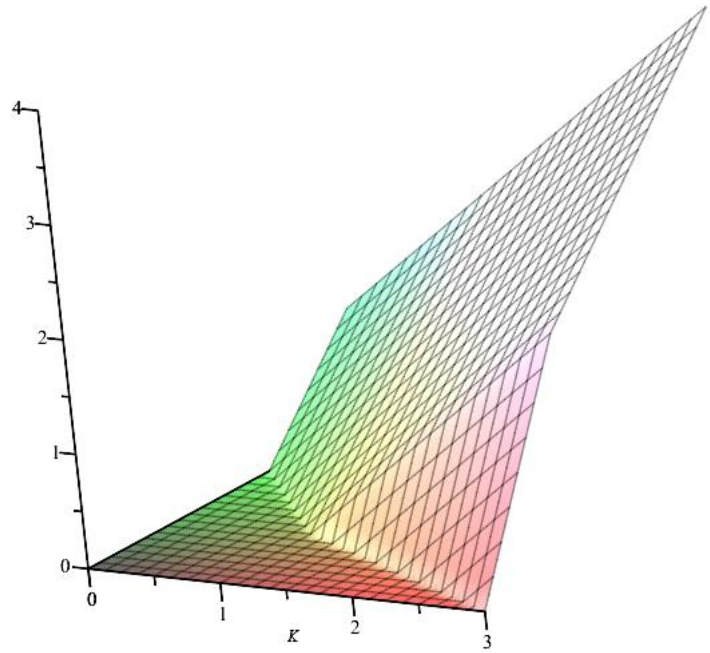
THE GLOBAL MAXIMUM OF THIS PROBLEM

$$Q = \begin{cases} \max(0, K+L-2) & \text{IF } L \geq 1 \\ \max(0, K+2L-3) & \text{IF } L \leq 1 \end{cases}$$

HENCE

$$F(K,L) = \begin{cases} K+L-2 & \text{IF } L \geq 1,\ K+L \geq 2 \\ K+2L-3 & \text{IF } L \leq 1,\ K+2L \geq 3 \\ 0 & \text{OTHERWISE} \end{cases} \qquad (6)$$

$$Q := \begin{cases} K+L-2 & 1 \leq L \textbf{ and } 2 \leq K+L \textbf{ and } 0 \leq K \\ K+2L-3 & L \leq 1 \textbf{ and } 3 \leq K+2L \textbf{ and } 0 \leq L \textbf{ and } 0 \leq K \\ 0 & \textit{otherwise} \end{cases}$$



ANSWER 4

BY (2) (3) (4) THE PATTERN OF QUESTION 4 OBTAINS IFF $0 \leq \theta < 1$. BY (1), ALL THESE VALUES ARE CONSISTENT WITH PRODUCER THEORY, AND IN PARTICULAR $\theta = 0, \alpha = 3$ IS CONSISTENT WITH PRODUCER THEORY. FOR THESE VALUES, THE SMALLEST PRODUCTION SET RATIONALIZING THE PATTERN OF QUESTION 4 IS

$$Y_{MIN} = \left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix} \right\} \quad (7)$$

TO FIND THE LARGEST PRODUCTION SET THAT RATIONALIZES THIS PATTERN, LET

$$Q(p^1, y^1) = \{ y \in R^3 : p^1 y^1 \geq p^1 y \} =$$

$$= \left\{ \begin{bmatrix} Q \\ -K \\ -L \end{bmatrix} \in R^3 : \quad 1 \geq Q - K - L, Q \geq 0, K \geq 0, L \geq 0 \right\}$$

$$Q(p^2, y^2) = \left\{ \begin{bmatrix} Q \\ -K \\ -L \end{bmatrix} : p^2 y^2 \geq p^2 \begin{bmatrix} Q \\ -K \\ -L \end{bmatrix} \right\} =$$

$$= \left\{ \begin{bmatrix} Q \\ -K \\ -L \end{bmatrix} : \quad 0 \geq Q - K - 2L, Q \geq 0, K \geq 0, L \geq 0 \right\}$$

HENCE

$$YO = Q(p^1, y^1) \cap Q(p^2, y^2) =$$

$$\left\{ \begin{bmatrix} Q \\ -K \\ -L \end{bmatrix} : \begin{array}{l} Q \leq K + L + 1 \\ Q \leq K + 2L \\ Q \geq 0, L \geq 0, K \geq 0 \end{array} \right\} \quad (B)$$

TO FIND THE PRODUCTION FUNCTION DESCRIBED BY THE SET YO IN (5), SOLVE THE PROBLEM

$$\max \; Q$$

SUBJECT TO $[Q, -K, -L] \in YO$

SUBJECT TO $[Q, -K, -L] \in Y_0$

VARIABLES: Q

PARAMETERS: K, L

CONDITIONS ON PARAMETERS: $K \geq 0, L \geq 0$

NOTICING THAT $K+L+1 \leq K+2L$ IFF $L \geq 1$

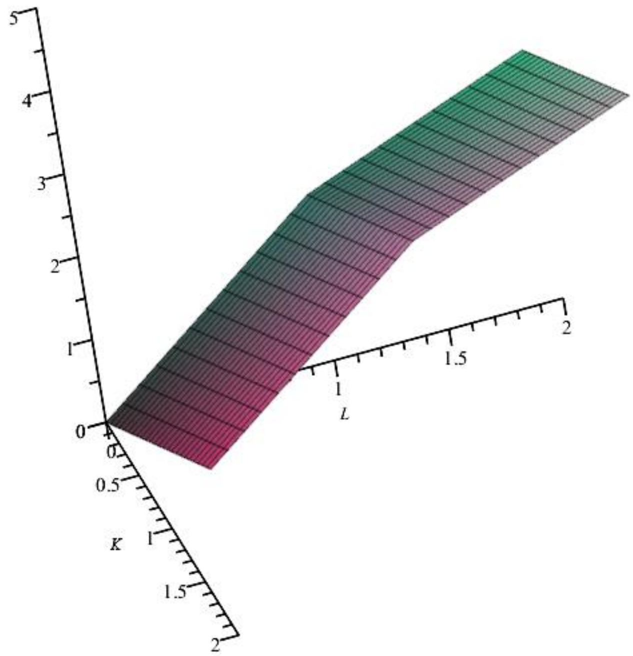
THE GLOBAL MAXIMUM OF THIS PROBLEM

$$Q = \begin{cases} K+L+1 & \text{IF } L \geq 1 \\ K+2L & \text{IF } L \leq 1 \end{cases}$$

HENCE

$$F(K,L) = \begin{cases} K+L+1 & \text{IF } L \geq 1 \\ K+2L & \text{IF } L \leq 1 \end{cases}$$

$$Q := \begin{cases} K+L+1 & 1 \leq L \textbf{ and } 0 \leq K \\ K+2L & 0 \leq L \leq 1 \textbf{ and } 0 \leq K \\ 0 & \textit{otherwise} \end{cases}$$



5
4
3
2
1
0
0.5
1
1.5
2
K
1
1.5
2
L