PROBLEM 1

Let $\mathbb{R}^n \xrightarrow{T} \mathbb{R}^m$ be an injective linear map and A a closed set in $\mathbb{R}^n$.

1.Show that $T(A)$ is a closed set.

2.Provide an example of a linear map T and a closed set A such that $T(A)$ is not closed

3.Generalize to injective affine maps

PROBLEM 2

Let A be a closed convex set and $x \in \mathbb{R}^n, x \notin A$.Let $\mathbb{R}^n \xrightarrow{\Pi} A$ be the nearest point map onto A.By the geometric form of the Hahn-Banach theorem ,there exists $p \in \mathbb{R}^n$ such that

$$\sup_{\alpha \in A}(p\alpha) = p\Pi\left(x\right) < px, \left|p\right| = 1 \qquad (1)$$

Provide a sharper version of (1) in the case that A is a closed convex **cone.**

PROBLEM 3

Let $\mathbb{R}^n \xrightarrow{T} \mathbb{R}$ be defined by

$$T(x_1,..,x_n) = \max(x_1,..,x_n) \qquad (2)$$

Show that T is a continuous nonlinear function.

PROBLEM 4

Let A be a set in $\mathbb{R}^n$.Show that

$$cl(A^c) = \left(\mathrm{int}(A)\right)^c \qquad (3)$$

PROBLEM 5

The perspective map $\mathbb{R}^n \times \mathbb{R}_{++} \xrightarrow{P} \mathbb{R}^n$ is defined by

$$P(x,t) = \frac{x}{t} \tag{4}$$

Show that

1.if A is a convex set in $\mathbb{R}^n \times \mathbb{R}_{++}$,then $P(A)$ is a convex set in $\mathbb{R}^n$

2.if B is a convex set in $\mathbb{R}^n$,then $P^{-1}(B)$ is a convex set in $\mathbb{R}^{n+1}$