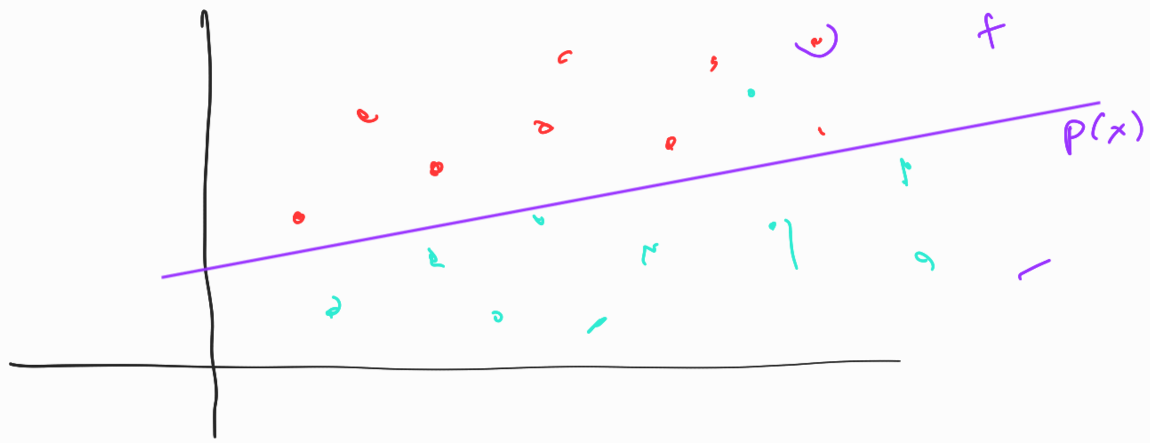
$p(x)$: Συνάρτηση που προέκυψε από την τεχνική ΕΤ

$$p(x^*) = y^* \approx y$$



SVD

Κάθε πίνακας $A \in \mathbb{R}^{n,m}$ $n \geq m$ μπορεί να γραφεί ως

$$A = U \Sigma V^T$$

$V^T \rightarrow \mathbb{R}^{m,m}$ ορθογ.

$U \rightarrow$ ορθ $\in \mathbb{R}^{n,m}$

$\Sigma \rightarrow$ διαγώνιος $\in \mathbb{R}^{m,m}$, $\sigma_i \geq 0$

$$U^T U = I \quad , \quad V^T V = I$$

$u_i^T u_j = 0$, $u_i^T u_i = \|u_i\|_2^2 = 1$

$v_i^T v_j = 0$, $v_i^T v_i = \|v_i\|_2^2 = 1$

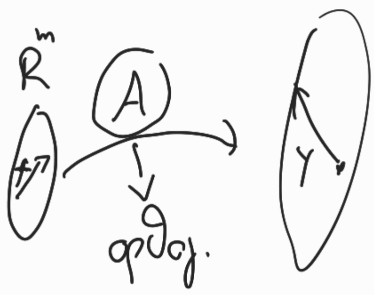
SVD & ET

$A \in \mathbb{R}^{n,m}$

$$\min_{x \in \mathbb{R}^m} \|r\|_2 = \min_{x \in \mathbb{R}^m} \|b - Ax\|_2^2 = \min_{x \in \mathbb{R}^m} \|b - U \Sigma V^T x\|_2^2 = (1)$$

$\rightarrow \mathbb{R}^{n,m} \; \mathbb{R}^{m,1}$

$$A \cdot x = y$$



$\rightarrow \quad \|Qx\|_2 = \|x\|_2$

(1) $\Rightarrow \quad \min \|\hat{U}^T(b - U\Sigma V^T x)\|_2^2 = \left\| \begin{bmatrix} U^T \\ \tilde{U}^T \end{bmatrix} (b - U\Sigma V^T x) \right\|_2^2$ (2)

U της SVD του A είναι ορθογ. $\in \mathbb{R}^{n,m}$

$$\hat{U} = {}_n\Big[\underbrace{U}_{m} \;\Big|\; \underbrace{\tilde{U}}_{n-m}\Big] \qquad \begin{array}{ccc} v_1 & v_2 & \\ \end{array} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \underbrace{}_{U} \;\; \tilde{U}$$

(2) $\left\| \begin{matrix} U^T b - U^T U \Sigma V^T x \\ \tilde{U}^T b - \underbrace{\tilde{U}^T U}_{0} \Sigma V^T x \end{matrix} \right\|_2^2 = \left\| \begin{matrix} U^T b - \Sigma V^T x \\ \tilde{U}^T b \end{matrix} \right\|_2^2 =$

$\|y\|_2 = \left\| \begin{matrix} y^1 \\ y^2 \end{matrix} \right\|$

$y \in \mathbb{R}^n$

$\|y\|_2^2 = \left(\sqrt{y_1^2 + y_2^2 + \dots y_n^2}\right)^2$

$= \underbrace{y_1^2 + y_2^2 +}\,\underbrace{\dots y_n^2}$

$\min\limits_x \|U^T b - \Sigma V^T x\|_2^2 + \min\limits_x \|\tilde{U}^T b\|_2^2 =$

$\min\limits_x \underbrace{\|U^T b - \Sigma V^T x\|_2^2} + \|\tilde{U}^T b\|_2^2$

$\exists x ? : \quad \underbrace{U^T b} = \underbrace{\Sigma V^T} x \quad ? \quad \xRightarrow{\sigma_i > 0} \quad \Sigma^{-1} U^T b = V^T x$

$\exists \Sigma^{-1}$

$\tilde{b} = X x$

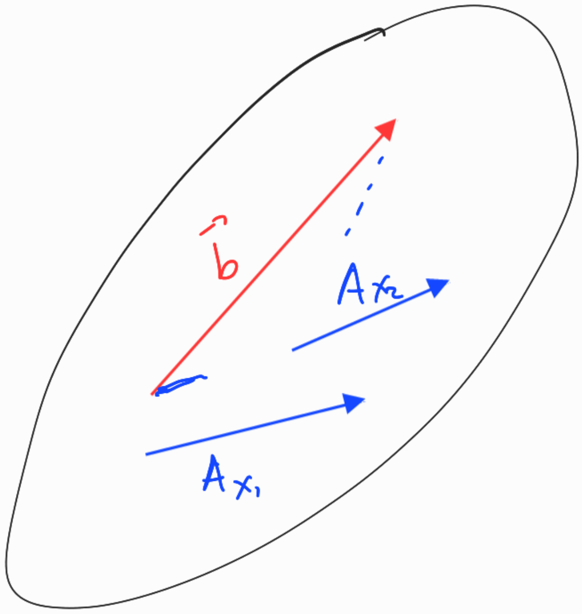
$\Leftrightarrow \; V \Sigma^{-1} U^T b = x$

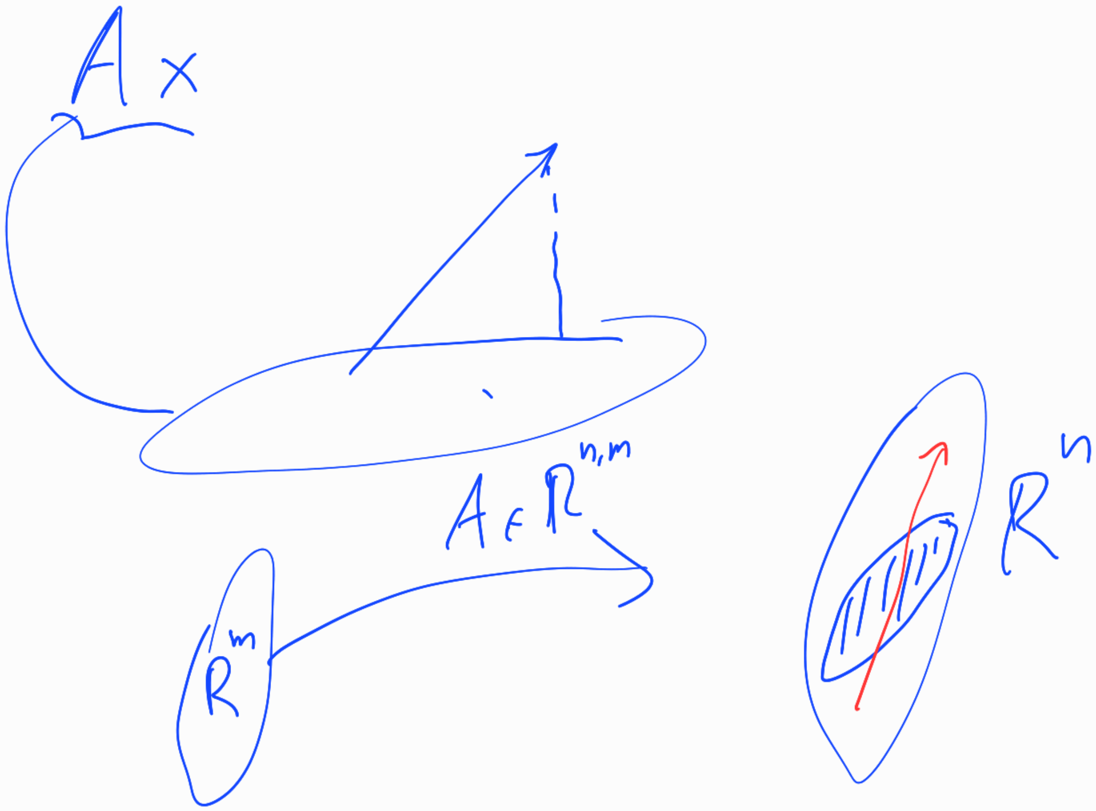
$\mathbb{R}^n$

$\|b - Ax\|_2^2$?

$A = (U \Sigma V^T)^{-1}$

$A^+ = (V \Sigma^{-1} U^T)$





Περίπτωση όπου $\sigma_1 \geq \sigma_2 \dots > \sigma_{m-k} \geq \underbrace{\sigma_{m-k+1} = \dots \sigma_m = 0}_{=0}$

$$\Sigma = \begin{bmatrix} \sigma_1 & & & & \\ & \sigma_2 & & & \\ & & \ddots & & \\ & & & 0 & \\ & & & & \ddots \; 0 \end{bmatrix} = \Sigma_{m-k} = \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 \ddots & \\ & & \sigma_{m-k} \end{bmatrix}$$

U_{m-k} : οι πρώτες $m-k$ στήλες του U

V_{m-k} : " $m-k$ γραμμές/στήλες του V

$$A_{m-k} = U_{m-k} \Sigma_{m-k} V_{m-k}^T$$

$$A_{m-k}^+ = V_{m-k} \Sigma_{m-k}^{-1} U_{m-k}^T$$

$$\cancel{x} = A_{m-k}^+ \cdot b$$

$$\min \| A - A_k \|_2 =$$

B

$$\sum_{i=1}^{k} \sigma_i u_i v_i^T$$

$$x = \boxed{\frac{v_i u_i^T}{\sigma_i}} b = \frac{u_i^T b}{\sigma_i + \varepsilon_i} \cdot v_i$$

$$X = c_1 v_1 + c_2 v_2 + \dots + c_k v_k$$

$$\tilde{c}_1 \qquad \tilde{c}_k$$

$$\begin{pmatrix} \sigma_1+\varepsilon_1 & & & \\ & \sigma_2+\varepsilon_2 & & \\ & & \ddots & \\ & & & \sigma_m+\varepsilon_m \end{pmatrix}$$

$$\varepsilon_i \sim N(0,1)$$
$$\sim N(\mu, \sigma^2)$$