Solutions of Practice Exercises
Preparatory Course
M.Sc. in ISFM

Andrianos E. Tsekrekos*

September, 2015

Exercise 1:
[a] $(f \circ g)(x) = \sqrt{x+1}$ and $(g \circ f)(x) = \sqrt{x} + 1$
[b] $(f \circ g)(x) = (\frac{2}{x-1})^2 + \frac{14}{x-1} + 1$ and $(g \circ f)(x) = \frac{2}{x^2+7x}$
[c] $(f \circ g)(x) = \frac{1}{|x+2|+1}$ and $(g \circ f)(x) = \sqrt{\frac{1}{x^2+1} + 2}$
[d] $(f \circ g)(x) = \left(\frac{4}{5}x^2 - 16x - 1\right)^2$ and $(g \circ f)(x) = \frac{1}{5}(4x-13)^4 - 4(4x-13)^2 + 3$
[e] $(f \circ g)(x) = (x^3 - 7)^{2/3}$ and $(g \circ f)(x) = x^2 - 7$

Exercise 2: Revenues are $Price \times Quantity = \$9.75 \times q$ and Costs are $\$4500 + \$4.25 \times q$, thus Profits must be $P(q) = \$9.75 \times q - \$4500 - \$4.25 \times q = \$5.50 \times q - \$4500$.

Exercise 3: The composite function $S(I(E))$ should be

$$S(I(E)) = 0.45\left[(7202 + 0.29E^{3.68}) - 1000\right]^{0.53} = 0.45\left(6202 + 0.29E^{3.68}\right)^{0.53},$$

which represents the statistical relationship between a person's numerical value of status with his/her number of years of education.

Exercise 4: Solve as follows:

*Department of Accounting & Finance, A.U.E.B. Tel./Fax: +30-210-8203928, e-mail: tsekrekos@aueb.gr.

[a] $x^2 - 4x + 4 = 0 \Leftrightarrow (x-2)^2 = 0 \Leftrightarrow x - 2 = 0 \Leftrightarrow x = 2$

[b] $x^2 - 8x + 15 = 0 \Leftrightarrow x = \frac{8 \pm \sqrt{(-8)^2 - 4\times 1 \times 15}}{2\times 1} \Leftrightarrow x = 3$ and $x = 5$

[c] $-x^2 + 3x + 10 = 0 \Leftrightarrow x = \frac{-3 \pm \sqrt{(3)^2 - 4\times(-1)\times 10}}{2\times(-1)} \Leftrightarrow x = -2$ and $x = 5$

[d] $x^2 = \frac{x+3}{2} \Leftrightarrow 2x^2 - x - 3 = 0 \Leftrightarrow x = \frac{1 \pm \sqrt{(-1)^2 - 4\times 2\times(-3)}}{2\times 2} \Leftrightarrow x = -1$ and $x = 3/2$

[e] $\frac{6x+7}{2x+1} - \frac{6x+1}{2x} = 1 \Leftrightarrow \frac{6x+7}{2x+1} = \frac{6x+1}{2x} + 1 \Leftrightarrow \frac{6x+7}{2x+1} = \frac{8x+1}{2x} \Leftrightarrow 2x(6x+7) = (8x+1)(2x+1) \Leftrightarrow 12x^2 + 14x = 16x^2 + 10x + 1 \Leftrightarrow 4x^2 - 4x + 1 = 0 \Leftrightarrow x = \frac{4 \pm \sqrt{(-4)^2 - 4\times 4\times 1}}{2\times 4} \Leftrightarrow x = 1/2$

[f] $\frac{2x-3}{2x+5} - \frac{2x}{3x+1} = 1 \Leftrightarrow \frac{2x-3}{2x+5} = \frac{2x}{3x+1} + 1 \Leftrightarrow \frac{2x-3}{2x+5} = \frac{5x+1}{3x+1} \Leftrightarrow (2x-3)(3x+1) = (2x+5)(5x+1) \Leftrightarrow 6x^2 - 7x - 3 = 10x^2 + 27x + 5 \Leftrightarrow 4x^2 + 34x + 8 = 0 \Leftrightarrow x = \frac{-34 \pm \sqrt{(34)^2 - 4\times 4\times 8}}{2\times 4} \Leftrightarrow x = \frac{1}{4}(-17 - \sqrt{257})$ and $x = \frac{1}{4}(-17 + \sqrt{257})$

[g] $5 - \frac{3(x+3)}{x^2+3x} = \frac{1-x}{x} \Leftrightarrow 5x(x^2+3x) - 3x(x+3) = (1-x)(x^2+3x) \Leftrightarrow 5x^3 + 15x^2 - 3x^2 - 9x = x^2 + 3x - x^3 - 3x^2 \Leftrightarrow 6x^3 + 14x^2 - 12x = 0 \Leftrightarrow x(6x^2 + 14x - 12) = 0$ so $x = 0$ and $x = -3$ and $x = 2/3$. But $x = 0$ and $x = -3$ cannot be solutions (why?), so just $x = 2/3$.

[h] $\left|\frac{5x-6}{13}\right| = 0 \Leftrightarrow 5x - 6 = 0 \Leftrightarrow x = \frac{6}{5}$

Exercise 5: From the equilibrium conditions for the three products,

$$\begin{aligned} Q_1^D &= Q_1^S \\ Q_2^D &= Q_2^S \\ Q_3^D &= Q_3^S \end{aligned}$$

we get

$$\begin{aligned} (a_1 - d_1)P_1 + (a_2 - d_2)P_2 + (a_3 - d_3)P_3 &= d_0 - a_0 \\ (b_1 - e_1)P_1 + (b_2 - e_2)P_2 + (b_3 - e_3)P_3 &= e_0 - b_0 \\ (c_1 - h_1)P_1 + (c_2 - h_2)P_2 + (c_3 - h_3)P_3 &= h_0 - c_0 \end{aligned}$$

that is just a 3×3 system of linear equations.

Exercise 6: The solutions should be:

[a] $f(x) = 3x + 7 \Leftrightarrow f(x) - 7 = 3x \Leftrightarrow x = \frac{f(x)-7}{3}$, thus $f^{-1}(x) = \frac{x-7}{3}$

[b] $f(x) = \frac{1}{2}x - 7 \Leftrightarrow f(x) + 7 = \frac{1}{2}x \Leftrightarrow x = 2f(x) + 14$, thus $f^{-1}(x) = 2x + 14$

[c] $F(Y) = AY^2 \Leftrightarrow \frac{F(Y)}{A} = Y^2 \Leftrightarrow Y = \sqrt{\frac{F(Y)}{A}}$, thus $F^{-1}(Y) = \sqrt{\frac{Y}{A}}$

Exercise 7: From the equilibrium conditions

$$\begin{aligned} Q_R^D &= Q_R^S \\ Q_C^D &= Q_C^S \end{aligned}$$

we get

$$\begin{aligned} 11P_R - 3P_C &= 7 \\ 2P_R - 11P_C &= -5 \end{aligned}$$

that is a 2×2 system, with unique solution $(P_R, P_C) = (0.8, 0.6)$.

Exercise 8: Profits must be exactly zero for the investment to break even. But profits are revenues minus costs, thus if m stands for flying miles, it must be

$$\begin{aligned} Profits(m) &= Revenues(m) - Costs(m) = 120m - 100m - (100000 + 700000) \\ &= 20m - 800000 \end{aligned}$$

Thus for $Profits(m^*) = 0 \Leftrightarrow m^* = 40000$ flying miles.

Exercise 9: Let x_s, x_{ss} and x_{sc} stand for the number of skilled, semi–skilled and shipping clerks that the company should hire. These can be determined by solving

$$\begin{aligned} x_s + x_{ss} + x_{sc} &= 70 \\ 16x_s + 9.50x_{ss} + 10x_{sc} &= 725 \\ 2x_s - x_{ss} &= 0 \end{aligned}$$

The above system has unique solution $(x_s, x_{ss}, x_{sc}) = (5, 10, 55)$.

Exercise 10: For the break–even quantity, we must have

$$\begin{aligned} TR(Q) = TC(Q) &\Leftrightarrow 100\sqrt{Q} = 2Q + 1200 \Leftrightarrow \\ &\Leftrightarrow 10000Q = 4Q^2 + 4800Q + 1440000 \Leftrightarrow \\ &\Leftrightarrow 4Q^2 - 5200Q + 1440000 = 0 \end{aligned}$$

This has solutions $Q = 400$ and $Q = 900$.

Exercise 11: The balance in three years will be:
(i) $FV = \$10000(1 + 0.053)^3 = \$11675.76$

(ii) $FV = \$10000(1 + 0.053/2)^{(}3 \times 2) = \$11699.13$

(iii) $FV = \$10000(1 + 0.053/4)^{(}3 \times 4) = \$11711.14$

(iv) $FV = \$10000(1 + 0.053/12)^{(}3 \times 12) = \$11719.28$

(v) $FV = \$10000(1 + 0.053/52)^{(}3 \times 52) = \$11722.43$

Exercise 12: This is given by

$$\begin{aligned} FV &= PV(1 + \frac{r}{m})^{n \times m} \Leftrightarrow £27000 = PV(1 + \frac{0.10}{2})^{11 \times 2} \Leftrightarrow \\ \Leftrightarrow PV &= £27000(1 + 0.05)^{-22} \Leftrightarrow PV = £9229.95 \end{aligned}$$

Exercise 13: Please refer to the accompanying excel.