Solutions of Practice Exercises
Preparatory Course
M.Sc. in ISFM

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Exercise 1:
[a] $f'(x) = 3e^x + 2e^{x^2}x + e^2 x^{-1+e^2}$

[b] $f'(x) = \frac{1+2x}{x+x^2}$

[c] $f'(x) = \frac{(9-x)(-6+x)+(9-x)(5+x)-(-6+x)(5+x)}{2\sqrt{(9-x)(-6+x)(5+x)}}$

[d] $f'(x) = -\frac{4e^{1+2x}}{x^2} + \frac{8e^{1+2x}}{x}$

[e] $f'(x) = \frac{e^{-x}}{x} - e^{-x}\ln(x)$

Exercise 2: The elasticity is

$$\epsilon = \frac{dq}{dp} \times \frac{p}{q} = \frac{e^p p \left(\frac{e^{-p}}{p} - e^{-p}\ln(p)\right)}{\ln(p)}$$

which at $p = 30$ gives $\epsilon = -29.71$. This implies that a $\frac{2}{3}\%$ decrease in the price will increase demand by $\frac{2}{3}(-29.71) = -19.81\%$.

Exercise 3: The fourth–order Taylor series expansion of $f(x)$ at $x_0 = 0$

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is given by

$$f(x) \approx 1 - 2x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24},$$

which at $x = 0.5$ gives $f(x) \approx 0.148438$.

Exercise 4: The profit function is

$$\Pi(q) = 5000 + 20q + (450 - 4q)q - 0.004q^3$$

and to find the maximum, consider the f.o.c.

$$\Pi'(q) = 0 \Leftrightarrow 470 - 8q - 0.012q^2 = 0 \Leftrightarrow q = 54.3234 \text{ and } q = -720.99$$

The negative solution is ruled out (as negative quantity produced cannot exist). Check the s.o.c.

$$\Pi''(q) = -8 - 0.024q$$

and verify that at $q = 54.3234$, $\Pi''(q) < 0$.

Exercise 5: The marginal cost function is

$$MC(q) = TC'(q) = 12 - 12q + 3q^2$$

For the marginal cost to be increasing, its first derivative should be positive, thus we look for the values of q for which

$$MC'(q) > 0 \Leftrightarrow -12 + 6q > 0 \Leftrightarrow q > 2.$$

Exercise 6: Performing the matrix multiplications yields

$$f(\alpha, \beta) = 0.05 - 0.01\alpha^2\lambda + \beta(0.15 - 0.09\beta\lambda) + \alpha(0.05 - 0.03\beta\lambda)$$

The f.o.c. for a maximum are

$$\begin{aligned} \frac{\partial f(\alpha, \beta)}{\partial \alpha} &= 0 \Leftrightarrow 0.05 - 0.02\alpha\lambda - 0.03\beta\lambda = 0 \\ \frac{\partial f(\alpha, \beta)}{\partial \beta} &= 0 \Leftrightarrow 0.15 - 0.03\alpha\lambda - 0.18\beta\lambda = 0 \end{aligned}$$

and yield

$$\alpha = \frac{1.66667}{\lambda} \text{ and } \beta = \frac{0.555556}{\lambda}$$

For the s.o.c., construct the Hessian matrix

$$H = \begin{bmatrix} f_{\alpha\alpha} & f_{\alpha\beta} \\ f_{\beta\alpha} & f_{\beta\beta} \end{bmatrix} = \begin{bmatrix} -0.02\lambda & -0.03\lambda \\ -0.03\lambda & -0.18\lambda \end{bmatrix}$$

and calculate its determinant $|H| = 0.0036\lambda^2 - 0.0009\lambda^2 = 0.0027\lambda^2$ which is always positive.