



Statistics for Business

Fall Semester 2023–2024

Assignment 1: Questions

Answer the following questions and hand in your answers by Wednesday, September 13, 2023. These should either be typed and sent electronically or handwritten, scanned and sent electronically (via E-mail at pkonstantinou.aueb@gmail.com). Please make sure that your files are in PDF format and that they do not exceed 5MB in size, otherwise the server might block them!

Calculating Probabilities

1. Suppose there are six items and use the letters $A, B, C, D, E,$ and F to identify them.
 - (a) How many ways can three items be selected from the above group of six items? List each of the different combinations.
 - (b) How many permutations of three items can be selected from the above group of six? List each of the permutations of items $B, D,$ and F .
2. Consider the experiment of rolling a pair of dice. Suppose that we are interested in the sum of the face values showing on the dice.
 - (a) How many sample points are possible?
 - (b) List the sample points.
 - (c) What is the probability of obtaining a value of 7?
 - (d) What is the probability of obtaining a value of 9 or greater?
 - (e) Because each roll has six possible even values (2, 4, 6, 8, 10, and 12) and only five possible odd values (3, 5, 7, 9, and 11). the dice should show even values more often than odd values. Do you agree with this statement? Explain.
3. AUEB surveyed alumni to learn more about what they think of AUEB. One part of the survey asked respondents to indicate whether their overall experience at AUEB fell short of expectations, met expectations, or surpassed expectations. The results showed that 4% of the respondents did not provide a response, 26% said that their experience fell short of expectations, and 65% of the respondents said that their experience met expectations.
 - (a) If we chose an alumnus at random, what is the probability that the alumnus would say their experience surpassed expectations?

(b) If we chose an alumnus at random. what is the probability that the alumnus would say their experience met or surpassed expectations?

4. The automobile industry sold 657,000 vehicles in the United States during January 2009. This volume was down 37% from January 2008 as economic conditions continued to decline. The Big Three U.S. automakers – General Motors, Ford, and Chrysler – sold 280,500 vehicles, down 48% from January 2008. A summary of sales by automobile manufacturer and type of vehicle sold is shown in the following table. Data are in thousands of vehicles. The non-U.S. manufacturers are led by Toyota, Honda, and Nissan. The category Light Truck includes pickup, minivan, SUV, and crossover models.

	Type	of Vehicle
	<u>Car</u>	<u>Light Truck</u>
U.S.	87.4	193.1
Non U.S.	228.5	148.0

- (a) Develop a joint probability table for these data and use the table to answer the following questions.
- (b) What are the marginal probabilities? What do they tell you about the probabilities associated with the manufacturer and the type of vehicle sold?
- (c) If a vehicle was manufactured by one of the U.S. automakers, what is the probability that the vehicle was a car? What is the probability that it was a light truck?
- (d) If a vehicle was not manufactured by one of the U.S. automakers, what is the probability that the vehicle was a car? What is the probability that it was a light truck?
- (e) If the vehicle was a light truck, what is the probability that it was manufactured by one of the U.S. automakers?
- (f) What does the probability information tell you about sales?

5. A survey of MSc students, provided the following data for 2018 students.

Age Group	Applied to More than One School	
	YES	NO
23 and under	207	201
24 – 26	299	379
27 – 30	185	268
31 – 35	66	193
36 and over	51	169

Table 1: Table: Survey Data

- (a) For a randomly selected MSc student, prepare a joint probability table for the experiment consisting of observing the student's age and whether the student applied to one or more schools.
- (b) What is the probability that a randomly selected applicant is 23 or under?
- (c) What is the probability that a randomly selected applicant is older than 26?
- (d) What is the probability that a randomly selected applicant applied to more than one school?
- (e) Given that a person applied to more than one school, what is the probability that the person is 24–26 years old?
- (f) Given that a person is in the 36-and-over age group, what is the probability that the person applied to more than one school?
- (g) What is the probability that a person is 24–26 years old or applied to more than one school?
- (h) Suppose a person is known to have applied to only one school. What is the probability that the person is 31 or more years old?
- (i) Is the number of schools applied to independent of age? Explain.

Discrete Random Variables

- 6. A technician services mailing machines at companies in the Athens area. Depending on the type of malfunction, the service call can take one, two, three, or four hours. The different types of malfunctions occur at about the same frequency.
 - (a) Develop a probability distribution for the duration of a service call.
 - (b) Show that your probability distribution satisfies the conditions required for a discrete probability function.
 - (c) What is the probability that a service call will take three hours?
 - (d) A service call has just come in, but the type of malfunction is unknown. It is 3:00 P.M. and service technicians usually get off work at 5:00 P.M. What is the probability that the service technician will have to work overtime to fix the machine today?
- 7. A survey reported that 25% of employees said their company is loyal to them. Suppose 10 employees are selected randomly and will be interviewed about company loyalty.
 - (a) Is the selection of 10 employees a binomial experiment? Explain.
 - (b) What is the probability that none of the 10 employees will say their company is loyal to them?

- (c) What is the probability that 4 of the 10 employees will say their company is loyal to them?
 - (d) What is the probability that at least 2 of the 10 employees will say their company is loyal to them?
8. AUEB found that 20% of its students withdraw without completing the introductory economics course. Assume that 20 students registered for the course.
- (a) Calculate the probability that two or fewer will withdraw.
 - (b) Calculate the probability that exactly four will withdraw.
 - (c) Calculate the probability that more than three will withdraw.
 - (d) Calculate the expected number of withdrawals.

Continuous Random Variables

9. Using “TABLE 3 Areas under the Normal Curve” (Appendix I Tables) find the following probabilities for a standard normal random variable Z :
- (a) $\Pr(0 \leq Z \leq 0.83)$
 - (b) $\Pr(-1.57 \leq Z \leq 0)$
 - (c) $\Pr(Z \geq 0.44)$
 - (d) $\Pr(Z \geq -0.23)$
 - (e) $\Pr(Z \leq 1.20)$
 - (f) $\Pr(Z \leq -0.71)$
 - (g) $\Pr(-1.98 \leq Z \leq 0.49)$
 - (h) $\Pr(0.52 \leq Z \leq 1.22)$
 - (i) $\Pr(-1.75 \leq Z \leq -1.04)$
10. In an article about the cost of health care, it was reported that a visit to a hospital emergency room for something as simple as a sore throat has a mean cost of €328. Assume that the cost for this type of hospital emergency room visit is normally distributed with a standard deviation of €92. Answer the following questions about the cost of a hospital emergency room visit for this medical service.
- (a) What is the probability that the cost will be more than €500?
 - (b) What is the probability that the cost will be less than €250?

- (c) What is the probability that the cost will be between €300 and €400?
- (d) If the cost to a patient is in the lower 8% of charges for this medical service, what was the cost of this patient's emergency room visit?
11. For a person to qualify for membership at SISA (a special society) he/she must score in the upper 2% of the population on an a special skills test. If test scores are normally distributed with a mean of 100 and a standard deviation of 15, what score must a person have to qualify SISA?
12. In May 2012, the Public sector worker spent an average of 98 hours logged on to the Internet while at work. Assume the population mean is 98 hours, the times are normally distributed, and that the standard deviation is 25 hours.
- (a) a. What is the probability that in May 2012 a randomly selected public sector worker spent fewer than 60 hours logged on to the Internet?
- (b) What percentage of workers spent more than 120 hours in May 2012 logged on to the Internet?
- (c) A person is classified as a heavy user if he or she is in the upper 20% of usage. In May 2012, how many hours did a worker have to be logged on to the Internet to be considered a heavy user?

Multivariate Probability Distributions

13. After analyzing monthly ales data, the marketing director of KOTSOVOLOS (Patisson Branch) produced the following joint probability distribution of the number of refrigerators and TVs sold daily.

Table 2: Joint Probability Distribution of Sales

TVs	Refrigerators		
	0	1	2
0	0.08	0.14	0.12
1	0.09	0.17	0.13
2	0.05	0.18	0.04

- (a) Find the marginal probability distribution of the number of refrigerators sold daily.
- (b) Find the marginal probability distribution of the number of TVs sold daily.
- (c) Compute the mean and variance of the number of refrigerators sold daily.

- (d) Compute the mean and variance of the number of **TVs** sold daily.
- (e) Compute the covariance and the coefficient of correlation.
- (f) Calculate $\Pr(1 \text{ refrigerator} | 0 \text{ TVs})$.
- (g) Calculate $\Pr(0 \text{ TVs} | 1 \text{ refrigerator})$.
- (h) Calculate $\Pr(2 \text{ refrigerators} | 2 \text{ TVs})$.

14. A portfolio is composed of two stocks. The proportion of each stock, their expected values, and standard deviations are listed in the following table. For each of the follow-

Table 3: Two Stock Portfolio

Stock	1	2
Proportion of portfolio	0.30	0.70
Mean	0.12	0.25
Standard deviation	0.02	0.15

ing coefficients of correlation, calculate the expected value and standard deviation of the portfolio:

- (a) $\rho = 0.5$
- (b) $\rho = 0.2$
- (c) $\rho = 0$