ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ
ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

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Statistics for Business

Assignment 1: Suggested Solutions

Probability Calculations

1. (a) There are $6! = 720$ possible itineraries.

   (b) In the 720 orderings, exactly 360 have Denver before San Francisco and 360 have San Francisco before Denver. So, the probability is 0.5.

2. There are $\binom{50}{3} = \frac{50!}{3!(50-3)!} = 19,600$ ways to choose the 3 winners. Each of these is equally likely.

   (a) There are $\binom{4}{3} = \frac{4!}{3!(4-3)!} = 4$ ways for the organizers to win all of the prizes. The probability is $4/19600$.

   (b) There are $\binom{4}{2}\binom{46}{1} = 276$ ways the organizers can win two prizes and one of the other 46 people to win the third prize. So, the probability is $276/19600$.

   (c) There are $\binom{4}{1}\binom{46}{2} = 4,140$ ways the organizers can win one prize and two of the other 46 people to win the second and third prize. The probability is $4140/19600$.

   (d) There are $\binom{46}{3} = 15,180$ ways the organizers can win none of the prizes. The probability is $15180/19600$.

3. (a) Given the first two cards drawn are spades, there are 11 spades left in the deck. Thus, the probability is

$$\frac{\binom{11}{3}}{\binom{50}{3}} = \frac{165}{19600} = 0.0084.$$

   (b) Given the first three cards drawn are spades, there are 10 spades left in the deck. Thus, the probability is

$$\frac{\binom{10}{2}}{\binom{49}{2}} = \frac{45}{1176} = 0.0383.$$

(c) Given the first four cards drawn are spades, there are 9 spades left in the deck. Thus, the probability is

$$\frac{\binom{9}{1}}{\binom{48}{1}} = \frac{9}{48} = 0.1875.$$

4. Define the events: P : positive response; M : male respondent; and F : female respondent. Then (given from the problem)

$$\Pr(P|F) = 0.7;\ \ \Pr(P|M) = 0.4;\ \text{and}\ \Pr(M) = 0.25.$$

Using Bayes' rule,

$$\Pr(M|\bar{P}) = \frac{\Pr(\bar{P}|M)\Pr(M)}{\Pr(\bar{P}|M)\Pr(M) + \Pr(\bar{P}|F)\Pr(F)} = \frac{.6(.25)}{.6(.25) + .3(.75)} = 0.4.$$

5. Define the events M : major airline; P : private airline; C : commercial airline, and B : travel for business. Then (given from the problem)

$$\Pr(M) = 0.6;\ \ \Pr(C) = 0.3;\ \ \Pr(B|M) = 0.5;\ \ \Pr(B|P) = 0.6;\ \ \Pr(B|C) = 0.9.$$

(a) (Using the rule of total probability)

$$\begin{aligned} \Pr(B) &= \Pr(B|M)\Pr(M) + \Pr(B|P)\Pr(P) + \Pr(B|C)\Pr(C) \\ &= .6(.5) + .3(.6) + .1(.9) = 0.57. \end{aligned}$$

(b) $\Pr(B \cap P) = \Pr(B|P)\Pr(P) = .3(.6) = 0.18.$

(c) $\Pr(P|B) = \Pr(B \cap P)/\Pr(B) = .18/.57 = 0.3158$

(d) $\Pr(B|C) = 0.90.$