SECOND EDITION

THE ESSENTIALS OF STATISTICS

A TOOL FOR SOCIAL RESEARCH

JOSEPH F. HEALEY

FREQUENTLY USED FORMULAS

CHAPTER 2

Proportion

$$p = \frac{f}{N}$$

Percentage

$$\% = \left(\frac{f}{N}\right) \times 100$$

CHAPTER 4

Mean

$$\overline{X} = \frac{\Sigma(X_i)}{N}$$

CHAPTER 5

Standard deviation

$$s = \sqrt{\frac{\Sigma(X_i - \overline{X})^2}{N}}$$

CHAPTER 6

Z scores

$$Z = \frac{X_i - \overline{X}}{s}$$

CHAPTER 7

Confidence interval for a sample mean

$$c.i. = \overline{X} \pm Z\left(\frac{s}{\sqrt{N-1}}\right)$$

Confidence interval for a sample proportion

$$\text{c.i.} = P_s \pm Z\sqrt{\frac{P_u(1 - P_u)}{N}}$$

CHAPTER 8

Means

$$Z(\text{obtained}) = \frac{\overline{X} - \mu}{s/\sqrt{N-1}}$$

Proportions

$$Z(\text{obtained}) = \frac{P_s - P_u}{\sqrt{P_u(1 + P_u)/N}}$$

CHAPTER 9

Means

$$Z(\text{obtained}) = \frac{(\overline{X}_1 - \overline{X}_2)}{\sigma_{\bar{x}-\bar{x}}}$$

Standard deviation of the sampling distribution for sample means

$$\sigma_{\bar{x}-\bar{x}} = \sqrt{\frac{s_1^2}{N_1 - 1} + \frac{s_2^2}{N_2 - 1}}$$

Pooled estimate of population proportion

$$P_u = \frac{N_1 P_{s1} + N_2 P_{s2}}{N_1 + N_2}$$

Standard deviation of the sampling distribution for sample proportions

$$\sigma_{p-p} = \sqrt{P_u(1 - P_u)}\ \sqrt{(N_1 + N_2)/N_1 N_2}$$

Proportions

$$Z(\text{obtained}) = \frac{(P_{s1} - P_{s2})}{\sigma_{p-p}}$$

CHAPTER 10

Total sum of squares

$$\text{SST} = \Sigma X^2 - N\overline{X}^2$$

Sum of squares between

$$\text{SSB} = \Sigma N_k(\overline{X}_k - \overline{X})^2$$

Sum of squares within

$$\text{SSW} = \text{SST} - \text{SSB}$$

Degrees of freedom for SSW

$$dfw = N - k$$

Degrees of freedom for SSB

$$dfb = k - 1$$

Mean square within

$$\text{MSW} = \frac{SSW}{dfw}$$

Mean square between

$$\text{MSB} = \frac{SSB}{dfb}$$

F ratio

$$F = \frac{MSB}{MSW}$$

(continued on inside back cover)

The Essentials of STATISTICS

A Tool for Social Research

Second Edition

Joseph F. Healey

Christopher Newport University

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***The Essentials of Statistics: A Tool for Social Research*, Second Edition**
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Cover Designer: RHDG
Cover Image: © istock.com
Compositor: Macmillan Publishing Solutions

Library of Congress Control Number: 2008940409

Student Edition:

ISBN-13: 978-0-495-60143-2

ISBN-10: 0-495-60143-8

Wadsworth
10 Davis Drive
Belmont, CA 94002-3098
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Printed in Canada
1 2 3 4 5 6 7 12 11 10 09

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Preface

Statistics are part of the everyday language of sociology and the other social sciences (including political science, social work, public administration, criminal justice, urban studies, and gerontology). These research-based disciplines routinely use statistics to express knowledge and to discuss theory and research. To join the conversation, you must be literate in the vocabulary of research, data analysis, and scientific thinking. Fluency in statistics will help you understand the research reports you encounter in everyday life and the professional research literature of your discipline. You will also be able to conduct quantitative research, contribute to the growing body of social science knowledge, and reach your full potential as a social scientist.

Although essential, learning (and teaching) statistics can be a challenge. Students in statistics courses typically bring with them a wide range of mathematical backgrounds and an equally diverse set of career goals. They are often puzzled about the relevance of statistics for them and, not infrequently, there is some math anxiety to deal with.

This text introduces statistical analysis for the social sciences while addressing these challenges. The text is an abbreviated version of *Statistics: A Tool for Social Research,* 8th edition, and presents only the most essential material from that larger volume. It makes minimal assumptions about mathematical background (the ability to read a simple formula is sufficient preparation for virtually all of the material in the text), and a variety of special features help students analyze data successfully. The theoretical and mathematical explanations are kept at an elementary level, as is appropriate in a first exposure to social statistics. This text has been written especially for sociology and social work programs but it is flexible enough to be used in any program with a social science base.

GOAL OF THE TEXT AND CHANGES IN THE *ESSENTIALS* VERSION

The goal of this text is to develop basic statistical literacy. The statistically literate person understands and appreciates the role of statistics in the research process, is competent to perform basic calculations, and can read and appreciate the professional research literature in their field as well as any research reports they may encounter in everyday life. These three aspects of statistical literacy provide a framework for discussing the features of this text:

1. An Appreciation of Statistics. A statistically literate person understands the relevance of statistics for social research, can analyze and interpret the meaning of a statistical test, and can select an appropriate statistic for a given purpose and a given set of data. This textbook develops these qualities, within the constraints imposed by the introductory nature of the course, in the following ways:

- *The relevance of statistics.* Chapter 1 includes a discussion of the role of statistics in social research and stresses their usefulness as ways of analyzing and manipulating data and answering research questions. Throughout the text,

example problems are framed in the context of a research situation. A question is posed and then, with the aid of a statistic, answered. The relevance of statistics for answering questions is thus stressed throughout the text. This central theme of usefulness is further reinforced by a series of Application boxes, each of which illustrates some specific way statistics can be used to answer questions.

Most all end-of-chapter problems are labeled by the social science discipline or subdiscipline from which they are drawn: SOC for sociology, SW for social work, PS for political science, CJ for criminal justice, PA for public administration, and GER for gerontology. By identifying problems with specific disciplines, students can more easily see the relevance of statistics to their own academic interests. (Not incidentally, they will also see that the disciplines have a large subject matter in common.)

- *Interpreting statistics.* For most students, interpretation—saying what statistics mean—is a big challenge. The ability to interpret statistics can be developed only by exposure and experience. To provide exposure, I have been careful, in the example problems, to express the meaning of the statistic in terms of the original research question. To provide experience, the end-of-chapter problems almost always call for an interpretation of the statistic calculated. To provide examples, many of the Answers to Odd-Numbered Computational Problems in the back of the text are expressed in words as well as numbers.
- *Using Statistics: You Are the Researcher.* In this new feature found at the end of chapters, students become researchers. They use SPSS (Statistical Package for the Social Sciences), the most widely used computerized statistical package, to analyze variables from a survey administered to a national sample of U.S. citizens, the 2006 General Social Survey. They will develop hypotheses, select variables to match their concepts, generate output, and interpret the results. In these mini-research projects, students learn to use SPSS, apply their statistical knowledge, and, most importantly, say what the results mean in terms of their original questions. For convenience, the report forms for these exercises are available at **www.cengage.com/sociology/healey.**
- *Using Statistics: Ideas for research projects.* Appendix E offers ideas for independent data-analysis projects for students. These projects build on the You Are the Researcher feature but are more open-ended and provide more choices to student researchers. These assignments can be scheduled at intervals throughout the semester or at the end of the course. Each project provides an opportunity for students to practice and apply their statistical skills and, above all, to exercise their ability to understand and interpret the meaning of the statistics they produce.

2. Computational Competence. Students should emerge from their first course in statistics with the ability to perform elementary forms of data analysis—to execute a series of calculations and arrive at the correct answer. To be sure, computers and calculators have made computation less of an issue today. Yet, computation is inseparable from statistics, and since social science majors frequently do not have strong quantitative backgrounds, I have included a number of features to help students cope with these challenges:

- *One Step at a Time computational algorithms* are provided for each statistic.
- *Extensive problem sets* are provided at the end of each chapter. Many of these problems use simplified, fictitious data, and all are designed for ease of computation.

- *Solutions* to odd-numbered computational problems are provided so that students may check their answers.
- *SPSS for Windows* is incorporated throughout the text to give students access to the computational power of the computer.

3. The Ability to Read the Professional Social Science Literature. The statistically literate person can comprehend and critically appreciate research reports written by others. The development of this quality is a particular problem at the introductory level since (1) the vocabulary of professional researchers is so much more concise than the language of the textbook, and (2) the statistics featured in the literature are more advanced than those covered at the introductory level. The text helps to bridge this gap by

- always expressing the meaning of each statistic in terms of answering a social science research question, and
- providing a new series of boxed inserts, Becoming a Critical Consumer, which help students to decipher the uses of statistics they are likely to encounter in everyday life as well as in the professional literature. Many of these inserts include excerpts from the popular media, the research literature, or both.

Additional Features. A number of other features make the text more meaningful for students and more useful for instructors:

- *Readability and clarity.* The writing style is informal and accessible to students without ignoring the traditional vocabulary of statistics. Problems and examples have been written to maximize student interest and to focus on issues of concern and significance. For the more difficult material (such as hypothesis testing), students are first walked through an example problem before being confronted by formal terminology and concepts. Each chapter ends with a summary of major points and formulas and a glossary of important concepts. Frequently used formulas are listed inside the front and back covers, and Appendix H provides a glossary of symbols inside the back cover can be used for quick reference.
- *Organization and coverage.* The text is divided into four parts, with most of the coverage devoted to univariate descriptive statistics, inferential statistics, and bivariate measures of association. The distinction between description and inference is introduced in the first chapter and maintained throughout the text. In selecting statistics for inclusion, I have tried to strike a balance between the essential concepts with which students must be familiar and the amount of material students can reasonably be expected to learn in their first (and perhaps only) statistics course, while bearing in mind that different instructors will naturally wish to stress different aspects of the subject. Thus, the text covers a full gamut of the usual statistics, with each chapter broken into subsections so that instructors may choose the particular statistics they wish to include.
- *Learning objectives.* Learning objectives are stated at the beginning of each chapter. These are intended to serve as study guides and to help students identify and focus on the most important material.
- *Review of mathematical skills.* A comprehensive review of all of the mathematical skills that will be used in this text is provided in the Prologue. Students who are inexperienced or out of practice with mathematics are

urged to study this review at the start of the semester and may refer back to it as needed. A self-test is included so students may check their level of preparation for the course.

- *Statistical techniques and end-of-chapter problems are explicitly linked.* After a technique is introduced, students are directed to specific problems for practice and review. The "how-to-do-it" aspects of calculation are reinforced immediately and clearly.
- *End-of-chapter problems are organized progressively.* Simpler problems with small data sets are presented first. Often, explicit instructions or hints accompany the first several problems in a set. The problems gradually become more challenging and require more decision making by the student (e.g., choosing the most appropriate statistic for a certain situation). Thus, each problem set develops problem-solving abilities gradually and progressively.
- *Computer applications.* To help students take advantage of the power of the computer to do statistical analysis, this text incorporates SPSS, the most widely used statistical package. Appendix F provides an introduction to SPSS and the You Are the Researcher exercises at the ends of chapters explain how to use the statistical package to produce the statistics presented in the chapter. The exercises require the student to frame hypotheses, select variables, generate output, and interpret results. Forms for writing up the exercises are available at **www.cengage.com/sociology/healey.** The student version of SPSS is available as a supplement to this text.
- *Realistic, up-to-date data.* The database for computer applications in the text is a shortened version of the 2006 General Social Survey. This database will give students the opportunity to practice their statistical skills on real-life data. The database is described in Appendix G and is available in SPSS format at **www.cengage.com/sociology/healey.**
- *Companion Website.* The website for this text, includes additional material, self-tests, and a number of other features.
- *Instructor's Manual/Testbank.* The *Instructor's Manual* includes chapter summaries, a test item file of multiple-choice questions, answers to even-numbered computational problems, and step-by-step solutions to selected problems. In addition, the *Instructor's Manual* includes cumulative exercises (with answers) that can be used for testing purposes.

Summary of Key Changes in the *Essentials* Edition. The most important changes in this edition include the following:

- A new feature called Becoming a Critical Consumer.
- A new feature called You Are the Researcher.
- A division of the chapter on basic descriptive statistics has been split. Chapter 2 covers percentages, ratios, rates, and frequency distributions, and the new Chapter 3 covers graphs and charts. This reorganization is a more logical grouping of the material and provides the room to present several new types of graphs, including population pyramids.
- An updated version of the data set used in the text, the 2006 General Social Survey.

The text has been thoroughly reviewed for clarity and readability. As with previous editions, my goal is to provide a comprehensive, flexible, and student-oriented text that will provide a challenging first exposure to social statistics.

ACKNOWLEDGMENTS

This text has been in development, in one form or another, for over 20 years. An enormous number of people have made contributions, both great and small, to this project, and at the risk of inadvertently omitting someone, I am bound to at least attempt to acknowledge my many debts.

This edition reflects the thoughtful guidance of Chris Caldeira of Cengage, and I thank her for her contributions. Much of whatever integrity and quality this book has is a direct result of the very thorough (and often highly critical) reviews that have been conducted over the years. I am consistently impressed by the sensitivity of my colleagues to the needs of the students, and, for their assistance in preparing this edition, I would like to thank Marion Manton, Christopher Newport University; Dennis Berg, California State University, Fullerton; Bradley Buckner, Cheyney University of Pennsylvania; Kwaku Twumasi-Ankrah, Fayetteville State University; Craig Tollini, Western Illinois University; H. David Hunt, University of Southern Mississippi; Karen Schaumann, Eastern Michigan University. Any failings contained in the text are, of course, my responsibility and are probably the results of my occasional decisions not to follow the advice of my colleagues.

I would like to thank the instructors who made statistics understandable to me (Professors Satoshi Ito, Noelie Herzog, and Ed Erikson) and all of my colleagues at Christopher Newport University for their support and encouragement. I would be very remiss if I did not acknowledge the constant support and excellent assistance of Iris Price, and I thank all of my students for their patience and thoughtful feedback. Also, I am grateful to the literary executor of the late Sir Ronald A. Fisher, F.R.S., to Dr. Frank Yates, F.R.S., and to Longman Group Ltd., London, for permission to reprint Appendices B, C, and D, from their book *Statistical Tables for Biological, Agricultural and Medical Research* (6th edition, 1974).

Finally, I want to acknowledge the support of my family and rededicate this work to them. I have the extreme good fortune to be a member of an extended family that is remarkable in many ways and that continues to increase in size. Although I cannot list everyone, I would like to especially thank the older generation (my mother, Alice T. Healey), the next generation (my sons Kevin and Christopher, my daughters-in-law Jennifer and Jessica), the new members (my wife Patricia Healey, Christopher Schroen, Jennifer Schroen, and Kate and Matt Cowell), and the youngest generation (Benjamin and Caroline Healey, Isabelle Healey, and Abagail Cowell).

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Prologue
Basic Mathematics Review

You will probably be relieved to hear that this text, your first exposure to statistics for social science research, is not particularly mathematical and does not stress computation per se. While you will encounter many numbers to work with and numerous formulas to use, the major emphasis will be on understanding the role of statistics in research and the logic by which we attempt to answer research questions empirically. You will also find that, in this text, the example problems and many of the homework problems have been intentionally simplified so that the computations will not unduly distract you from the task of understanding the statistics themselves.

On the other hand, you may regret to learn that there is, inevitably, some arithmetic that you simply cannot avoid if you want to master this material. It is likely that some of you haven't had any math in a long time, others have convinced themselves that they just cannot do math under any circumstances, and still others are just rusty and out of practice. All of you will find that mathematical operations that might seem complex and intimidating can be broken down into simple steps. If you have forgotten how to cope with some of these steps or are unfamiliar with these operations, this prologue is designed to ease you into the skills you will need in order to do all of the computations in this textbook.

CALCULATORS AND COMPUTERS

A calculator is a virtual necessity for this text. Even the simplest, least expensive model will save you time and effort and is definitely worth the investment. However, I recommend that you consider investing in a more sophisticated calculator with memory and preprogrammed functions, especially the statistical models that can compute means and standard deviations automatically. Calculators with these capabilities are available for less than $20.00 and will almost certainly be worth the small effort it takes to learn to use them.

In the same vein, there are several computerized statistical packages (or *statpaks*) commonly available on college campuses that you may use to further enhance your statistical and research capabilities. The most widely used of these is the Statistical Package for the Social Sciences (SPSS). This program comes in a student version, which is available bundled with this text (for a small fee). Statistical packages such as SPSS are many times more powerful than even the most sophisticated handheld calculators, and it will be well worth your time to learn how to use them because they will eventually save you time and effort. SPSS is introduced in Appendix F of this text, and at the end of almost every chapter there are exercises that will show you how to use the program to generate and interpret the statistics just covered.

There are many other programs that are probably available to you that will help you accomplish the goal of generating accurate statistical results with a minimum of effort and time. Even spreadsheet programs such as Microsoft

Excel, which is included in many versions of Microsoft Office, have some statistical capabilities. You should be aware that all of these programs (other than the simplest calculators) will require some effort to learn, but the rewards will be worth the effort.

In summary, you should find a way at the beginning of this course—with a calculator, a statpak, or both—to minimize the tedium and hassle of mere computing. This will permit you to devote maximum effort to the truly important goal of increasing your understanding of the meaning of statistics in particular and social research in general.

VARIABLES AND SYMBOLS

Statistics are a set of techniques by which we can describe, analyze, and manipulate *variables*. A variable is a trait that can change value from case to case or from time to time. Examples of variables would include height, weight, level of prejudice, and political party preference. The possible values or scores associated with a given variable might be numerous (for example, income) or relatively few (for example, gender). I will often use symbols, usually the letter X, to refer to variables in general or to a specific variable.

Sometimes we will need to refer to a specific value or set of values of a variable. This is usually done by using subscripts. So, the symbol X_1 (read "X-sub-one") would refer to the first score in a set of scores, X_2 ("X-sub-two") to the second score, and so forth. Also, we will use the subscript i to refer to all the scores in a set. Thus, the symbol X_i ("X-sub-eye") refers to all of the scores associated with a given variable (for example, the test grades of a particular class).

OPERATIONS

You are all familiar with the four basic mathematical operations of addition, subtraction, multiplication, and division and the standard symbols ($+$, $-$, $\times$, $\div$) used to denote them. The latter two operations can be symbolized in a variety of ways. For example, the operation of multiplying some number a by some number b may be symbolized in (at least) six different ways:

$a \times b$
$a \cdot b$
$a * b$
ab
$a(b)$
$(a)(b)$

In this text, we will commonly use the "adjacent symbols" format (that is, ab), the conventional times sign ($\times$), or adjacent parentheses to indicate multiplication. On most calculators and computers, the asterisk (*) is the symbol for multiplication.

The operation of division can also be expressed in several different ways. In this text, we will use either of these two methods:

$$a/b \quad \text{or} \quad \frac{a}{b}$$

Several of the formulas with which we will be working require us to find the square of a number. To do this, simply multiply the number by itself. This

operation is symbolized as X^2 (read "X squared"), which is the same thing as $(X)(X)$. If X has a value of 4, then

$$X^2 = (X)(X) = (4)(4) = 16$$

or we could say, "4 squared is 16."

The square root of a number is the value that, when multiplied by itself, results in the original number. So the square root of 16 is 4 because (4)(4) is 16. The operation of finding the square root of a number is symbolized as

$$\sqrt{X}$$

A final operation with which you should be familiar is summation, or the addition of the scores associated with a particular variable. When a formula requires the addition of a series of scores, this operation is usually symbolized as ΣX_i. Σ is the uppercase Greek letter sigma and stands for "the summation of." Thus, the combination of symbols ΣX_i means "the summation of all the scores" and directs us to add the value of all the scores for that variable. If four people had family sizes of 2, 4, 5, and 7, then the summation of these four scores for this variable could be symbolized as

$$\Sigma X_i = 2 + 4 + 5 + 7 = 18$$

The symbol Σ is an operator, just like the + or × signs. It directs us to add all of the scores on the variable indicated by the X symbol.

There are two other common uses of the summation sign. Unfortunately, the symbols denoting these uses are not, at first glance, sharply different from each other or from the symbol used above. A little practice and some careful attention to these various meanings should minimize the confusion. The first set of symbols is ΣX_i^2, which means "the sum of the squared scores." This quantity is found by *first* squaring each of the scores and *then* adding the squared scores together. A second common set of symbols will be $(\Sigma X_i)^2$, which means "the sum of the scores, squared." This quantity is found by *first* summing the scores and *then* squaring the total.

These distinctions might be confusing at first, so let's see if an example helps to clarify the situation. Suppose we had a set of three scores: 10, 12, and 13. So,

$$X_i = 10, 12, 13$$

The sum of these scores would be indicated as

$$\Sigma X_i = 10 + 12 + 13 = 35$$

The sum of the squared scores would be

$$(\Sigma X_i)^2 = (10)^2 + (12)^2 + (13)^2 = 100 + 144 + 169 = 413$$

Take careful note of the order of operations here. First, the scores are squared one at a time, and then the squared scores are added. This is a completely different operation from squaring the sum of the scores:

$$(\Sigma X_i)^2 = (10 + 12 + 13)^2 = (35)^2 = 1{,}225$$

To find this quantity, first the scores are summed and then the total of all the scores is squared. The value of the sum of the scores squared (1,225) is not the same as the value of the sum of the squared scores (413). In summary,

the operations associated with each set of symbols can be summarized as follows.

Symbols	Operations
ΣX_i	Add the scores.
ΣX_i^2	First square the scores and then add the squared scores.
$(\Sigma X_i)^2$	First add the scores and then square the total.

OPERATIONS WITH NEGATIVE NUMBERS

A number can be either positive (if it is preceded by a + sign or by no sign at all) or negative (if it is preceded by a − sign). Positive numbers are greater than zero, and negative numbers are less than zero. It is very important to keep track of signs because they will affect the outcome of virtually every mathematical operation. This section will briefly summarize the relevant rules for dealing with negative numbers. First, adding a negative number is the same as subtraction. For example,

$$3 + (-1) = 3 - 1 = 2$$

Second, subtraction changes the sign of a negative number:

$$3 - (-1) = 3 + 1 = 4$$

Note the importance of keeping track of signs here. If you neglected to change the sign of the negative number in the second expression, you would arrive at the wrong answer.

For multiplication and division, you should be aware of various combinations of negative and positive numbers. For purposes of this text, you will rarely have to multiply or divide more than two numbers at a time, and we will confine our attention to this situation. Ignoring the case of all positive numbers, this leaves several possible combinations. A negative number times a positive number results in a negative value:

$$(-3)(4) = -12$$

or

$$(3)(-4) = -12$$

A negative number multiplied by a negative number is always positive:

$$(-3)(-4) = 12$$

Division follows the same patterns. If there is a single negative number in the calculations, the answer will be negative. If both numbers are negative, the answer will be positive. So,

$$(-4)/(2) = -2$$

and

$$(4)/(-2) = -2$$

but

$$(-4)/(-2) = 2$$

Negative numbers do not have square roots, since multiplying a number by itself cannot result in a negative value. Squaring a negative number always results in a positive value (see the multiplication rules above).

ACCURACY AND ROUNDING OFF

A possible source of confusion in computation involves the issues of accuracy and rounding off. People work at different levels of accuracy and precision and, for this reason alone, may arrive at different answers to problems. This is important because, if you work at one level of precision and I (or your instructor or your study partner) work at another, we can arrive at solutions that are at least slightly different. You may sometimes think you've gotten the wrong answer when all you've really done is rounded off at a different place in the calculations or in a different way.

There are two issues here: when to round off and how to round off. In this text, I have followed the convention of working in as much accuracy as my calculator or statistics package will allow and then rounding off to two places of accuracy (two places beyond or to the right of the decimal point) only at the very end. If a set of calculations is lengthy and requires the reporting of intermediate sums or subtotals, I will round the subtotals off to two places also.

In terms of how to round off, begin by looking at the digit immediately to the right of the last digit you want to retain. If you want to round off to 100ths (two places beyond the decimal point), look at the digit in the 1,000ths place (three places beyond the decimal point). If that digit is 5 or more, round up. For example, 23.346 would round off to 23.35. If the digit to the right is less than 5, round down. So, 23.343 would become 23.34.

Let's look at some more examples of how to follow the rounding rules stated above. If you are calculating the mean value of a set of test scores and your calculator shows a final value of 83.459067, and you want to round off to two places beyond the decimal point, look at the digit three places beyond the decimal point. In this case the value is 9 (greater than 5), so we would round the second digit beyond the decimal point up and report the mean as 83.46. If the value had been 83.453067, we would have reported our final answer as 83.45.

FORMULAS, COMPLEX OPERATIONS, AND THE ORDER OF OPERATIONS

A mathematical formula is a set of directions, stated in general symbols, for calculating a particular statistic. To "solve a formula," you replace the symbols with the proper values and then manipulate the values through a series of calculations. Even the most complex formula can be rendered manageable if it is broken down into smaller steps. Working through these steps requires some knowledge of general procedure and the rules of precedence of mathematical operations. This is because the order in which you perform calculations may affect your final answer. Consider the following expression:

$$2 + 3(4)$$

Note that if you do the addition first, you will evaluate the expression as

$$5(4) = 20$$

but if you do the multiplication first, the expression becomes

$$2 + 12 = 14$$

Obviously, it is crucial to complete the steps of a calculation in the correct order.

The basic rules of precedence are to find all squares and square roots first, then do all multiplication and division, and finally complete all addition and subtraction. So the following expression:

$$8 + 2 \times 2^2/2$$

would be evaluated as

$$8 + 2 \times 4/2 = 8 + 8/2 = 8 + 4 = 12$$

The rules of precedence may be overridden when an expression contains parentheses. Solve all expressions within parentheses before applying the rules stated above. For most of the complex formulas in this text, the order of calculations will be controlled by the parentheses. Consider the following expression:

$$(8 + 2) - 4(3)^2/(8 - 6)$$

Resolving the parenthetical expressions first, we would have

$$(10) - 4 \times 9/(2) = 10 - 36/2 = 10 - 18 = -8$$

Without the parentheses, the same expression would be evaluated as

$$\begin{aligned} 8 + 2 - 4 \times 3^2/8 - 6 &= 8 + 2 - 4 \times 9/8 - 6 = 8 + 2 - 36/8 - 6 \\ &= 8 + 2 - 4.5 - 6 = 10 - 10.5 = -0.5 \end{aligned}$$

A final operation you will encounter in some formulas in this text involves denominators of fractions that themselves contain fractions. In this situation, solve the fraction in the denominator first and then complete the division. For example,

$$\frac{15 - 9}{6/2}$$

would become

$$\frac{15 - 9}{6/2} = \frac{6}{3} = 2$$

When you are confronted with complex expressions such as these, don't be intimidated. If you are patient with yourself and work through them step by step, beginning with the parenthetical expression, even the most imposing formulas can be managed.

EXERCISES

You can use the problems below as a self-test on the material presented in this review. If you can handle these problems, you are ready to do all of the arithmetic in this text. If you have difficulty with any of these problems, please review the appropriate section of this prologue. You might also want to use this section as an opportunity to become more familiar with your calculator. Answers are given on the next page, along with some commentary and some reminders.

1. Complete each of the following:

a. $17 \times 3 =$

b. $17\ (3) =$

c. $(17)\ (3) =$

d. $17/3 =$

e. $(42)^2 =$

f. $\sqrt{113} =$

2. For the set of scores (X_i) of 50, 55, 60, 65, and 70, evaluate each of the expressions below:

$\Sigma X_i =$

$\Sigma X_i^2 =$

$(\Sigma X_i)^2 =$

3. Complete each of the following:

a. $17 + (-3) + (4) + (-2) =$

b. $15 - 3 - (-5) + 2 =$

c. $(-27)(54) =$

d. $(113)(-2) =$

e. $(-14)(-100) =$

f. $-34/-2 =$

g. $322/-11 =$

h. $\sqrt{-2} =$

i. $(-17)^2 =$

4. Round off each of the following to two places beyond the decimal point:

a. 17.17532

b. 43.119

c. 1,076.77337

d. 32.4651152301

e. 32.4751152301

5. Evaluate each of the following:

a. $(3 + 7)/10 =$

b. $3 + 7/10 =$

c. $\dfrac{(4 - 3) + (7 + 2)(3)}{(4 + 5)(10)} =$

d. $\dfrac{22 + 44}{15/3} =$

ANSWERS TO EXERCISES

1. **a.** 51 **b.** 51 **c.** 51
(The obvious purpose of these first three problems is to remind you that there are several different ways of expressing multiplication.)

d. 5.67 (Note the rounding off.) **e.** 1,764

f. 10.63

2. The first expression translates to "the sum of the scores," so this operation would be

$$\Sigma X_i = 50 + 55 + 60 + 65 + 70 = 300$$

The second expression is the "sum of the squared scores." So

$$\Sigma X_i^2 = (50)^2 + (55)^2 + (60)^2 + (65)^2 + (70)^2$$
$$\Sigma X_i^2 = 2{,}500 + 3{,}025 + 3{,}600 + 4{,}225 + 4{,}900$$
$$\Sigma X_i^2 = 18{,}250$$

The third expression is "the sum of the scores, squared":

$$(\Sigma X_i)^2 = (50 + 55 + 60 + 65 + 70)^2$$
$$(\Sigma X_i)^2 = (300)^2$$
$$(\Sigma X_i)^2 = 90{,}000$$

Remember that ΣX_i^2 and $(\Sigma X_i)^2$ are two completely different expressions with very different values.

3. **a.** 16 **b.** 19 (Remember to change the sign of −5.)

c. −1,458 **d.** −226 **e.** 1,400

f. 17 **g.** −29.27

h. Your calculator probably gave you some sort of error message for this problem, since negative numbers do not have square roots.

i. 289

4. **a.** 17.17 **b.** 43.12 **c.** 1,076.77

d. 32.47 **e.** 32.48

5. **a.** 1 **b.** 3.7 (Note again the importance of parentheses.)

c. 0.31 **d.** 13.2

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1 Introduction

LEARNING OBJECTIVES

By the end of this chapter, you will be able to:

1. Describe the limited but crucial role of statistics in social research.
2. Distinguish between three applications of statistics (univariate descriptive, bivariate descriptive, and inferential) and identify situations in which the use of each is appropriate.
3. Identify and describe three levels of measurement and cite examples of variables from each.

1.1 WHY STUDY STATISTICS?

Students sometimes approach their first course in statistics with questions about the value of the subject matter. What, after all, do numbers and statistics have to do with understanding people and society? In a sense, this entire book will attempt to answer this question, and the value of statistics will become clear as we move from chapter to chapter. For now, the importance of statistics can be demonstrated by reviewing the process of research in the social sciences—sociology, political science, psychology, and related disciplines such as social work and public administration. These disciplines are scientific in the sense that social scientists attempt to verify their ideas and theories through **research.** Broadly conceived, research is any process by which information is carefully gathered in order to answer questions, examine ideas, or test theories. Research is a disciplined inquiry that can take numerous forms. Statistical analysis is relevant only for research projects in which information is represented as numbers. Numerical information—like age, income, or level of prejudice—is called **data. Statistics are mathematical techniques used to examine data in order to answer questions and test theories.**

What is so important about learning how to analyze data? On one hand, some of the most important and enlightening works in the social sciences do not use any statistical techniques. There is nothing magical about data and statistics. The mere presence of numbers guarantees nothing about the quality of a research project. On the other hand, data can be the most trustworthy information available to the researcher, and, consequently, they deserve special attention. Data that have been carefully collected and thoughtfully analyzed are the strongest, most objective foundations for building theory and enhancing understanding. Without a firm base in data, the social sciences would be less scientific and less valuable.

Thus, the social sciences rely heavily on data analysis for the advancement of knowledge, but even the most carefully collected data do not (and cannot) speak for themselves. The researcher must be able to use statistics effectively to organize, evaluate, and analyze the data. Without a good understanding of the principles of statistical analysis, the researcher will be unable to make sense of the data. Without the appropriate application of statistical techniques, the data will remain mute and useless.

Statistics are an indispensable tool for the social sciences. They provide the scientist with some of the most useful techniques for evaluating hypotheses and testing theory. The next section describes the relationships between theory, research, and statistics in more detail.

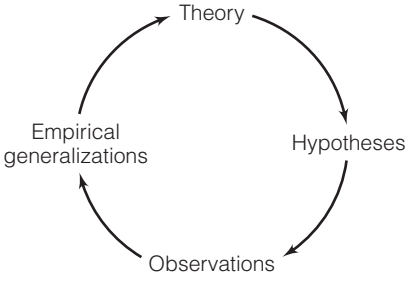
1.2 THE ROLE OF STATISTICS IN SCIENTIFIC INQUIRY

Figure 1.1 graphically represents the role of statistics in the research process. The diagram is based on the thinking of Walter Wallace and illustrates how the knowledge base of any scientific enterprise grows and develops. One point the diagram makes is that scientific theory and research continually shape each other. Statistics are one of the most important means by which research and theory interact. Let's take a closer look at the wheel.

Since the figure is circular, it has no beginning or end, and we could begin our discussion at any point. For the sake of convenience, let's begin at the top and follow the arrows around the circle. A **theory** is an explanation of the relationships between phenomena. People naturally (and endlessly) wonder about problems in society (like prejudice, poverty, child abuse, or serial murders) and, in their attempt to understand these phenomena, they develop explanations (lack of education causes prejudice). This kind of informal "theorizing" about society is no doubt very familiar to you. A major difference between our informal, everyday explanations of social phenomena and scientific theory is that the latter is subject to a rigorous testing process. Let's take the problem of racial prejudice as an example to illustrate how the research process works.

What causes racial prejudice? One possible answer to this question is provided by a theory called the *contact hypothesis*. This theory was stated over 40 years ago by the social psychologist Gordon Allport, and it has been tested on a number of occasions since that time.[1] The theory links prejudice to the volume and nature of interaction between members of different racial groups. Specifically, the hypothesis asserts that contact situations in which the members

FIGURE 1.1 THE WHEEL OF SCIENCE



Source: Adapted from Walter Wallace, *The Logic of Science in Sociology* (Chicago: Aldine-Atherton, 1971).

[1]Allport, Gordon, 1954. *The Nature of Prejudice*. Reading, Massachusetts: Addison-Wesley. For recent attempts to test this theory, see: McLaren, Lauren, 2003. "Anti-Immigrant Prejudice in Europe: Contact, Threat Perception, and Preferences for the Exclusion of Migrants." *Social Forces*. 81: 909–937; Pettigrew, Thomas, 1997. "Generalized Intergroup Contact Effects on Prejudice." *Personality and Social Psychology Bulletin*. 23:173–185, and Sigelman, Lee and Susan Welch, 1993. "The Contact Hypothesis Revisited: Black-White Interaction and Positive Racial Attitudes." *Social Forces*. 71:781–795.

of different groups have equal status and are engaged in cooperative behavior will reduce prejudice for all. The greater the extent to which contact is equal and cooperative, the more likely people will see each other as individuals and not as representatives of a particular group. For example, the contact hypothesis predicts that members of a racially mixed athletic team that cooperate with each other to achieve victory would tend to experience a decline in prejudice. On the other hand, when different groups compete for jobs, housing, or other valuable resources, prejudice will increase.

The contact hypothesis is not a complete explanation of prejudice, of course, but it will serve to illustrate a sociological theory. This theory offers an explanation for the relationship between two social phenomena: (1) prejudice and (2) equal-status, cooperative contact between members of different groups. People who have little contact will tend to be more prejudiced, and those who experience more contact will tend to be less prejudiced.

Before moving on, let's examine theory in a little more detail. The contact hypothesis, like most theories, is stated in terms of causal relationships between variables. A **variable** is any trait that can change values from case to case. Examples of variables would be gender, age, income, or political party affiliation. In any specific theory, some variables will be identified as causes and others will be identified as effects or results. In the language of science, the causes are called **independent variables** and the effects or result variables are called **dependent variables**. In our theory, contact would be the independent variable (or the cause) and prejudice would be the dependent variable (the result or effect). In other words, we are arguing that equal-status contact is a cause of prejudice or that an individual's level of prejudice depends on the extent to which he or she participates in equal-status, cooperative contacts with other groups.

How can you tell which variables in a theory are causes (independent variables) and which are effects (dependent variables)? Most importantly, this can be determined from the wording of the theory: the contact hypothesis argues that level of prejudice *depends on* the frequency of equal-status contacts and this tells us that prejudice is the dependent variable. If we argued that prejudice was the result of low levels of education, the words *the result of*) tells us that prejudice is a dependent variable and education is an independent variable.

Figuring out which variable is cause and which is effect can be especially confusing because most variables can play either role, depending on the situation. For example, consider these statements:

- Equal-status contact leads to (causes) lower prejudice.
- Lower levels of prejudice lead to (cause) higher levels of interaction with other groups.

In the first statement, prejudice is the dependent variable or effect, but in the second, it has become the independent or causal variable. Both statements seem reasonable: prejudice can be either a cause or an effect.

In some cases, we can use time to help us decide which variable is cause and which is effect. For example, variables such as sex and race are (pretty much) always independent: they are determined at birth and could only be causal variables in a theory (with the exceptions, of course, of transgendered people and people who "pass" as members of a race or group other than the

one they were born into). Using the same logic, level of education is usually thought of as a cause of income or occupation prestige since it comes first in the typical life course.

So far, we have a theory of prejudice and an independent and a dependent variable. What we don't know yet is whether the theory is true or false. To find out, we need to compare our theory with the facts: we need to do some research. The next steps in the process would be to define our terms and ideas more specifically and exactly. One problem we often face in doing research is that scientific theories are too complex and abstract to be fully tested in a single research project. To conduct research, one or more hypotheses must be derived from the theory. A **hypothesis** is a statement about the relationship between variables that, while logically derived from the theory, is much more specific and exact.

For example, if we wished to test the contact hypothesis, we would have to say exactly what we mean by prejudice and we would need to describe "equal-status, cooperative contact" in great detail. There has been a great deal of research on the effect of contact on prejudice, and we would consult the research literature to develop and clarify our definitions of these concepts.

As our definitions develop and the hypotheses take shape, we begin the next step of the research process during which we will decide exactly how we will gather our data. We must decide how cases will be selected and tested, how exactly the variables will be measured, and a host of related matters. Ultimately, these plans will lead to the observation phase (the bottom of the wheel of science), where we actually measure social reality. Before we can do this, we must have a very clear idea of what we are looking for and a well-defined strategy for conducting the search.

To test the contact hypothesis, we would begin with people from different racial or ethnic groups. We might place some subjects in situations that required them to cooperate with members of other groups and other subjects in situations that feature intergroup competition. We would need to measure levels of prejudice before and after each type of contact. We might do this by administering a survey that asked subjects to agree or disagree with statements such as, "Greater efforts must be made to racially integrate the public school system" or "Skin color is irrelevant and people are just people." Our goal would be to see if the people exposed to the cooperative contact situation actually become less prejudiced.

Now, finally, we come to statistics. As the observation phase of our research project comes to an end, we will be confronted with a large collection of numerical information or data. If our sample consisted of 100 people, we would have 200 completed surveys measuring prejudice: 100 completed before the contact situation and 100 filled out afterwards. Try to imagine dealing with 200 completed surveys. If we had asked each respondent just five questions to measure his or her prejudice, we would have a total of 1,000 separate pieces of information to deal with. What do we do? We have to have some systematic way to organize and analyze this information, and at this point, statistics will become very valuable. Statistics will supply us with many ideas about what to do with the data, and we will begin to look at some of the options in the next chapter. For now, let me stress two points about statistics.

First, statistics are crucial. Statistics give social scientists the ability to conduct **quantitative research:** research based on the analysis of numerical

information or data.[2] Researchers use statistical techniques to organize and manipulate data so that hypotheses can be tested, theories can be shaped and refined, and our understanding of the social world can be improved. Second, and somewhat paradoxically, the role of statistics is rather limited. As figure 1.1 makes clear, scientific research proceeds through multiple, mutually interdependent stages, and statistics become directly relevant only at the end of the observation stage. Before any statistical analysis can be legitimately applied, the preceding phases of the process must have been successfully completed. If the researcher has asked poorly conceived questions or has made serious errors of design or method, then even the most sophisticated statistical analysis is valueless. As useful as they can be, statistics cannot substitute for rigorous conceptualization, detailed and careful planning, or creative use of theory. Statistics cannot salvage a poorly conceived or designed research project. They cannot make sense out of garbage.

On the other hand, inappropriate statistical applications can limit the usefulness of an otherwise carefully done project. Only by successfully completing *all* phases of the process can a quantitative research project hope to contribute to understanding. A reasonable knowledge of the uses and limitations of statistics is as essential to the education of the social scientist as is training in theory and methodology.

As the statistical analysis comes to an end, we would begin to develop empirical generalizations. While we would be primarily focused on assessing our theory, we would also look for other trends in the data. Assuming that we found that equal-status, cooperative contact reduces prejudice in general, we might go on to ask if the pattern applies to males as well as females, to the well educated as well as the poorly educated, to older respondents as well as to the younger. As we probed the data, we might begin to develop some generalizations based on the empirical patterns we observe. For example, what if we found that contact reduced prejudice for younger respondents but not for older respondents? Could it be that younger people are less "set in their ways" and have attitudes and feelings that are more open to change? As we developed tentative explanations, we would begin to revise or elaborate our theory.

If we change the theory to take account of these findings, however, a new research project designed to test the revised theory is called for, and the wheel of science would begin to turn again. We (or perhaps some other researchers) would go through the entire process once again with this new—and, we hope, improved—theory. This second project might result in further revisions and elaboration that would require still more research projects, and the wheel of science would continue turning as long as scientists were able to suggest additional revisions or develop new insights. Every time the wheel turned, our understandings of the phenomena under consideration would (we hope) improve.

Fully testing a theory can take a very long time—sociologists are still arguing about the contact hypothesis 55 years after Allport's classic statement. In the normal course of science, it is a rare occasion when we can say with absolute certainty that a given theory or idea is definitely true or false. Rather,

[2]Social science researchers also do *qualitative research,* or research in which information is expressed in a form other than numbers. Interviews, participant observation, and content analysis are examples of research methodologies that are often qualitative.

evidence for (or against) a theory will gradually accumulate over time, and ultimate judgments of truth will likely be the result of many years of hard work, research, and debate.

Let's briefly review our imaginary research project. We began with an idea or theory about intergroup contact and racial prejudice. We imagined some of the steps we would have to take to test the theory and took a quick look at the various stages of the research project. We wound up back at the level of theory, ready to begin a new project guided by a revised theory. We saw how theory can motivate a research project and how our observations might cause us to revise the theory and thus motivate a new research project. Wallace's wheel of science illustrates how theory stimulates research and how research shapes theory. This constant interaction between theory and research is the lifeblood of science and the key to enhancing our understandings of the social world.

The dialogue between theory and research occurs at many levels and in multiple forms. Statistics are one of the most important links between these two realms. Statistics permit us to analyze data, to identify and probe trends and relationships, to develop generalizations, and to revise and improve our theories. As you will see throughout this text, statistics are limited in many ways. They are also an indispensable part of the research enterprise. Without statistics, the interaction between theory and research would become extremely difficult and the progress of our disciplines would be severely retarded. *(For practice in describing the relationship between theory and research and the role of statistics in research, see Problems 1.1 and 1.2.)*

1.3 THE GOALS OF THIS TEXT

In the preceding section, I argued that statistics are a crucial part of the process by which scientific investigations are carried out and that, therefore, some training in statistical analysis is a crucial component in the education of every social scientist. In this section, we will address the questions of how much training is necessary and what the purposes of that training are.

First, this textbook takes the point of view that statistics are tools. They can be very useful as part of the process by which we increase our knowledge of the social world, but they are not ends in themselves. Thus, we will not take a "mathematical" approach to the subject. Statistical techniques will be presented as a set of tools that can be used to answer important questions. This emphasis does not mean that we will dispense with arithmetic entirely, of course. This text includes enough mathematical material so that you can develop a basic understanding of why statistics "do what they do." Our focus, however, will be on how these techniques are applied in the social sciences.

Second, all of you will soon become involved in advanced coursework in your major fields of study, and you will find that much of the literature used in these courses assumes at least basic statistical literacy. Furthermore, many of you, after graduation, will find yourselves in positions—either in a career or in graduate school—where some understanding of statistics will be very helpful or perhaps even required. Very few of you will become statisticians per se (and this text is not intended for the preprofessional statistician), but you must have a grasp of statistics in order to read and critically appreciate your own professional literature. As a student in the social sciences and in many careers related to the social sciences, you simply cannot realize your full potential without a background in statistics.

Within these constraints, this textbook is an introduction to statistics as they are used in the social sciences. The general goal of the text is to develop an appreciation—a "healthy respect"—for statistics and their place in the research process. You should emerge from this experience with the ability to use statistics intelligently and to know when other people have done so. You should be familiar with the advantages and limitations of the more commonly used statistical techniques, and you should know which techniques are appropriate for a given set of data and a given purpose. Lastly, you should develop sufficient statistical and computational skills and enough experience in the interpretation of statistics to be able to carry out some elementary forms of data analysis by yourself.

1.4 DESCRIPTIVE AND INFERENTIAL STATISTICS

As noted earlier, the general function of statistics is to manipulate data so that a research question(s) can be answered. There are two general classes of statistical techniques that, depending on the research situation, are available to accomplish this task, and each are introduced in this section.

Descriptive Statistics. The first class of techniques is called **descriptive statistics** and is relevant in several different situations:

1. When a researcher needs to summarize or describe the distribution of a single variable. These statistics are called *univariate* (one variable) descriptive statistics.
2. When the researcher wishes to describe the relationship between two or more variables. These statistics are called *bivariate* (two variable) or *multivariate* (more than two variable) descriptive statistics.

To describe a single variable, we would arrange the values or scores of that variable so that the relevant information can be quickly understood and appreciated. Many of the statistics that might be appropriate for this summarizing task are probably familiar to you. For example, percentages, graphs, and charts can all be used to describe single variables.

To illustrate the usefulness of univariate descriptive statistics, consider the following problem. Suppose you wanted to summarize the distribution of the variable family income for a community of 10,000 families. How would you do it? Obviously, you couldn't simply list all incomes in the community and let it go at that. Imagine trying to make sense of a listing of 10,000 different incomes! Presumably, you would want to develop some summary measures of the overall income distributions—perhaps an arithmetic average or the proportions of incomes that fall in various ranges (such as low, middle, and high). Or perhaps a graph or a chart would be more useful. Whatever specific method you choose, its function is the same: to reduce these thousands of individual items of information into a few easily understood numbers. The process of allowing a few numbers to summarize many numbers is called **data reduction** and is the basic goal of univariate descriptive statistical procedures. Part I of this text is devoted to these statistics, the primary goal of which is simply to report, clearly and concisely, essential information about a variable.

The second type of descriptive statistics is designed to help the investigator understand the relationship between two or more variables. These statistics, called **measures of association,** allow the researcher to quantify the strength

and direction of a relationship. These statistics are very useful because they enable us to investigate two matters of central theoretical and practical importance to any science: causation and prediction. These techniques help us disentangle and uncover the connections between variables. They help us trace the ways in which some variables might have causal influences on others, and, depending on the strength of the relationship, they enable us to predict scores on one variable from the scores on another. Note that measures of association cannot, by themselves, prove that two variables are causally related. However, these techniques can provide valuable clues about causation and are therefore extremely important for theory testing and theory construction.

For example, suppose you were interested in the relationship between time spent studying statistics (the independent variable or cause) and the final grade in statistics (the dependent variable or effect) and had gathered data on these two variables from a group of college students. By calculating the appropriate measure of association, you could determine the strength of the bivariate relationship and its direction. Suppose you found a relationship that was strong and positive. This would indicate that study time and grade were closely related (strength of the relationship) and that as one increased in value, the other also increased (direction of the relationship). You could make predictions from one variable to the other (the longer the study time, the higher the grade).

As a result of finding this strong, positive relationship, you might be tempted to make causal inferences. That is, you might jump to such conclusions as longer study time leads to (causes) higher grades. Such a conclusion might make a good deal of common sense and would certainly be supported by your statistical analysis. However, the causal nature of the relationship cannot be proven by the statistical analysis. Measures of association can be important clues about causation, but the mere existence of a relationship can never be taken as conclusive proof of causation: causation and correlation are two different things and must not be confused.

In fact, other variables might have an effect on the relationship. In the example above, we probably would not find a perfect relationship between study time and final grade. That is, we will probably find some individuals who spend a great deal of time studying but receive low grades and some individuals who fit the opposite pattern. We know intuitively that other variables besides study time affect grades (such as efficiency of study techniques, amount of background in mathematics, and even random chance). Fortunately, researchers can incorporate these other variables into the analysis and measure their effects. Part III of this text is devoted to bivariate (two variables) and part IV to multivariate (more than two variables) descriptive statistics.

Inferential Statistics. This second class of statistical techniques becomes relevant when we wish to generalize our findings from a **sample** to a **population.** A population is the total collection of all cases in which the researcher is interested and wishes to understand better. Examples of possible populations would be voters in the United States, all parliamentary democracies, unemployed Puerto Ricans in Atlanta, or sophomore college football players in the Midwest.

Populations can theoretically range from inconceivable in size (all humanity) to quite small (all 35-year-old red-haired belly dancers currently residing in downtown Cleveland) but are usually fairly large. In fact, they are almost always too large to be measured. To put the problem another way, social scientists

almost never have the resources or time to test every case in a population, hence the need for **inferential statistics,** which involve using information from a sample (a carefully chosen subset of the population) to make inferences about a population. Since they have fewer cases, samples are much cheaper to assemble, and—if the proper techniques are followed—generalizations based on these samples can be very accurate representations of the population.

Many of the concepts and procedures involved in inferential statistics may be unfamiliar. However, most of us are experienced consumers of inferential statistics—most familiarly, perhaps, in the form of public-opinion polls and election projections. When a public-opinion poll reports that 42% of the American electorate plans to vote for a certain presidential candidate, it is essentially reporting a generalization to a population (the American electorate, which numbers about over 120 million people) from a carefully drawn sample (usually about 1,500 respondents). Matters of inferential statistics will occupy our attention in Part II of this book. *(For practice in describing different statistical applications, see Problems 1.3 and 1.7.)*

1.5 LEVEL OF MEASUREMENT

In the next chapter, you will begin to encounter some of the broad array of statistics available to the social scientist. One aspect of using statistics that can be puzzling is deciding when to use which statistic. You will learn specific guidelines as you go along, but we will consider the most basic and important guideline at this point: the **level of measurement,** or the mathematical nature of the variables under consideration. Variables at the highest level of measurement have numerical scores and can be analyzed with a broad range of statistics. Variables at lower levels of measurement have "scores" that are really just labels, not numbers at all. Statistics that require numerical variables are inappropriate and, usually, completely meaningless when used with nonnumerical variables. When selecting statistics, you must be sure that the level of measurement of the variable justifies the mathematical operations required to compute the statistic.

For example, consider these variables: age (measured in years) and income (measured in dollars). Both of these variables have numerical scores and could be summarized with a statistic such as the mean or average (e.g., The average income of this city is $43,000. The average age of students on this campus is 19.7.). In contrast, the arithmetic average would be meaningless as a way of describing religious affiliation or zip codes, variables with nonnumerical scores. Your personal zip code might *look* like a number, but it is merely an arbitrary label that happens to be expressed in digits. The numerals in your zip code cannot be added or divided, and statistics such as the average cannot be applied to this variable: the average zip code of a group of people is a meaningless statistic.

Determining the level at which a variable has been measured is one of the first steps in any statistical analysis, and we will consider this matter at some length. I will make it a practice throughout this text to introduce level-of-measurement considerations for each statistical technique.

There are three levels of measurement. In order of increasing sophistication, they are nominal, ordinal, and interval-ratio. Each is discussed separately.

The Nominal Level of Measurement. Variables measured at the nominal level have "scores" or categories that are not numerical. Examples of variables at the nominal level include gender, zip code, race, religious affiliation, and place

BECOMING A CRITICAL CONSUMER: Introduction

The most important goal of this text is to develop your ability to understand, analyze, and appreciate statistical information. To assist in reaching this goal, I have included a series of boxed inserts called Becoming a Critical Consumer to help you exercise your statistical expertise. In this feature, we will examine the everyday statistics you might encounter in the media and in casual conversations with friends, as well as in the professional social science research literature. In this first installment, I briefly outline the activities that will be included in this feature. We'll start with social science research and then examine statistics in everyday life.

As you probably already know, articles published in social science journals are often mathematically sophisticated and use statistics, symbols, formulas, and numbers that may be, at this point in your education, completely indecipherable. Compared to my approach in this text, the language of the professional researcher is more compact and dense. This is partly because space in research journals and other media is expensive and partly because the typical research project requires the analysis of many variables. Thus, a large volume of information must be summarized in very few words. Researchers may express in just a word or two a result or an interpretation that will take us a paragraph or more to state in this text. Also, professional researchers assume a certain level of statistical knowledge in their audience: they write for colleagues, not for undergraduate students.

How can you bridge the gap that separates you from this literature? It is essential to your education that you develop an appreciation for this knowledge base, but how can you understand the articles that seem so challenging? The (unfortunate but unavoidable) truth is that a single course in statistics will not close the gap entirely. However, the information and skills developed in this text will enable you to read much of the social science research literature and give you the ability to critically analyze statistical information. I will help you decode research articles by explaining their typical reporting style and illustrating with actual examples from a variety of social science disciplines.

As you develop your ability to read professional research reports, you will simultaneously develop your ability to critically analyze the statistics you encounter in everyday life. *In this age of information, statistical literacy is not just for academics or researchers*. A critical perspective on statistics in everyday life–as well as in the social science research literature–can help you think more critically and carefully, assess the torrent of information, opinion, facts and factoids that wash over us every day, and make better decisions on a broad range of issues. Therefore, these boxed inserts will also examine how to analyze the statistics you are likely to encounter in your everyday, nonacademic life. What (if anything) do statements like the following really mean?

- Candidate X will get 55% of the vote in the next election.
- The average life expectancy has reached 77 years.
- The number of cohabiting couples in this town has increased by 300% since 1980.
- There is a strong correlation between church attendance and vulnerability to divorce: the more frequent the church attendance, the lower the divorce rate.

Which of these statements sounds credible? How would you evaluate the statistical claims in each? The truth is elusive and multifaceted: how can we know it when we see it? The same skills that help you read the professional research literature can also be used to sort out everyday statistical information, and these boxed inserts will help you develop a more critical and informed approach at this level as well.

Statistical literacy will not always lead you to the truth, of course, but it will enhance your ability to analyze and evaluate information and thus enhance your ability to sort through claims and counterclaims and appraise them sensibly.

of birth. At this lowest level of measurement, the only mathematical operation permitted is comparing the relative sizes of the categories (e.g., there are more females than males in this dorm). The categories or scores of nominal level variables cannot be ranked with respect to each other and cannot be added, divided, or otherwise manipulated mathematically. Even when the scores are expressed in digits (like zip codes or street addresses), all we can do is compare relative sizes of categories (e.g., the most common zip code on this campus is 22033). The scores of nominal level variables do not form a mathematical scale: the scores are different from each other, but not more or less or higher or lower than each other. Males and females differ in terms of gender, but neither category has more or less gender than the other. In the same way, a zip code of 54398 is different from but not "more than" a zip code of 13427.

Nominal variables are rudimentary, but there are criteria and procedures that we need to observe in order to assure adequate measurement. In fact, these criteria apply to variables measured at all levels, not just nominal variables. First, the categories of nominal level variables must be mutually exclusive so that no ambiguity exists concerning classification of any given case. There must be one and only one category for each case. Second, the categories must be exhaustive. In other words, there must be a category—at least an "other" or miscellaneous category—for every possible score that might be found.

Third, the categories of nominal variables should be relatively homogeneous. That is, our categories should include cases that are truly comparable or, to put it another way, we need to avoid categories that lump apples with oranges. There are no hard and fast guidelines for judging if a set of categories is appropriately homogeneous. The researcher must make that decision in terms of the specific purpose of the research, and categories that are too broad for some purposes may be perfectly adequate for others.

Table 1.1 demonstrates some errors of measurement in four different schemes for measuring the nominal level variable religious preference. Scale A in the table violates the criterion of mutual exclusivity because of overlap between the categories Protestant and Episcopalian. Scale B is not exhaustive because it does not provide a category for people with no religious preference (None) or people who belong to religions other than the three listed. Scale C uses a category (Non-Protestant) that would be too broad for many research purposes. Scale D is the way religious preference is often measured in North America, but note that these categories may be too general for some research projects and not comprehensive enough for others. For example, an investigation of issues that have strong moral and religious content (assisted suicide, abortion, or capital

TABLE 1.1 FOUR SCALES FOR MEASURING RELIGIOUS PREFERENCE

Scale A (not mutually exclusive)	Scale B (not exhaustive)	Scale C (not homogeneous)	Scale D (an adequate scale)
Protestant	Protestant	Protestant	Protestant
Episcopalian	Catholic	Non-Protestant	Catholic
Catholic	Jew		Jew
Jew			None
None			Other
Other			

punishment, for example) might need to distinguish between the various Protestant denominations, and an effort to document religious diversity would need to add categories for Buddhists, Muslims, and other religious preferences that are less common in North America.

As is the case with zip codes, numerical labels are often used to identify the categories or scores of nominal level variables, especially when the data are being prepared for computer analysis. For example, the various religions might be labeled with a 1 indicating Protestant, a 2 signifying Catholic, and so on. Remember that these numbers are merely labels or names and have no numerical quality to them. They cannot be added, subtracted, multiplied, or divided. The only mathematical operation permissible with nominal variables is counting and comparing the number of cases in each category of the variable.

The Ordinal Level of Measurement. Variables measured at the ordinal level are more sophisticated than nominal level variables. They have scores or categories that can be ranked from high to low, so in addition to classifying cases into categories, we can describe the categories in terms of "more or less" with respect to each other. Thus, with variables measured at this level, not only can we say that one case is different from another, we can also say that one case is higher or lower, more or less than another.

For example, the variable socioeconomic status (SES) is usually measured at the ordinal level. The categories of the variable are often ordered according to the following scheme:

4. Upper class
3. Middle class
2. Working class
1. Lower class

Individuals can be compared in terms of the categories into which they are classified: a person classified as a 4 (upper class) would be ranked higher than someone classified as a 2 (working class), and a lower-class person (1) would rank lower than a middle-class person (3). Other variables that are usually measured at the ordinal level include attitude and opinion scales such as those that measure prejudice, alienation, or political conservatism.

The major limitation of the ordinal level of measurement is that a particular score only represents position with respect to some other score. We can distinguish between high and low scores, but the distance between the scores cannot be described in precise terms. Although we know that a score of 4 is more than a score of 2, we do not know if it is twice as much as 2.

Since we don't know what the exact distances are from score to score on an ordinal scale, our options for statistical analysis are limited. For example, addition (and most other mathematical operations) assumes that the intervals between scores are exactly equal. If the distances from score to score are not equal, 2 + 2 might equal 3 or 5 or even 15. Thus, strictly speaking, statistics such as the average or mean (which requires that the scores be added together and then divided by the number of scores) are not permitted with ordinal level variables. The most sophisticated mathematical operation fully justified with an ordinal variable is the ranking of categories and cases (although, as we will see, it is common for social scientist to take some liberties with this criterion).

The Interval-Ratio Level of Measurement. The categories of nominal level variables have no numerical quality to them. Ordinal level variables have categories that can be arrayed along a scale from high to low, but the exact distances between categories or scores are undefined. Variables measured at the interval-ratio level not only permit classification and ranking but also allow the distance from category to category (or score to score) to be exactly defined.[3]

Interval-ratio variables have two characteristics. First, they are measured in units that have equal intervals. For example, asking people how old they are will produce an interval-ratio level variable (age) because the unit of measurement (years) has equal intervals (the distance from year to year is 365 days). Similarly, if we ask people how many siblings they have, we would produce a variable with equal intervals: two siblings are one more than 1 and 13 is one more than 12.

The second characteristic of interval-ratio variables is that they have a true zero point. That is, the score of zero for these variables is not arbitrary: it indicates the absence or complete lack of whatever is being measured. For example, the variable "number of siblings" has a true zero point because it is possible to have no siblings at all. Similarly, it is possible to have zero years of education, no income at all, a score of zero on a multiple-choice test, and to be zero years old (although not for very long). Other examples of interval-ratio variables would be number of children, life expectancy, and years married. All mathematical operations are permitted for data measured at the interval-ratio level.

Table 1.2 summarizes this discussion by presenting the basic characteristics of the three levels of measurement. Note that the number of permitted mathematical operations increases as we move from nominal to ordinal to interval-ratio levels of measurement. Ordinal level variables are more sophisticated and flexible than nominal level variables, and interval-ratio level variables permit the broadest range of mathematical operations.

TABLE 1.2 BASIC CHARACTERISTICS OF THE THREE LEVELS OF MEASUREMENT

Levels	Examples	Measurement Procedures	Mathematical Operations Permitted
Nominal	Sex, race, religion, marital status	Classification into categories	Counting number in each category, comparing sizes of categories
Ordinal	Social class, attitude and opinion scales	Classification into categories plus ranking of categories with respect to each other	All above plus statements of "greater than" and "less than"
Interval-ratio	Age, number of children, income	All above plus description of distances between scores in terms of equal units	All above plus all other mathematical operations (addition, subtraction, multiplication, division, square roots, etc.)

[3]Many statisticians distinguish between the interval level (equal intervals) and the ratio level (true zero point). I find the distinction unnecessarily cumbersome in an introductory text and will treat these two levels as one.

ONE STEP AT A TIME **Determining the Level of Measurement of a Variable**

Step ***Operation***

1. Inspect the scores or values of the variable *as they are actually stated*, keeping in mind the basic definition of the three levels of measurement (see Table 1.2).
2. Change the order of the scores. Do the scores still make sense? If the answer is *yes*, the variable is *nominal*. If the answer is *no*, proceed to Step 3.

   Illustration: Gender is a nominal level variable, and its scores can be stated in any order:

   1. Male
   2. Female

   or

   1. Female
   2. Male

   Each statement of the scores is just as sensible as the other. On a nominal level variable, no score is higher or lower than any other score, and the order in which they are stated is arbitrary.
3. Is the distance between the scores unequal or undefined? If the answer is *yes*, the variable is *ordinal*. If the answer is *no*, proceed to Step 4.

   Illustration: Consider the following scale, which measures support for capital punishment:

   1. Strongly support
   2. Somewhat support
   3. Neither support or oppose
   4. Somewhat oppose
   5. Strongly oppose

   People who "strongly support" the death penalty are more in favor than people who "somewhat support" it, but the distance from one level of support to the next (from a score of 1 to a score of 2) is undefined. We do not have enough information to ascertain how much more or less one score is than another.
4. If you answered *no* in Steps 2 and 3, the variable is *interval-ratio*.

   Variables at this level have scores that are actual numbers: they have an order with respect to each other and are a defined, equal distance apart. For example, income is an interval-ratio variable, and the distance from one income to the next is always $1. Interval-ratio variables also have a true zero point (it is possible to have an income of $0). Other examples of interval-ratio variables include age, years of education, and number of siblings.

Source: This system for determining level of measurement was suggested by Professor Michael R. Bisciglia, Louisiana State University.

Level of Measurement: Final Points. Let us end this section by making three points. The first stresses the importance of level of measurement, and the next two discuss some common points of confusion in applying this concept.

First, knowing the level of measurement of a variable is crucial because it tells us which statistics are appropriate and useful. Not all statistics can be used with all variables. As displayed in Table 1.2, different statistics require different mathematical operations. For example, computing an average requires addition and division, and finding a median (or middle score) requires that the

TABLE 1.3 MEASURING INCOME AT THE ORDINAL LEVEL

Score	Income Ranges
1	Less than $24,999
2	$25,000 to $49,999
3	$50,000 to $99,999
4	$100,000 or more

scores be ranked from high to low. Addition and division are appropriate only for interval-ratio level variables, and ranking is possible only for variables that are at least ordinal in level of measurement. Your first step in dealing with a variable and selecting appropriate statistics is *always* to determine its level of measurement.

Second, in determining level of measurement, always examine the way in which the scores of the variable are *actually stated*. This is particularly a problem with interval-ratio variables that have been measured at the ordinal level. To illustrate, consider income as a variable. If we asked respondents to list their exact income in dollars, we will generate scores that are interval-ratio in level of measurement. Measured in this way, the variable would have a true zero point (an income of $0) and equal intervals from score to score ($1). It is more convenient for respondents, however, to simply check the appropriate category from a broad list, as in Table 1.3.

The four scores or categories in Table 1.3 are ordinal in level of measurement because they are unequal in size. It is common for researchers to sacrifice precision (income in actual dollars) for the convenience of the respondents in this way. You should be careful to look at the way in which the variable is measured before making a decision about its level of measurement.

Third, there is a mismatch between the variables that are usually of most interest to social scientists (race, sex, marital status, attitudes, and opinions) and the most powerful and interesting statistics (such as the mean). The former are typically nominal or, at best, ordinal in level of measurement, but more sophisticated statistics require measurement at the interval-ratio level. This mismatch creates some very real difficulties for social science researchers. On one hand, researchers will want to measure variables at the highest, most precise level of measurement. If income is measured in exact dollars, for example, researchers can make very precise descriptive statements about the differences between people, for example, “Ms. Smith earns $12,547 more than Mr. Jones.” If the same variable is measured in broad, unequal categories, such as those in Table 1.3, comparisons between individuals would be less precise and provide less information: “Ms. Smith earns more than Mr. Jones.”

On the other hand, given the nature of the disparity, researchers are more likely to treat variables as if they were higher in level of measurement than they actually are. In particular, variables measured at the ordinal level, especially when they have many possible categories or scores, are often treated as if they were interval-ratio and analyzed with the more powerful, flexible, and interesting statistics available at the higher level. This practice is common, but the researcher should be cautious in assessing statistical results and

developing interpretations when the level of measurement criterion has been violated.

In conclusion, level of measurement is a very basic characteristic of a variable, and we will always consider it when presenting statistical procedures. Level of measurement is also a major organizing principle for the material that follows, and you should make sure that you are familiar with these guidelines. *(For practice in determining the level of measurement of a variable, see Problems 1.4 through 1.8.)*

SUMMARY

1. Within the context of social research, the purpose of statistics is to organize, manipulate, and analyze data so that researchers can test their theories and answer their questions. Along with theory and methodology, statistics are a basic tool by which social scientists attempt to enhance their understanding of the social world.
2. There are two general classes of statistics. Descriptive statistics are used to summarize the distribution of a single variable and the relationships between two or more variables. Inferential statistics provide us with techniques by which we can generalize to populations from random samples.
3. Variables may be measured at any of three different levels. At the nominal level, we can compare category sizes. At the ordinal level, categories and cases can be ranked with respect to each other. At the interval-ratio level, all mathematical operations are permitted.

GLOSSARY

Data. Any information collected as part of a research project and expressed as numbers.

Data reduction. Summarizing many scores with a few statistics. A major goal of descriptive statistics.

Dependent variable. A variable that is identified as an effect, result, or outcome variable. The dependent variable is thought to be caused by the independent variable.

Descriptive statistics. The branch of statistics concerned with (1) summarizing the distribution of a single variable or (2) measuring the relationship between two or more variables.

Hypothesis. A statement about the relationship between variables that is derived from a theory. Hypotheses are more specific than theories, and all terms and concepts are fully defined.

Independent variable. A variable that is identified as a causal variable. The independent variable is thought to cause the dependent variable.

Inferential statistics. The branch of statistics concerned with making generalizations from samples to populations.

Level of measurement. The mathematical characteristic of a variable and the major criterion for selecting statistical techniques. Variables can be measured at any of three levels, each permitting certain mathematical operations and statistical techniques. The characteristics of the three levels are summarized in Table 1.2.

Measures of association. Statistics that summarize the strength and direction of the relationship between variables.

Population. The total collection of all cases in which the researcher is interested.

Quantitative research: Research based on the analysis of numerical information or data.

Research. Any process of gathering information systematically and carefully to answer questions or test theories. Statistics are useful for research projects in which the information is represented in numerical form or as data.

Sample. A carefully chosen subset of a population. In inferential statistics, information is gathered from a sample and then generalized to a population.

Statistics. A set of mathematical techniques for organizing and analyzing data.

Theory. A generalized explanation of the relationship between two or more variables.

Variable. Any trait that can change values from case to case.

PROBLEMS

1.1 In your own words, describe the role of statistics in the research process. Using the "wheel of science" in Figure 1.1 as a framework, explain how statistics link theory with research.

1.2 Find a research article in any social science journal. Choose an article on a subject of interest to you, and don't worry about being able to understand all of the statistics that are reported.

- **a.** How much of the article is devoted to statistics per se (as distinct from theory, ideas, discussion, and so on)?
- **b.** Is the research based on a sample from some population? How large is the sample? How were subjects or cases selected? Can the findings be generalized to some population?
- **c.** What variables are used? Which are independent and which are dependent? For each variable, determine the level of measurement.
- **d.** What statistical techniques are used? Try to follow the statistical analysis and see how much you can understand. Save the article and read it again after you finish this course to see if you do any better.

1.3 Distinguish between descriptive and inferential statistics. Describe a research situation that would use each type.

1.4 Below are some items from a public opinion survey. For each item, indicate the level of measurement.

- **a.** What is your occupation? _____
- **b.** How many years of school have you completed? _____
- **c.** If you were asked to use one of these four names for your social class, which would you say you belonged in?
  _____ Upper _____Middle
  _____ Working _____Lower
- **d.** What is your age? _____
- **e.** In what country were you born? _____
- **f.** What is your grade point average? _____
- **g.** What is your major? _____
- **h.** The only way to deal with the drug problem is to legalize all drugs.
  _____ Strongly agree
  _____ Agree
  _____ Undecided
  _____ Disagree
  _____ Strongly disagree
- **i.** What is your astrological sign? _____
- **j.** How many brothers and sisters do you have? _____

1.5 Below are brief descriptions of how researchers measured a variable. For each situation, determine the variable's level of measurement.

- **a. Race.** Respondents were asked to select a category from the following list:
  _____ Black
  _____ White
  _____ Asian
  _____ American Indian
  _____ Other (Please specify: _____)
- **b. Honesty.** Subjects were observed as they passed by a spot on campus where an apparently lost wallet was lying. The wallet contained money and complete identification. Subjects were classified into one of the following categories:
  _____ Returned the wallet with money
  _____ Returned the wallet but kept the money
  _____ Did not return wallet
- **c. Social class.** Subjects were asked about their family situation when they were 16 years old. Was their family
  _____ very well off compared to other families?
  _____ about average?
  _____ not so well off?
- **d. Education.** Subjects were asked how many years of schooling they and each parent had completed.
- **e. Racial integration on campus.** Students were observed during lunchtime at the cafeteria for a month. The number of students sitting with students of other races was counted for each meal period.
- **f. Number of children.** Subjects were asked, "How many children have you had? Please include any that may have passed away."
- **g. Student seating patterns in classrooms.** On the first day of class, instructors noted where each student sat. Seating patterns were remeasured every two weeks until the end of the semester. Each student was classified as
  _____ same seat as last measurement;
  _____ adjacent seat;
  _____ different seat, not adjacent;
  _____ absent.

h. Physicians per capita. The number of practicing physicians was counted in each of 50 cities, and the researchers used population data to compute the number of physicians per capita.

i. Physical attractiveness. A panel of 10 judges rated each of 50 photos of a mixed-race sample of males and females for physical attractiveness on a scale from 0 to 20, with 20 being the highest score.

j. Number of accidents. The number of traffic accidents for each of 20 busy intersections in a city was recorded. Also, each accident was rated as

_____ minor damage, no injuries;
_____ moderate damage, personal injury requiring hospitalization;
_____ severe damage and injury.

1.6 What is the level of measurement of each of the first 20 items in the General Social Survey (see Appendix G)?

1.7 For each research situation summarized below, identify the level of measurement of all variables. Also, decide which statistical applications are used: descriptive statistics (single variable), descriptive statistics (two or more variables), or inferential statistics. Remember that it is quite common for a given situation to require more than one type of application.

a. The administration of your university is proposing a change in parking policy. You select a random sample of students and ask each one if he or she favors or opposes the change.

b. You ask everyone in your social research class to tell you the highest grade he or she ever received in a math course and his or her grade on a recent statistics test. You then compare the two sets of scores to see if there is any relationship.

c. Your aunt is running for mayor and hires you (for a huge fee, incidentally) to question a sample of voters about their concerns in local politics. Specifically, she wants a profile of the voters that will tell her what percentage belong to each political party, what percentage are male or female, and what percentage favor or oppose the widening of the main street in town.

d. Several years ago, a state reinstituted the death penalty for first-degree homicide. Supporters of capital punishment argued that this change would reduce the homicide rate. To investigate this claim, a researcher has gathered information on the number of homicides in the state for the two-year periods before and after the change.

e. A local automobile dealer is concerned about customer satisfaction. He wants to mail a survey form to all customers who purchased cars during the past year and ask them if they are satisfied, very satisfied, or not satisfied with their purchases.

1.8 For each research situation below, identify the independent and dependent variables. Classify the level of measurement of each variable.

a. A graduate student is studying sexual harassment on college campuses and asks 500 female students if they personally have experienced any such incidents. Each student is asked to estimate the frequency of these incidents as either often, sometimes, rarely, or never. The researcher also gathers data on age and major to see if there is any connection between these variables and frequency of sexual harassment.

b. A supervisor in the solid waste management division of a city government is attempting to assess two different methods of trash collection. One area of the city is served by trucks with two-person crews who do "backyard" pickups, and the rest of the city is served by "hi-tech" single-person trucks with curbside pickup. The assessment measures include the number of complaints received from the two different areas over a six-month period, the amount of time per day required to service each area, and the cost per ton of trash collected.

c. The adult bookstore near campus has been raided and closed by the police. Your social research class has decided to poll the student body and get their reactions and opinions. The class decides to ask each student if he or she supports or opposes the closing of the store, how many times each one has visited the store, and if he or she agrees or disagrees that "pornography is a direct cause of sexual assaults on women." The class also collects information on the sex, age, religious and political philosophy, and major of each student to see if opinions are related to these characteristics.

d. For a research project in a political science course, a student has collected information

about the quality of life and the degree of political democracy in 50 nations. Specifically, she used infant mortality rates to measure quality of life and the percentage of all adults who are permitted to vote in national elections as a measure of democratization. Her hypothesis is that quality of life is higher in more democratic nations.

e. A highway engineer wonders if a planned increase in speed limit on a heavily traveled local avenue will result in any change in number of accidents. He plans to collect information on traffic volume, number of accidents, and number of fatalities for the six-month periods before and after the change.

f. Students are planning a program to promote safe sex and awareness of a variety of other health concerns for college students. To measure the effectiveness of the program, they plan to give a survey measuring knowledge about these matters to a random sample of the student body before and after the program.

g. Several states have drastically cut their budgets for mental health care. Will this increase the number of homeless people in these states? A researcher contacts a number of agencies serving the homeless in each state and develops an estimate of the size of the population before and after the cuts.

h. Does tolerance for diversity vary by race, ethnicity, or gender? Samples of white, black, Asian, Hispanic, and Native Americans have been given a survey that measures their interest in and appreciation of cultures and groups other than their own.

YOU ARE THE RESEARCHER: Introduction

The best way—maybe the only way—to learn statistics and to appreciate their importance is to apply and use them. This means that you must actually select the correct statistic for a given situation and purpose, do the calculations and compute the statistics correctly, and interpret their meaning. I have included extensive end-of-chapter problems to give you multiple opportunities to select and calculate statistics and say what they mean.

Most of these problems have been written so that they can be solved with just a simple hand calculator. I've purposely kept the number of cases involved unrealistically low so that the tedium of mere calculation would not interfere unduly with the learning process. These problems thus present an important and useful opportunity for you to develop your statistical skills.

As important as they are, these end-of-chapter problems are artificial, simplified, and several steps removed from the reality of conducting social science research. To provide a more realistic statistical experience, I have included a feature called You Are the Researcher in which you will walk through many of the steps of a research project, make decisions about how to apply your growing knowledge of research and statistics, and interpret the statistical output you generate.

To conduct these research projects, you will analyze a shortened version of the 2006 General Social Survey (GSS). This database can be downloaded from our Web site (www.cengage.com/sociology/healey). The GSS is a public opinion poll that has been conducted on nationally representative samples of citizens of the United States since 1972. The full survey includes hundreds of questions covering a broad range of social and political issues. The version supplied with this text has a limited number of variables and cases but is still actual, "real-life" data, so you have the opportunity to practice your statistical skills in a more realistic context.

Even though the version of the GSS we use for this text is shortened, it is still a large data set with almost 1,500 respondents and almost 50 different variables, too large for even the most advanced hand calculator. To analyze the GSS, you will learn how to use a computerized statistical package called Statistical Package for the Social Sciences (SPSS). A statistical package is a set of computer programs designed to analyze data. The advantage of these packages is that, since the programs are already written, you can capitalize on the power of the computer even though you may have minimal computer literacy and virtually no programming experience. Be sure to read Appendix F before attempting any data analysis.

We will begin these exercises in Chapter 2. In most of these exercises, you will make the same kinds of decisions as would a professional researcher and move through some of the steps of a research project: selecting variables and appropriate statistics, generating and analyzing output, and expressing your results and conclusions. When you finish these exercises, you will be well prepared to conduct your own research project (within limits, of course) and perhaps make a contribution to the ever-growing social science research literature.

Part I Descriptive Statistics

Part I consists of five chapters, each devoted to a different application of univariate descriptive statistics. Chapter 2 covers basic descriptive statistics, including percentages, ratios, rates, and frequency distributions, and Chapter 3 covers graphs and charts. Although the statistics covered in these chapters are "basic," they are not necessarily simple or obvious, and the explanations and examples should be considered carefully before attempting the end-of-chapter problems or using them in actual research.

Chapters 4 and 5 cover measures of central tendency and dispersion, respectively. Measures of central tendency describe the typical case or average score (e.g., the mean), and measures of dispersion describe the amount of variety or diversity among the scores (e.g., the range or the distance from the high score to the low score). These two types of statistics are presented in separate chapters to stress the point that centrality and dispersion are independent, separate characteristics of a variable. You should realize, however, that *both* measures are necessary and commonly reported together (along with some of the statistics presented in Chapter 2 and the graphs in Chapter 3). To reinforce the idea that measures of centrality and dispersion are complimentary descriptive statistics, many of the problems at the end of Chapter 5 require the computation of one of the measures of central tendency discussed in Chapter 4.

Chapter 6 is a pivotal chapter in the flow of the text. It takes some of the statistics from Chapters 2 through 5 and applies them to the normal curve, a concept of great importance in statistics. The normal curve is a type of line chart or frequency polygon (see Chapter 3) that can be used to describe the position of scores using means (Chapter 4) and standard deviations (Chapter 5). Chapter 6 also uses proportions (discussed in Chapter 2) to introduce the concept of probability, a central component of social science research.

In addition to its role in descriptive statistics, the normal curve is a central concept in inferential statistics, the topic of Part II of this text. Thus, Chapter 6 serves a dual purpose: it ends the presentation of univariate descriptive statistics and lays essential groundwork for the material to come.

2

Basic Descriptive Statistics
Percentages, Ratios and Rates, Frequency Distributions

LEARNING OBJECTIVES

By the end of this chapter, you will be able to:

1. Explain the purpose of descriptive statistics in making data comprehensible.
2. Compute and interpret percentages, proportions, ratios, rates, and percentage change.
3. Construct and analyze frequency distributions for variables at each of the three levels of measurement.

Research results do not speak for themselves. They must be organized and manipulated so that whatever meaning they have can be quickly and easily understood by the researcher and his or her readers. Researchers use statistics to clarify their results and communicate effectively. In this chapter, we will consider some commonly used techniques for presenting research results: percentages and proportions; ratios and rates; percentage change. Mathematically speaking, these univariate descriptive statistics are not very complex (although they are not as simple as they may appear at first glance), but as you will see, they are extremely useful for presenting research results clearly and concisely.

2.1 PERCENTAGES AND PROPORTIONS

Consider the following statement: Of the 269 cases handled by the court, 167 resulted in prison sentences of five years or more. While there is nothing wrong with this statement, the same fact could have been more clearly conveyed if it had been reported as a percentage: About 62% of all cases resulted in prison sentences of five or more years.

Percentages and proportions supply a frame of reference for reporting research results in the sense that they standardize the raw data, percentages to the base 100 and proportions to the base 1.00. The mathematical definitions of **proportions** and **percentages** are

FORMULA 2.1

$$\text{Proportion: } p = \frac{f}{N}$$

FORMULA 2.2

$$\text{Percentage: } \% = \left(\frac{f}{N}\right) \times 100$$

Where: f = frequency, or the number of cases in any category
N = the number of cases in all categories

To illustrate the computation of percentages, consider the data presented in Table 2.1. Note that there are 167 cases in the category ($f = 167$) and a total of 269 cases in all ($N = 269$). So,

TABLE 2.1 DISPOSITION OF 269 CRIMINAL CASES (fictitious data)*

Sentence	Frequency (f)	Proportion (p)	Percentage (%)
Five years or more	167	0.6208	62.08
Less than five years	72	0.2677	26.77
Suspended	20	0.0744	7.44
Acquitted	10	0.0372	3.72
Totals =	269	1.0001	100.01%

*The slight discrepancies in the totals of the proportion and percentage columns are due to rounding error.

$$\text{Percentage (\%)} = \left(\frac{f}{N}\right) \times 100 = \left(\frac{167}{269}\right) \times 100 = (0.6208) \times 100 = 62.08\%$$

Using the same procedures, we can also find the percentage of cases in the second category:

$$\text{Percentage (\%)} = \left(\frac{f}{N}\right) \times 100 = \left(\frac{72}{269}\right) \times 100 = (0.2677) \times 100 = 26.77\%$$

Both results could have been expressed as proportions. For example, the proportion of cases in the third category is 0.0744.

$$\text{Proportion } (p) = \frac{f}{N} = \frac{20}{269} = 0.0744$$

Percentages and proportions are easier to read and comprehend than are frequencies. This advantage is particularly obvious when attempting to compare groups of different sizes. For example, based on the information presented in Table 2.2, which college has the higher relative number of social science majors?

Because the total enrollments are so different, comparisons are difficult to make using the raw frequencies. Computing percentages eliminates the size difference of the two campuses by standardizing both distributions to the base of 100. The same data are presented in percentages in Table 2.3.

The percentages in Table 2.3 make it easier to identify both differences and similarities between the two colleges. College A has a much higher percentage of social science majors (even though the absolute number of social science majors is less than at College B) and about the same percentage of humanities majors. How would you describe the differences in the remaining two major fields? *(For practice in computing and interpreting percentages and proportions, see Problems 2.1 and 2.2.)*

TABLE 2.2 DECLARED MAJOR FIELDS ON TWO COLLEGE CAMPUSES (fictitious data)

Major	College A	College B
Business	103	312
Natural sciences	82	279
Social sciences	137	188
Humanities	93	217
	$N = 415$	996

TABLE 2.3 DECLARED MAJOR FIELDS ON TWO COLLEGE CAMPUSES (fictitious data)

Major	College A	College B
Business	24.82	31.33
Natural sciences	19.76	28.01
Social sciences	33.01	18.88
Humanities	22.41	21.79
	100.00%	100.01%
	(415)	(996)

Application 2.1

In Table 2.2, 237 of the 415 students enrolled in College A and 458 of the 996 students enrolled in College B are males. What percentage of each student body is male?

College A

$$\% = \left(\frac{237}{415}\right) \times 100 = (0.5711) \times 100 = 57.11\%$$

College B

$$\% = \left(\frac{237}{415}\right) \times 100 = (0.5711) \times 100 = 57.11\%$$

College B has the greater number of men, but College A has the larger percentage.

Here are some further guidelines on the use of percentages and proportions:

1. When working with a small number of cases (say, fewer than 20), it is usually preferable to report the actual frequencies rather than percentages or proportions. With a small number of cases, the percentages can change drastically with relatively minor changes in the data. For example, if you begin with a data set that includes 10 males and 10 females (that is, 50% of each sex) and then add another female, the percentage distributions will change noticeably to 52.38% female and 47.62% male. Of course, as the number of observations increases, each additional case will have a smaller impact. If we started with 500 males and females and then added one more female, the percentage of females would change by only a tenth of a percent (from 50% to 50.10%).
2. Always report the number of observations along with proportions and percentages. This permits the reader to judge the adequacy of the sample size and, conversely, helps to prevent the researcher from lying with statistics. Statements like "two out of three people questioned prefer courses in statistics to any other course" might sound impressive, but the claim would lose its gloss if you learned that only three people were tested. You should be extremely suspicious of reports that fail to report the number of cases tested.
3. Percentages and proportions can be calculated for variables at the ordinal and nominal levels of measurement, in spite of the fact that they require division. This is not a violation of the level of measurement guideline (see Table 1.2). Percentages and proportions do not require

ONE STEP AT A TIME **Finding Percentages and Proportions**

Step	*Operation*
1.	Determine the values for *f* (number of cases in a category) and *N* (number of cases in all categories). Remember that *f* will be the number of cases in a *specific category* (e.g., males on your campus), *N* will be the number of cases in *all* categories (e.g., all students, males and females, on your campus), and *f* will be smaller than *N*, except when the category and the entire group are the same (e.g., when all students are male). Proportions cannot exceed 1.00, and percentages cannot exceed 100.00%.
2.	For a proportion, divide *f* by *N*.
3.	For a percentage, multiply the value you calculated in Step 2 by 100.

the division of the *scores* of the variable, as would be the case in computing the average score on a test, for example. Instead, we divide the *number of cases* in a particular category (f) of the variable by the *total number of cases* in the sample (N). When we make a statement like "43% of the sample is female," we are merely expressing the relative size of a category (female) of the variable (sex) in a convenient way.

2.2 RATIOS, RATES, AND PERCENTAGE CHANGE

Ratios, rates, and percentage change provide some additional ways of summarizing results simply and clearly. Although they are similar to each other, each statistic has a specific application and purpose.

Ratios. Ratios are especially useful for comparing the number of cases in the categories of a variable. Instead of standardizing the distribution of the variable to the base 100 or 1.00, as we did in computing percentages and proportions, we determine **ratios** by dividing the frequency of one category by the frequency in another. Mathematically, a ratio can be defined as

FORMULA 2.3

$$\text{Ratio} = \frac{f_1}{f_2}$$

Where: f_1 = the number of cases in the first category.
f_2 = the number of cases in the second category.

To illustrate the use of ratios, suppose that you were interested in the relative sizes of the various religious denominations and found that a particular community included 1,370 Protestant families and 930 Catholic families. To find the ratio of Protestants (f_1) to Catholics (f_2), divide 1,370 by 930:

$$\text{Ratio} = \frac{f_1}{f_2} = \frac{1{,}370}{930} = 1.47$$

The resultant ratio is 1.47, which means that for every Catholic family, there are 1.47 Protestant families.

Ratios can be very economical ways of expressing the relative predominance of two categories. That Protestants outnumber Catholics in our example is obvious from the raw data. Percentages or proportions could have been used to summarize the overall distribution (e.g., 59.56% of the families were

Protestant, 40.44% were Catholic). In contrast to these other methods, ratios express the relative size of the categories: they tell us exactly how much one category outnumbers the other.

Ratios are often multiplied by some power of 10 to eliminate decimal points. For example, the ratio computed above might be multiplied by 100 and reported as 147 instead of 1.47. This would mean that for every 100 Catholic families, there are 147 Protestant families in the community. To ensure clarity, the comparison units for the ratio are often expressed as well. Based on a unit of ones, the ratio of Protestants to Catholics would be expressed as 1.47:1. Based on hundreds, the same statistic might be expressed as 147:100. *(For practice in computing and interpreting ratios, see Problems 2.1 and 2.2.)*

Rates. Rates provide still another way of summarizing the distribution of a single variable. **Rates** are defined as the number of actual occurrences of some phenomenon divided by the number of possible occurrences per some unit of time. Rates are usually multiplied by some power of 10 to eliminate decimal points. For example, the crude death rate for a population is defined as the number of deaths in that population (actual occurrences) divided by the number of people in the population (possible occurrences) per year. This quantity is then multiplied by 1,000. The formula for the crude death rate can be expressed as

$$\text{Crude death rate} = \frac{\text{Number of deaths}}{\text{Total population}} \times 1{,}000$$

If there were 100 deaths during a given year in a town of 7,000, the crude death rate for that year would be

$$\text{Crude death rate} = \frac{100}{7{,}000} \times 1{,}000 = (0.01429) \times 1{,}000 = 14.29$$

Or, for every 1,000 people, there were 14.29 deaths during this particular year. In the same way, if a city of 237,000 people experienced 120 auto thefts during a particular year, the auto theft rate would be

$$\text{Auto theft rate} = \frac{120}{237{,}000} \times 100{,}000 = (0.00005063) \times 100{,}000 = 50.63$$

Or, for every 100,000 people, there were 50.63 auto thefts during the year in question. *(For practice in computing and interpreting rates, see Problems 2.3 and 2.4a.)*

Application 2.2

How many natural science majors are there compared to social science majors at College B? This question could be answered with frequencies, but a more easily understood way of expressing the answer would be with a ratio. The ratio of natural science to social science majors would be

$$\text{Ratio} = \frac{f_1}{f_2} = \frac{279}{188} = 1.48$$

For every social science major, there are 1.48 natural science majors at College B.

Application 2.3

In 2000, there were 2,500 births in a city of 167,000. In 1960, when the population of the city was only 133,000, there were 2,700 births. Is the birthrate rising or falling? Although this question can be answered from the preceding information, the trend in birthrates will be much more obvious if we compute birthrates for both years. Like crude death rates, crude birthrates are usually multiplied by 1,000 to eliminate decimal points. For 1960,

$$\text{Crude birth rate} = \frac{2{,}700}{133{,}000} \times 1{,}000 = 20.30$$

In 1960, there were 20.30 births for every 1,000 people in the city. For 2000,

$$\text{Crude birthrate} = \frac{2{,}500}{167{,}000} \times 1{,}000 = 14.97$$

In 2000, there were 14.97 births for every 1,000 people in the city. With the help of these statistics, the decline in the birthrate is clearly expressed.

Percentage change. Measuring social change, in all its variety, is an important task for all social sciences. One very useful statistic for this purpose is the **percentage change,** which tells us how much a variable has increased or decreased over a certain span of time.

To compute this statistic, we need the scores of a variable at two different points in time. The scores could be in the form of frequencies, rates, or percentages. The percentage change will tell us how much the score has changed at the later time relative to the earlier time. Using death rates as an example once again, imagine a society suffering from a devastating outbreak of disease in which the death rate rose from 16.00 per 1,000 population in 1995 to 24.00 per 1,000 in 2000. Clearly, the death rate is higher in 2000, but by how much relative to 1995?

The formula for the percentage change is

FORMULA 2.4

$$\text{Percentage change} = \left(\frac{f_2 - f_1}{f_1}\right) \times 100$$

Where: f_1 = first score, frequency, or value
f_2 = second score, frequency, or value

In our example, f_1 is the death rate in 1995 (16.00) and f_2 is the death rate in 2000 (24.00). The formula tells us to subtract the earlier score from the later and then divide by the earlier score. The resultant value expresses the size of the change in scores ($f_2 - f_1$) relative to the score at the earlier time (f_1). The value is then multiplied by 100 to express the change in the form of a percentage:

$$\text{Percentage change} = \left(\frac{24 - 16}{16}\right) \times 100 = \left(\frac{8}{16}\right) \times 100 = (0.50) \times 100 = 50\%$$

The death rate in 2000 is 50% higher than in 1995. This means that the 2000 rate was equal to the 1995 rate *plus* half of the earlier score. If the rate had risen to 32 per 1,000, the percentage change would have been 100% (the rate would have doubled), and if the death rate had fallen to 8 per 1,000, the percentage

TABLE 2.4 PROJECTED POPULATION GROWTH FOR SIX NATIONS, 2000–2050

Nation	Population, 2000 (f_1)	Population, 2050 (f_2)	Increase/ Decrease ($f_2 - f_1$)	Percentage Change $\left(\frac{f_2 - f_1}{f_1}\right) \times 100$
China	1,268,853,362	1,424,161,948	155,308,586	12.24
United States	282,338,631	420,080,587	137,741,956	48.79
Canada	31,099,561	41,135,648	10,036,087	32.27
Mexico	99,926,620	147,907,650	47,981,030	48.02
Italy	57,719,337	50,389,841	−7,329,496	−12.70
Nigeria	123,178,818	264,262,405	141,083,587	114.54

Source: http://www.census.gov/cgi-bin/ipc/idbrank.pl.

change would have been −50%. Note the negative sign: it means that the death rate has decreased by 50%. The 2000 rate would have been half the size of the 1995 rate.

An additional example should make the computation and interpretation of the percentage change clearer. Suppose we wanted to compare the projected population growth rates for various nations for a 50-year period starting in 2000. The necessary information is presented in Table 2.4. Casual inspection will give us some information. For example, compare China and Nigeria. These societies are projected to add roughly similar numbers of people (about a 155 million for China, a little less for Nigeria), but since China's 2000 population is 10 times the size of Nigeria's, its percentage change will be much lower (about 12% vs. over 100%).

Calculating percentage change will make these comparisons more precise. Table 2.4 shows the actual population for each nation in 2000 and the projected population for 2050. The "increase/decrease" column shows how many people will be added or lost. The right-hand column shows the percentage change in projected population for each nation. These values were computed by subtracting the 2000 population (f_1) from the 2050 population (f_2), dividing by the 2000 population, and multiplying by 100.

Application 2.4

The American family has been changing rapidly over the past several decades. One major change has been an increase in the number of married women and mothers with jobs outside the home. For example, in 1975, 36.7% of women with children under the age of six worked outside the home. In 2001, this percentage had risen to 62.5%. How large has this change been?

It is obvious that the 2001 percentage is much higher, and calculating the percentage change will give us an exact idea of the magnitude of the change. The 1975 percentage is f_1 and the 2000 figure is f_2, so

$$\text{Percentage change} = \left(\frac{62.5 - 36.7}{36.7}\right) \times 100$$

$$= \left(\frac{25.8}{36.7}\right) \times 100 = (0.70299) \times 100 = 70.30\%$$

Between 1975 and 2001, the percentage of women with children younger than six who worked outside the home increased by 70.30%.

U.S. Bureau of the Census. 2003. *Statistical Abstract of the United States, 2002*. Washington, DC: Government Printing Office. p. 373.

ONE STEP AT A TIME **Finding Ratios, Rates, and Percentage Change**

Step	*Operation*
Ratios	
1.	Determine the values for f_1 and f_2. The value for f_1 will be the number of cases in the first category (e.g., the number of males on your campus), and the value for f_2 will be the number of cases in the second category (e.g., the number of females on your campus).
2.	Divide the value of f_1 by the value of f_2.
Rates	
1.	Determine the number of actual occurrences (e.g., births, deaths, homicides, assaults). This value will be the numerator.
2.	Determine the number of possible occurrences. This value will usually be the total population for the area in question.
3.	Divide the number of actual occurrences by the number of possible occurrences.
4.	Multiply the value you calculated in Step 3 by some power of 10. Conventionally, birth and death rates are multiplied by 1,000 and crime rates are multiplied by 100,000.
Percentage Change	
1.	Determine the values for f_1 and f_2. The former will be the score at time 1 (the earlier time), and the latter will be the score at time 2 (the later time).
2.	Subtract f_1 from f_2.
3.	Divide the quantity you found in Step 2 by f_1.
4.	Multiply the quantity you found in Step 3 by 100.

Although China has the largest population of these six nations, it will grow at the slowest rate (12.24%). The United States and Mexico will increase by about 50% (in 2050, their populations will be half again larger than in 2000), and Canada will grow by about one-third. Italy's population will actually decline by almost 13%. Nigeria has by far the highest growth rate: it will increase in size by over 100%. This means that, in 2050, the population of Nigeria will be more than two times its 2000 size. *(For practice in computing and interpreting percentage change, see Problem 2.4b.)*

2.3 FREQUENCY DISTRIBUTIONS: INTRODUCTION

Frequency distributions are tables that summarize the distribution of a variable by reporting the number of cases contained in each category of the variable. They are very helpful and commonly used ways of organizing and working with data. In fact, the construction of frequency distributions is almost always the first step in any statistical analysis.

To illustrate the usefulness of frequency distributions and to provide some data for examples, assume that the counseling center at a university is assessing the effectiveness of its services. Any realistic evaluation research would collect a variety of information from a large group of students, but for the sake of this example, we will confine our attention to just four variables and 20 students. The data are reported in Table 2.5.

TABLE 2.5 DATA FROM COUNSELING CENTER SURVEY

Student	Sex	Marital Status	Satisfaction with Services*	Age
A	Male	Single	4	18
B	Male	Married	2	19
C	Female	Single	4	18
D	Female	Single	2	19
E	Male	Married	1	20
F	Male	Single	3	20
G	Female	Married	4	18
H	Female	Single	3	21
I	Male	Single	3	19
J	Female	Divorced	3	23
K	Female	Single	3	24
L	Male	Married	3	18
M	Female	Single	1	22
N	Female	Married	3	26
O	Male	Single	3	18
P	Male	Married	4	19
Q	Female	Married	2	19
R	Male	Divorced	1	19
S	Female	Divorced	3	21
T	Male	Single	2	20

*Key: 4 = Very satisfied 3 = Satisfied 2 = Dissatisfied 1 = Very dissatisfied

Note that, even though the data in Table 2.5 represent an unrealistically low number of cases, it is difficult to discern any patterns or trends. For example, try to ascertain the general level of satisfaction of the students from Table 2.5. You may be able to do so with just 20 cases, but it will take some time and effort. Imagine the difficulty with 50 cases or 100 cases presented in this fashion. Clearly, the data need to be organized in a format that allows the researcher (and his or her audience) to understand easily the distribution of the variables.

One general rule that applies to all frequency distributions is that the categories of the frequency distribution must be exhaustive and mutually exclusive. In other words, the categories must be stated in a way that permits each case to be counted in one and only one category. This basic principle applies to the construction of frequency distributions for variables measured at all three levels of measurement.

Beyond this rule, there are only guidelines to help you construct useful frequency distributions. As you will see, the researcher has a fair amount of discretion in stating the categories of the frequency distribution (especially with variables measured at the interval-ratio level). I will identify the issues to consider as you make decisions about the nature of any particular frequency distribution. Ultimately, however, the guidelines I state are aids for decision making, nothing more than helpful suggestions. As always, the researcher has the final responsibility for making sensible decisions and presenting his or her data in a meaningful way.

2.4 FREQUENCY DISTRIBUTIONS FOR VARIABLES MEASURED AT THE NOMINAL AND ORDINAL LEVELS

Nominal-Level Variables. For nominal level variables, construction of the frequency distribution is typically very straightforward. For each category of the variable being displayed, the occurrences are counted and the subtotals, along with the total number of cases (N), are reported. Table 2.6 displays a frequency distribution for the variable of sex from the counseling center survey. For purposes of illustration, a column for tallies has been included in this table to illustrate how the cases would be sorted into categories. (This column would not be included in the final form of the frequency distribution.) Take a moment to notice several other features of the table. Specifically, the table has a descriptive title, clearly labeled categories (male and female), and a report of the total number of cases at the bottom of the frequency column. These items must be included in all tables regardless of the variable or level of measurement.

The meaning of the table is quite clear. There are 10 males and 10 females in the sample, a fact that is much easier to comprehend from the frequency distribution than from the unorganized data presented in Table 2.5.

For some nominal variables, the researcher might have to make some choices about the number of categories he or she wishes to report. For example, the distribution of the marital status variable could be reported using the categories listed in Table 2.5. The resultant frequency distribution is presented in Table 2.7. Although this is a perfectly fine frequency distribution, it may be too detailed for some purposes. For example, the researcher might want to focus solely on nonmarried as distinct from married students. That is, the researcher might not be concerned with the difference between single and divorced respondents, but may want to treat both as simply "not married." In that case, these categories could be grouped together and treated as a single entity, as in Table 2.8. Notice that when categories are collapsed like this, information and detail will be lost. This latter version of the table would not allow the researcher to discriminate between the two unmarried states.

TABLE 2.6 SEX OF RESPONDENTS, COUNSELING CENTER SURVEY

Sex	Tallies	Frequency (f)
Male	///// /////	10
Female	///// /////	10
		$N = 20$

TABLE 2.7 MARITAL STATUS OF RESPONDENTS, COUNSELING CENTER SURVEY

Status	Frequency (f)
Single	10
Married	7
Divorced	3
	$N = 20$

TABLE 2.8 MARITAL STATUS OF RESPONDENTS, COUNSELING CENTER SURVEY

Status	Frequency (*f*)
Married	7
Not married	13
	$N = 20$

TABLE 2.9 SATISFACTION WITH SERVICES, COUNSELING CENTER SURVEY

Satisfaction	Frequency (*f*)	Percentage (%)
(4) Very satisfied	4	20
(3) Satisfied	9	45
(2) Dissatisfied	4	20
(1) Very dissatisfied	3	15
	$N = 20$	100%

TABLE 2.10 SATISFACTION WITH SERVICES, COUNSELING CENTER SURVEY

Satisfaction	Frequency (*f*)	Percentage (%)
Satisfied	13	65
Dissatisfied	7	35
	$N = 20$	100%

Ordinal Level Variables. Frequency distributions for ordinal level variables are constructed following the same routines used for nominal level variables. Table 2.9 reports the frequency distribution of the satisfaction variable from the counseling center survey. Note that a column of percentages by category has been added to this table. Such columns heighten the clarity of the table (especially with larger samples) and are common adjuncts to the basic frequency distribution for variables measured at all levels.

This table reports that most students were either satisfied or very satisfied with the services of the counseling center. The most common response (nearly half the sample) was "satisfied." If the researcher wanted to emphasize this major trend, the categories could be collapsed as in Table 2.10. Again, the price paid for this increased compactness is that some information (in this case, the exact breakdown of degrees of satisfaction and dissatisfaction) is lost. *(For practice in constructing and interpreting frequency distributions for nominal and ordinal level variables, see Problem 2.5.)*

2.5 FREQUENCY DISTRIBUTIONS FOR VARIABLES MEASURED AT THE INTERVAL-RATIO LEVEL

Basic Considerations. In general, the construction of frequency distributions for variables measured at the interval-ratio level is more complex than for nominal and ordinal variables. Interval-ratio variables usually have a large number of possible scores (that is, a wide range from the lowest to the highest score). The large number of scores requires some collapsing or grouping of categories

to produce reasonably compact frequency distributions. To construct frequency distributions for interval-ratio level variables, you must decide how many categories to use and how wide these categories should be.

For example, suppose you wished to report the distribution of the variable age for a sample drawn from a community. Unlike the college data reported in Table 2.5, a community sample would have a very broad range of ages. If you simply reported the number of times that each year of age (or score) occurred, you could easily wind up with a frequency distribution that contained 70, 80, or even more categories. Such a large frequency distribution would not present a concise picture. The scores (years) must be grouped into larger categories to heighten clarity and ease of comprehension. How large should these categories be? How many categories should be included in the table? Although there are no hard-and-fast rules for making these decisions, they always involve a trade-off between more detail (a greater number of narrow categories) or more compactness (a smaller number of wide categories).

Constructing the Frequency Distribution. To introduce the mechanics and decision-making processes involved, we will construct a frequency distribution to display the ages of the students in the counseling center survey. Because of the narrow age range of a group of college students, we can use categories of only one year (these categories are often called class intervals when working with interval-ratio data). The frequency distribution is constructed by listing the ages from youngest to oldest, counting the number of times each score (year of age) occurs, and then totaling the number of scores for each category. Table 2.11 presents the information and reveals a concentration or clustering of scores in the 18 and 19 class intervals.

Even though the picture presented in this table is fairly clear, assume for the sake of illustration that you desire a more compact (less-detailed) summary. To achieve this, you will have to group scores into wider class intervals. By increasing the interval width (say, to two years), you reduce the number of intervals and achieve a more compact expression. The grouping of scores in Table 2.12 clearly emphasizes the relative predominance of younger respondents. This trend in the data can be stressed even more by adding a column to display the percentage of cases in each category.

TABLE 2.11 AGE OF RESPONDENTS, COUNSELING CENTER SURVEY
(interval width = 1 year of age)

Class Intervals	Frequency (*f*)
18	5
19	6
20	3
21	2
22	1
23	1
24	1
25	0
26	1
	$N = 20$

TABLE 2.12 AGE OF RESPONDENTS, COUNSELING CENTER SURVEY (interval width = 2 years of age)

Class Intervals	Frequency (*f*)	Percentage (%)
18–19	11	55
20–21	5	25
22–23	2	10
24–25	1	5
26–27	1	5
	$N = 20$	100%

Note that the class intervals in Table 2.12 have been stated with an apparent gap between them (that is, the class intervals are separated by a distance of one unit). At first glance, these gaps may appear to violate the principle of exhaustiveness; but, since age has been measured in whole numbers, the gaps actually are not a problem. Given the level of precision of the measurement (in years, as opposed to 10ths or 100ths of a year), no case could have a score falling between these class intervals. In fact, for these data, the set of class intervals contained in Table 2.12 constitutes a scale that is exhaustive and mutually exclusive. Each of the 20 respondents in the sample can be sorted into one and only one age category.

However, consider the difficulties that might have been encountered if age had been measured with greater precision. If age had been measured in 10ths of a year, into which class interval in Table 2.12 would a 19.4-year-old subject be placed? You can avoid this ambiguity by always stating the limits of the class intervals at the same level of precision as the data. Thus, if age were being measured in 10ths of a year, the limits of the class intervals in Table 2.12 would be stated in 10ths of a year. For example:

17.0–18.9

19.0–20.9

21.0–22.9

23.0–24.9

25.0–26.9

To maintain mutual exclusivity between categories, do not overlap the class intervals. If you state the limits of the class intervals at the same level of precision as the data (which might be in whole numbers, 10ths, 100ths, etc.) and maintain a gap between intervals, you will always produce a frequency distribution in which each case can be assigned to one and only one category.

Midpoints. On occasion, you will need to work with the midpoints of the class intervals, for example, when constructing or interpreting certain graphs. **Midpoints** are defined as the points exactly halfway between the upper and lower limits and can be found for any interval by dividing the sum of the upper and lower limits by two. Table 2.13 displays midpoints for two different sets of class intervals. *(For practice in finding midpoints, see Problems 2.8b and 2.9b.)*

TABLE 2.13 MIDPOINTS

Class interval width = 3	
Class Intervals	Midpoints
0–2	1
3–5	4
6–8	7
9–11	10
Class Interval width = 6	
Class Intervals	Midpoints
100–105	102.5
106–111	108.5
112–117	114.5
118–123	120.5

ONE STEP AT A TIME — Finding Midpoints

Step	*Operation*
1.	Find the upper and lower limits of the lowest interval in the frequency distribution. For any interval, the upper limit is the highest score included in the interval and the lower limit is the lowest score included in the interval. For example, for the top set of intervals in Table 2.13, the lowest interval (0–2) includes scores of 0, 1, and 2. The upper limit of this interval is 2, and the lower limit is 0.
2.	Add the upper and lower limits and divide by 2. For the interval 0–2: 0 + 2/2 = 1. The midpoint for this interval is 1.
3.	Midpoints for other intervals can be found by repeating steps 1 and 2 for each interval. As an alternative, you can find the midpoint for any interval by adding the value of the interval width to the midpoint of the next lower interval. For example, the lowest interval in Table 2.13 is 0–2, and the midpoint is 1. Intervals are three units wide (that is, they each include three scores), so the midpoint for the next higher interval (3–5) is 1 + 3 or 4. The midpoint for the interval 6–8 is 4 + 3 or 7, and so forth.

Cumulative Frequency and Cumulative Percentage. Two commonly used adjuncts to the basic frequency distribution for interval-ratio data are the **cumulative frequency** and **cumulative percentage** columns. Their primary purpose is to allow the researcher (and his or her audience) to tell at a glance how many cases fall below a given score or class interval in the distribution.

To construct a cumulative frequency column, begin with the lowest class interval (i.e., the class interval with the lowest scores) in the distribution. The entry in the cumulative frequency columns for that interval will be the same as the number of cases in the interval. For the next higher interval, the cumulative frequency will be all cases in the interval plus all the cases in the first interval. For the third interval, the cumulative frequency will be all cases in the interval plus all cases in the first two intervals. Continue adding (or accumulating) cases until you reach the highest class interval, which will have a cumulative

TABLE 2.14 AGE OF RESPONDENTS, COUNSELING CENTER SURVEY

Class Intervals	Frequency (*f*)	Cumulative Frequency
18–19	11	11
20–21	5	16
22–23	2	18
24–25	1	19
26–27	1	20
	$N = 20$	

frequency of all the cases in the interval plus all cases in all other intervals. For the highest interval, cumulative frequency equals the total number of cases. Table 2.14 shows a cumulative frequency column added to Table 2.12.

The cumulative percentage column is quite similar to the cumulative frequency column. Begin by adding a column to the basic frequency distribution for percentages as in Table 2.14. This column shows the percentage of all cases in each class interval. To find cumulative percentages, follow the same addition pattern explained above for cumulative frequency. That is, the cumulative percentage for the lowest class interval will be the same as the percentage of cases in the interval. For the next higher interval, the cumulative percentage is the percentage of cases in the interval plus the percentage of cases in the first interval, and so on. Table 2.15 shows the age data with a cumulative percentage column added.

These cumulative columns are quite useful in situations where the researcher wants to make a point about how cases are spread across the range of scores. For example, Tables 2.14 and 2.15 show quite clearly that most students in the counseling center survey are less than 21 years of age. If the researcher wishes to impress this feature of the age distribution on his or her audience, then these cumulative columns are quite handy. Most realistic research situations will be concerned with many more than 20 cases and/or many more categories than our tables have. Since the cumulative percentage column is clearer and easier to interpret in such cases, it is normally preferred to the cumulative frequencies column.

Using Unequal Class Intervals. As a general rule, the class intervals of frequency distributions should be equal in size to maximize clarity and ease of comprehension. For example, note that all of the class intervals in Tables 2.14 and 2.15 are the same width (two years). There are several situations, however, in which the researcher may choose to use open-ended class intervals or

TABLE 2.15 AGE OF RESPONDENTS, COUNSELING CENTER SURVEY

Class Intervals	Frequency (*f*)	Cumulative Frequency	Percentage (%)	Cumulative Percentage
18–19	11	11	55	55
20–21	5	16	25	80
22–23	2	18	10	90
24–25	1	19	5	95
26–27	1	20	5	100
	$N = 20$		100%	

TABLE 2.16 AGE OF RESPONDENTS, COUNSELING CENTER SURVEY ($N = 21$)

Class Intervals	Frequency (f)	Cumulative Frequency
18–19	11	11
20–21	5	16
22–23	2	18
24–25	1	19
26–27	1	20
28 and older	1	21
	$N = 21$	

intervals of unequal size. Open-ended intervals have an unspecified upper or lower limit and can be used when there are a few cases with extremely high or extremely low scores. Intervals of unequal size can be used to collapse a variable with a wide range of scores into more easily comprehended groupings. We will examine each situation separately.

Open-Ended Intervals. What would happen to the frequency distribution in Tables 2.14 and 2.15 if we added one more student who was 47 years old? We would now have 21 cases, and there would be a large gap between the oldest respondent (now 47) and the second oldest (age 26). If we simply added the older student to the frequency distribution, we would have to include nine new class intervals (28–30, 31–32, 32–33, etc.) with zero cases in them before we got to the 46–47 interval. This would waste space and probably be unclear and confusing. An alternative way to handle the situation would be to add an open-ended interval to the frequency distribution, as in Table 2.16.

The open-ended interval in Table 2.16 presents the distribution more compactly and efficiently than listing all of the empty intervals between 28–29 and 46–47. Note also that we could handle an extremely low score by adding an open-ended interval as the lowest class interval (e.g., 17 and younger). There is a small price to pay for this efficiency (there is no information in Table 2.16 about the value of the scores included in the open-ended interval), so this technique should not be used indiscriminately.

Intervals of Unequal Size. Some variables have a few cases with scores very different from the bulk of the cases. Consider, for example, the distribution of income in the United States. Some households will have lower incomes (for example, less than $20,000), and many will have moderate incomes (say, $20,000–$60,000). There will also be many incomes spread between $60,000 and $100,000 and some over $100,000, and very few in the high six-figure or seven-figure range.

If we tried to summarize income with a frequency distribution with equal intervals of, say, $10,000, the table would have to have 20 or 30 (or more) intervals to include all the scores, and many of the intervals in the higher income ranges—those over $100,000—would have few or zero cases. In situations such as this, researchers sometimes use intervals of unequal size to summarize the distribution of the variable more efficiently. To illustrate, Table 2.17 uses unequal intervals for both the lowest and highest scores to summarize the distribution of income in the United States as of 2006. *(For practice in constructing and interpreting frequency distributions for interval-ratio level variables, see Problems 2.5 to 2.9.)*

TABLE 2.17 DISTRIBUTION OF INCOME BY HOUSEHOLD, UNITED STATES, 2006

Income	Households (Frequency)	Households (Percent)
Less than $20,000	21,760,690	19.5
$20,000–$29,999	12,661,512	11.3
$30,000–$39,999	12,018,154	10.8
$40,000–$49,999	10,778,124	9.7
$50,000–$74,999	21,221,889	19.0
$75,000–$99,999	13,214,551	11.8
$100,000–$149,999	12,164,206	10.9
$150,000–$199,999	3,981,276	3.6
$200,000 or more	3,817,000	3.4
	111,617,402	100.0%

Source: U.S. Census Bureau, American Fact Finder. http://factfinder.census.gov/servlet/DTTable?_bm=y&-geo_id=01000US&-ds_name=ACS_2006_EST_G00_&-_lang=en&-mt_name=ACS_2006_EST_G2000_B19001&-format=&-CONTEXT=dt.

ONE STEP AT A TIME — Constructing Frequency Distributions for Interval-Ratio Variables

Step ***Operation***

1. Decide how many class intervals (k) you wish to use. One reasonable convention suggests that the number of intervals should be about 10. Many research situations may require fewer than 10 intervals, and it is common to find frequency distributions with as many as 15 intervals. Only rarely will more than 15 intervals be used because the resultant frequency distribution would not be very concise.
2. Find the range (R) of the scores by subtracting the low score from the high score.
3. Find the size of the class intervals (i) by dividing R (from Step 2) by k (from Step 1).
$$i = R/k$$
Round the value of i to a convenient whole number. This will be the interval size or width.
4. State the lowest interval so that its lower limit is equal to or below the lowest score. By the same token, your highest interval will be the one that contains the highest score. Generally, intervals should be equal in size, but unequal and open-ended intervals may be used when convenient.
5. State the limits of the class intervals at the same level of precision as you have used to measure the data. Do not overlap intervals. You will thereby define the class intervals so that each case can be sorted into one and only one category.
6. Count the number of cases in each class interval and report these subtotals in a column labeled "frequency." Report the total number of cases (N) at the bottom of this column. The table may also include a column for percentages, cumulative frequencies, and cumulative percentages.
7. Inspect the frequency distribution carefully. Has too much detail been lost? If so, reconstruct the table with a greater number of class intervals (or smaller interval size). Is the table too detailed? If so, reconstruct the table with fewer class intervals (or use wider intervals). Are there too many intervals with no cases in them? If so, consider using open-ended intervals or intervals of unequal size. Remember that the frequency distribution results from a number of decisions you make in a rather arbitrary manner. If the appearance of the table seems less than optimal given the purpose of the research, redo the table until you are satisfied that you have struck the best balance between detail and conciseness.
8. Give your table a clear, concise title, and number the table if your report contains more than one. All categories and columns must also be clearly labeled.

2.6 CONSTRUCTING FREQUENCY DISTRIBUTIONS FOR INTERVAL-RATIO LEVEL VARIABLES: A REVIEW

We covered a lot of ground in the preceding section, so let's pause and review these principles by considering a specific research situation. Below are the numbers of visits received over the past year by 90 residents of a retirement community.

0	52	21	20	21	24	1	12	16	12
16	50	40	28	36	12	47	1	20	7
9	26	46	52	27	10	3	0	24	50
24	19	22	26	26	50	23	12	22	26
23	51	18	22	17	24	17	8	28	52
20	50	25	50	18	52	46	47	27	0
32	0	24	12	0	35	48	50	27	12
28	20	30	0	16	49	42	6	28	2
16	24	33	12	15	23	18	6	16	50

Listed in this format, the data are a hopeless jumble from which no one could derive much meaning. The function of the frequency distribution is to arrange and organize these data so that their meanings will be made obvious.

First, we must decide how many class intervals to use in the frequency distribution. Following the guidelines established in the previous section, let's use about 10 intervals. By inspecting the data, we can see that the lowest score is 0 and the highest is 52. The range of these scores is 52 to 0, or 52. To find the approximate interval size, divide the range (52) by the number of intervals (10). Since 52/10 = 5.2, we can set the interval size at 5.

The lowest score is 0, so the lowest class interval will be 0–4. The highest class interval will be 50–54, which will include the high score of 52. All that remains is to state the intervals in table format, count the number of scores that fall into each interval, and report the totals in a frequency column. These steps have been taken in Table 2.18, which also includes columns for the

TABLE 2.18 NUMBER OF VISITS PER YEAR, 90 RETIREMENT COMMUNITY RESIDENTS

Class Intervals	Frequency (*f*)	Cumulative Frequency	Percentage (%)	Cumulative Percentage
0–4	10	10	11.11	11.11
5–9	5	15	5.56	16.67
10–14	8	23	8.89	25.26
15–19	12	35	13.33	38.89
20–24	18	53	20.00	58.89
25–29	12	65	13.33	72.22
30–34	3	68	3.33	75.55
35–39	2	70	2.22	77.77
40–44	2	72	2.22	79.99
45–49	6	78	6.67	86.66
50–54	12	90	13.33	99.99
	$N = 90$		99.99%*	

*Percentage columns will occasionally fail to total 100% because of rounding error. If the total is between 99.90% and 100.10%, ignore the discrepancy. Discrepancies of greater than ±.10% may indicate mathematical errors, and the entire column should be computed again.

percentages and cumulative percentages. Note that this table is the product of several relatively arbitrary decisions. The researcher should remain aware of this fact and inspect the frequency distribution carefully. If the table is unsatisfactory for any reason, it can be reconstructed with a different number of categories and interval sizes.

Now, with the aid of the frequency distribution, some patterns in the data can be discerned. There are three distinct clusterings of scores in the table. Ten residents were visited rarely, if at all (the 0–4 visits per year interval). The single largest interval, with 18 cases, is 20–24. Combined with the intervals immediately above and below, this represents quite a sizable grouping of cases (42 out of 90, or 46.66% of all cases) and suggests that the dominant visiting rate is about twice a month, or approximately 24 visits per year. The third grouping is in the 50–54 class interval with 12 cases, reflecting a visiting rate of about once a week. The cumulative percentage column indicates that the majority of the residents (58.89%) were visited 24 or fewer times a year.

Application 2.5

The following list shows the ages of 50 prisoners enrolled in a work-release program. Is this group young or old? A frequency distribution will provide an accurate picture of the overall age structure.

18	60	57	27	19
20	32	62	26	20
25	35	75	25	21
30	45	67	41	30
37	47	65	42	25
18	51	22	52	30
22	18	27	53	38
27	23	32	35	42
32	37	32	40	45
55	42	45	50	47

To construct the frequency distribution we will follow the steps listed in the box One Step at a Time: Constructing Frequency Distributions for Interval-Ratio Variables, which appears at the end of Section 2.5:

1. Set number of categories at 10 ($k = 10$).
2. By inspection, we see that the youngest prisoner is 18 and the oldest is 75. The range is thus 57 ($R = 57$). Interval size will be 57/10, or 5.7, which we can round off to either 5 or 6. Let's use a six-year interval beginning at 18.
3. The limits of the lowest interval will be 18–23 and the highest will be 72–77.
4. The intervals will be stated in whole numbers (like the scores) and are presented in the table below.
5. The table presents the number of cases in each interval and the total number of cases (N), and it includes a percentage column.

Ages	Frequency	Percentage
18–23	10	20
24–29	7	14
30–35	9	18
36–41	5	10
42–47	8	16
48–53	4	8
54–59	2	4
60–65	3	6
66–71	1	2
72–77	1	2
	$N = 50$	100%

The prisoners seem to be fairly evenly spread across the age groups up to the 48–53 interval. There is a noticeable lack of prisoners in the oldest age groups and a concentration of prisoners in their 20s and 30s.

BECOMING A CRITICAL CONSUMER: Urban Legends, Road Rage, and Context

The statistics covered in this chapter may seem simple, even humble. However, as with any tool, they can be misunderstood and applied inappropriately. Here, we will examine some ways in which these statistics can be misused and abused and also reinforce some points about their usefulness in communicating information. We will finish by examining the ways in which percentages and rates are used in the professional research literature.

First of all, by themselves, statistics guarantee nothing about the accuracy or validity of a statement. False information, such as so-called urban legends, can be expressed statistically, and this may enhance their credibility in the eyes of many people. Consider, for example, the legend that the rate (number of incidents per 100,000 population) of domestic violence increases on Super Bowl Sunday, the day the championship game in American football is played. You may have heard a variation of this report that used specific percentages (for example, admissions to domestic abuse shelters rise by 50% in the city of the team that loses the Super Bowl). The credibility of reports such as these stems partly from the close association between football "macho" values, aggression, and violence. Also, people celebrate the Super Bowl with parties and gatherings during which large quantities of alcohol and other substances may be consumed. It seems quite reasonable that this heady mixture of macho values and alcohol would lead to higher rates of domestic violence.

The problem is that there is no evidence of a connection between the Super Bowl and spouse abuse. Two different studies, conducted at different times and locations, found no increase in spouse abuse on Super Bowl Sunday.[1,2] Of course, a connection may still exist at some level or in some form between football and domestic violence: maybe we just haven't found it. My point is that the mere presence of seemingly exact percentages (or any other statistic) is no guarantee of accuracy or validity. Incorrect, even outrageously wrong information can (and does) seep into everyday conversation and become part of what "everyone knows" to be true. The best one can do to guard against these false reports is to follow the scientific method and evaluate the evidence (if there is any) used to support the claim.

Of course, even "true" statistics and solid evidence can be misused. This brings up a second point: the need to carefully examine the context in which the statistic is reported. Sometimes, exactly the same statistical fact can be made to sound alarming and scary or trivial and uninteresting simply by changing the context in which the information is embedded. To illustrate, consider the phenomena of road rage, or aggressive driving. Angry drivers have been around since the invention of the automobile (and maybe since the invention of the wheel), but the term *road rage* entered the language in the mid-1990s, sparked by several violent incidents on the nation's highways and a frenzy of media coverage. We will follow sociologist Barry Glassner's analysis of road rage and look first at what the media reported at that time, then look at the realities.[3]

Beginning in the mid-1990s, the media began to characterize road rage as a "growing American danger," "an exploding phenomenon," and a "plague." One widely cited statistic was that incidents of road rage rose almost 60% between 1990 and 1996. This percentage change was based on two numbers: there were 1,129 road rage incidents in 1990 and 1800 in 1996. These values yield a percentage increase of 59.43%:

$$\text{Percentage change} = \left(\frac{f_2 - f_1}{f_1}\right) \times 100$$

$$= \left(\frac{1{,}800 - 1{,}129}{1{,}129}\right) \times 100 = \left(\frac{671}{1{,}129}\right) \times 100 = 59.43\%$$

The media reported the *percentage increase*—not the *frequency* of incidents—and a 60% increase certainly seems to justify the characterization of road rage as "an exploding phenomenon." However, in this case, it's the raw frequencies that are actually the crucial pieces of information.

Note how the perception of the percentage increase in road rage changes when it is framed in a broader context:

- Between 1990 and 1996, there were 20 *million* injuries from traffic accidents and about 250,000 fatalities on U.S. roadways.
- In this same period, there were a *total* of 11,000 acts of road rage.
- Alcohol is involved in about half of all traffic fatalities, road rage in about 1 in a 1,000.

(continued next page)

BECOMING A CRITICAL CONSUMER *(continued)*

In the context of the total volume of traffic mayhem, injury, and death (and alcohol-related incidents), is it reasonable to label road rage a "plague"?

As Professor Glassner says, "big percentages don't always have big numbers behind them." Road rage represents a miniscule danger compared to drunk driving. In fact, concern about road rage actually may be harmful if it deflects attention from more serious problems. Considered in isolation, the increase in road rage seems very alarming. When viewed against the total volume of traffic injury and death, the problem fades in significance.

In a related point, you should be aware of the time frame used to report changes in statistical trends. Consider that the homicide rate (number of homicides per 100,000 population) in the United States went up by 3.6% between 2000 and 2006, a change that could cause concern, fear, even panic in the general public. However, over a different time frame, from 1991 to 2006, the homicide rate actually declined by over 40%. Thus, different time frames can lead to different conclusions about the dangers of living in the United States.

Finally, let us consider the proper use of these statistics as devices for communicating facts clearly and simply. We'll use an example from professional social science research to make this final point. Social scientists rely heavily on the U.S. census for information about the characteristics and trends of change in American society, including age composition, birth and death rates, residential patterns, educational levels, and a host of other variables. Census data is readily available (at www.census.gov), but since it presents information about the entire population (over 300 million people), the numbers are often large, cumbersome, and awkward to use or understand. Percentages and rates are extremely useful statistical devices when analyzing or presenting census information.

Suppose, for example, that a report on voter turnout in the United States included the following information:

> In 1996, 127,648,000 of the 193,700,000 eligible voters actually turned out to cast their ballots in the national election. In 2004, on the other hand, 142,146,300 of the 215,700,000 Americans over 18 voted. Also, 66,432,000 of 103,800,000 males and 77,131,600 of 111,900,000 females voted in 2004.[4]

Can you distill any meaningful understanding about American politics from these sentences? Raw information simply does not speak for itself, and these facts have to be organized or placed in some context to reveal their meaning. Thus, social scientists almost always use percentages or rates to present this kind of information so that they can understand it themselves, assess the meaning, and convey their interpretations to others.

In contrast with the raw information above, consider the following short paragraph:

> Between 1996 and 2004, the percentage of voters in national elections who actually went to the polls remained unchanged at 65.9%. Furthermore, in 2004, women were more likely to turn out and vote, 67.6% versus 64.0% for men.

The second paragraph actually contains less information—because it omits the raw numbers and these are very important—but is much easier to comprehend.

Finally, remember that most research projects analyze interrelationships among many variables. Because the statistics covered in this chapter summarize variables one at a time, they are unlikely to be included in such research reports (or perhaps, included only as background information). Even when they are not reported, you can be sure that the research began with an inspection of percentages and frequency distributions for each variable.

[1]See: Oths, Kathryn, and Robertson, Tara. 2007. "Give Me Shelter: Temporal Patterns of Women Fleeing Domestic Abuse." *Human Organization*: 66: 249–260.
[2]Sachs, Carolyn and Chu, Lawrence. 2000. "The Association Between Professional Football Games and Domestic Violence in Los Angeles County." *Journal of Interpersonal Violence*. 15: 1192–1201.
[3]Barry Glassner. 1999. *The Culture of Fear: Why Americans Are Afraid of the Wrong Things*. Basic Books: New York.
[4]U.S. Bureau of the Census. 2008. *Statistical Abstract of the United States, 2008*. Washington, DC: Government Printing Office.

SUMMARY

1. We considered several different ways of summarizing the distribution of a single variable and, more generally, reporting the results of our research. Our emphasis throughout was on the need to communicate our results clearly and concisely. You will often find that as you strive to communicate statistical information to others, the meanings of the information will become clearer to you as well.
2. Percentages and proportions, ratios, rates, and percentage change represent several different techniques for enhancing clarity by expressing our results in terms of relative frequency. Percentages and proportions report the relative occurrence of some category of a variable compared with the distribution as a whole. Ratios compare two categories with each other, and rates report the actual occurrences of some phenomenon compared with the number of possible occurrences per some unit of time. Percentage change shows the relative increase or decrease in a variable over time.
3. Frequency distributions are tables that summarize the entire distribution of some variable. It is very common to construct these tables for each variable of interest as the first step in a statistical analysis. Columns for percentages, cumulative frequency, and/or cumulative percentages often enhance the readability of frequency distributions.

SUMMARY OF FORMULAS

FORMULA 2.1	Proportions:	$p = \frac{f}{N}$
FORMULA 2.2	Percentage:	$\% = \left(\frac{f}{N}\right) \times 100$
FORMULA 2.3	Ratios:	$\text{Ratio} = \frac{f_1}{f_2}$
FORMULA 2.4	Percentage change:	$\text{Percentage change} = \left(\frac{f_2 - f_1}{f_1}\right) \times 100$

GLOSSARY

Cumulative frequency. An optional column in a frequency distribution that displays the number of cases within an interval and all preceding intervals.

Cumulative percentage. An optional column in a frequency distribution that displays the percentage of cases within an interval and all preceding intervals.

Frequency distribution. A table that displays the number of cases in each category of a variable.

Midpoint. The point exactly halfway between the upper and lower limits of a class interval.

Percentage. The number of cases in a category of a variable divided by the number of cases in all categories of the variable, the entire quantity multiplied by 100.

Percentage change. A statistic that expresses the magnitude of change in a variable from time 1 to time 2.

Proportion. The number of cases in one category of a variable divided by the number of cases in all categories of the variable.

Rate. The number of actual occurrences of some phenomenon or trait divided by the number of possible occurrences per some unit of time.

Ratio. The number of cases in one category divided by the number of cases in some other category.

PROBLEMS

(Problems are labeled with the social science discipline from which they are drawn: SOC for sociology, SW for social work, PS for political science, CJ for criminal justice, PA for public administration, and GER for gerontology.)

2.1 SOC The tables that follow report the marital status of 20 respondents in two different apartment complexes. (*HINT: Make sure that you have the correct numbers in the numerator and denominator before solving the following problems. For*

example, Problem 2.1a asks for the percentage of respondents in each complex who are married, and the denominators will be 20 for these two fractions. Problem 2.1d, on the other hand, asks for the percentage of the single respondents who live in Complex B, and the denominator for this fraction will be 4 + 6, or 10.)

Status	Complex A	Complex B
Married	5	10
Unmarried (living together)	8	2
Single	4	6
Separated	2	1
Widowed	0	1
Divorced	1	0
	20	20

a. What percentage of the respondents in each complex are married?
b. What is the ratio of single-to-married respondents at each complex?
c. What proportion of each sample is widowed?
d. What percentage of the single respondents live in Complex B?
e. What is the ratio of the unmarried/living together to the married at each complex?

2.2 At St. Algebra College, the numbers of males and females in the various major fields of study are as follows.

Major	Males	Females	Totals
Humanities	117	83	200
Social sciences	97	132	229
Natural sciences	72	20	92
Business	156	139	295
Nursing	3	35	38
Education	30	15	45
Totals	475	424	899

Read each of the following problems carefully before constructing the fraction and solving for the answer. (*HINT: Be sure you place the proper number in the denominator of the fractions. For example, some problems use the total number of males or females as the denominator, but others use the total number of majors.*)

a. What percentage of social science majors are male?
b. What proportion of business majors are female?
c. For the humanities, what is the ratio of males to females?
d. What percentage of the total student body is males?
e. What is the ratio of males to females for the entire sample?
f. What proportion of the nursing majors are male?
g. What percentage of the sample are social science majors?
h. What is the ratio of humanities majors to business majors?
i. What is the ratio of female business majors to female nursing majors?
j. What proportion of the males are education majors?

2.3 CJ The town of Shinbone, Kansas, has a population of 211,732 and experienced 47 bank robberies, 13 murders, and 23 auto thefts during the past year. Compute a rate for each type of crime per 100,000 population. (*HINT: Make sure that you set up the fraction with size of population in the denominator.*)

2.4 CJ The numbers of homicides in five states and five Canadian provinces for the years 1997 and 2005 are reported below.

	1997		2005	
State	Homicides	Population	Homicides	Population
New Jersey	338	8,053,000	417	8,717,925
Iowa	52	2,852,000	38	2,966,334
Alabama	426	4,139,000	374	4,557,808
Texas	1,327	19,439,000	1,407	22,859,968
California	2,579	32,268,000	2,503	36,132,147

Source: http://www.fbi.gov/ucr/05cius/.

	1997		2005	
Province	Homicides	Population	Homicides	Population
Nova Scotia	24	936,100	20	936,100
Quebec	132	7,323,600	100	7,597,800
Ontario	178	11,387,400	218	12,558,700
Manitoba	31	1,137,900	49	1,174,100
British Columbia	116	3,997,100	98	4,257,800

Source: http://www.statcan.ca.

a. Calculate the homicide rate per 100,000 population for each state and each province for each year. Relatively speaking, which state and which province had the highest homi-

cide rates in each year? Which society seems to have the higher homicide rate? Write a paragraph describing these results.

b. Using the rates you calculated in part a, calculate the percentage change between 1997 and 2001 for each state and each province. Which states and provinces had the largest increase and decrease? Which society seems to have the largest change in homicide rates? Summarize your results in a paragraph.

2.5 SOC The scores of 15 respondents on four variables are reported below. These scores were taken from a public opinion survey called the General Social Survey, or the GSS. This data set, which is described in Appendix G, is used for the computer exercises in this text. Small subsamples from the GSS will be used throughout the text to provide "real" data for problems. For the actual questions and other details, see Appendix G. The numerical codes for the variables are as follows.

Sex	Support for Gun Control	Level of Education	Age
1 = Male	1 = In favor	0 = Less than high school	Actual years
2 = Female	2 = Opposed	1 = High school	
		2 = Junior college	
		3 = Bachelor's degree	
		4 = Graduate degree	

Case Number	Sex	Support for Gun Control	Level of Education	Age
1	2	1	1	45
2	1	2	1	48
3	2	1	3	55
4	1	1	2	32
5	2	1	3	33
6	1	1	1	28
7	2	2	0	77
8	1	1	1	50
9	1	2	0	43
10	2	1	1	48
11	1	1	4	33
12	1	1	4	35
13	1	1	0	39
14	2	1	1	25
15	1	1	1	23

Construct a frequency distribution for each variable. Include a column for percentages.

2.6 SW A local youth service agency has begun a sex education program for teenage girls who have been referred by the juvenile courts. The girls were given a 20-item test for general knowledge about sex, contraception, and anatomy and physiology upon admission to the program and again after completing the program. The scores of the first 15 girls to complete the program are listed below.

Case	Pretest	Posttest
A	8	12
B	7	13
C	10	12
D	15	19
E	10	8
F	10	17
G	3	12
H	10	11
I	5	7
J	15	12
K	13	20
L	4	5
M	10	15
N	8	11
O	12	20

Construct frequency distributions for the pretest and posttest scores. Include a column for percentages. (*HINT: There were 20 items on the test, so the maximum range for these scores is 20. If you use 10 class intervals to display these scores, the interval size will be 2. Since there are no scores of 0 or 1 for either test, you may state the first interval as 2–3. To make comparisons easier, both frequency distributions should have the same intervals.*)

2.7 SOC Sixteen high school students completed a class to prepare them for the College Board exams. Their scores are reported below.

420	345	560	650
459	499	500	657
467	480	505	555
480	520	530	589

These same 16 students were given a test of math and verbal ability to measure their readiness for college-level work. Scores are reported below in terms of the percentage of correct answers for each test.

Math Test			
67	45	68	70
72	85	90	99
50	73	77	78
52	66	89	75
Verbal Test			
89	90	78	77
75	70	56	60
77	78	80	92
98	72	77	82

Display each of these variables in a frequency distribution with columns for percentages and cumulative percentages.

2.8 GER The number of times 25 residents of a community for senior citizens left their homes for any reason during the past week is reported below.

0	2	1	7	3
7	0	2	3	17
14	15	5	0	7
5	21	4	7	6
2	0	10	5	7

a. Construct a frequency distribution to display these data.
b. What are the midpoints of the class intervals?
c. Add columns to the table to display the percentage distribution, cumulative frequency, and cumulative percentages.
d. Write a paragraph summarizing this distribution of scores.

2.9 SOC Twenty-five students completed a questionnaire that measured their attitudes toward interpersonal violence. Respondents who scored high believed that in many situations a person could legitimately use physical force against another person. Respondents who scored low believed that in no situation (or very few situations) could the use of violence be justified.

52	47	17	8	92
53	23	28	9	90
17	63	17	17	23
19	66	10	20	47
20	66	5	25	17

a. Construct a frequency distribution to display these data.
b. What are the midpoints of the class intervals?
c. Add columns to the table to display the percentage distribution, cumulative frequency, and cumulative percentage.
d. Write a paragraph summarizing this distribution of scores.

2.10 PA The city's department of transportation has been keeping track of accidents on a particularly dangerous stretch of highway. Early in the year, the city lowered the speed limit on this highway and increased police patrols. Data on number of accidents before and after the changes are presented below. Did the changes work? Is the highway safer?

Month	12 Months Before	12 Months After
January	23	25
February	25	21
March	20	18
April	19	12
May	15	9
June	17	10
July	24	11
August	28	15
September	23	17
October	20	14
November	21	18
December	22	20

YOU ARE THE RESEARCHER: Is There a "Culture War" in the United States?

One of the early steps in a research project is to inspect the variables by producing frequency distributions. If nothing else, an understanding of how the variables break down will be excellent background information, and sometimes you can use the tables to begin to answer research questions. In this installment of You Are the Researcher, you will use SPSS to produce summary tables for several variables that measure attitudes about controversial issues in U.S. society and that may map the battlefronts in what many call the American culture wars.

There is a great deal of disagreement about a number of issues and values that seem to divide the United States along religious, political, and cultural lines. We might characterize the opposing sides in terms of liberal versus conservative, modern versus traditional, or progressive versus old school, and some of the most bitter debates along these lines include the topics of abortion, gay marriage, and gun control, along with many other issues. As you know, debates over issues like these can be intense, bitter, and even violent: adherents of one position may view their opponents with utter contempt, blast them with insults, demonize them, and dismiss their arguments. How deep is this fault line in U.S. society? How divided are the American people?

We can begin to investigate these questions by examining variables from the 2006 GSS. Pick three of these variables that seem to differentiate the sides in the American culture war (see Appendix A or click **Utilities → Variables** on the menu bar of the **Data Editor Window** of SPSS for a list of variables).

Before continuing, let's take a moment to consider this process of picking variables. Technically, selecting a variable to represent or stand for a concept is called *operationalization,* and this can be one of the most difficult steps in a research project. On one hand, we have a concept that, as in the case of culture wars, can be quite abstract and subject to a variety of perspectives. What exactly is a culture war, and what positions are liberal, traditional, conservative, progressive, and so forth? In order to do research, we must use concrete, specific variables to represent our abstract and general concepts, but which variables relate to which concepts?

Any pairing we make between variables and concepts is bound to be at least a little arbitrary. In many cases, the best strategy is to use several variables to represent the concept: if our operationalizations are reasonable, our selected variables will behave similarly, and each will behave as the abstract concept would if we could measure it directly. This is why I ask you to select three different variables to represent the culture wars. Each of you may select different variables, but if everyone makes reasonable decisions, the chosen variables should be close representations of the concept.

After you have made your selections, complete the following steps. Forms for recording your decision are available at the Web site for this text (www.cengage.com/sociology/healey).

STEP 1: Identify Your Three Variables

Variable 1: SPSS name _____

Explain exactly what this variable measures:

Variable 2: SPSS name _____

Explain exactly what this variable measures:

Variable 3: SPSS name _____

Explain exactly what this variable measures:

STEP 2: Operationalization

Explain why you selected each variable to represent an issue in the culture wars. How is the issue measured by the variable related to the debate? Which value or response of the variable indicates that the respondent is liberal or progressive, and which indicates conservative or traditional?

SPSS name of variable 1: _____

How does this variable relate to or exemplify the culture war?

Which value or response (e.g., "agree" or "support") is

Liberal: _____

Conservative: _____

SPSS name of variable 2: _____

How does this variable relate to or exemplify the culture war?

Which value or response (e.g., "agree" or "support") is

Liberal: _____

Conservative: _____

SPSS name of variable 3: _____

How does this variable relate to or exemplify the culture war?

Which value or response (e.g., "agree" or "support") is

Liberal: _____

Conservative: _____

STEP 3: Using SPSS for Windows to Produce Frequency Distributions

Now we are ready to generate some output and get some background on the nature of disagreements over values and issues among Americans. If necessary, click the SPSS icon on your monitor screen to start SPSS for Windows. Load the 2006 GSS by clicking the file name on the first screen or by clicking **File, Open,** and **Data** on the **SPSS Data Editor screen.** You may have to change the drive specification to locate the 2006 GSS data supplied with this text (probably named **GSS2006.sav**). Double-click the file name to open the data set. When you see the message "SPSS Processor is Ready" on the bottom of the screen, you are ready to proceed.

Generating Frequency Distributions

We produced and examined a frequency distribution for the variable *sex* in Appendix F. Use the same procedures to produce frequency distributions for the three variables you used to represent the American culture wars. From the menu bar, click **Analyze.** From the menu that drops down, click **Descriptive Statistics** and **Frequencies.** The **Frequencies** window appears with the variables listed in alphabetical order in the left-hand box. The window may display variables by name (e.g., *abany*, *abhlth*) or by label (e.g., ABORTION IF WOMAN WANTS FOR ANY REASON). If labels are displayed, you may switch to variable names by clicking **Edit, Options,** and then making the appropriate selections on the **General** tab. See Appendix F and Table F.2 for further information.

Find the first of your three variables, click on its name to highlight it, and then click the arrow button in the middle of the screen to move it to the right-hand window. Find your other two variables and follow the same procedure to move their names to the right-hand window. SPSS will process all variables listed in the right-hand box together. Click **OK** in the upper-right-hand corner of the **Frequencies** window, and SPSS will rush off to create the frequency distributions you requested.

The tables will be in the **SPSS Viewer** window that will now be closest to you on the screen. The tables, along with other information, will be in the right-hand

box of the window. To change the size of the output window, click the middle symbol (shaped like either a square or two intersecting squares) in the upper-right-hand corner of the **Output** window.

Reading SPSS Frequency Distributions

I will illustrate how to decipher the SPSS output using the variable *marital,* which measures current marital status. I chose *marital* so as not to duplicate any of the variables you selected in Step 1. The output looks like this.

MARITAL STATUS

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	MARRIED	686	48.1	48.1	48.1
	WIDOWED	119	8.3	8.4	56.5
	DIVORCED	222	15.6	15.6	72.1
	SEPARATED	39	2.7	2.7	74.8
	NEVER MARRIED	359	25.2	25.2	100.0
	Total	1425	99.9	100.0	
	Missing NA	1	.1		
	Total	1426	100.0		

Let's examine the elements of this table. The variable name is printed at the top of the output (MARITAL STATUS). The various categories are printed on the left. Moving one column to the right, we find the actual frequencies or the number of times each score of the variable occurred. We see that 686 of the respondents were married, 119 were widowed, and so forth. Next are two columns that report percentages. The entries in the Percent column are based on all respondents who were asked this question and include the scores NA (No Answer), DK (Don't Know), or NAP (Not applicable). The Valid Percent column eliminates all cases with missing values. Since we almost always ignore missing values, we will pay attention only to the Valid Percent column (even though, in this case, only one respondent did not supply this information and the columns are virtually identical). The final column is a Cumulative Percentage column (see Table 2.14). For nominal level variables like *marital,* this information is not meaningful, since the order in which the categories are stated is arbitrary.

The three frequency distributions you generated will have the same format and can be read in the same way. Use these tables—especially the Valid Percent column—to complete Step 4.

STEP 4: Interpreting Results

Characterize your results by reporting the percentage (not the frequencies) of respondents who endorsed each response. How large are the divisions in American values? Is there consensus on the issue measured by your variable (do the great majority endorse the same response) or is there considerable disagreement? The lower the consensus, the greater the opportunity for the issue to be included in the culture war.

SPSS name of variable 1: _____

Summarize the frequency distribution in terms of the percentage of respondents who endorsed each position:

Are these results consistent with the idea that there is a "war" over Americans values? How?

SPSS name of variable 2: _____

Summarize the frequency distribution in terms of the percentage of respondents who endorsed each position:

Are these results consistent with the idea that there is a "war" over Americans values? How?

SPSS name of variable 3: _____

Summarize the frequency distribution in terms of the percentage of respondents who endorsed each position:

Are these results consistent with the idea that there is a "war" over American values? How?

3 Charts and Graphs

LEARNING OBJECTIVES

By the end of this chapter, you will be able to:

1. Explain the usefulness of graphs and charts as descriptive statistics.
2. Identify which types of graphs should be used with variables at different levels of measurement.
3. Analyze and interpret the meaning of all graphs and charts presented in this chapter.

Researchers frequently use charts and graphs to present their data in ways that are visually more dramatic than tables and frequency distributions. These devices are particularly useful for conveying an impression of the overall shape of a distribution and for highlighting any clustering of cases in a particular range of scores. Many graphing techniques are available, but we will examine just five. The first two, pie charts and bar charts, are appropriate for variables with a limited number of categories at any level of measurement. The next two graphs, histograms and line charts, are used with ordinal and interval-ratio variables, particularly variables that have a large number of scores. Finally, we will consider population pyramids, graphs that are used to display the sex and age characteristics of large groups of people.

These days, computer programs such as Microsoft Excel are almost always used to produce graphs and charts. Graphing software is sophisticated and flexible and also relatively easy to use. If such programs are available to you, you should familiarize yourself with them; the effort required to learn these programs will be repaid in the quality of the final product. The section on computer applications at the end of this chapter explains how to produce charts and graphs using SPSS.

3.1 GRAPHS FOR NOMINAL LEVEL VARIABLES

Two types of charts are widely used for nominal level variables with few scores: pie charts and bar charts. Both are essentially a visual display of the frequency distribution for the variable. We will consider each in turn.

Pie Charts. **Pie charts** are generally used to display the percentage of cases in each category of a variable. Each segment or slice of the "pie" or circle represents the percentage of cases in that category: the bigger the slice, the larger the relative size of the category.

The pie chart in Figure 3.1 displays the marital status of the counseling center survey respondents from Chapter 2. The frequency distribution in Chapter 2's Table 2.7 is reproduced here as Table 3.1, with a column added for the percentage distribution. Since a circle's circumference is 360°, we apportion 180° (or 50%) for the first category, 126° (35%) for the second, and 54° (15%) for the last category. The pie chart visually reinforces the relative preponderance of single respondents and the relative absence of divorced students in the counseling center survey.

For additional examples, consider Figures 3.2 and 3.3, which show the relative size of racial and ethnic groups in the United States in 2000 and the projected

TABLE 3.1 MARITAL STATUS OF RESPONDENTS, COUNSELING CENTER SURVEY

Status	Frequency (f)	Percentage (%)
Single	10	50
Married	7	35
Divorced	3	15
	$N = 20$	100%

FIGURE 3.1 MARITAL STATUS OF RESPONDENTS, COUNSELING CENTER SURVEY

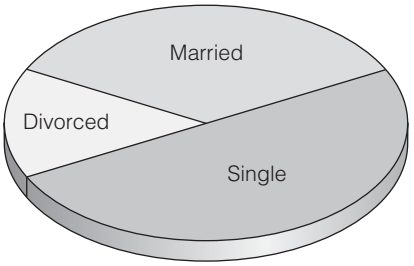


FIGURE 3.2 RACIAL AND ETHNIC GROUPS IN U.S. SOCIETY (PERCENT OF TOTAL POPULATION) IN 2000

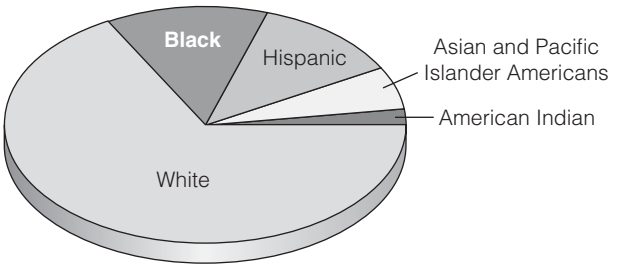


FIGURE 3.3 PROJECTED SIZES OF RACIAL AND ETHNIC GROUPS IN U.S. SOCIETY (PERCENT OF TOTAL POPULATION) IN 2050

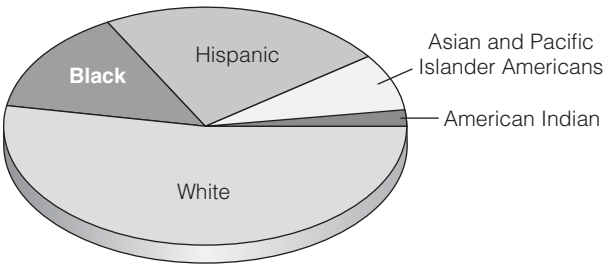


TABLE 3.2 RACIAL AND ETHNIC GROUPS IN U.S. SOCIETY IN 2000 AND PROJECTIONS FOR 2050 (Percent of Total Population)

Group	2000	2050
White Americans	71%	53%
Black Americans	12%	13%
Hispanic Americans	12%	24%
Asian and Pacific Islander Americans	4%	9%
American Indians	1%	1%

size of those groups in 2050. Note how these figures dramatically and clearly illustrate the growing diversity of U.S. society. The data are presented in Table 3.2.

Both Table 3.2 and Figures 3.2 and 3.3 tell the same story: the relative population of white Americans will shrink; Hispanic, Asian, and Pacific Island American populations will grow as a percentage of the total population; and black Americans and American Indian populations will stay the same relative size.

Bar Charts. Like pie charts, **bar charts** are relatively straightforward. Conventionally, the categories of the variable are arrayed along the horizontal axis (or abscissa) and frequencies, or percentages if you prefer, along the vertical axis (or ordinate). For each category of the variable, construct (or draw) a rectangle of constant width and a height that corresponds to the number of cases in the category. The bar chart in Figure 3.4 reproduces the marital status data from Table 3.1 and Figure 3.1.

This chart would be interpreted in exactly the same way as the pie chart in Figure 3.1, and researchers are free to choose between these two methods of displaying data. However, if a variable has more than five or six categories, the bar chart would be preferred to a pie chart. With too many categories, the pie chart gets very crowded and loses its visual clarity. To illustrate, Figure 3.5 uses a bar chart to display the data on visiting rates for the retirement community presented in Chapter 2 (Table 2.18). A pie chart for this same data would have had 11 different "slices," a more complex or busier picture than that presented by the bar chart. In Figure 3.5, the clustering of scores in the 20 to 24 range (approximately two visits a month) is readily apparent, as are the groupings in the 0 to 4 and 50 to 54 ranges.

FIGURE 3.4 MARITAL STATUS OF RESPONDENTS, COUNSELING CENTER SURVEY

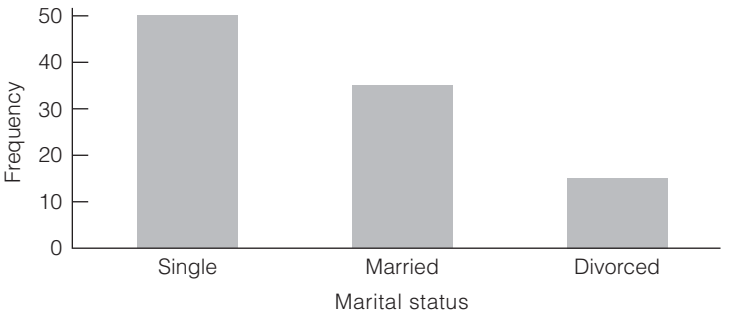


FIGURE 3.5 VISITS PER YEAR, RETIREMENT COMMUNITY RESPONDENTS

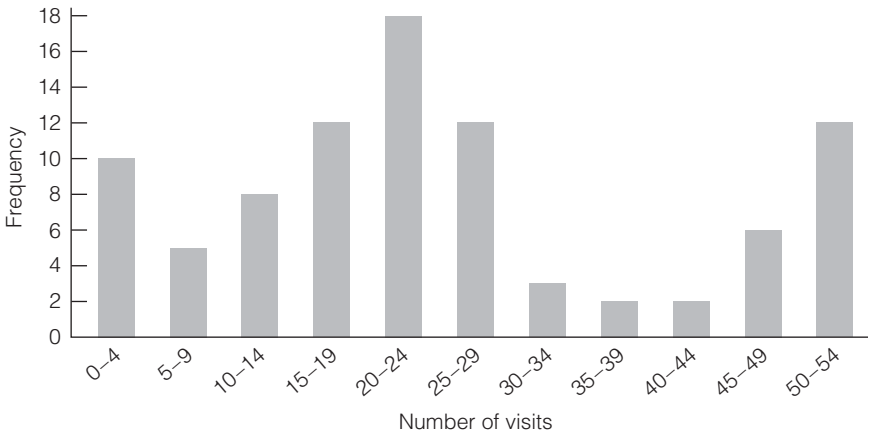
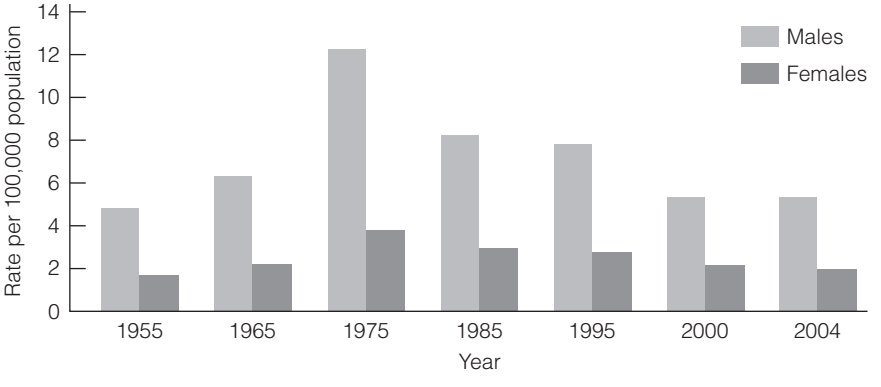


FIGURE 3.6 HOMICIDE VICTIMIZATION RATES FOR MALES AND FEMALES, SELECTED YEARS (WHITES ONLY)



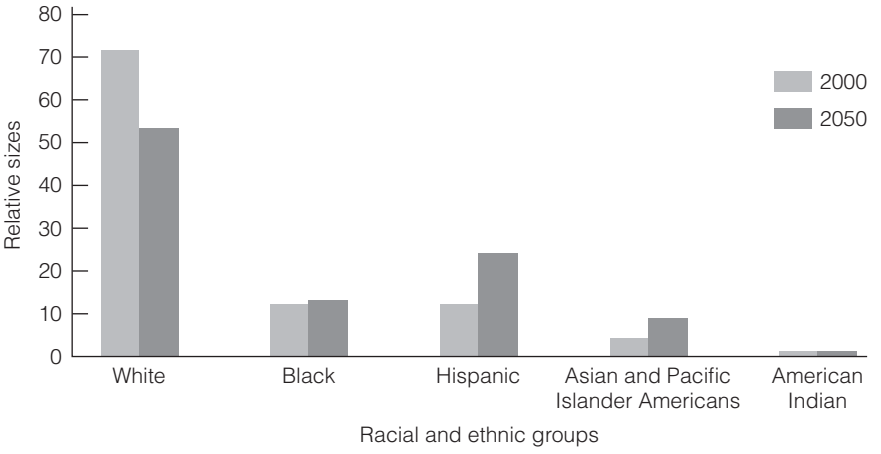
Source: U.S. Bureau of the Census. 2008. *Statistical Abstract of the United States, 2008.* Washington, DC: Government Printing Office. P. 196.

Bar charts are particularly effective ways to display the relative frequencies for two or more categories of a variable when you want to emphasize some comparisons. Suppose, for example, that you wished to make a point about changing rates of homicide victimization for white males and females since 1955. Figure 3.6 displays the data in a dramatic and easily comprehended way. The bar chart shows that

- rates for males are much higher than rates for females,
- rates for both sexes were highest in 1975, and
- rates have been generally declining since 1975, with a leveling off for males in the most recent time periods.

As a final example, consider Figure 3.7, which places the information in the pie charts in Figures 3.2 and 3.3 into a grouped bar format. This format clarifies

FIGURE 3.7 RELATIVE SIZES OF U.S. RACIAL AND ETHNIC GROUPS, 2000 AND 2050



both the relative group sizes in both years and the changes that are projected to take place over the half century. It is particularly easy to see the relative numerical decline of white Americans and the growth of Hispanic, Asian, and Pacific Islander Americans. *(For practice in constructing and interpreting pie and bar charts, see Problems 3.1 and 3.5.)*

3.2 GRAPHS FOR INTERVAL-RATIO LEVEL VARIABLES

In this section, we consider two types of graphs that can be used with ordinal and interval-ratio variables that have many scores. Like bar and pie charts, these graphs can be used interchangeably at the discretion of the researcher.

Histograms. **Histograms** look a lot like bar charts and, in fact, are constructed in much the same way. However, in histograms, the bars representing the frequency of each score are contiguous, their borders touching as if they merge in a continuous series from the lowest to highest scores. Histograms are particularly appropriate for interval-ratio variables (such as income or age) that have many scores covering a wide range.

Let's examine the anatomy of a histogram. The class intervals or scores of the variable are arrayed along the horizontal axis, and the frequencies are arrayed along the vertical axis. A bar is drawn over the scores of each interval. The height of the bar corresponds to the number of cases in the category: the higher the bar, the more common the score.

For example, Figure 3.8 uses a histogram to display the distribution of ages for a sample of respondents to a national public opinion poll. The graph shows that the respondents in the sample are concentrated in their late 20s, mid-30s, and early 40s and that the number of respondents declines with age. Note also that there are no people in the sample younger than age 18, the usual cutoff point for respondents to public opinion polls.

We will consider an additional example of a histogram before moving on. Table 3.3 shows the distribution of household income for the United States in 2006, using more detail than seen in Table 2.17 in Chapter 2. Note that the highest interval in the table is open-ended since it would be extremely difficult (perhaps impossible and probably unnecessary) to state all of the intervals between

FIGURE 3.8 AGE FOR A NATIONAL SAMPLE OF RESPONDENTS

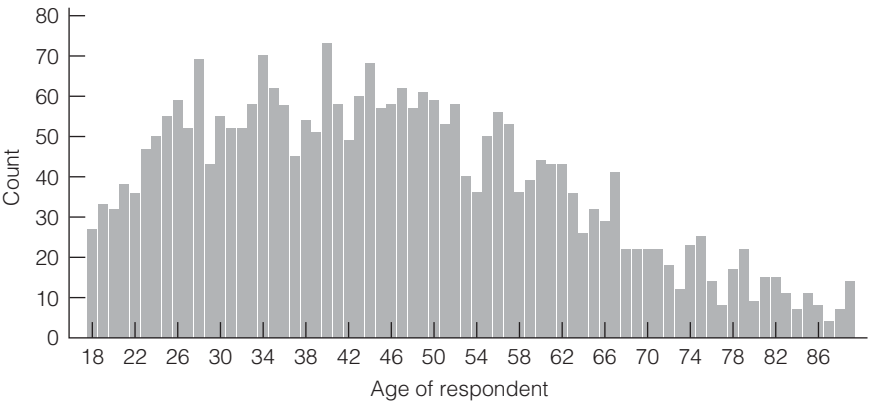


TABLE 3.3 DISTRIBUTION OF INCOME FOR HOUSEHOLDS, UNITED STATES, 2006

Income	Percent of All Households in Bracket	Cumulative Percent
Less than $10,000	7.97	7.97
$10,000–$14,999	5.95	13.92
$15,000–$19,999	5.57	19.50
$20,000–$24,999	5.82	25.32
$25,000–$29,999	5.52	30.84
$30,000–$34,999	5.63	36.47
$35,000–$39,999	5.14	41.61
$40,000–$44,999	5.11	46.71
$45,000–$49,999	4.55	51.26
$50,000–$59,999	8.48	59.74
$60,000–$74,999	10.53	70.28
$75,000–$99,999	11.84	82.12
$100,000–$124,999	7.11	89.22
$125,000–$149,999	3.79	93.01
$150,000–$199,999	3.57	96.58
$200,000 or more	3.42	100.00
	100.00%	

an income of $200,000 and the highest income in the nation. The bottom interval is also stated as an open-ended interval (although, given the nature of the variable, we know that the actual lower limit is zero). The intervals are unequal in size, and a column for cumulative percentages has been included. Using the latter, we can see that about a third of all households have incomes below $30,000, and about half of all households have incomes higher than $50,000.

Even though Table 3.3 is straightforward, the histogram presented in Figure 3.9 makes it easier to see and comprehend the basic shape of the distribution of income in the United States. We can see a noticeable grouping of cases (indicated by high bars) in the lowest income interval, a very large grouping of cases in the $50,000 to $100,000 range (we might call these middle-income Americans), and a gradual decline of cases in the highest income brackets.

FIGURE 3.9 DISTRIBUTION OF HOUSEHOLD INCOME, UNITED STATES, 2006

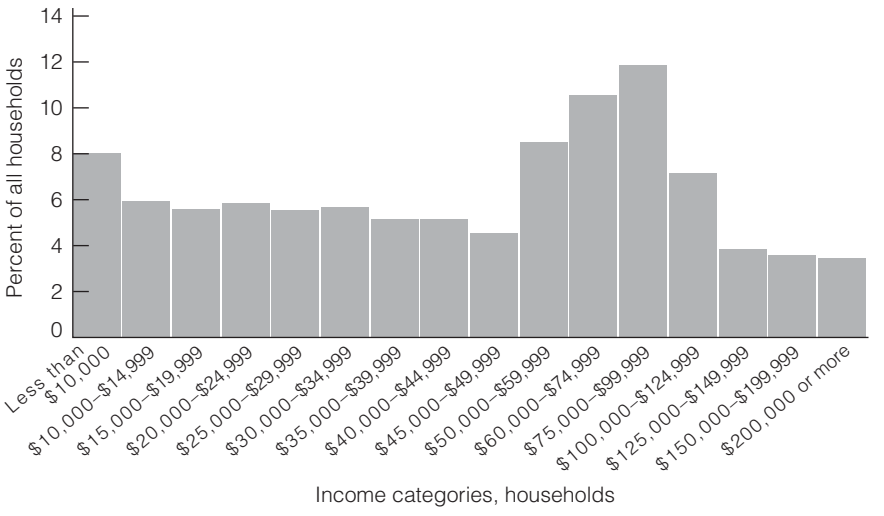
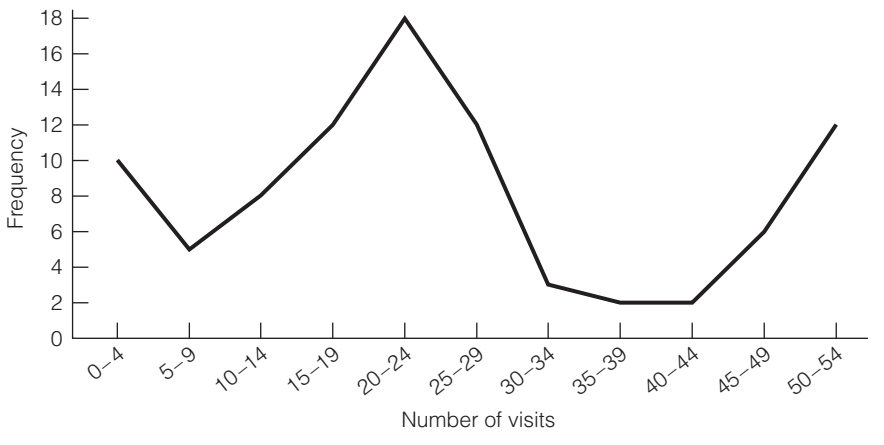


FIGURE 3.10 NUMBER OF VISITS PER YEAR. RETIREMENT COMMUNITY RESIDENTS



Line Charts. Construction of a **line chart** or **frequency polygon** is similar to construction of a histogram. Instead of using bars to represent the frequencies, however, use a dot at the midpoint of each interval. Straight lines then connect the dots. Figure 3.10 displays a line chart for the visiting data previously displayed in the bar chart in Figure 3.5.

Line charts are very effective ways of displaying trends across time. Figure 3.11 shows both marriage and divorce rates per 1,000 population for the United States since 1950. Note that both rates rose until the early 1980s and have been falling since, with the marriage rate falling slightly faster.

Line charts can use multiple lines and even multiple axes to convey a great deal of information in a compact space. Consider Figure 3.12, which shows the changing income gap between the genders over a 50-year period. The data include only people who worked full time for the entire year. This eliminates

FIGURE 3.11 MARRIAGE AND DIVORCE RATES PER 1,000 POPULATION, UNITED STATES, 1950–2006

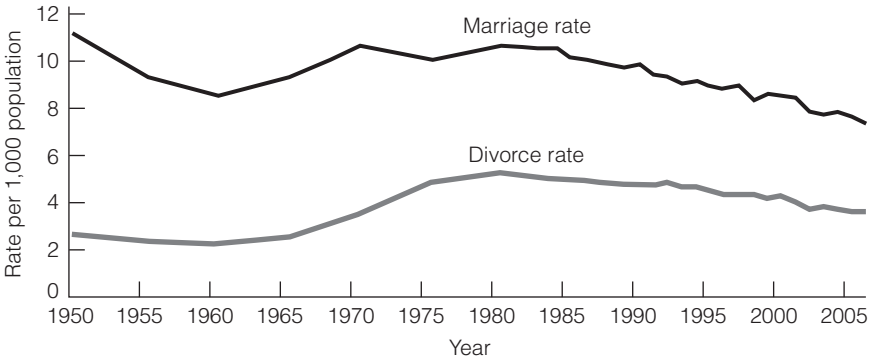
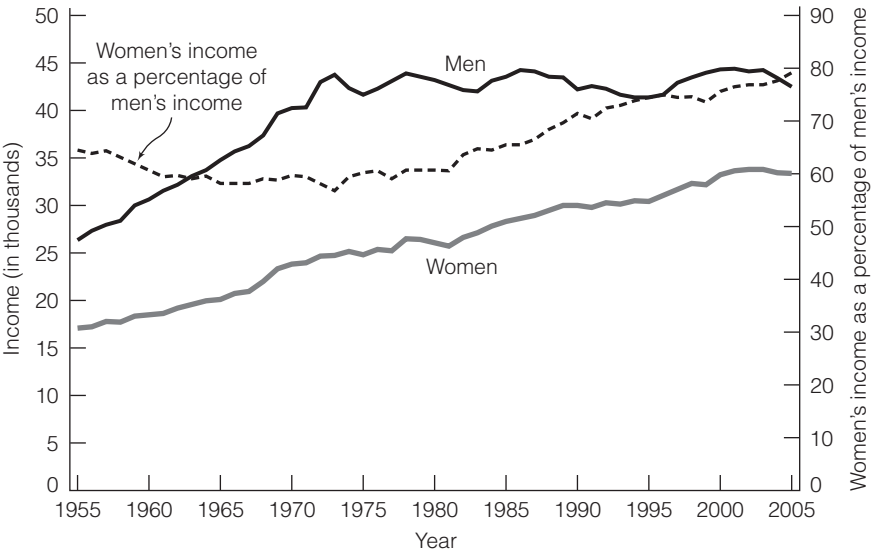


FIGURE 3.12 INCOME FOR FULL-TIME, YEAR-ROUND WORKERS BY GENDER, 1955–2005, IN 2005 DOLLARS



(*Note:* Read income on left-hand axis and percentages on right-hand axis.)

any differences in income between men and women created by differences in their participation in the paid labor force. Also, incomes are expressed in 2005 dollars so as to eliminate the effect of the changing value of the dollar.

Begin by inspecting the horizontal and vertical axes of the graph. The horizontal axis is calibrated in years, marked off in intervals of five years. There are two vertical axes, each measuring a different variable. The left vertical axis is calibrated in dollars and shows average income, whereas the right vertical axis is expressed in percentages and shows women's income as a percentage of men's income.

The body of the graph has three lines, two solid and one dashed. The top and bottom solid lines represent average incomes for men and women, respectively, and the values for these lines are read on the left-hand vertical axis (the specific

statistic used is median income, which we will consider in the next chapter). As you can see, men's income rose until the 1970s, when it leveled off and actually fell in some years. This pattern is due to many factors, including the loss of well-paid manual labor and factory jobs to other nations with cheaper work forces.

The bottom solid line shows average income for women and shows a steady increase throughout the time period. Again, many factors lie behind the comparatively good fortune of women, including the fact that they were generally less employed in the sectors of the economy most affected by the movement of good factory jobs offshore.

The rise of women's wages relative to that of men is captured by the third line in the graph, the dashed line that runs between the other two lines. This line represents women's income relative to men's, and the values for this line are read from the right-hand vertical axis. At the start of the time period, women earned about 65% of what men earned. This figure rose to almost 80% by the end of the time period.

Figure 3.12 shows that U.S. society has moved closer to gender equity in pay, but also that a gap of about 20% still persists between men and women's wages.

Histograms and frequency polygons are alternative ways of displaying essentially the same message. Thus, the choice between the two techniques is left to the aesthetic pleasure of the researcher. *(For practice in constructing and interpreting histograms and line charts, see Problems 3.2, 3.3, 3.4, 3.6, and 3.7.)*

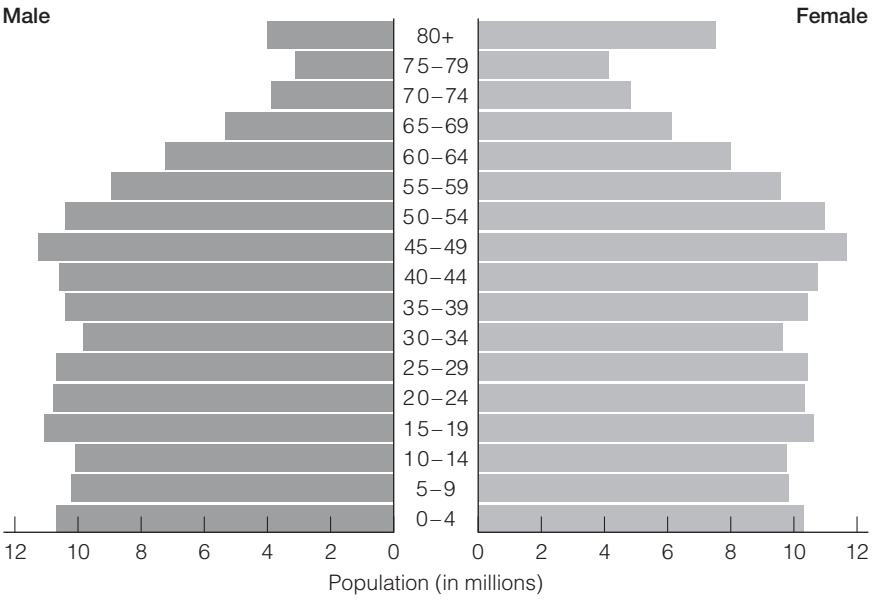
3.3 POPULATION PYRAMIDS

One very commonly used graph in the social sciences is the **population pyramid.** This graph displays basic demographic information about a society or community in a very compact and easily understood format. The anatomy of the population pyramid is straightforward. It has two axes. The horizontal axis is calibrated in terms of numbers of people or the percentage of the total population. Percentages are generally preferred for this axis because it allows us to easily compare populations of different size. The vertical axis represents age groups, usually in intervals (or cohorts) of five years. Males are counted to the left and females to the right. Each bar in the figure represents the number or percentage of the total population in each age-sex group.

To illustrate, consider Figure 3.13, which displays the population pyramid for the United States for 2008. Note that the horizontal axis is calibrated in raw numbers, not percentages. The bottom bar on the left shows that there were about 11 million boys aged 0–4 in 2008. The comparable bar for girls shows a slightly smaller population of a little more than 10 million. At the top of the figure, the effect of the higher life expectancy for females is clearly displayed: the bar for females aged 80 and up is much wider than the bar for males in the same age group. Also note the prominent "bulge" in the pyramid for the age groups 40–59: the people in these age cohorts were born between 1949 (2008 minus 59) and 1968 (2008 minus 40) and compose the so-called baby boom that was produced by the relatively high birth rate in American society during these years. There is a second bulge in the pyramid at the bar denoting the age group that is 20 years younger than the baby boom generation: these are the children of the boomers.

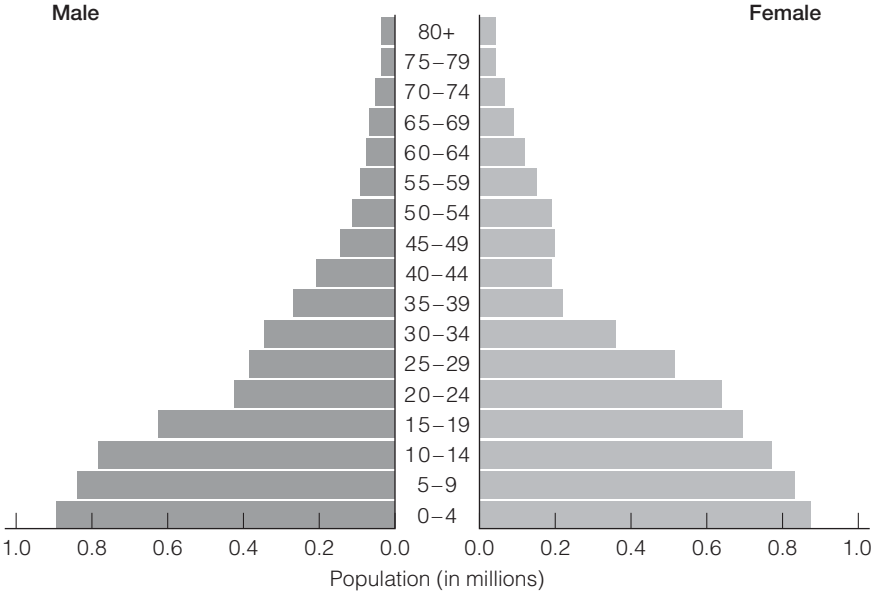
Finally, note that the sides of the U.S. pyramid are relatively straight and don't begin to slope inward until age 60 or so. This indicates that the death rate in the United States is relatively low and that most people survive until a relatively old age. In contrast, consider Figure 3.14, which presents the population pyramid for Zimbabwe, an impoverished nation in southern Africa. Perhaps

FIGURE 3.13 POPULATION PYRAMID FOR THE UNITED STATES, 2008



Source: U.S. Census Bureau. International Data Base.

FIGURE 3.14 POPULATION PYRAMID FOR ZIMBABWE, 2008



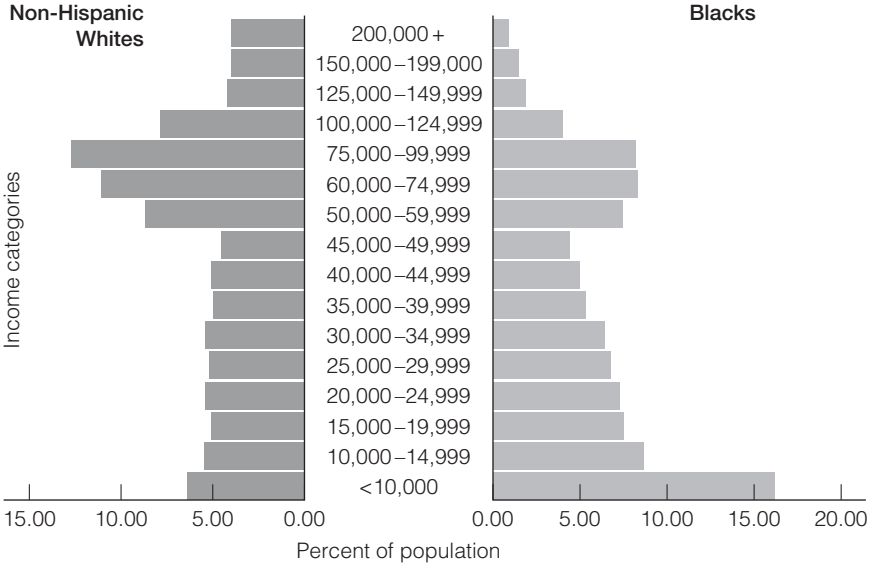
Source: U.S. Census Bureau. International Data Base.

the most noticeable difference between the two pyramids is that Zimbabwe's pyramid is much more triangular in shape. The broad base, relatively narrow top, and sharply sloping sides are the hallmarks of a less-developed nation with high birthrate and high death rate. Relatively many children are born (hence the broad base), but relatively few survive to old age (or even middle age), indicating a high death rate. What other contrasts and similarities can you identify?

The shape and logic of the population pyramid can be used for other purposes. For example, the pyramid provides a useful way to graph income inequality between blacks and whites in the United States. Like the gender gap, the racial income gap has shrunk over the years, but huge differences remain. Some of these differences are illustrated in Figure 3.15, which shows the distribution of income for full-time year-round black and white workers in 2006.

The horizontal axis is calibrated by the percentage of the population, and blacks are counted on the right and whites on the left. The vertical axis displays income rather than age, and instead of age-sex groups, the bars in this pyramid show the distribution of race-income groups. The higher rate of poverty in the black community is reflected in the wider bars in the right-hand side of the bottom of the figure, whereas the shorter bars at the top right-hand side of the figure reflect the lower levels of black affluence. For both groups, there is a noticeable grouping in the middle income areas ($50,000–$100,000), but again, the greater relative affluence of the white community is reflected in their wider bars. Taken as a whole, Figure 3.15 shows that both groups include people who are poor, middle income, and rich, but the relative sizes of the bars in the different income ranges clearly refutes the common idea that there are no longer any important racial differences in income in the United States.

FIGURE 3.15 DISTRIBUTION OF HOUSEHOLD INCOME FOR NON-HISPANIC WHITES AND BLACKS, 2006



BECOMING A CRITICAL CONSUMER: Graphing Social Trends

Pictures can paint a thousand words, but they can also be used to shape perceptions, encourage viewers to come to a certain conclusion, or simply deceive and mislead. This might be done subtly, with the use of color or shading on a bar or pie chart. For example, red is more dramatic and noticeable than other colors and may be used to influence perception and make a certain bar or "slice" on a graph stand out. For example, reconsider Figures 3.2 and 3.3. How could color or shading be used in these figures to alarm prejudiced white Americans and create or increase a sense of threat among them?

Another obvious technique for shaping the perception of the reader is to change the scales of either axis of a graph to stress or minimize some pattern or trend. To illustrate, reconsider the divorce rate presented in Figure 3.10. Suppose you wanted to argue that there has been little change in divorce over the years. One way to visually reinforce this point would be to increase the scale of the vertical axis, a change that will flatten the lines in the graph and minimize the sense of change. The first of the two figures below (Figure A) shows the divorce rate with the original scale (0 to 6 per 100,000 population) used in Figure 3.10. The second

FIGURE A DIVORCE RATE PER 100,000 POPULATION, 1950–2006 (ORIGINAL SCALE)

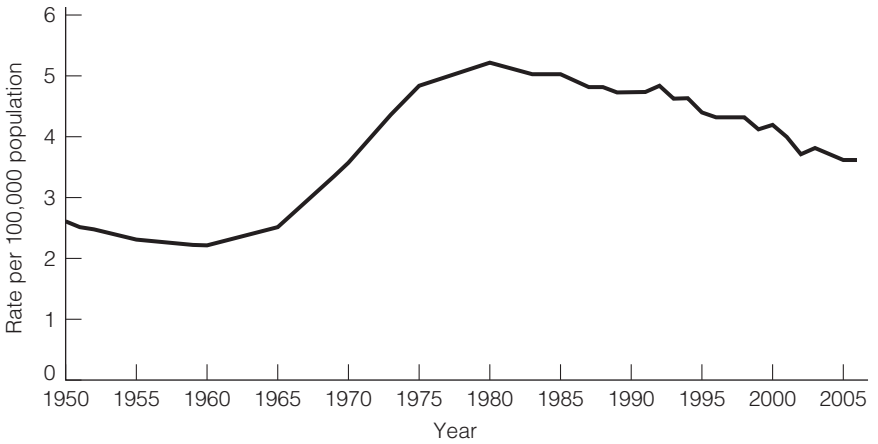
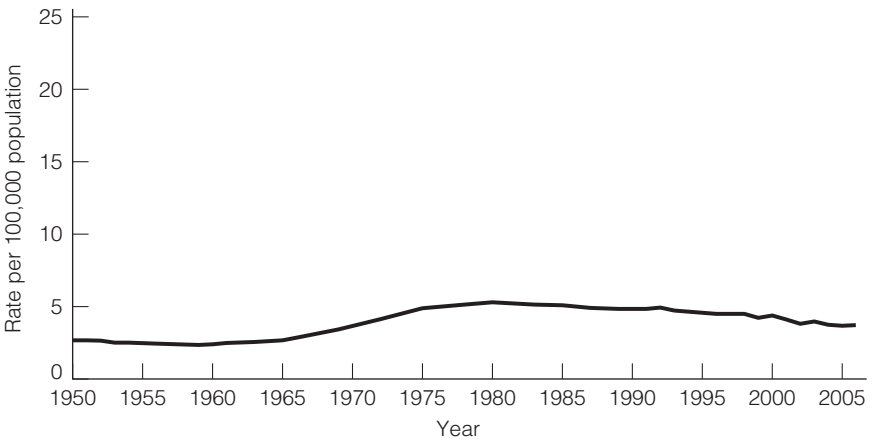


FIGURE B DIVORCE RATE PER 100,000 POPULATION, 1950–2006 (VERTICAL AXIS CALIBRATED WITH A MUCH BROADER RANGE OF SCORES)



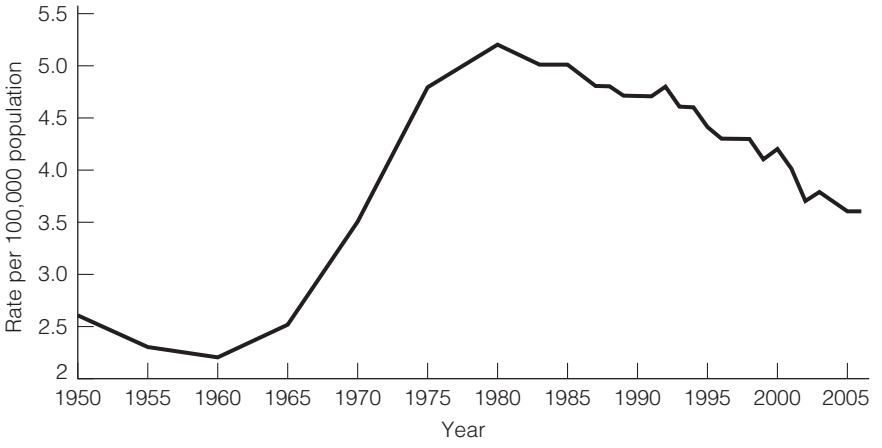
(continued next page)

BECOMING A CRITICAL CONSUMER *(continued)*

(Figure B) shows exactly the same data, but with the vertical axis calibrated with a much broader range of scores (0–25 divorces per 100,000 population). Note how the line in Figure B hardly changes, suggesting that the divorce rate has not fluctuated much over the 50-year period. You should be immediately suspicious of any graph that uses a large scale or includes a lot of blank or white space.

You can also change the scale of the vertical axis to exaggerate or emphasize the degree of change. Smaller units on the vertical axis will make the trends seem much more dramatic, as in Figure C below. The smaller scale (0–5.5) emphasizes both the increase in the divorce rate from the late 1960s through the late 1970s and the decline since.

FIGURE C DIVORCE RATE PER 100,000 POPULATION, 1950–2006 (SMALLER UNITS ON THE VERTICAL AXIS)



Although, when exaggerated, these techniques can be used to "lie" with statistics, they are also part of the tool kit that researchers use to support their points. We need to remember that charts and graphs—like all statistical techniques—are pieces of an argument, bits of evidence presented in a form that the author believes best expresses his or her ideas and supports his or her conclusions. At the same time, we should also use our own critical faculties to independently assess charts and graphs and see if we agree with the author.

SUMMARY

1. Pie and bar charts, histograms, line charts, and population pyramids are graphic devices used to express the basic information contained in the frequency distribution in a compact and visually dramatic way.

2. Population pyramids are generally used to display the sex-age structure of a population and can also be used to display the distribution of interval-ratio variables (such as income) by a nominal variable (such as racial groups).

GLOSSARY

Bar chart. A graphic display device for nominal or ordinal variables with few categories. Categories are represented by bars of equal width, the height of each corresponding to the number (or percentage) of cases in the category.

Frequency polygon. A graphic display device for interval-ratio variables. Class intervals are represented by dots placed over the midpoints, the height of each corresponding to the number (or percentage) of cases in the interval. All dots are connected by straight lines. Same as a line chart.

Histogram. A graphic display device for interval-ratio variables. Class intervals are represented by contiguous bars of equal width (equal to the class limits), the height of each corresponding to the number (or percentage) of cases in the interval.

Line chart. *See* Frequency polygon.

Pie chart. A graphic display device especially for nominal or ordinal variables with few categories. A circle (the pie) is divided into segments proportional in size to the percentage of cases in each category of the variable.

Population pyramid. A graph used to display the age-sex distribution of a population. This type of graph can be used to display other variables as well.

PROBLEMS

(Problems are labeled with the social science discipline from which they are drawn: SOC for sociology, SW for social work, PS for political science, CJ for criminal justice, PA for public administration, and GER for gerontology.)

The data sets for these problems are small, and you can draw all of these graphs by hand. See the SPSS exercises for instructions on drawing graphs with computer software.

3.1 a. SOC Construct pie and bar charts to display the distributions of sex, support for gun control, and level of education for the data presented in Chapter 2, Problem 2.5. The data are reproduced here for convenience.

Sex	Support for Gun Control	Level of Education	Age
1 = Male	1 = In favor	0 = Less than High school	Actual years
2 = Female	2 = Opposed	1 = High school	
		2 = Junior college	
		3 = Bachelor's degree	
		4 = Graduate degree	

Case Number	Sex	Support for Gun Control	Level of Education	Age
1	2	1	1	45
2	1	2	1	48
3	2	1	3	55
4	1	1	2	32

Continued on next column

Case Number	Sex	Support for Gun Control	Level of Education	Age
5	2	1	3	33
6	1	1	1	28
7	2	2	0	77
8	1	1	1	50
9	1	2	0	43
10	2	1	1	48
11	1	1	4	33
12	1	1	4	35
13	1	1	0	39
14	2	1	1	25
15	1	1	1	23

b. Construct a histogram and frequency polygon for age.

3.2. a. SOC Construct a histogram and frequency polygon for the College Board scores presented in Chapter 2, Problem 2.7. The data are reproduced here.

420	345	560	650
459	499	500	657
467	480	505	555
480	520	530	589

b. Construct a histogram and frequency polygon for the math and verbal scores presented in Chapter 2, Problem 2.7. The data are reproduced here. Compare the distributions of these scores. How are students distributed across the range of scores? Are the scores higher in math or verbal ability?

MATH TEST

67	45	68	70
72	85	90	99
50	73	77	78
52	66	89	75

VERBAL TEST

89	90	78	77
75	70	56	60
77	78	80	92
98	72	77	82

3.3 GER Construct a histogram and a frequency polygon to display the distribution of data presented in Chapter 2, Problem 2.8 (number of times residents of a senior citizen community left their home for any reason). The data are reproduced here.

0	2	1	7	3
7	0	2	3	17
4	15	5	0	7
5	21	4	7	6
2	0	10	5	7

3.4 SOC Construct a histogram and a frequency polygon to display these data presented in Chapter 2, Problem 2.9. The data are reproduced here.

52	47	17	8	92
53	23	28	9	90
17	63	17	17	23
19	66	10	20	47
20	66	5	25	17

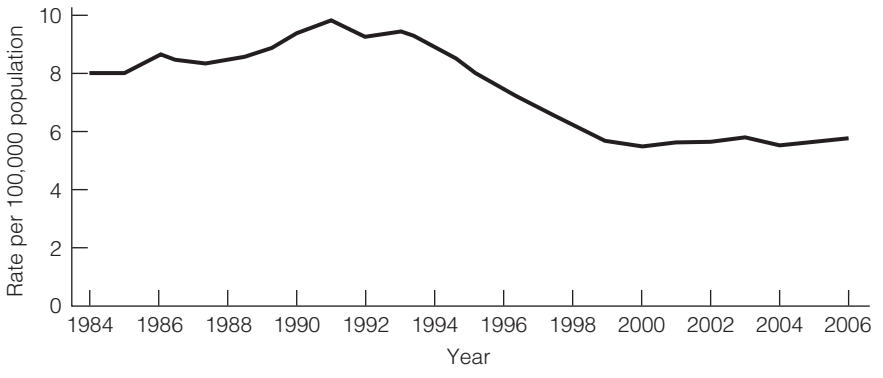
3.5 PA/CJ As part of an evaluation of the efficiency of your local police force, you have gathered the following data on police response time to calls for assistance during two different years. (Response times were rounded off to whole minutes.) Convert both frequency distributions into percentages and construct pie charts and bar charts to display the data. Write a paragraph comparing the changes in response time between the two years.

Response Time, 1990	Frequency (*f*)
21 minutes or more	35
16–20 minutes	75
11–15 minutes	180
6–10 minutes	375
Less than 6 minutes	210
	875

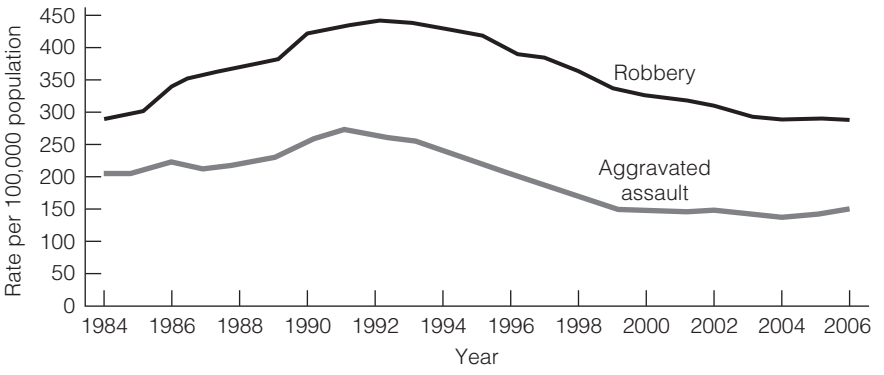
Response Time, 2000	Frequency (*f*)
21 minutes or more	45
16–20 minutes	95
11–15 minutes	155
6–10 minutes	350
Less than 6 minutes	250
	895

3.6 SOC The next three figures display trends in crime in the United States. Write a paragraph describing each of these graphs. What similarities and differences can you observe among the three graphs? (For example, do crime rates always change in the same direction?) Note the differences in the vertical axes from chart to chart—for homicide, the axis ranges from 0 to 12, while for burglary and auto theft the range is from 0 to 1,600. The latter crimes are far more common, and a scale with smaller intervals is needed to display the rates.

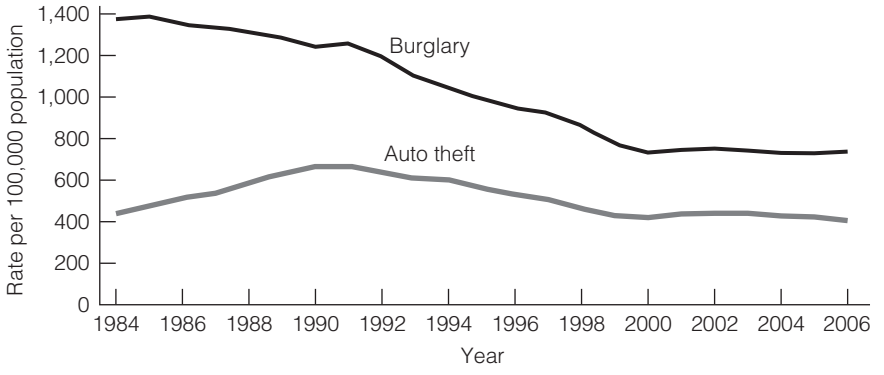
HOMICIDE RATE (NUMBER OF HOMICIDES PER 100,000 POPULATION), 1984–2006



RATES OF ROBBERY AND AGGRAVATED ASSAULT PER 100,000 POPULATION, 1984–2006



RATES OF BURGLARY AND AUTO THEFT PER 100,000 POPULATION, 1984–2006



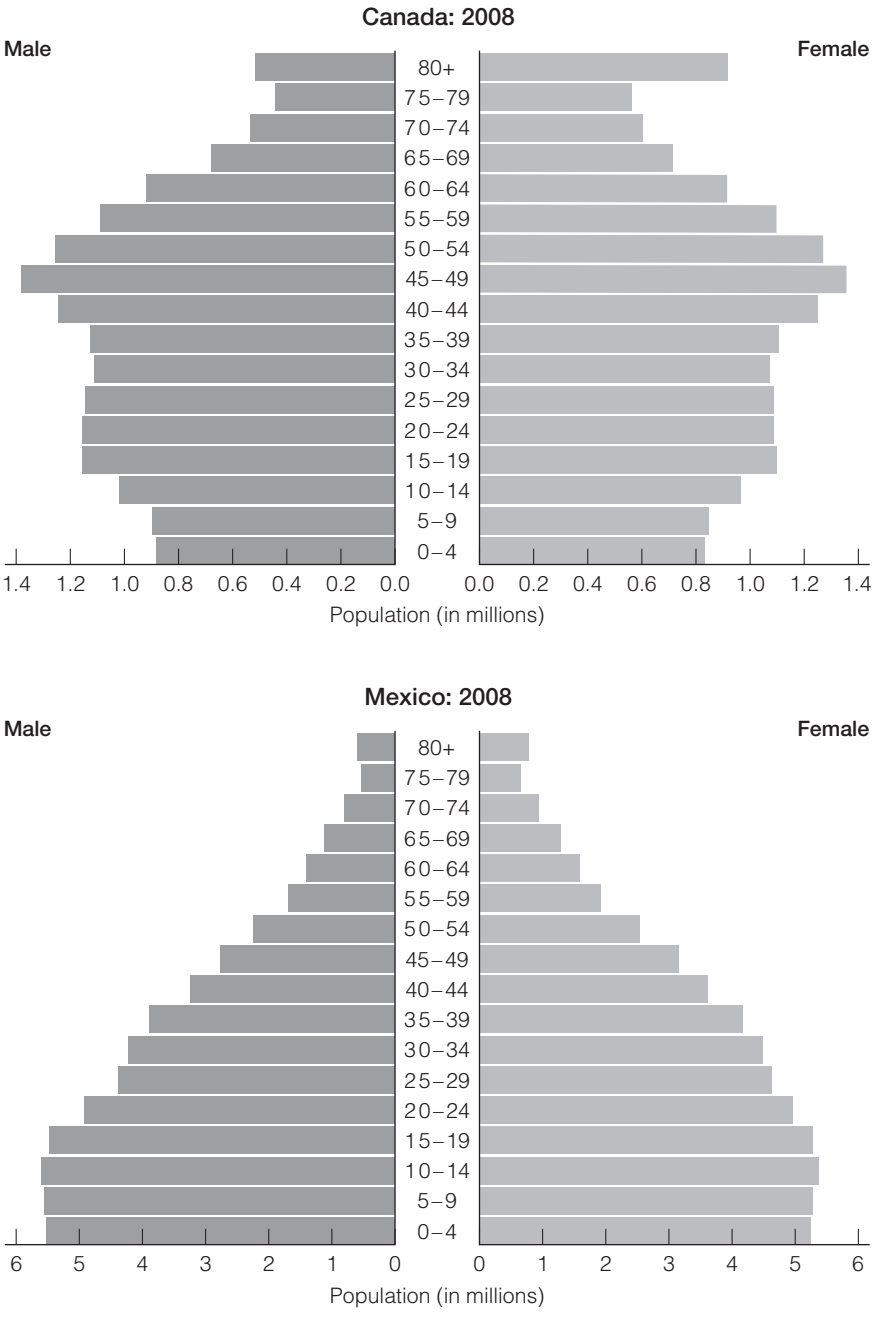
3.7 PA In Problem 2.10 in Chapter 2, you analyzed frequency distributions that showed the number of accidents on a particular highway before and after the speed limit was lowered. The data are reproduced below. Draw histograms for each time period. Do these pictures clarify the changes in the pattern of accidents? How?

Month	12 Months Before	12 Months After
January	23	25
February	25	21
March	20	18
April	19	12
May	15	9
June	17	10
July	24	11
August	28	15
September	23	17
October	20	14
November	21	18
December	22	20

3.8 SOC Read and analyze each of the following pairs of population pyramids. For each pair, answer the following questions:

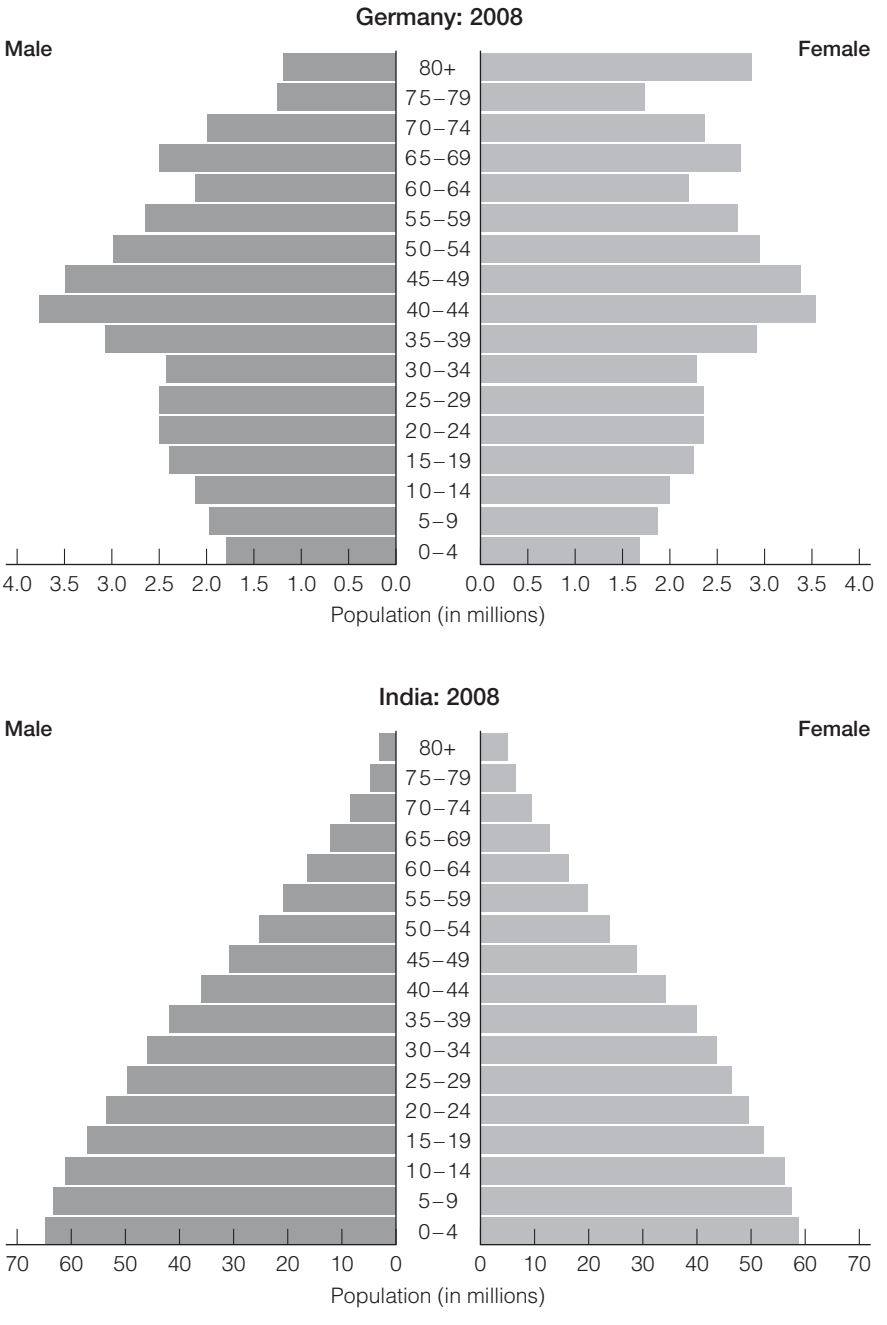
a. Which nation has the higher birthrate (indicated by a broader base)?
b. Which nation has the lower birthrate (look for a narrower base)?
c. Which nation has the higher death rate (indicated by sides that slope inwards; nations with high death rates have pyramids that look like equilateral triangles)?
d. Which nation has the lower death rate (the more perpendicular the sides of the pyramid, the lower the death rate)?

MEXICO VS. CANADA



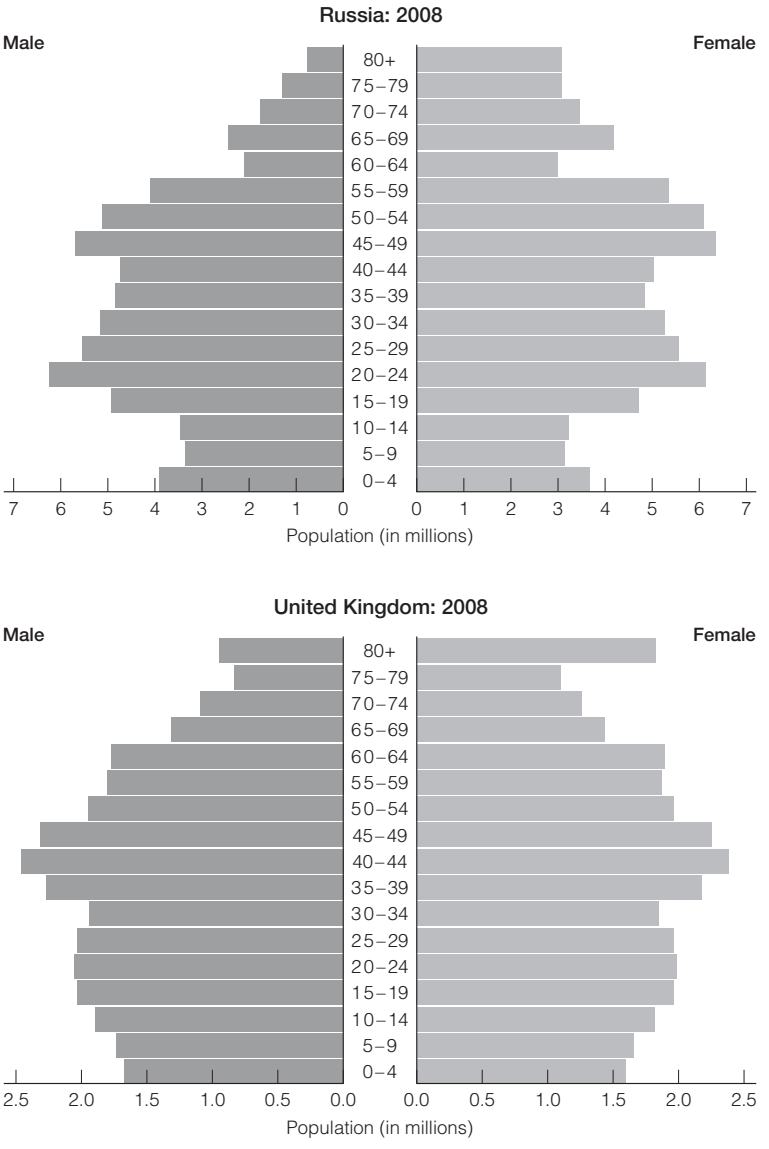
Source: U.S. Census Bureau, International Data Base.

GERMANY VS. INDIA



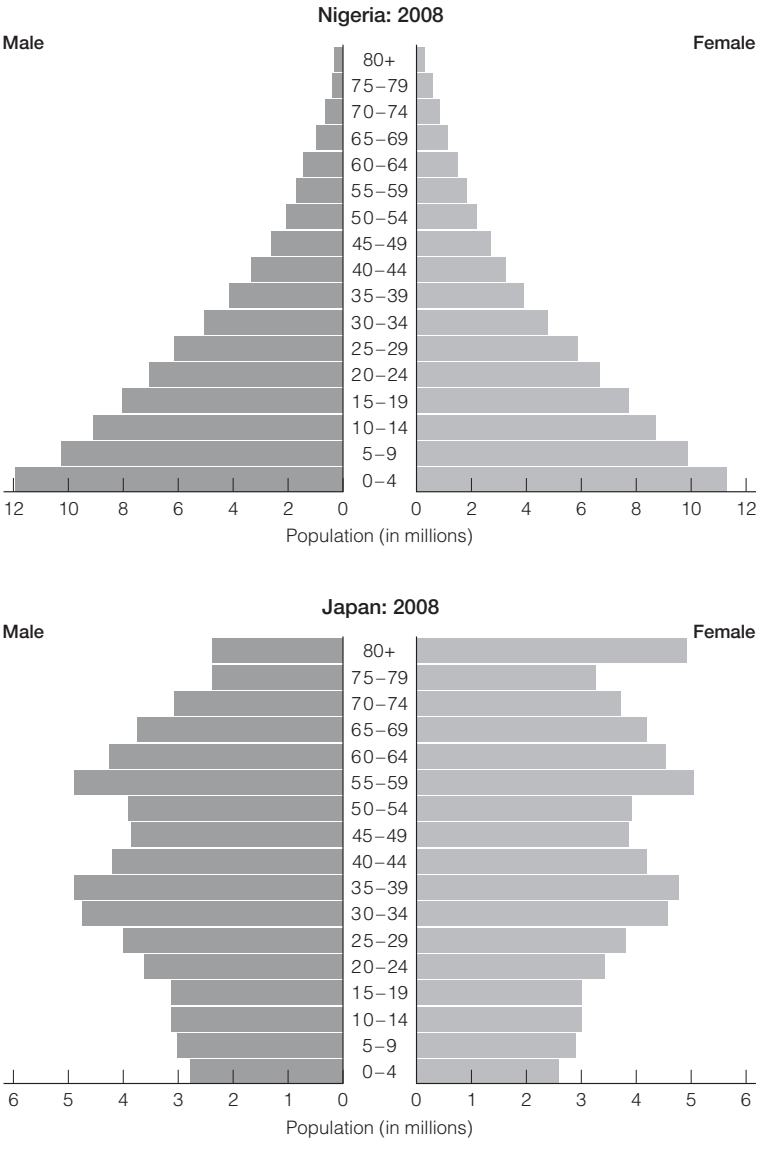
Source: U.S. Census Bureau, International Data Base.

RUSSIA VS. UNITED KINGDOM



Source: U.S. Census Bureau, International Data Base.

NIGERIA VS. JAPAN



Source: U.S. Census Bureau, International Data Base.

3.9. SOC The tables below present the age-sex population breakdowns for five different nations. The nations vary by geographical location and level of development. Construct population pyramids for each nation using graph paper (or graphing software if available). Write a brief description comparing and contrasting the nations and answer these questions:

a. Which has the highest birthrate (indicated by the broadest base)?
b. Which has the lowest birthrate (look for the narrowest base)?
c. Which has the highest death rate (indicated by sides that slope inwards; nations with high death rates have pyramids that look like equilateral triangles)?
d. Which nation has the lowest death rate (the more perpendicular the sides of the pyramid, the lower the death rate)?

BRAZIL—PERCENTAGE OF TOTAL POPULATION IN EACH AGE-SEX GROUP

Age Group	Males	Females
0–4	9.46%	8.89%
5–9	9.34%	8.80%
10–14	8.99%	8.47%
15–19	8.65%	8.19%
20–24	8.98%	8.59%
25–29	9.08%	8.81%
30–34	8.25%	8.11%
35–39	7.60%	7.52%
40–44	6.93%	6.98%
45–49	5.88%	6.04%
50–54	4.81%	5.06%
55–59	3.82%	4.14%
60–64	2.87%	3.23%
65–69	2.10%	2.50%
70–74	1.51%	1.93%
75–79	0.97%	1.38%
80+	0.76%	1.36%
Totals	100.00%	100.00%

SWEDEN—PERCENTAGE OF TOTAL POPULATION IN EACH AGE-SEX GROUP

Age Group	Males	Females
0–4	5.35%	4.96%
5–9	5.35%	4.93%
10–14	5.92%	5.54%
15–19	7.30%	6.79%
20–24	6.56%	6.17%
25–29	6.04%	5.75%
30–34	6.26%	5.95%
35–39	6.86%	6.56%
40–44	7.41%	7.03%
45–49	6.78%	6.48%
50–54	6.48%	6.26%
55–59	6.44%	6.30%
60–64	6.97%	6.89%
65–69	5.53%	5.56%
70–74	3.93%	4.29%
75–79	2.98%	3.72%
80+	3.83%	6.81%
Totals	99.99%	99.99%

CHINA—PERCENTAGE OF TOTAL POPULATION IN EACH AGE-SEX GROUP

Age Group	Males	Females
0–4	6.54%	6.16%
5–9	6.59%	6.09%
10–14	7.63%	7.16%
15–19	8.75%	8.41%
20–24	8.44%	8.39%
25–29	7.44%	7.47%
30–34	7.46%	7.60%
35–39	9.39%	9.46%
40–44	8.89%	8.98%
45–49	5.99%	6.03%
50–54	6.58%	6.60%
55–59	5.27%	5.37%
60–64	3.63%	3.69%
65–69	2.77%	2.87%
70–74	2.22%	2.45%
75–79	1.41%	1.71%
80+	0.99%	1.56%
Totals	99.99%	100.00%

NIGER—PERCENTAGE OF TOTAL POPULATION IN EACH AGE-SEX GROUP

Age Group	Males	Females
0–4	19.20%	19.49%
5–9	15.02%	15.08%
10–14	12.56%	12.55%
15–19	10.57%	10.47%
20–24	8.77%	8.59%
25–29	7.19%	6.99%
30–34	5.91%	5.71%
35–39	4.88%	4.77%
40–44	4.00%	3.98%
45–49	3.27%	3.30%
50–54	2.61%	2.68%
55–59	2.07%	2.17%
60–64	1.59%	1.69%
65–69	1.13%	1.21%
70–74	0.70%	0.75%
75–79	0.36%	0.38%
80+	0.17%	0.18%
Totals	100.00%	99.99%

UKRAINE—PERCENTAGE OF TOTAL POPULATION IN EACH AGE-SEX GROUP

Age Group	Males	Females
0–4	5.16%	4.17%
5–9	4.65%	3.76%
10–14	5.64%	4.60%
15–19	7.47%	6.12%
20–24	9.15%	7.57%
25–29	8.46%	7.13%
30–34	7.81%	6.81%
35–39	7.32%	6.55%
40–44	7.09%	6.56%
45–49	8.03%	7.82%
50–54	7.19%	7.41%
55–59	6.42%	7.02%
60–64	3.87%	4.67%
65–69	4.31%	5.97%
70–74	3.74%	5.51%
75–79	2.10%	3.88%
80+	1.58%	4.45%
Totals	99.99%	100.00%

3.10 Draw "income pyramids" to compare Hispanic Americans and Asian Americans to white Americans.

INCOME DISTRIBUTION FOR NON-HISPANIC WHITES AND HISPANIC AMERICANS, 2006

	Non-Hispanic Whites	Hispanic Americans
<10,000	6.38%	9.33%
10,000–14,999	5.43%	6.87%
15,000–19,999	5.07%	7.32%
20,000–24,999	5.38%	7.67%
25,000–29,999	5.17%	7.02%
30,000–34,999	5.36%	6.92%
35,000–39,999	5.00%	6.24%
40,000-–44,999	5.05%	5.76%
45,000–49,999	4.54%	5.00%
50,000–59,999	8.69%	8.62%
60,000–74,999	11.04%	9.55%
75,000–99,999	12.74%	9.51%
100,000–124,999	7.83%	4.82%
125,000–149,999	4.23%	2.28%
150,000–199,000	4.02%	1.84%
200,000+	4.06%	1.25%
	99.99%	100.00%

INCOME DISTRIBUTION FOR NON-HISPANIC WHITES AND ASIAN AMERICANS, 2006

	Non-Hispanic Whites	Asian Americans
<10,000	6.38%	7.28%
10,000–14,999	5.43%	3.88%
15,000–19,999	5.07%	3.78%
20,000–24,999	5.38%	4.18%
25,000–29,999	5.17%	3.78%
30,000–34,999	5.36%	4.17%
35,000–39,999	5.00%	4.00%
40,000–44,999	5.05%	4.25%
45,000–49,999	4.54%	3.96%
50,000–59,999	8.69%	7.35%
60,000–74,999	11.04%	11.07%
75,000–99,999	12.74%	13.37%
100,000–124,999	7.83%	10.11%
125,000–149,999	4.23%	6.08%
150,000–199,000	4.02%	6.77%
200,000+	4.06%	5.97%
	99.99%	100.00%

YOU ARE THE RESEARCHER: Graphing the Culture War

In Chapter 2, you used SPSS to produce tables that measured the amount of disagreement in U.S. society about certain values and moral issues. In this installment of You Are the Researcher, you will create graphs for each of the three variables you used in Chapter 2.

STEP 1: Choosing An Appropriate Graph

What criteria should you use to choose a type of graph? Generally, the level of measurement and the number of values or scores will be the most important criteria. Consider your three variables and classify them by level of measurement. If your variable is nominal or ordinal and has four or fewer values, you will generally use pie or bar charts. Histograms or line charts are used for ordinal or interval-ratio variables with many values. Use the table below to help decide which graphs to use for each of your variables.

Variable	SPSS Name	Briefly Describe What the Variable Measures	Level of Measurement	Graph
1				
2				
3				

The next section explains how to use SPSS to generate pie and bar graphs, line charts, and histograms.

STEP 2: Using SPSS for Windows to Produce Graphs

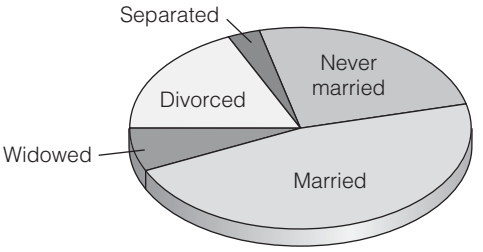
Now we are ready to generate some output and get some background on the nature of disagreements over values and issues among Americans. If necessary, click the SPSS icon on your monitor screen to start SPSS for Windows. Load the 2006 GSS by clicking the file name on the first screen or by clicking **File, Open,** and **Data** on the **SPSS Data Editor screen.** You may have to change the drive specification to locate the 2006 GSS data supplied with this text (probably named **GSS2006.sav**). Double-click the file name to open the data set. When you see the message "SPSS Processor is Ready" on the bottom of the screen, you are ready to proceed.

Generating Graphs

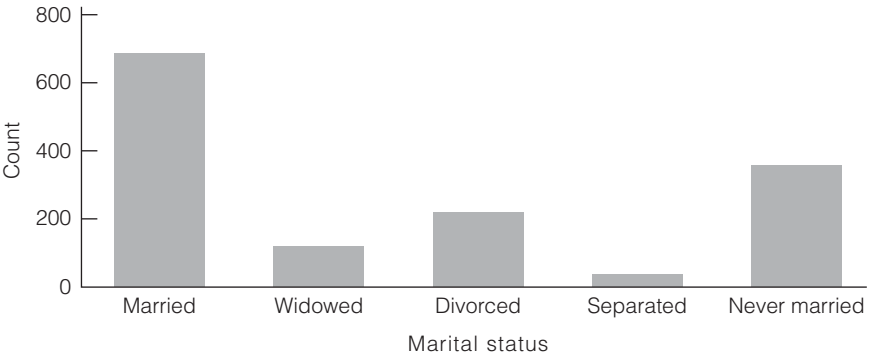
You produced frequency distributions for your variables in Chapter 2. Now you will use SPSS to produce an appropriate graph for each variable that you chose to represent the American culture wars. From the menu bar, click **Graphs** and then click **Legacy Dialogs.** The drop-down menu lists the graphs that are available in SPSS, including bar and pie graphs, line charts, and histograms.

Pie and Bar Charts. I will use *marital* to illustrate the SPSS graphing procedure. This variable is nominal in level of measurement and has five values

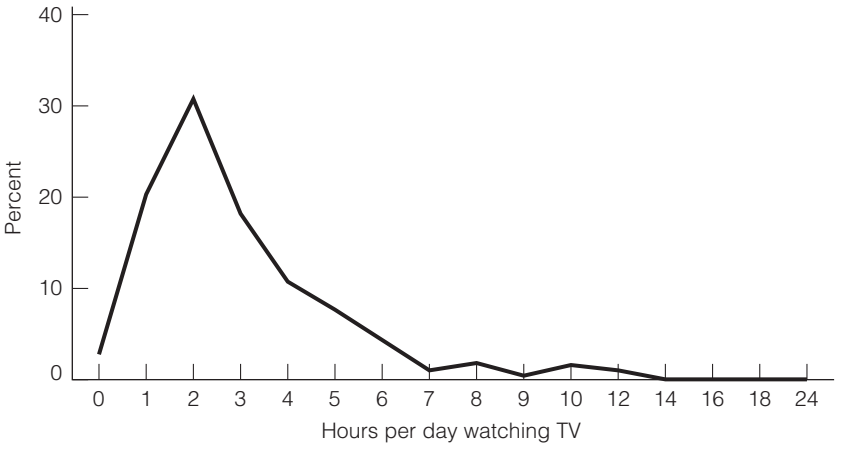
or scores, so a pie chart seems an appropriate choice. Click **Pie** in the list of types of graphs and the **Pie Charts** dialog box opens and gives us three choices for types of pie charts. The **Summaries for groups of cases** choice is checked, and this is the one we want, so click **Define**. The **Define Pie: Summaries for Groups of Cases** dialog box opens. Find your variable on the list and click the arrow pointing to the **Define Slices by** box, then click **OK**. The pie chart clearly shows that the most common marital status is married, followed by never married.



We could easily have chosen to produce a bar chart for this variable, and the SPSS commands we would have used are quite similar to those for the pie chart. When we select **Bar** from the list of graphs, the **Bar Chart** dialog box gives us three choices. We want the **Simple** bar chart, which is already checked, so just click **Define.** Find your variable (*marital* in this case) on the list and click on the arrow to move the variable name to the **Category Axis** box; then click **OK,** and the bar chart will appear.

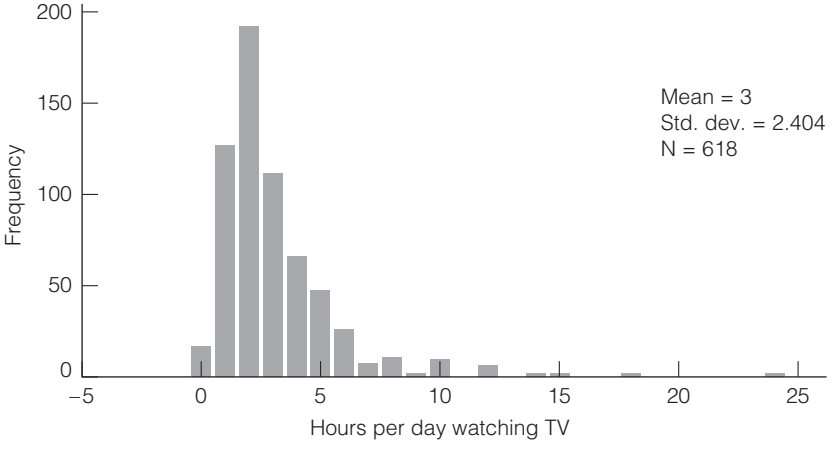


Line Charts and Histograms. Line charts and histograms are created in a similar way. For line charts, select **Line** from the **Graphs** submenu. The **Simple** option—the one we want—will be preselected, so click **Define** and then find your variable on the list at the left. For purposes of illustration, we will use *tvhours* (number of hours per day spent watching TV) to illustrate. We will also select **% of cases** in the **Line Represents** box at the top of the window. Press **OK,** and the following line chart will be produced.



A few respondents watch no TV at all, and the line peaks at two hours of TV viewing a day, the most common score with over 30% of the sample. Also note that the line declines gradually, and many people watch many more than just two hours a day (including one who claims to watch every single hour of the day).

To generate a histogram, click on **Graphs, Legacy Dialogs,** and then **Histograms.** Find your variable name on the list at the left and transfer it to the **Variables:** box on the **Histogram** dialog window. Click OK, and the following histogram will appear.



Although the axes of the graph are scaled in an unusual way (since it is not possible to view for less than 0 hours a day), this graph conveys essentially the same information about tvhours as the line chart.

Now it is your turn.

STEP 3: Interpreting Results

Get appropriate graphs for each of your variables and write a sentence or two of description for each graph. What do the graphs add to the

descriptions you produced for these same variables in Chapter 2 using frequency distributions?

SPSS name of variable 1: _____ Summarize the graph

SPSS name of variable 2: _____ Summarize the graph

SPSS name of variable 3: _____ Summarize the graph

Are these results consistent with the idea that there is a "war" over Americans values? How?

4 Measures of Central Tendency

LEARNING OBJECTIVES

By the time you finish this chapter, you will be able to:

1. Explain the purposes of measures of central tendency and interpret the information they convey.
2. Calculate, explain, and compare and contrast the mode, median, and mean.
3. Explain the mathematical characteristics of the mean.
4. Select an appropriate measure of central tendency according to level of measurement.

4.1 INTRODUCTION

One clear benefit of frequency distributions, graphs, and charts is that they summarize the overall shape of a distribution of scores in a way that can be quickly understood. Often, however, you will need to report more detailed information about the distribution. Specifically, two additional kinds of statistics are almost always useful. First, we will want to have some idea of the typical or average case or value in the distribution (e.g., the average starting salary for social workers is $40,000 per year) and some idea of how much variety or heterogeneity there is in the distribution (e.g., in this state, starting salaries for social workers range from $28,000 per year to $47,000 per year). Statistics that give us the first type of information are called **measures of central tendency** and they will be covered in this chapter. Statistics that give us the second type of information are called *measures of dispersion,* and they will be covered in Chapter 5.

The three commonly used measures of central tendency—the mode, median, and mean—are all probably familiar to you. All three summarize an entire distribution of scores by describing the most common score (the **mode**), the score of the middle case (the **median**), or the average score (the **mean**). These statistics are powerful because they can reduce huge arrays of data to a single, easily understood number. Remember that the central purpose of descriptive statistics is to summarize or "reduce" data.

Even though they share a common purpose, the three measures of central tendency are quite different from each other. In fact, they will have the same value only under specific and limited conditions. As we shall see, they vary in terms of level-of-measurement considerations and, perhaps more importantly, they also vary in terms of how they define central tendency—they will not necessarily identify the same score or case as "typical." Thus, your choice of an appropriate measure of central tendency will depend in part on the way you measure the variable and in part on the purpose of the research.

4.2 THE MODE

The mode of any distribution is the value that occurs most frequently. For example, in the set of scores 58, 82, 82, 90, 98, the mode is 82 since it occurs twice and the other scores occur only once.

The mode is a simple statistic, most useful when you want a quick-and-easy indicator of central tendency and when you are working with nominal level

TABLE 4.1 RELIGIOUS PREFERENCE (fictitious data)

Protestant	128
Catholic	57
Jew	10
None	32
Other	15
	$N = 242$

TABLE 4.2 DISTRIBUTION OF TEST SCORES

Scores (% correct)	Frequency
93	5
70	1
69	1
68	1
67	4
66	3
64	2
62	3
60	2
58	2
	$N = 24$

variables. In fact, the mode is the only measure of central tendency that can be used with nominal level variables. Such variables do not, of course, have numerical "scores" per se, and the mode of a nominally measured variable would be its largest category. For example, Table 4.1 reports the religious affiliations of a fictitious sample of 242 respondents. The mode of this distribution, the single largest category, is Protestant.

If a researcher desires to report only the most popular or common value of a distribution, or if the variable under consideration is nominal, then the mode is the appropriate measure of central tendency. However, keep in mind that the mode does have several limitations. First, some distributions have no mode at all (see Chapter 2, Table 2.6), or they have so many modes that the statistic loses all meaning. Second, with ordinal and interval-ratio data, the modal score may not be central to the distribution as a whole. That is, *most common* does not necessarily mean "typical" in the sense of identifying the center of the distribution. For example, consider the rather unusual (but not impossible) distribution of scores on a statistics test presented in Table 4.2. The mode of the distribution is 93. Is this score very close to the majority of the scores? If the instructor summarized this distribution by reporting only the modal score, would he or she be conveying an accurate picture of the distribution as a whole?

Remember that the mode is the most common *category* or *score* (e.g., Protestant, or 93) of the variable, not the *frequency* in the modal category (e.g., 128 or 5). In a frequency distribution, the mode is found by looking in the frequency

column for the largest number of cases, but the mode itself is the score or the name of the category. *(For practice in finding and interpreting the mode, see Problems 4.1–4.7.)*

4.3 THE MEDIAN

Unlike the mode, the median (*Md*) is always at the exact center of a distribution of scores. The median is the score of the case that is in the middle of a distribution: half the cases have scores higher and half the cases have scores lower than the case with the median score. Thus, if the median family income for a community is $45,000, half the families earn more than $45,000 and half earn less.

Before the median can be found, the cases must be placed in order from the highest to the lowest (or lowest to highest). Once this is done, the median is found by locating the case that divides the sample into two equal halves. For example, if five students received grades of 93, 87, 80, 75, and 61 on a test, the median would be 80, the score that splits the distribution into two equal halves.

When the number of cases (*N*) is odd, the value of the median is unambiguous because there will always be a middle case. With an even number of cases, however, there will be two middle cases; in this situation, the median is defined as the score exactly halfway between the scores of the two middle cases.

To illustrate, assume that seven students were asked to indicate their level of support for the intercollegiate athletic program at their universities on a scale ranging from 10 (indicating great support) to 0 (no support). After arranging their responses from high to low, you can find the median by locating the case that divides the distribution into two equal halves. With a total of seven cases, the middle case would be the fourth case, since there will be three cases above and three cases below this case. In Table 4.3, the cases are listed in order and the median is identified. With a total of seven cases, the median is the score of the fourth case.

To summarize, when *N* is odd, find the middle case by adding 1 to *N* and then dividing that sum by 2. With an *N* of 7, the median is the score associated with the (7 + 1)/2, or 4th, case. If *N* had been 25, the median would be the score associated with the (25 + 1)/2, or 13th, case.

Now suppose we added one more student whose support for athletics was measured as a 1 to the sample. This would make *N* an even number (8), and we would no longer have a single middle case. Table 4.4 presents the new

TABLE 4.3 FINDING THE MEDIAN WITH SEVEN CASES

Case no.	Score	
1	10	
2	10	
3	8	
4	7	← *Md*
5	5	
6	4	
7	2	

TABLE 4.4 FINDING THE MEDIAN WITH EIGHT CASES

Case no.	Score	
1	10	
2	10	
3	8	
4	7	
		$Md = 6$
5	5	
6	4	
7	2	
8	1	

distribution of scores and, as you can see, any value between 7 and 5 would technically satisfy the definition of a median (that is, would split the distribution into two equal halves of four cases each). We resolve the ambiguity created by having an even number of cases by defining the median as the average of the scores of the two middle cases. In this example, the median would be defined as (7 + 5)/2, or 6.

We can now state these procedures in general terms:

- *When N is odd*, find the middle case by adding 1 to *N* and then dividing that sum by 2. This gives you the number of the middle case. The median is the score of that case. With an *N* of 7, the median is the score associated with the (7 + 1)/2, or 4th, case. If *N* had been 25, the median would be the score associated with the (25 + 1)/2, or 13th, case.
- *When N is even*, divide *N* by 2 to find the first middle case, and then increase that number by 1 to find the second middle case. The median is the average of the scores of the two middle cases.[1] With an *N* of 8 cases, the first middle case would be the 4th case (*N*/2 = 4), and the second middle case would be the (*N*/2) + 1, or 5th, case. If *N* had been 142, the first middle case would have been the 71st case and the second the 72nd case. Remember that the median is defined as the average of the scores associated with the two middle cases.

The median cannot be calculated for variables measured at the nominal level because it requires that scores be ranked from high to low. Remember that the scores of nominal level variables cannot be ordered or ranked: the scores are different from each other but do not form a mathematical scale of any sort. The median can be found for either ordinal or interval-ratio data, but is generally more appropriate for the former. *(The median may be found for any problem at the end of this chapter. Remember that this statistic can be computed for ordinal or interval-ratio variables.)*

[1]If the middle cases have the same score, that score is defined as the median. In the distribution 10, 10, 8, 6, 6, 4, 2, 1, the middle cases both have scores of 6, and thus the median would be defined as 6.

ONE STEP AT A TIME **Finding the Median**

Step	*Operation*
1.	Array the scores in order from high score to low score.
2.	Count the number of cases to see if *N* is odd or even. *If* N *is odd: The median will be the score of the middle case.*
3.	To find the middle case, add 1 to *N* and divide by 2.
4.	The value you calculated in Step 3 is the number of the middle case. The median is the score of this case. For example, if $N = 13$, the median will be the score of the $(13 + 1)/2$, or seventh case. *If* N *is even: The median is the score halfway between the two middle cases.*
3.	To find the first middle case, divide *N* by 2.
4.	To find the second middle case, increase the value you computed in Step 3 by 1.
5.	Find the scores of the two middle cases. Add the scores together and divide by 2. The result is the median. For example, if $N = 14$, the median is the score halfway between the scores of the seventh and eighth cases.

4.4 THE MEAN

The mean ($\overline{X}$; read this as "ex-bar"),[2] or arithmetic average, is by far the most commonly used measure of central tendency. It reports the average score of a distribution, and its calculation is straightforward: to compute the mean, add the scores and then divide by the number of scores (N). To illustrate, a birth control clinic administered a 20-item test of general knowledge about contraception to 10 clients. The number of correct responses was 2, 10, 15, 11, 9, 16, 18, 10, 11, 7. To find the mean of this distribution, add the scores (total = 109) and divide by the number of scores (10). The result (10.9) is the average score on the test.

The mathematical formula for the mean is

FORMULA 4.1

$$\overline{X} = \frac{\Sigma(X_i)}{N}$$

Where: $\overline{X}$ = the mean
$\Sigma(X_i)$ = the summation of the scores
N = the number of cases.

Since this formula introduces some new symbols, let us take a moment to consider it. First, the symbol Σ (uppercase Greek letter sigma) is a mathematical operator just as are the plus sign (+) or divide sign (÷). It stands for "the summation of" and directs us to add whatever quantities are stated immediately following it. The second new symbol is X_i (read as "X sub i"), which refers to any single score—the "ith" score. If we wished to refer to a particular score in the distribution, the specific number of the score could replace the subscript. Thus, X_1 would refer to the 1st score, X_2 to the 2nd, X_{26} to the 26th, and so forth. The operation of adding all the scores is symbolized as $\Sigma(X_i)$. This combination of symbols directs us to sum the scores, beginning with the first score and ending with the last score in the distribution. Thus,

[2]This is the symbol for the mean of a sample. The mean of a population is symbolized with the Greek letter mu (μ, pronounced "mew").

Application 4.1

Ten students have been asked how many hours they spent in the college library during the past week. What is the average library time for these students? The hours are reported in the following list, and we will find the mode, the median, and the mean for these data.

Student	Number of Visits to the Library Last Week (X_i)
1	0
2	2
3	5
4	5
5	7
6	10
7	14
8	14
9	20
10	30
	$N = 107$

By scanning the scores, we can see that two scores, 5 and 14, occurred twice, and no other score occurred more than once. This distribution has two modes, 5 and 14.

Because the number of cases is even, the median will be the average of the two middle cases. Note that the scores have been ordered from low to high. With 10 cases, the first middle case will be the (N/2), or (10/2), or fifth case. The second middle case is the (N/2) + 1, or (10/2) + 1, or sixth case. The median will be the score halfway between the scores of the fifth and sixth cases. The score of the fifth case is 7 and the score of the sixth case is 10. The median is (7 + 10)/2, or (17/2), or 8.5.

The mean is found by first adding all the scores (X_i) and then dividing by the number of scores. The sum of the scores is 107, so the mean is

$$\bar{X} = \frac{\Sigma(X_i)}{N} = \frac{107}{10} = 10.7$$

These 10 students spent an average of 10.7 hours in the library during the week in question.

Note that the mean is a higher value than the median. This indicates a positive skew in the distribution (a few extremely high scores). By inspection, we can see that the positive skew is caused by the two students who spent many more hours (20 hours and 30 hours) in the library than the other 8 students.

Formula 4.1 states in symbols what has already been stated in words (to calculate the mean, add the scores and divide by the number of scores), but in a very succinct and precise way. *(For practice in computing the mean, use any Problem at the end of this chapter.)*

Since computation of the mean requires addition and division, it should be used with variables measured at the interval-ratio level. However, researchers often calculate the mean for variables measured at the ordinal level, because the mean is much more flexible than the median and is a central feature of many interesting and powerful advanced statistical techniques. Thus, if the researcher plans to do any more than merely describe his or her data, the mean will probably be the preferable measure of central tendency, even for ordinal level variables.

ONE STEP AT A TIME — Computing the Mean

Step	Operation
1.	Add up the scores (X_i)
2.	Divide the quantity you found in Step 1 by N.

TABLE 4.5 A DEMONSTRATION SHOWING THAT ALL SCORES CANCEL OUT AROUND THE MEAN

X_i	$(X_i - \bar{X})$
65	$65 - 78 = -13$
73	$73 - 78 = -5$
77	$77 - 78 = -1$
85	$85 - 78 = 7$
90	$90 - 78 = 12$
$\sum(X_i) = 390$	$\sum(X_i - \bar{X}) = 0$
$\bar{X} = 390/5 = 78$	

4.5 THREE CHARACTERISTICS OF THE MEAN

The mean is the most commonly used measure of central tendency, and we will consider its mathematical and statistical characteristics in some detail. First, the mean is an excellent measure of central tendency because it acts as a fulcrum that "balances" all the scores in the sense that the mean is the point around which all of the scores cancel out. We can express this property symbolically:

$$\sum(X_i - \bar{X}) = 0$$

This expression says that if we subtract the mean ($\bar{X}$) from each score (X_i) in a distribution and then sum the differences, the result will always be zero.

To illustrate, consider the test scores presented in Table 4.5. The mean of these five scores is 390/5, or 78. The difference between each score and the mean is listed in the right-hand column ($X_i - \bar{X}$), and the sum of these differences is zero. The total of the negative differences (−19) is exactly equal to the total of the positive differences (+19), as will always be the case. Thus, the mean "balances" the scores and is at the center of the distribution.

A second characteristic of the mean is called the **least squares principle,** a characteristic that is expressed in the statement

$$\sum(X_i - \bar{X})^2 = \text{minimum}$$

or, the mean is the point in a distribution around which the variation of the scores (as indicated by the squared differences) is minimized. If the differences between the scores and the mean are squared and then added, the resulting sum will be less than the sum of the squared differences between the scores and any other point in the distribution.

To illustrate this principle, consider the distribution of five scores mentioned above: 65, 73, 77, 85, and 90. The differences between the scores and the mean have already been found. As illustrated in Table 4.6, if we square and sum these differences, we would get a total of 388. If we performed those same mathematical operations with any number other than the mean—say the value 77—the resultant sum would be greater than 388. Table 4.6 illustrates this point by showing that the sum of the squared differences around 77 is 393, a value greater than 388.

This least-squares principle underlines the fact that the mean is closer to all of the scores than the other measures of central tendency. Also, this characteristic

TABLE 4.6 DEMONSTRATION SHOWING THAT THE MEAN IS THE POINT OF MINIMIZED VARIATION

X_i	$(X_i - \bar{X})$	$(X_i - \bar{X})^2$	$(X_i - 77)^2$
65	$65 - 78 = -13$	$(-13)^2 = 169$	$(65 - 77)^2 = (-12)^2 = 144$
73	$73 - 78 = -5$	$(-5)^2 = 25$	$(73 - 77)^2 = (-4)^2 = 16$
77	$77 - 78 = -1$	$(-1)^2 = 1$	$(77 - 77)^2 = (0)^2 = 0$
85	$85 - 78 = 7$	$(7)^2 = 49$	$(85 - 77)^2 = (8)^2 = 64$
90	$90 - 78 = 12$	$(12)^2 = 144$	$(90 - 77)^2 = (13)^2 = 169$
$\Sigma(X_i) = 390$	$\Sigma(X_i - \bar{X}) = 0$	$\Sigma(X_i - \bar{X})^2 = 388$	$\Sigma(X_i - 77)^2 = 393$

TABLE 4.7 DEMONSTRATION SHOWING THAT THE MEAN IS AFFECTED BY EVERY SCORE

1	2	3	4
Scores	Measures of Central Tendency	Scores	Measures of Central Tendency
15	$\bar{X} = 125/5 = 25$	15	$\bar{X} = 3{,}590/5 = 718$
20		20	
25	$Md = 25$	25	$Md = 25$
30		30	
35		3,500	
$\Sigma(X_i) = 125$		$\Sigma = 3{,}590$	

of the mean is important for the statistical techniques of correlation and regression, topics we will take up toward the end of this book.

The final important characteristic of the mean is that every score in the distribution affects it. The mode (which is only the most common score) and the median (which deals only with the score of the middle case or cases) are not so affected. This quality is both an advantage and a disadvantage. On one hand, the mean uses all the available information—every score in the distribution affects the mean. On the other hand, when a distribution has a few very high or very low scores, the mean may become a very misleading measure of centrality.

To illustrate, consider Table 4.7. The five scores listed in column 1 have a mean and median of 25 (see column 2). In column 3, the scores are listed again with one score changed: 35 is changed to 3,500. Look in column 4 and you will see that this change has no effect on the median; it remains at 25. This is because the median is based *only* on the score of the middle case and is not affected by changes in the scores of other cases in the distribution.

The mean, in contrast, is very much affected by the change because it takes *all* scores into account. The mean changes from 25 to 718 solely because of the one extreme score of 3,500. Note also that the mean in column 4 is very different from four of the five scores listed in column 3. In this case, is the mean or the median a better representation of the scores? For distributions that have a few very high or very low scores, the mean may present a very misleading picture of the typical or central score. In these cases, the median may be the preferred measure of central tendency for interval-ratio variables. *(For practice in dealing with the effects of extreme scores on means and medians, see Problems 4.7, 4.10–4.14.)*

FIGURE 4.1 A POSITIVELY SKEWED DISTRIBUTION (The mean is greater in value than the median.)

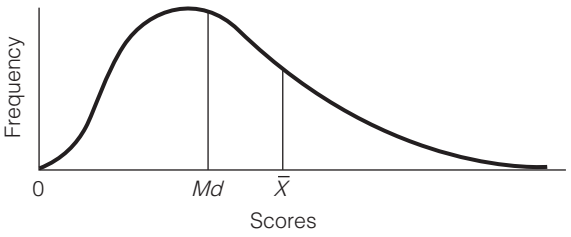


FIGURE 4.2 A NEGATIVELY SKEWED DISTRIBUTION (The mean is less than the median.)

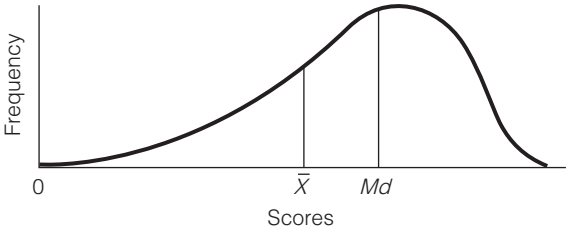
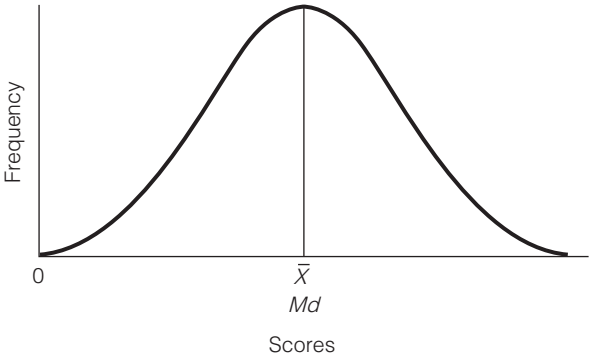


FIGURE 4.3 AN UNSKEWED, SYMMETRICAL DISTRIBUTION (The mean and median are equal)



The general principle to remember is that, relative to the median, the mean is always pulled in the direction of extreme scores (i.e., scores that are much higher or lower than other scores); that is, when the data show a **skew.** The mean and median will have the same value when and only when a distribution is symmetrical. When a distribution has some extremely high scores (this is called a positive skew), the mean will always have a greater numerical value than the median. If the distribution has some very low scores (a negative skew), the mean will be lower in value than the median. Figures 4.1 to 4.3 depict three different frequency polygons that demonstrate these relationships.

These relationships between medians and means also have a practical value. For one thing, a quick comparison of the median and mean will always tell you if a distribution is skewed and, if so, the direction of the skew. If the

BECOMING A CRITICAL CONSUMER: Using an Appropriate Measure of Central Tendency

Consider the following recent headlines:

"Median Price of Housing Falls"
"Average Price of Gas at Record High"
"Non-whites Will Soon Become Majority in Area"

By now, you recognize that each of these reports cites a different measure of central tendency, and we can analyze how appropriately these choices were made.

When selecting a measure of central tendency, our first concern is the level of measurement of the variable being summarized. The first two variables—the cost of houses and the price of gas—are interval-ratio. Why does one report use the median and the other the mean or average? Consider the nature of the variables. Housing costs, like almost any variable related to money, will be positively skewed (see Figure 4.1). Why? Any community will have a wide range of housing available. Some houses will be tumbledown shacks ("handyman specials"), many will be modest in quality and price, some will be upscale, and a few will cost millions; that is, most cases (houses) will cluster in the low to middle range, but there will be a few with extremely high scores or prices. When a variable is positively skewed, the mean will have a higher value than the median and may not present an accurate picture of what is "typical" or average. When a variable is highly skewed, the median is the preferred measure of central tendency, even for interval-ratio level variables. Generally speaking, careful description of the central tendency of any variable related to money or price will use the median, and you should be immediately suspicious of reports that do not.

What about gas prices? Why are they reported using the mean (or "average")? Like housing prices or income, the price of gas fluctuates over time and across the country (see http://www.gasbuddy.com/gb_gastemperaturemap.aspx for a visual representation), and a few service stations will have very high prices. However, the range of variation in price per gallon is small, at least compared to housing prices. While there can be a difference of millions of dollars in the price of two houses, the maximum difference in the cost of a gallon of regular gas is 60 to 80 cents. The smaller range means that gas prices can't be very skewed (relatively speaking), and it is reasonable to use the mean to report the average.

The final headline uses the mode to report information about the changing racial and ethnic composition of U.S. society. Race and ethnicity are nominal level variables, and the mode is the only measure of central tendency available for these variables. The majority category of any variable is its mode, or most common score.

The fact that there are three different measures of central tendency in common use can lead to some confusion and misunderstanding, especially since the word *average* can be used as a synonym for any of the three. Always check to see which of the three is being used when an average is reported, and use extra caution when the exact statistic is not identified.

mean is less than the median, the distribution has a negative skew. If the mean is greater than the median, the distribution has a positive skew.

Second, these characteristics of the mean and median also provide a simple and effective way to "lie" with statistics. For example, if you want to maximize the average score of a positively skewed distribution, report the mean. Income data usually have a positive skew (there are only a few very wealthy people). If you want to impress someone with the general affluence of a mixed-income community, report the mean. If you want a lower figure, report the median.

Which measure is most appropriate for skewed distributions? This will depend on what point the researcher wishes to make, but as a rule, either both measures of central tendency or the median alone should be reported.

TABLE 4.8 RELATIONSHIP BETWEEN LEVEL OF MEASURE AND MEASURES OF CENTRAL TENDENCY

	Level of Measurement		
Measure of Central Tendency:	Nominal	Ordinal	Interval-Ratio
Mode	**YES**	Yes	Yes
Median	No	**YES**	Yes
Mean	No	Yes (?)	**YES**

4.6 CHOOSING A MEASURE OF CENTRAL TENDENCY

You should consider two main criteria when choosing a measure of central tendency. First, make sure that you know the level of measurement of the variable in question. This will generally tell you whether you should report the mode, median, or mean. Table 4.8 shows the relationship between the level of measurement and measures of central tendency. The capitalized, boldface **"YES"** identifies the most appropriate measure of central tendency for each level of measurement, and the lowercase "yes" indicates the levels of measurement for which the measure is also permitted. An entry of "no" in the table means that the statistic cannot be computed for that level of measurement. Finally, the "Yes (?)" entry in the bottom row indicates that the mean is often used with ordinal level variables, even though, strictly speaking, this practice violates level of measurement guidelines.

Second, consider the definitions of the three measures of central tendency and remember that they provide different types of information. They will be the same value only under certain, specific conditions (that is, for symmetrical distributions with one mode), and each has its own message to report. In many circumstances, you might want to report all three.

The guidelines in Table 4.9 stress both selection criteria and may be helpful when choosing a specific measure of central tendency.

TABLE 4.9 CHOOSING A MEASURE OF CENTRAL TENDENCY

Use the mode when:	1. The variable is measured at the nominal level.
	2. You want a quick and easy measure for ordinal and interval-ratio variables.
	3. You want to report the most common score.
Use the median when:	1. The variable is measured at the ordinal level.
	2. A variable measured at the interval-ratio level has a highly skewed distribution.
	3. You want to report the central score. The median always lies at the exact center of a distribution.
Use the mean when:	1. The variable is measured at the interval-ratio level (except when the variable is highly skewed).
	2. You want to report the typical score. The mean is the fulcrum that exactly balances all of the scores.
	3. You anticipate additional statistical analysis.

SUMMARY

1. Measures of central tendency provide information about the most typical or representative value in a distribution. These statistics permit the researcher to report important information about an entire distribution of scores in a single, easily understood number.
2. The mode reports the most common score and is used most appropriately with nominally measured variables.
3. The median (*Md*) reports the score that is the exact center of the distribution. It is most appropriately used with variables measured at the ordinal level and with variables measured at the interval-ratio level when the distribution is skewed.
4. The mean ($\overline{X}$), the most frequently used of the three measures, reports the most typical score. It is used most appropriately with variables measured at the interval-ratio level (except when the distribution is highly skewed).
5. The mean has a number of mathematical characteristics that are significant for statisticians. First, it is the point in a distribution of scores around which all other scores cancel out. Second, the mean is the point of minimized variation. Last, in contrast to the mode or median, the mean is affected by every score in the distribution and is therefore pulled in the direction of extreme scores.

SUMMARY OF FORMULAS

FORMULA 4.1 Mean: $$\overline{X} = \frac{\Sigma(X_i)}{N}$$

GLOSSARY

Least squares principle. This principle states that the mean is a good measure of central tendency because it is the point of minimized variation of the scores, as measured by the squared differences between the mean and all the scores.

Mean. The arithmetic average of the scores. $\overline{X}$ represents the mean of a sample and μ refers to the mean of a population.

Measures of central tendency. Statistics that summarize a distribution of scores by reporting the most typical or representative value of the distribution.

Median (*Md*). The point in a distribution of scores above and below which exactly half of the cases fall.

Mode. The most common value in a distribution or the largest category of a variable.

Skew. The extent to which a distribution of scores has a few scores that are extremely high (positive skew) or extremely low (negative skew).

$\boldsymbol{X_i}$ (Read "*X* sub *i*"). Any score in a distribution.

PROBLEMS

(Problems are labeled with the social science discipline from which they are drawn: SOC for sociology, SW for social work, PS for political science, CJ for criminal justice, PA for public administration, and GER for gerontology.)

4.1 SOC A variety of information has been gathered from a sample of college freshmen and seniors, including their region of birth; the extent to which they support legalization of marijuana (measured on a scale for which 7 = strong support, 4 = neutral, and 1 = strong opposition); the amount of money they spend each week out of pocket for food, drinks, and entertainment; how many movies they watched in their dorm rooms last week; their opinion of cafeteria food (10 = excellent, 0 = very bad); and their religious affiliation. Some results are presented below. Find the *most appropriate* measure of central tendency for each variable for freshmen and then for seniors. Report both the measure you selected as well as its value for each variable (e.g., mode = 3 or median = 3.5). (*HINT: Determine the level of measurement for each variable first. In general, this will tell you which statistic is appropriate. See Section 4.6 to review the relationship between measure of central tendency and level of measurement.*)

FRESHMEN

Student	Region of Birth	Legalization	Out-of-Pocket Expenses	Movies	Cafeteria Food	Religion
A	North	7	33	0	10	Protestant
B	North	4	39	14	7	Protestant
C	South	3	45	10	2	Catholic
D	Midwest	2	47	7	1	None
E	North	3	62	5	8	Protestant
F	North	5	48	1	6	Jew
G	South	1	52	0	10	Protestant
H	South	4	65	14	0	Other
I	Midwest	1	51	3	5	Other
J	West	2	43	4	6	Catholic

SENIORS

Student	Region of Birth	Legalization	Out-of-Pocket Expenses	Movies	Cafeteria Food	Religion
K	North	7	65	0	1	None
L	Midwest	6	62	5	2	Protestant
M	North	7	60	11	8	Protestant
N	North	5	90	3	4	Catholic
O	South	1	62	4	3	Protestant
P	South	5	57	14	6	Protestant
Q	West	6	40	0	2	Catholic
R	West	7	49	7	9	None
S	North	3	45	5	4	None
T	West	5	85	3	7	Other
U	North	4	78	5	4	None

4.2 A variety of information has been collected for each of the nine high schools in a district. Find the most appropriate measure of central tendency for each variable and summarize this information in a paragraph. (*HINT: The level of measurement of the variable will generally tell you which statistic is appropriate. Remember to organize the scores from high to low before finding the median.*)

High School	Enrollment	Largest Racial/Ethnic Group	Percent College Bound	Most Popular Sport	Condition of Physical Plant (Scale of 1–10, 10 = High)
1	1,400	White	25	Football	10
2	1,223	White	77	Baseball	7
3	876	Black	52	Football	5
4	1,567	Hispanic	29	Football	8
5	778	White	43	Basketball	4
6	1,690	Black	35	Basketball	5
7	1,250	White	66	Soccer	6
8	970	White	54	Football	9
9	1,109	Hispanic	64	Soccer	3

4.3 PS You have been observing the local Democratic Party in a large city and have compiled some information about a small sample of party regulars. Find the appropriate measure of central tendency for each variable.

Respondent	Sex	Social Class	No. of Years in Party	Education	Marital Status	No. of Children
A	M	High	32	High school	Married	5
B	M	Medium	17	High school	Married	0
C	M	Low	32	High school	Single	0
D	M	Low	50	8th grade	Widowed	7
E	M	Low	25	4th grade	Married	4
F	M	Medium	25	High school	Divorced	3
G	F	High	12	College	Divorced	3
H	F	High	10	College	Separated	2
I	F	Medium	21	College	Married	1
J	F	Medium	33	College	Married	5
K	M	Low	37	High school	Single	0
L	F	Low	15	High school	Divorced	0
M	F	Low	31	8th grade	Widowed	1

4.4 SOC You have compiled the information below on each of the graduates voted "most likely to succeed" by a local high school for a 10-year period. For each variable, find the appropriate measures of central tendency.

Case	Present Income ($)	Marital Status	Owns a BMW	Years of Education Post-High School
A	24,000	Single	No	8
B	48,000	Divorced	No	4
C	54,000	Married	Yes	4
D	45,000	Married	No	4
E	30,000	Single	No	4
F	35,000	Separated	Yes	8
G	30,000	Married	No	3
H	17,000	Married	No	1
I	33,000	Married	Yes	6
J	48,000	Single	Yes	4

4.5 SOC For 15 respondents, data have been gathered on four variables (table below). Find and report the appropriate measure of central tendency for each variable.

Respondent	Marital Status	Racial/Ethnic Group	Age	Attitude on Abortion Scale (High Score = Strong Opposition)
A	Single	White	18	10
B	Single	Hispanic	20	9
C	Widowed	White	21	8
D	Married	White	30	10
E	Married	Hispanic	25	7
F	Married	White	26	7
G	Divorced	Black	19	9
H	Widowed	White	29	6
I	Divorced	White	31	10
J	Married	Black	55	5
K	Widowed	Asian American	32	4
L	Married	American Indian	28	3
M	Divorced	White	23	2
N	Married	White	24	1
O	Divorced	Black	32	9

4.6 SOC Following are four variables for 30 cases from the General Social Survey. Age is reported in years. The variable "happiness" consists of answers to the following question: Taken all together, would you say that you are (1) very happy, (2) pretty happy, or (3) not too happy? Respondents were asked how many sex partners they had over the past five years. Responses were measured on the following scale: 0–4 = actual numbers; 5 = 5–10 partners; 6 = 11–20 partners; 7 = 21–100 partners; 8 = more than 100. For each variable, find the appropriate measure of central tendency and write a sentence reporting this statistical information as you would in a research report.

Respondent	Age	Happiness	No. of Partners	Religion
1	20	1	2	Protestant
2	32	1	1	Protestant
3	31	1	1	Catholic
4	34	2	5	Protestant
5	34	2	3	Protestant
6	31	3	0	Jew
7	35	1	4	None
8	42	1	3	Protestant
9	48	1	1	Catholic
10	27	2	1	None
11	41	1	1	Protestant
12	42	2	0	Other
13	29	1	8	None
14	28	1	1	Jew
15	47	2	1	Protestant
16	69	2	2	Catholic
17	44	1	4	Other
18	21	3	1	Protestant
19	33	2	1	None
20	56	1	2	Protestant
21	73	2	0	Catholic
22	31	1	1	Catholic
23	53	2	3	None
24	78	1	0	Protestant
25	47	2	3	Protestant
26	88	3	0	Catholic
27	43	1	2	Protestant
28	24	1	1	None
29	24	2	3	None
30	60	1	1	Protestant

4.7 SOC The table to the right lists the median family incomes of 13 Canadian provinces and territories in 2000 and 2006. Compute the mean and median for each year. Use these statistics to compare the two years, and describe the changes you observe. Which measure of central tendency is greater for each year? Are the distributions skewed? If so, in which direction?

Province	2000	2006
Newfoundland and Labrador	38,800	50,500
Prince Edward Island	44,200	56,100
Nova Scotia	44,500	56,400
New Brunswick	43,200	54,000
Quebec	47,700	59,000
Ontario	55,700	66,600
Manitoba	47,300	58,700
Saskatchewan	45,800	60,500
Alberta	55,200	78,400
British Columbia	49,100	62,600
Yukon	56,000	76,000
Northwest Territories	61,000	88,800
Nunavut	37,600	54,300

Source: Statistics Canada. http://www40.statcan.ca/l01/cst01/famil108a.htm?sdi=income.

4.8 SOC The school administration is considering a total ban on student automobiles. You have conducted a poll on this issue by surveying 20 fellow students and 20 neighbors who live around the campus and have calculated scores for your respondents. On the scale you used, a high score indicates strong opposition to the proposed ban. The scores are presented below for both groups. Calculate an appropriate measure of central tendency and compare the two groups in a sentence or two.

Students		Neighbors	
10	11	0	7
10	9	1	6
10	8	0	0
10	11	1	3
9	8	7	4
10	11	11	0
9	7	0	0
5	1	1	10
5	2	10	9
0	10	10	0

4.9 SW As the head of a social services agency, you believe that your staff of 20 social workers is very much overworked compared to 10 years ago. The caseloads for each worker are reported below for each of the two years in question. Has the average caseload increased? What measure of central tendency is most appropriate to answer this question? Why?

1997		2007	
52	55	42	82
50	49	75	50
57	50	69	52
49	52	65	50
45	59	58	55
65	60	64	65
60	65	69	60
55	68	60	60
42	60	50	60
50	42	60	60

4.10 SOC The following table lists the approximate number of cars per 100 population for eight nations in 1999. Compute the mean and median for this data. Which measure of central tendency is greater in value? Is there a positive skew in this data? How do you know?

Nation	Number of Cars per 100 Population
United States	50
Canada	45
France	46
Germany	51
Japan	39
Mexico	10
Sweden	44
United Kingdom	37

4.11 SW For the test scores first presented in Chapter 2, Problem 2.6 and reproduced below, compute a median and mean for both the pretest and posttest. Interpret these statistics.

Case	Pretest	Posttest
A	8	12
B	7	13
C	10	12
D	15	19
E	10	8
F	10	17
G	3	12
H	10	11
I	5	7
J	15	12
K	13	20
L	4	5
M	10	15
N	8	11
O	12	20

4.12 SOC A sample of 25 freshmen at a major university completed a survey that measured their degree of racial prejudice (the higher the score, the greater the prejudice).

a. Compute the median and mean for these data.

10	43	30	30	45
40	12	40	42	35
45	25	10	33	50
42	32	38	11	47
22	26	37	38	10

b. These same 25 students completed the same survey during their senior year. Compute measures of central tendency for this second set of scores and compare them to the earlier set. What happened?

10	45	35	27	50
35	10	50	40	30
40	10	10	37	10
40	15	30	20	43
23	25	30	40	10

4.13 PA The table below presents the annual person hours of time lost due to traffic congestion for a group of cities for 2003. This statistic is a measure of traffic congestion.

City	Annual Person-Hours of Time Lost to Traffic Congestion per Year
Baltimore	27
Boston	25
Buffalo	6
Chicago	31
Cleveland	6
Dallas	35
Detroit	30
Houston	36
Kansas City	9
Los Angeles	50
Miami	29
Minneapolis	23
New Orleans	10
New York	23
Philadelphia	21
Pittsburgh	8
Phoenix	26
San Antonio	18
San Diego	28
San Francisco	37
Seattle	25
Washington, DC	34

Source: U.S. Bureau of the Census. 2008. *Statistical Abstract of the United States,* 2008. *Washington,* DC: Government Printing Office. p. 682.

a. Calculate the mean and median of this distribution.
b. Compare the mean and median. Which is the higher value? Why?
c. If you removed Los Angeles from this distribution and recalculated, what would happen to the mean? To the median? Why?
d. Report the mean and median as you would in a formal research report.

4.14 SOC Professional athletes are threatening to strike because they claim that they are underpaid. The team owners have released a statement that says, in part, "the average salary for players was $1.2 million last year." The players counter by issuing their own statement that says, in part, "the average player earned only $753,000 last year." Is either side necessarily lying? If you were a sports reporter and had just read this chapter, what questions would you ask about these statistics?

YOU ARE THE RESEARCHER: The Typical American

Is there such a thing as a "typical" American? In this exercise, you will develop a profile of the average American based on measures of central tendency for 10 variables that you will choose from the GSS. Choose the variables that you think are the most important in defining what it means to be a member of this society. You will choose appropriate measures of central tendency for the variables you select and use this information to write a description of the typical American. We will also use this opportunity to introduce a new SPSS program.

STEP 1: Choosing Your Variables

Scroll through the list of variables available in the GSS using either Appendix G or the **Utilities** → **Variables** command in SPSS. Select 10 variables that, in your view, are central to defining or describing the "typical American" and list them in the table on the next page. Select at least one variable from each level of measurement.

Variable	SPSS Name	Explain Exactly What This Variable Measures	Level of Measurement
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

STEP 2: Getting the Statistics

Start *SPSS for Windows* by clicking the SPSS icon on your monitor screen. Load the 2006 GSS, and when you see the message "SPSS Processor is Ready" on the bottom of the closest screen, you are ready to proceed.

Using the Frequencies Procedure for the Mode and Median

The only procedure in SPSS that will produce all three commonly used measures of central tendency (mode, median, and mean) is **Frequencies.** We used this procedure to produce frequency distributions in Chapter 2 and in Appendix F. Here you will use **Frequencies** to get modes and medians for the nominal and ordinal level variables you selected in Step 1.

Begin by clicking **Analyze** on the menu bar, and then click **Descriptive Statistics** and **Frequencies**. In the **Frequencies** dialog box, find the names of your nominal and ordinal level variables in the list on the left and click the arrow button in the middle of the screen to move the names to the **Variables** box on the right.

To request specific statistics, click the **Statistics** button in the **Frequencies** dialog box, and the **Frequencies: Statistics** dialog box will open. Find the **Central Tendency** box on the right and click **Median** and **Mode.** Click **Continue,** and you will be returned to the **Frequencies** dialog box, where you might want to click the **Display Frequency Tables** box. When this box is *not* checked, SPSS will *not* produce frequency distribution tables, and only the statistics we request (mode, median, mean) will appear in the **Output** window. Click **OK,** and SPSS will produce your output.

Report the mode for all nominal level variables and the median for ordinal level variables in the table below, using as many lines as necessary.

Variable	SPSS Name	Level of Measurement	Mode	Median
1				
2				
3				
4				
5				
6				
7				
8				
9				

Using the Descriptives Procedure for the Mean

The **Descriptives** command in *SPSS for Windows* is designed to provide summary statistics for continuous interval-ratio level variables. By default (i.e., unless you tell it otherwise), **Descriptives** produces the mean, the minimum and maximum scores (i.e., the lowest and highest scores, which can be used to compute the range), and the standard deviation. We will consider the range and standard deviation in Chapter 5.

To use **Descriptives,** click **Analyze, Descriptive Statistics,** and **Descriptives.** The **Descriptives** dialog box will open. This dialog box looks just like the **Frequencies** dialog box and works in the same way. Find the names of your interval-ratio level variables and any ordinal level variables with many scores in the list on the left, and once they are highlighted, click the arrow button in the middle of the screen to transfer them to the **Variables** box on the right. Click **OK** to produce your output and record your results in the table below, using as many lines as necessary.

Variable	SPSS Name	Level of Measurement	Mean
1			
2			
3			
4			
5			
6			
7			
8			
9			

STEP 3: Interpreting Results

Examine the two tables displaying your results and write a summary paragraph or two describing the typical American. Be sure to report all 10 variables and, as appropriate, describe the most common case (the mode), the typical case (the median), or the typical score (mean). Use the median for ordinal level variables with relatively few scores (say, less than 8 or 10) and the mean for ordinal level variables with many scores (more than 8 or 10). Use the mean for interval-ratio variables except when there is a strong skew. Write as if you are reporting in a newspaper: your goal should be clarity and accuracy. For example, you might report that the typical American is Protestant and voted for President Bush in 2004.

5 Measures of Dispersion

LEARNING OBJECTIVES

By the end of this chapter, you will be able to:

1. Explain the purpose of measures of dispersion and the information they convey.
2. Compute and explain the range (R), the interquartile range (Q), the standard deviation (s), and the variance (s^2).
3. Select an appropriate measure of dispersion and correctly calculate and interpret the statistic.
4. Describe and explain the mathematical characteristics of the standard deviation.

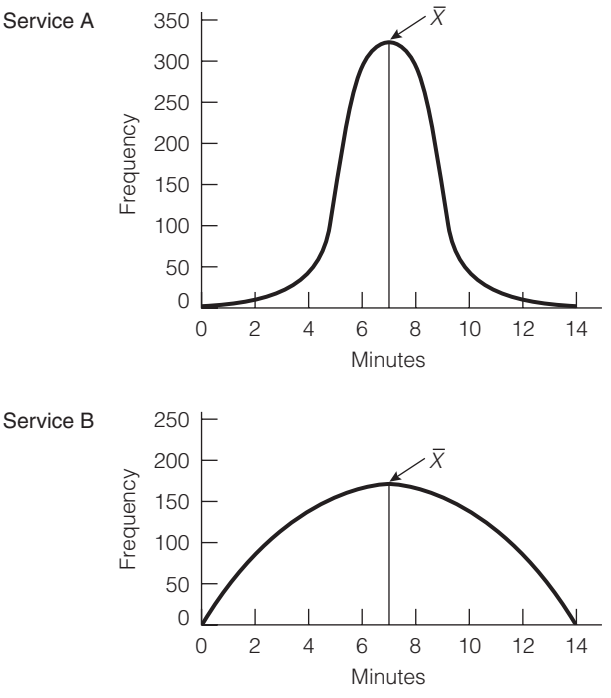
5.1 INTRODUCTION

The last three chapters presented a variety of ways to describe a variable, including frequency distributions, charts and graphs, and measures of central tendency. For a complete description of a distribution of scores, these statistics must be combined with **measures of dispersion,** the subject of this chapter. While measures of central tendency describe the typical, average, or central score, measures of dispersion describe the variety, diversity, or heterogeneity of a set of scores.

The importance of the concept of **dispersion** might be easier to grasp if we consider a brief example. Suppose that the director of public safety wants to evaluate two ambulance services that have contracted with her city to provide emergency medical aid. As a part of the investigation, for both services she has collected data on the response time to calls for assistance. Data collected for the past year show that the average response time is 7.4 minutes for Service A and 7.6 minutes for Service B. These averages or means (calculated by adding up the response times to all calls and dividing by the number of calls) are so close to each other that they provide no basis for judging one service as more efficient than the other. Measures of dispersion, however, can reveal substantial differences, even when the measures of central tendency are equivalent. For example, consider Figure 5.1, which displays the distribution of response times for the two services in the form of line charts (see Chapter 3).

Compare the shapes of these two figures. Note that the line chart for Service B is much flatter than that for Service A. This is because the scores (or response times) for Service B are more spread out or more diverse than the scores for Service A. In other words, Service B was much more variable in response time and, compared to Service A, had more scores in the high and low ranges and fewer in the middle. Service A was more consistent in its response time, and its scores are more clustered or grouped around the mean. Both distributions have essentially the same *average* response time, but there is considerably more variation or *dispersion* in the response times for Service B. If you were the director of public safety, would you be more likely to select an ambulance service that was always on the scene of an emergency in about the same amount of time (Service A) or one that was sometimes very slow and sometimes very quick to respond (Service B)? Note that if you had not considered dispersion, an

FIGURE 5.1 RESPONSE TIME FOR TWO AMBULANCE SERVICES



important difference in the performance of the two ambulance services might have gone unnoticed.

Keep the two shapes in Figure 5.1 in mind as visual representations of the concept of dispersion. The greater clustering of scores around the mean in the distribution for Service A indicates *less* dispersion, and the flatter curve of the distribution for Service B indicates *more* variety or dispersion. Measures of dispersion decrease in value as the scores become less dispersed and the distribution becomes more peaked (or as the distribution looks more and more like that of Service A) and increase in value as the scores become more dispersed and the distribution becomes flatter (as the distribution looks more and more like that of Service B).

This example, along with Figure 5.1, may give you a general notion of what is meant by *dispersion,* but the concept is not easily described in words alone. In this chapter, we will introduce some of the more common measures of dispersion, each of which provides a quantitative indication of variety in a set of scores.

5.2 THE RANGE (*R*) AND INTERQUARTILE RANGE (*Q*)

We begin our treatment of measures of dispersion with the **range (*R*),** which is defined as the distance between the highest and lowest scores in a distribution.

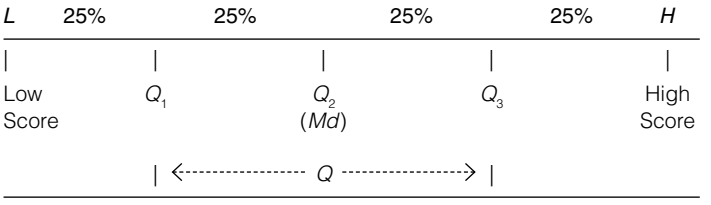
FORMULA 5.1

$$R = \text{High score} - \text{Low score}$$

The range is easy to calculate and most useful as a quick and general indicator of dispersion. However, since it is based on only two scores, the range tells us nothing about the scores between the two extremes. Furthermore, the range

might be quite misleading as a measure of dispersion since almost any sizable distribution will contain some atypically high and low scores.

The **interquartile range (*Q*)** avoids some of the problems associated with *R* by considering only the middle 50% of the cases in a distribution. To find *Q,* arrange the scores from highest to lowest and then divide the distribution into quarters (as distinct from halves, which we used to locate the median). The first quartile (Q_1) is the point below which 25% of the cases fall and above which 75% of the cases fall. The second quartile (Q_2) divides the distribution into halves (thus, Q_2 is equal in value to the median). The third quartile (Q_3) is the point below which 75% of the cases fall and above which 25% of the cases fall. Thus, if line *LH* represents a distribution of scores, the quartiles are located as shown.



The interquartile range is the distance from the third to the first quartile, as stated in Formula 5.2.

FORMULA 5.2

$$Q = Q_3 - Q_1$$

The interquartile range essentially extracts the middle 50% of the distribution and, like *R,* is based on only two scores. While *Q* avoids the problem of being based on the most extreme scores, it has all the other disadvantages associated with *R.* Most importantly, *Q* also fails to yield any information about the variation of the scores other than the two upon which it is based.

5.3 COMPUTING THE RANGE AND INTERQUARTILE RANGE[1]

Table 5.1 presents per capita school expenditures for 20 states. What are the range and interquartile range of these data?

Note that the scores have already been ordered from high to low. This makes the range easy to calculate, and it is a necessary step for finding the interquartile range. Of these 20 states, New Jersey spent the most per capita on public education ($2,221) and Arizona spent the least ($1,152). The range is therefore $2,221 − $1,152, or $1,069 ($R$ = $1,069).

To find *Q,* we must locate the first and third quartiles (Q_1 and Q_3). As we did when finding the median, we can define both of these points in terms of the scores associated with certain cases. Q_1 is determined by multiplying *N* by (0.25). Since (20) × (0.25) is 5, Q_1 is the score associated with the fifth case, counting up from the lowest score. The fifth case is Idaho, with a score of 1,288. So, Q_1 = 1,288. The case that lies at the third quartile (Q_3) is given by multiplying *N* by (0.75), and (20) × (0.75) = 15th case. The 15th case, again counting up from the lowest score, is Texas, with a score of 1,759 (Q_3 = 1,759). Therefore

$$Q = Q_3 - Q_1$$
$$Q = 1{,}759 - 1{,}288$$
$$Q = 471$$

[1]This section is optional.

TABLE 5.1 PER CAPITA EXPENDITURES ON PUBLIC SCHOOLS, 2001

Rank	State	Expenditure ($)
20 (highest)	New Jersey	2,221
19	Wyoming	2,045
18	Michigan	1,911
17	Maine	1,793
16	California	1,770
15	Texas	1,759
14	Ohio	1,754
13	Virginia	1,740
12	Illinois	1,740
11	Pennsylvania	1,710
10	New Hampshire	1,673
9	Louisiana	1,431
8	Oregon	1,430
7	Florida	1,374
6	Nebraska	1,338
5	Idaho	1,288
4	Alabama	1,286
3	North Carolina	1,280
2	Mississippi	1,226
1 (lowest)	Arizona	1,152

Source: U.S. Bureau of the Census. *Statistical Abstract of the United States, 2008*. p. 163. Washington, DC: Government Printing Office. http://www.census.gov/prod/2007pubs/08abstract/educ.pdf.

In most situations, the locations of Q_1 and Q_3 will not be as obvious as they are when $N = 20$. For example, if N had been 157, then Q_1 would be (157)(0.25), or the score associated with the 39.25th case, and Q_3 would be (157)(0.75), or the score associated with the 117.75th case. Since fractions of cases are impossible, these numbers present some problems. The easy solution to this difficulty is to round off and take the score of the closest case to the numbers that mark the quartiles. Thus, Q_1 would be defined as the score of the 39th case and Q_3 as the score of the 118th case.

The more accurate solution would be to take the fractions of cases into account. For example, Q_1 could be defined as the score that is one-quarter of the distance between the scores of the 39th and 40th cases, and Q_3 could be defined as the score that is three-quarters of the distance between the scores of the 117th and 118th cases. (This procedure could be analogous to defining the median—which is Q_2—as halfway between the two middle scores when N is even.) In most cases, the differences in the values of Q for these two methods would be quite small. *(For practice in finding and interpreting Q, see Problems 5.8 and 5.9. The range may be found for any variable in the problems at the end of this chapter.)*

5.4 THE STANDARD DEVIATION AND VARIANCE

Both Q and R are limited in their usefulness because they are based on only two scores. They do not use all the scores in the distribution, and thus, they do not capitalize on all the available information. Also, neither statistic provides any

information on how far the scores are from each other or from some central point such as the mean. How can we design a measure of dispersion that would correct these faults? We can begin with some specifications. A good measure of dispersion should do the following:

1. Use all the scores in the distribution. The statistic should use all the information available.
2. Describe the average or typical distance between the scores and some central point such as the mean. The statistic should give us an idea about how far the scores are from each other or from the center of the distribution.
3. Increase in value as the distribution of scores becomes more diverse. This feature would permit us to tell at a glance which distribution was more variable: the higher the numerical value of the statistic, the greater the dispersion.

One way to develop a statistic to meet these criteria would be to start with the distances between each score and the mean. The distances between the scores and the mean $(X_i - \bar{X})$ are called **deviations,** and this quantity will increase in value as the scores increase in their variety or heterogeneity. If the scores are more clustered around the mean (remember the graph for Service A in Figure 5.1), the deviations would be small. If the scores are more spread out or more varied (like the scores for Service B in Figure 5.1), the deviations would be greater in value. How can we use the deviations of the scores around the mean to develop a useful statistic?

One course of action would be to use the sum of the deviations, $\Sigma(X_i - \bar{X})$, as the basis for a statistic, but as we saw in Section 4.5, the sum of deviations will always be zero. To illustrate, consider a distribution of five scores: 10, 20, 30, 40, and 50 (see Table 5.2). If we sum the deviations of the scores from the mean, we would always wind up with a total of zero.

Still, the sum of the deviations is a logical basis for a statistic that measures the amount of variety in a set of scores, and statisticians have developed two ways around the fact that the positive deviations always equal the negative deviations. Both solutions eliminate the negative signs. The first does so by using the absolute values or by ignoring signs when summing the deviations. This is the basis for a statistic called the *average deviation,* a measure of dispersion that is rarely used and will not be mentioned further.

TABLE 5.2 A DISTRIBUTION OF FIVE SCORES

Scores (X_i)	Deviations ($X_i - \bar{X}$)
10	$(10 - 30) = -20$
20	$(20 - 30) = -10$
30	$(30 - 30) = 0$
40	$(40 - 30) = 10$
50	$(50 - 30) = 20$
$\Sigma(X_i) = 150$	$\Sigma(X_i - \bar{X}) = 0$
$\bar{X} = 150/5 = 30$	

The second solution squares each of the deviations. This makes all values positive because a negative number multiplied by a negative number becomes positive. For example: $(-20) \times (-20) = 400$. In the example above, the sum of the squared deviations would be (400 + 100 + 0 + 100 + 400) or 1,000. Thus, a statistic based on the sum of the squared deviations will have the properties we want in a good measure of dispersion.

Before we finish designing our measure of dispersion, we must deal with another problem. The sum of the squared deviations will increase with the size of the sample: the larger the *number* of scores, the greater the value of the measure. This would make it very difficult to compare the relative variability of distributions based on samples of different size. We can solve this problem by dividing the sum of the squared deviations by N (sample size) and thus standardizing for samples of different sizes.

These procedures yield a statistic known as the **variance,** which is symbolized as s^2. The variance is used primarily in inferential statistics, although it is a central concept in the design of some measures of association. For purposes of describing the dispersion of a distribution, a closely related statistic called the **standard deviation** (symbolized as s) is typically used, and this statistic will be our focus for the remainder of the chapter.[2]

The formulas for the variance and standard deviation are as follows.

FORMULA 5.3

$$s^2 = \frac{\Sigma(X_i - \bar{X})^2}{N}$$

FORMULA 5.4

$$s = \sqrt{\frac{\Sigma(X_i - \bar{X})^2}{N}}$$

Strictly speaking, Formulas 5.3 and 5.4 are for the variance and standard deviation of a population (see Chapter 1 for a discussion of populations and samples). Slightly different formulas, with $N - 1$ instead of N in the denominator, should be used when working with random samples rather than entire populations. This is an important point because many electronic calculators and statistical software packages (including SPSS) use $N - 1$ in the denominator and, thus, produce results that are at least slightly different from those derived from Formulas 5.3 and 5.4. The size of the difference will decrease as sample size increases; however, the problems and examples in this chapter use small samples, and the differences between using N and $N - 1$ in the denominator can be considerable in such cases. Some calculators offer the choice of $N - 1$ or N in the denominator. If you use the latter, the values calculated for the standard deviation should match the values in this text.

To compute the standard deviation, you should construct a table such as Table 5.3 to organize computations. The five scores used in the previous example are listed in the left-hand column, the deviations are in the middle column, and the squared deviations are in the right-hand column.

[2]The symbols for the variance and standard deviation of a sample are s^2 and s, respectively. The symbols for the variance and standard deviation of a population are σ^2 and σ, respectively.

TABLE 5.3 COMPUTING THE STANDARD DEVIATION

Scores (X_i)	Deviations	Deviations Squared $(X_i - X)^2$
10	$(10 - 30) = -20$	$(-20)^2 = 400$
20	$(20 - 30) = -10$	$(-10)^2 = 100$
30	$(30 - 30) = 0$	$(-0)^2 = 0$
40	$(40 - 30) = 10$	$(-10)^2 = 100$
50	$(50 - 30) = 20$	$(-20)^2 = 400$
$\Sigma(X_i) = 150$	$\Sigma(X_i - \bar{X}) = 0$	$\Sigma(X_i - \bar{X})^2 = 1{,}000$

The sum of the last column in Table 5.3 is the sum of the squared deviations and can be substituted into the numerator of Formula 5.4:

$$s = \sqrt{\frac{\Sigma(X_i - \bar{X})^2}{N}}$$

$$s = \sqrt{\frac{1{,}000}{5}}$$

$$s = \sqrt{200}$$

$$s = 14.14$$

To solve Formula 5.4, divide the sum of the squared deviations by N and take the square root of the result. To find the variance, square the standard deviation. For this problem, the variance is $s^2 = (14.14)^2 = 200$.

Application 5.1

At a local preschool, 10 children were observed for one hour, and the number of aggressive acts committed by each was recorded in the following list. What is the standard deviation of this distribution? We will use Formula 5.4 to compute the standard deviation. If you use the preprogrammed function on a hand calculator to check these computations, remember to choose "divide by N" not "divide by $N - 1$."

NUMBER OF AGGRESSIVE ACTS

(X_i)	$(X_i - \bar{X})$	$(X_i - X)^2$
1	$1 - 4 = -3$	9
3	$3 - 4 = -1$	1
5	$5 - 4 = 1$	1
2	$2 - 4 = -2$	4
7	$7 - 4 = 3$	9
11	$11 - 4 = 7$	49
1	$1 - 4 = -3$	9
8	$8 - 4 = 4$	16
2	$2 - 4 = -2$	4
0	$0 - 4 = -4$	16
$\Sigma(X_i) = 40$	$\Sigma(X_i - \bar{X}) = 0$	$\Sigma(X_i - X)^2 = 118$

$$\bar{X} = \frac{\Sigma(X_1)}{N} = \frac{40}{10} = 4.0$$

Substituting into Formula 5.4, we have

$$s = \sqrt{\frac{\Sigma(X_i - \bar{X})^2}{N}} = \sqrt{\frac{118}{10}} = \sqrt{11.8} = 3.44$$

The standard deviation for these data is 3.44.

ONE STEP AT A TIME — Computing the Standard Deviation

Step	Operation
1.	Construct a computing table like Table 5.3 with columns for the scores (X_i), the deviations ($X_i - \bar{X}$), and the deviations squared $(X_i - X)^2$.
2.	Place the scores (X_i) in the left-hand column. Add up the scores and divide by N to find the mean.
3.	Find the deviations ($X_i - \bar{X}$) by subtracting the mean from each score, one at a time.
4.	Add up the deviations. The sum should equal zero (within rounding error). If the sum of the deviations does not equal zero, you have made a computational error and need to repeat Steps 2 and 3.
5.	Square each deviation, one by one.
6.	Add up the squared deviations and transfer this sum to the numerator in Formula 5.4.
7.	Divide the sum of the squared deviations (see Step 6) by N.
8.	Take the square root of the quantity you computed in Step 7. This is the standard deviation.

Application 5.2

Five western and five eastern states were compared as part of a study of traffic safety. The states vary in size, and comparisons are made in terms of the rate of fatal accidents (number of fatal accidents per 1,000 million vehicle miles traveled). Columns for the computation of the standard deviation have already been added to the tables below. Which group of states varies the most in terms of this variable? Computations for both the mean and standard deviation are shown below.

FATALITIES PER 1,000 MILLION VEHICLE MILES TRAVELED FOR 2005, EASTERN STATES

State	Fatalities (X_i)	Deviations ($X_i - \bar{X}$)	Deviations Squared $(X_i - X)^2$
Pennsylvania	15	15 − 10.4 = 4.6	21.16
New York	10	10 − 10.4 = −0.4	0.16
New Jersey	10	10 − 10.4 = −0.4	0.16
Connecticut	9	9 − 10.4 = −1.4	1.96
Massachusetts	8	8 − 10.4 = −2.4	5.76
	$\Sigma(X_i) = 52$	$\Sigma(X_i - \bar{X}) = 0.00$	$\Sigma(X_i - X)^2 = 29.20$

$$\bar{X} = \frac{\Sigma X_i}{N} = \frac{52}{5} = 10.4$$

$$s = \sqrt{\frac{\Sigma(X_i - \bar{X})^2}{N}} = \sqrt{\frac{29.2}{5}} = \sqrt{5.84} = 2.42$$

FATALITIES PER 100,000 LICENSED DRIVERS FOR 2001, WESTERN STATES

State	Fatalities (X_i)	Deviations ($X_i - \bar{X}$)	Deviations Squared $(X_i - X)^2$
Montana	19	19 − 18 = 1	1
Nevada	21	21 − 18 = 3	9
Wyoming	23	23 − 18 = 5	25
Oregon	14	14 − 18 = −4	16
California	13	13 − 18 = −5	25
	$\Sigma(X_i) = 90$	$\Sigma(X_i - \bar{X}) = 0.00$	$\Sigma(X_i - \bar{X})^2 = 76$

(continued next page)

Application 5.2 *(continued)*

$$\bar{X} = \frac{X_i}{N} = \frac{90}{5} = 18$$

$$s = \sqrt{\frac{\Sigma(X_i - \bar{X})^2}{N}} = \sqrt{\frac{76}{5}} = \sqrt{15.20} = 3.90$$

With such small groups, you can tell by simply inspecting the scores that the western states have higher fatality rates. This impression is confirmed by both the median, which is 10 for the eastern states (New Jersey is the middle case) and 19 for the western states (Wyoming is the middle case), and the mean, which is 10.4 for the eastern states and 18.0 for the western states.

The five western states are also more variable and diverse than the eastern states. The range for the western states is 23 − 13, or 10, and for the eastern states it is 15 − 8, or 7. Similarly, the standard deviation for the western states (3.90) is greater than the standard deviation for the eastern states (2.41).

In summary, the five western states average higher fatality rates and are also more variable than the five eastern states.

5.5 COMPUTING THE STANDARD DEVIATION: AN ADDITIONAL EXAMPLE

An additional example will help to clarify the procedures for computing and interpreting the standard deviation. A researcher is comparing the student bodies of two campuses. One college is located in a small town, and almost all students reside on campus. The other is located in a large city, and the students are almost all part-time commuters. The researcher wishes to compare the age structure of the two campuses and has compiled the information presented in Table 5.4. Which student body is older and more diverse in age? (Obviously, these very small groups are much too small to be used for serious research and are used here only to simplify computations.)

We see from the means that the students from the residential campus are quite a bit younger than are the students from the urban campus (19 vs. 23 years of age). Which group is more diverse on this variable? Computing the standard deviation will answer this question.

To solve Formula 5.4, substitute the sum of the right-hand column ("deviations squared") in the numerator and N (5 in this case) in the denominator:

Residential Campus:

$$s = \sqrt{\frac{\Sigma(X_i - \bar{X})^2}{N}} = \sqrt{\frac{4}{5}} = \sqrt{0.8} = 0.89$$

Urban Campus:

$$s = \sqrt{\frac{\Sigma(X_i - \bar{X})^2}{N}} = \sqrt{\frac{88}{5}} = \sqrt{17.6} = 4.20$$

The higher value of the standard deviation for the urban campus means that it is much more diverse. As you can see by scanning the scores, the students at the residential college are within a narrow age range ($R = 20 - 18 = 2$), whereas the students at the urban campus are more mixed and include students of age 25 and 30 ($R = 30 - 18 = 12$). *(For practice in computing and interpreting the standard deviation, see any of the problems at the end of this chapter. Problems with smaller data sets, such as 5.1 and 5.2, are recommended for practicing computations until you are comfortable with these procedures.)*

TABLE 5.4 COMPUTING THE STANDARD DEVIATION FOR TWO CAMPUSES

Residential Campus		
Ages (X_i)	Deviations ($X_i - \bar{X}$)	Deviations Squared $(X_i - X)^2$
18	$(18 - 19) = -1$	$(-1)^2 = 1$
19	$(19 - 19) = 0$	$(-0)^2 = 0$
20	$(20 - 19) = 1$	$(-1)^2 = 1$
18	$(18 - 19) = -1$	$(-1)^2 = 1$
20	$(20 - 19) = 1$	$(-1)^2 = 1$
$\Sigma(X_i) = 95$	$\Sigma(X_i - \bar{X}) = 0$	$\Sigma(X_i - \bar{X})^2 = 4$

$$\bar{X} = \frac{\Sigma(X_1)}{N} = \frac{95}{5} = 19$$

Urban Campus		
Ages (X_i)	Deviations ($X_i - \bar{X}$)	Deviations Squared $(X_i - X)^2$
20	$(20 - 23) = -3$	$(-3)^2 = 9$
22	$(22 - 23) = -1$	$(-1)^2 = 1$
18	$(18 - 23) = -5$	$(-5) = 25$
25	$(25 - 23) = 2$	$(2)^2 = 4$
30	$(30 - 23) = 7$	$(7)^2 = 49$
$\Sigma(X_i) = 115$	$\Sigma(X_i - \bar{X}) = 0$	$\Sigma(X_i - \bar{X})^2 = 88$

$$\bar{X} = \frac{\Sigma(X_1)}{N} = \frac{115}{5} = 23$$

Application 5.3

If you could live anywhere in the United States, where would it be? What criteria would you use to make your selection? Suppose you place a very high value on climate and consider mild temperatures (around 70° F) to be especially desirable. You decide to use average (or mean) temperature to select your next place of residence.

So far, so good. But your decision about the "nicest place to live" could be sadly mistaken if you consider *only* the average temperature. What if the cities you examined had roughly the same mean temperature of 70° but differed in the range or in the extremes of temperature they experienced? What if some cities had temperatures that ranged from blistering (a high of 103°) to freezing (a low of −10°), but others had nothing but mild, balmy temperatures year round. Clearly, you would want to consider dispersion as well as central tendency when choosing your residence.

What American city comes closest to the ideal of a constant 70° temperature year round? The table below presents data for five cities.

AVERAGE MONTHLY TEMPERATURE DATA FOR FIVE CITIES*

	Miami	Chicago	New York City	Dallas	San Diego
Mean	82.79	58.63	62.32	76.28	70.79
Std. Dev.	5.12	19.90	17.31	15.21	4.55
Range	13.80	54.70	47.60	42.50	11.90

*Statistics are computed from average monthly temperatures for the past 30 years.
Source: Data are available at http://www.met.utah.edu/jhorel/html/wx/climate/maxtemp.html.

(continued next page)

Application 5.3 *(continued)*

Compared to your ideal average temperature of 70°, Chicago and New York City are too cold and Dallas and Miami are too hot. This leaves San Diego, which matches your ideal average daily temperature almost exactly.

Considering dispersion, temperature fluctuations are greatest for Chicago (with a standard deviation of almost 20°) and are also considerable for New York City and Dallas. These cities experience all four seasons, and this is not what you want in your ideal residence. Miami and San Diego have much smaller temperature swings throughout the year, and the latter city, with its 71° average temperature and small standard deviation, comes closest to matching your ideal climate. Although there are many more places to investigate in the search for the perfect place to live (and, no doubt, other criteria to consider besides climate), San Diego is the clear winner among these five cities.

5.6 INTERPRETING THE STANDARD DEVIATION

It is very possible that the importance of the standard deviation (i.e., why we calculate it) is not completely obvious to you at this point. You might be asking, What do I know after I've gone to the trouble of calculating the standard deviation? The meaning of this measure of dispersion can be expressed in three ways. The first and most important involves the normal curve, but we will defer this interpretation until the next chapter.

A second way of thinking about the standard deviation is as an index of variability that increases in value as the distribution becomes more variable. In other words, the standard deviation is higher for more diverse distributions and lower for less diverse distributions. The lowest value the standard deviation can have is zero, and this would occur for distributions with no dispersion (i.e., if every single case in the sample had exactly the same score). Thus, zero is the lowest value possible for the standard deviation (although there is no upper limit).

A third way to get a feel for the meaning of the standard deviation is to compare one distribution with another. We already did this when we compared the residential and urban campuses in Section 5.5. You might also do this when comparing one group to another (e.g., men vs. women, black vs. whites) or when comparing the same variable at two different times. For example, suppose we found that the ages of the students on a particular campus had changed over time, as indicated by the following summary statistics.

1975	2005
$\bar{X} = 21$	$\bar{X} = 25$
$s = 1$	$s = 3$

In 1975, students were, on the average, 21 years of age. By 2005, the average age had risen to 25. Clearly, the student body has grown older, and according to the standard deviation, it has also grown more diverse in terms of age. The lower standard deviation for 1975 indicates that the distribution of ages in that year would be more clustered around the mean (remember the distribution for Service A in Figure 5.1), whereas in 2005, the distribution would be flatter and more spread out, like the distribution for Service B in Figure 5.1. In other words, compared to 2005, the students in 1975 were more similar to each other and more clustered in a narrower age range. The standard deviation is extremely useful for making comparisons of this sort between distributions of scores.

BECOMING A CRITICAL CONSUMER: Getting the Whole Picture

As we have seen, measures of dispersion provide important information about variables. In fact, we can say that in order to completely understand a variable, we need three different kinds of information: overall shape (a table or graph; see Chapters 2 and 3); central tendency (see Chapter 4); and dispersion or variability (the subject of this chapter). Reporting only one type of information can be misleading and lead to completely incorrect impressions. This is particularly a problem in the popular media, which typically reports only central tendency (see the headlines about housing and gas prices in Becoming a Critical Consumer in Chapter 4). Here, we will consider several examples of problems stemming from incomplete reporting and take a look at the reporting styles you will find in the professional research literature.

As our first example, consider the following scenario. Assume that the economy is performing poorly and that, as a consequence, the real estate market is also doing poorly; every single house in your area is losing value.* You would naturally expect that average housing prices would fall, right? Maybe not. It is possible that the median or mean price of a house will be rising even during a recession. How? The answer relates to the overall shape of the distribution, not just the central point indentified by the mean or median. For example, the measures of central tendency may rise if expensive houses—say houses costing $2 million—are still selling, even though at a lower price (say $1.6 million), whereas houses in the lower and middle ranges (houses costing less than $250,000) are not selling at all. This pattern would exaggerate the positive skew in the distribution of sale prices, and measures of central tendency could rise even when the housing market is suffering.

The table below presents a simplified picture of how this might work. The left-hand panel of the table is a "balanced" market, where houses in all price ranges are selling, and the right-hand panel is the "skewed" market, where only high-end houses are selling (but for less than they were previously). As you can see, the two measures of central tendency both rise from time 1 to time 2, giving the false impression that the housing market is doing well. The measures of dispersion, on the other hand, both decline over the time period. This indicates that the distribution is becoming less variable and diverse: the sales are concentrated in the high end of the market. Taken together, we can see what these statistics are telling us: the rising mean and median reflect the unbalanced nature of sales, not an increase in housing values. The point, of course, is that partial information about a variable can be dangerously misleading.

Time 1		**Time 2**	
Price	Number of Sales	Price	Number of Sales
$2,000,000	1	$1,600,000	2
750,000	1	650,000	1
500,000	1	500,000	0
250,000	1	250,000	0
100,000	1	100,000	0
Mean sale price = $720,000		Mean sale price = $1,283,333	
Median sale price = $500,000		Median sale price = $1,600,000	
Range of sale prices = $1,900,00		Range of sale prices = $950,000	
Standard deviation of sale prices = $677,200.10		Standard deviation of sale prices = $447,834.29	

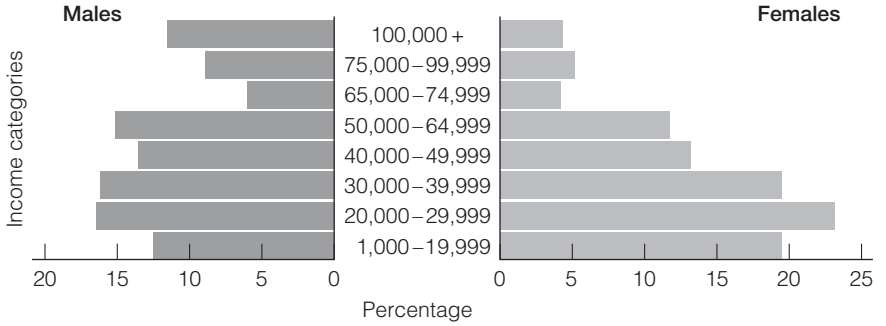
*We are grateful to Felix Salmon at http://seekingalpha.com/article/81663-lies-damn-lies-and-median-house-prices for this example.

(continued next page)

BECOMING A CRITICAL CONSUMER *(continued)*

Let's consider another example, this time using real data. In Chapter 3, we looked at the shrinking income gap between the genders. Looking only at full-time, year-round workers, Figure 3.12 showed that the difference in median income between men and women had shrunk from 65% in 1955 to 78% in 2005. This trend may be heartening to people who advocate gender equality, but it does not present the full picture. The income pyramid below (see Chapter 3) shows the overall shape of the distribution of incomes by gender for 2006. The figure shows that women are much more concentrated in the lower income brackets and men are disproportionately concentrated in the higher income groups. In fact, in the three highest income categories, men outnumber women 2 to 1, and in the very highest, almost 3 to 1. Looking only at the trends in median income may lead to a rosier impression of the extent of income inequality than a consideration of the entire distribution would merit.

DISTRIBUTION OF INCOME FOR MALE AND FEMALE FULL-TIME, YEAR-ROUND WORKERS IN THE UNITED STATES, 2006



READING THE PROFESSIONAL LITERATURE

Our point about the need to describe variables fully—their shape, central tendency, and dispersion—may not be honored in the professional research literature of the social sciences. These projects typically include many variables, and space in journals is expensive, so there may not be room to describe each variable fully. Furthermore, virtually all research reports focus on *relationships* between variables rather than the distribution of single variables. In this sense, univariate descriptive statistics will be irrelevant to the main focus of the report.

This does not mean, of course, that univariate descriptive statistics are irrelevant to the research project. They will be calculated and interpreted for virtually every variable, in virtually every research project. However, these statistics are less likely to be included in final research reports than are the more analytical statistical techniques presented in the remainder of this book.

When included in research reports, measures of central tendency and dispersion will most often be presented in summary form—often in the form of a table. To look at an example of how this is done, we will take a brief look at a project conducted by Professors Bill McCarthy, Diane Felmlee, and John Hagan.[1] They were concerned with the effects of friendship networks on involvement in crime for teenagers. Specifically, they hypothesized that friendships with females would provide better social control and less motivation for criminal behavior than friendships with males. While the popular media has raised awareness of "mean girls" and female aggression in recent years, these researchers believed that teens with strong

[1]McCarthy, Bill, Felmlee, Diane, and Hagan, John. 2004. "Girl Friends Are Better: Gender, Friends, and Crime Among School and Street Youth." *Criminology,* 42: 805–835.

(continued next page)

BECOMING A CRITICAL CONSUMER *(continued)*

relationships with female peers would be less involved in delinquency and would have lower scores on a number of other risk factors associated with deviant behavior.

McCarthy, Felmlee, and Hagan also believed that the relationship between friendships and delinquency would be affected by the social context in which those friendships arose. They had access to two samples of teenagers: (1) males and females who lived at home and attended school and (2) a group of homeless youth who spent their days and nights on the streets. The researchers believed that female friendships that developed in the less-conventional, homeless context would have weaker effects on involvement in delinquency. The researchers tested their ideas on a sample of 563 youths who lived in Toronto, Canada, and attended high school and a sample of street youth from the same city. The researchers reported information on means and standard deviation as background on the respondents. The actual hypotheses are tested with more advanced statistics.

MEANS AND STANDARD DEVIATIONS ON SELECTED VARIABLES FOR FOUR SAMPLES

	Females				Males			
	School		Street		School		Street	
	$\bar{X}$	s	$\bar{X}$	S	$\bar{X}$	S	$\bar{X}$	s
Friendship Variables:								
No. of friends	2.50	0.83	2.31	0.80	2.60	0.86	2.34	1.10
Confide in friends	4.00	1.25	4.18	1.53	4.00	1.36	4.07	1.82
Proportion of friends arrested	0.51	1.00	2.60	1.54	1.00	1.20	2.76	1.53
Background Variables:								
Parental attachment	13.4	4.1	7.0	4.0	12.1	4.1	7.4	4.1
Positive school experience	11.1	2.00	9.0	3.0	10.2	2.4	8.4	3.0
Dependent Variable:								
Property crime	4.9	35.1	36.2	95.4	5.4	23.8	71.7	132.3

There is little difference among the four groups in the total number of friends or in their average scores on a scale that measures how much they can confide in friends. Comparing males with females, females in both the school and street samples were less likely to have deviant friends and to be involved in property crimes. The researchers also found that context mattered: males and females from the street sample are more likely to have deviant friends and are much more likely to be involved in property crime than the males and females from the school sample. Also, "street" males and females had lower scores on scales that measured their attachment to parents and how positive their school experiences were.

SUMMARY

1. Measures of dispersion present information about the heterogeneity or variety in a distribution of scores. When combined with an appropriate measure of central tendency, these statistics convey a large volume of information in just a few numbers. While measures of central tendency locate the central points of the distribution, measures of dispersion indicate the amount of diversity in the distribution.

2. The range (R) is the distance from the highest to the lowest score in the distribution. The interquartile range (Q) is the distance from the third to the first quartile (the "range" of the middle 50% of the scores). These two ranges can be used with variables measured at either the ordinal or interval-ratio level.

3. The standard deviation (s) is the most important measure of dispersion because of its central role in many more-advanced statistical applications. The standard deviation has a minimum value of zero (indicating no variation in the distribution) and increases in value as the variability of the distribution increases. It is used most appropriately with variables measured at the interval-ratio level.

SUMMARY OF FORMULAS

FORMULA 5.1 Range: $R = \text{High score} - \text{Low score}$

FORMULA 5.2 Interquartile range: $Q = Q_3 - Q_1$

FORMULA 5.3 Variance: $s^2 = \dfrac{\Sigma(X_i - \bar{X})^2}{N}$

FORMULA 5.4 Standard deviation: $s = \sqrt{\dfrac{\Sigma(X_i - \bar{X})^2}{N}}$

GLOSSARY

Deviations. The distances between the scores and the mean.

Dispersion. The amount of variety or heterogeneity in a distribution of scores.

Interquartile range (*Q*). The distance from the third quartile to the first quartile.

Measures of dispersion. Statistics that indicate the amount of variety or heterogeneity in a distribution of scores.

Range (*R*). The highest score minus the lowest score.

Standard deviation. The square root of the squared deviations of the scores around the mean, divided by N. The most important and useful descriptive measure of dispersion. s represents the standard deviation of a sample, and σ the standard deviation of a population.

Variance. The squared deviations of the scores around the mean divided by N. A measure of dispersion used primarily in inferential statistics and also in correlation and regression techniques; s^2 represents the variance of a sample and σ^2 the variance of a population.

PROBLEMS

(Problems are labeled with the social science discipline from which they are drawn: SOC for sociology, SW for social work, PS for political science, CJ for criminal justice, PA for public administration, and GER for gerontology.)

5.1 Compute the range and standard deviation of 10 scores reported below. (*HINT: It will be helpful to organize your computations as in Tables 5.3 or 5.4.*)

10, 12, 15, 20, 25, 30, 32, 35, 40, 50

5.2 Compute the range and standard deviation of the 10 test scores below.

77, 83, 69, 72, 85, 90, 95, 75, 55, 45

5.3 In Problem 4.1 at the end of Chapter 4, you calculated measures of central tendency for six variables for freshman and seniors. Three of those variables are reproduced below. Calculate mean, range, and standard deviation for each variable. Write a paragraph summarizing the differences between freshman and seniors.

Out-of-Pocket Expenses		Number of Movies		Rating of Cafeteria Food	
Freshman	Seniors	Freshman	Seniors	Freshman	Seniors
33	65	0	0	10	1
39	62	14	5	7	2
45	60	10	11	2	8
47	90	7	3	1	4
62	62	5	4	8	3
48	57	1	14	6	6
52	40	0	0	10	2
65	49	14	7	0	9
51	45	3	5	5	4
43	85	4	3	6	7
	78		5		4

5.4 In Problem 4.5 at the end of Chapter 4, you calculated measures of central tendency for four variables for 15 respondents. Two of those variables are reproduced below. Calculate mean, range, and standard deviation for each variable. Write a paragraph summarizing this statistical information.

Respondent	Age	Attitude on Abortion Scale (High Score = Strong Opposition)
A	18	10
B	20	9
C	21	8
D	30	10
E	25	7
F	26	7
G	19	9
H	29	6
I	31	10
J	55	5
K	32	4
L	28	3
M	23	2
N	24	1
O	32	9

5.5 SOC In Problem 4.7, you calculated means and medians for the median family incomes for 13 Canadian provinces in 2000 and 2006. Calculate the range and standard deviation for each year separately. What information do the measures of dispersion add to the measures of central tendency you calculated previously? Summarize this information in a paragraph. The data are reproduced here.

Province	2000	2006
Newfoundland and Labrador	38,800	50,500
Prince Edward Island	44,200	56,100
Nova Scotia	44,500	56,400
New Brunswick	43,200	54,000
Quebec	47,700	59,000
Ontario	55,700	66,600
Manitoba	47,300	58,700
Saskatchewan	45,800	60,500
Alberta	55,200	78,400
British Columbia	49,100	62,600
Yukon	56,000	76,000
Northwest Territories	61,000	88,800
Nunavut	37,600	54,300

Source: Statistics Canada: http://www40.statcan.ca/l01/cst01/famil108a.htm?sdi=income.

5.6 SOC Data on several variables measuring overall heath and well-being for ten nations are reported below for 2000, with projections to 2010. Are these nations becoming more or less diverse on these variables? Calculate the mean, range, and standard deviation for each year for each variable. Summarize the results in a paragraph.

	Life Expectancy (years)		Infant Mortality Rate*		Fertility Rate#	
	2000	2010	2000	2010	2000	2010
Canada	80	81	5.0	4.5	1.6	1.6
U.S.	77	79	6.8	6.2	2.0	2.1
Mexico	72	74	25.4	18.5	2.6	2.3
Colombia	71	73	24.0	17.8	2.7	2.4
Japan	80	82	3.9	3.6	1.4	1.5
China	72	74	28.1	20.5	1.8	1.8
Sudan	57	61	68.7	55.2	5.4	4.2
Kenya	48	44	68.0	60.4	3.5	2.4
Italy	79	80	5.8	5.1	1.2	1.2
Germany	78	79	4.7	4.2	1.4	1.4

* Number of deaths of children under one year of age per 1,000 live births.

Average number of children per female.

Source: U.S. Bureau of the Census. 2003. *Statistical Abstract of the United States, 2002.* p. 829. Washington, DC: Government Printing Office.

5.7 SOC Labor force participation rates (percentage employed), percentage high school graduates, and mean income for males and females in 10 states are reported below. Calculate a mean and a standard deviation for both groups for each variable and describe the differences. Are males and females unequal on any of these variables? How great is the gender inequality?

	Labor Force Participation		High School Graduates (%)		Mean Income	
State	Male	Female	Male	Female	Male	Female
A	74	54	65	67	35,623	27,345
B	81	63	57	60	32,345	28,134
C	81	59	72	76	35,789	30,546
D	77	60	77	75	38,907	31,788
E	80	61	75	74	42,023	35,560
F	74	52	70	72	34,000	35,980
G	74	51	68	66	25,800	19,001
H	78	55	70	71	29,000	26,603
I	77	54	66	66	31,145	30,550
J	80	75	72	75	34,334	29,117

5.8 CJ **a.** Per capita expenditures for police protection for 20 cities are reported below for 1995 and 2000. Compute a mean and standard deviation for each year, and describe the differences in expenditures for the five-year period.

b. Find the Interquartile Range (Q) for both years. Does Q change in the same way as the standard deviation?

City	1995	2000
A	180	210
B	95	110
C	87	124
D	101	131
E	52	197
F	117	200
G	115	119
H	88	87
I	85	125
J	100	150
K	167	225
L	101	209
M	120	201
N	78	141
O	107	94
P	55	248
Q	78	140
R	92	131
S	99	152
T	103	178

5.9 SOC **a.** Below are listed the rates of abortion per 100,000 women for 20 states in 1973 and 1975. Describe what happened to these distributions over the two-year period. Did the average rate increase or decrease? What happened to the dispersion of this distribution? What happened between 1973 and 1975 that might explain these changes in central tendency and dispersion? *(HINT: It was a Supreme Court decision.)*

b. Find the interquartile range (Q) for both years. Does Q change in the same way as the standard deviation?

State	1973	1975
Maine	3.5	9.5
Massachusetts	10.0	25.7
New York	53.5	40.7
Pennsylvania	12.1	18.5
Ohio	7.3	17.9
Michigan	18.7	20.3
Iowa	8.8	14.7
Nebraska	7.3	14.3
Virginia	7.8	18.0
South Carolina	3.8	10.3
Florida	15.8	30.5
Tennessee	4.2	19.2
Mississippi	0.2	0.6
Arkansas	2.9	6.3
Texas	6.8	19.1
Montana	3.1	9.9
Colorado	14.4	24.6
Arizona	6.9	15.8
California	30.8	33.6
Hawaii	26.3	31.6

Source: U.S. Bureau of the Census. 1977. *Statistical Abstract of the United States, 1977.* 98th ed. Washington, DC: Government Printing Office.

5.10 SW One of your goals as the new chief administrator of a large social service bureau is to equalize workloads within the various divisions of the agency. You have gathered data on caseloads per worker within each division. Which division comes closest to the ideal of an equalized workload? Which is farthest away?

A	B	C	D
50	60	60	75
51	59	61	80
55	58	58	74
60	55	59	70
68	56	59	69
59	61	60	82
60	62	61	85
57	63	60	83
50	60	59	65
55	59	58	60

5.11 At St. Algebra College, the math department created some special sections of the freshman math course that used a variety of innovative teaching techniques. Students were randomly assigned to either the traditional sections or the experimental sections, and all students were given the same final exam. The results of the final are summarized below. What was the effect of the experimental course?

Traditional	Experimental
$\bar{X}$ = 77.8	$\bar{X}$ = 76.8
s = 12.3	s = 6.2
N = 478	N = 465

5.12 You are the governor of the state and must decide which of four metropolitan police departments will win the annual award for efficiency. The performance of each department is summarized in monthly arrest statistics as reported below. Which department will win the award? Why?

	Departments			
	A	B	C	D
$\bar{X}$ =	601.30	633.17	592.70	599.99
s =	2.30	27.32	40.17	60.23

YOU ARE THE RESEARCHER: The Typical American and U.S. Culture Wars Revisited

Two projects are presented below. The first is a follow-up for the project presented at the end of Chapter 4, and the second presents a new SPSS command. You are urged to complete both.

PROJECT 1: The Typical American

In Chapter 4, you described the typical American by using 10 variables selected from the 2006 GSS. Now you will examine variation or dispersion of some of the variables you selected.

STEP 1: Choosing the Variables

Select at least five of the ordinal and interval-ratio variables you used in Chapter 4. Add more variables if you had fewer than five variables at this level of measurement.

STEP 2: Getting the Statistics

Use **Descriptives** to find the range and standard deviation for each of your selected variables. With the 2006 GSS loaded, click **Analyze, Descriptive Statistics,** and **Descriptives** from the Main Menu of SPSS. The **Descriptives** dialog box will open. Find the names of your variables in the list on the left and click the right arrow button to transfer them to the **Variables** box. Click **OK,** and SPSS will produce the same output you analyzed in Chapter 4. Now, however, we will consider dispersion rather than central tendency.

The standard deviation for each variable is reported in the column labeled "Std. Deviation," and the range can be computed from the values given in the Minimum and Maximum columns. At this point, the range is probably easier to understand and interpret than is the standard deviation. As we saw in Section 5.6, the latter is more meaningful when we have a point of comparison. For example, suppose we were interested in the variable *tvhours* and how television-viewing habits have changed over the years. The **Descriptives** output for 2006 shows that people watched an average of 3.00 hours a day with a standard deviation of 2.40. Suppose that data from 1976 showed an average of 3.70 hours of television viewing a day with a standard deviation of 1.1? You could conclude that television watching had, on the average, decreased over the 30-year period, but that Americans had also become much more diverse in their viewing habits.

Record your results in the table below.

Variable	SPSS Name	Mean	Range	Standard Deviation
1				
2				
3				
4				
5				

STEP 3: Interpreting Results

Start with the descriptions of the variables you wrote in Chapter 4 and add information about dispersion, referring both to the range and the standard deviation. Your job now is to describe both the "typical" American *and* to suggest the amount of variation around this central point.

PROJECT 2: The Culture Wars Revisited

In Chapters 2 and 3, you examined some of the dimensions of the so-called culture wars in U.S. society. In this project, you will reexamine the topic and learn how to use a new SPSS command to combine existing variables into new summary variables. These created variables can be used to summarize feelings and attitudes in general and to explore new dimensions of the issue.

STEP 1: Creating a Scale to Summarize Attitudes Toward Abortion

One of the most controversial issues in the American culture wars is abortion: under what conditions, if any, should abortion be legal? The GSS data set supplied with this text includes two variables that measure support for legal abortion. The variables differ in context: one asks specifically if an abortion should be available when the pregnancy is the result of rape (*abrape*), and the other is more open-ended, asking if an abortion should be available when the woman wants it "for any reason" (*abany*). Since these two situations are distinct, each item should be analyzed in its own right. Suppose, however, that you wanted to create a summary scale that indicated a person's *overall* feelings about abortion.

One way to do this would be to add the scores on the two variables together. This would create a new variable, which we will call *abscale,* with three possible

scores. If a respondent was consistently pro-abortion and answered "yes" (coded as 1) to both items, the respondent's score on the summary variable would be 2. A score of 3 would occur when a respondent answered "yes" to one item and "no" to the other. This might be labeled an intermediate or moderate position. The final possibility would be a score of 4 if the respondent answered "no" (coded as 2) to both items. This would be a consistent anti-abortion position. The table below summarizes the scoring possibilities.

If Response on *abany* Is:	And Response on *abrape* Is:	Then Score on *abscale* Will Be:
1 (Yes)	1 (Yes)	2 (Pro-abortion)
1 (Yes)	2 (No)	3 (Moderate)
2 (No)	1 (Yes)	3 (Moderate
2 (No)	2 (No)	4 (Anti-abortion)

The new variable, *abscale,* summarizes each respondent's overall position on the issue. Once created, *abscale* could be analyzed, transformed, and manipulated exactly like a variable actually recorded in the data file.

Using Compute

We will create our new variable using an SPSS command called **Compute**. To use this command, click **Transform** and then **Compute** from the main menu. The **Compute Variable** window will appear. Find the **Target Variable** box in the upper-left-hand corner of this window. The first thing we need to do is assign a name (*abscale*) to the new variable we are about to compute and type that name in this box. Do this now. Next, we need to tell SPSS how to compute the new variable. In this case, *abscale* will be computed by adding the scores of *abany* and *abrape.* Find *abany* in the variable list on the left and click the arrow button in the middle of the screen to transfer the variable name to the **Numeric Expression** box. Next, click the plus sign (+) on the calculator pad under the **Numeric Expression** box, and the sign will appear next to *abany.* Finally, highlight *abrape* in the variable list and click the arrow button to transfer the variable name to the **Numeric Expression** box.

The expression in the **Numeric Expression** box should now read

abany + *abrape*

Click **OK,** and *abscale* will be created and added to the data set. If you want to keep this new variable permanently, click **Save** from the **File** menu, and the updated data set with *abscale* added will be saved to disk. If you are using the student version of SPSS, remember that your data set is limited to 50 variables.

Examining the Variables

We now have three variables that measure attitudes toward abortion—two items referring to specific situations and a more general summary item. It is always a good idea to check the frequency distribution for computed variables to make sure that the computations were carried out as we intended. Use the **Frequencies** procedure (click **Analyze**, **Descriptive Statistics**, and **Frequencies**) to get tables for *abany, abrape,* and *abscale.* Your output will look like this.

ABORTION IF WOMAN WANTS FOR ANY REASON (*abany*)*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	YES	276	19.4	43.8	43.8
	NO	354	24.8	56.2	100.0
	Total	630	44.2	100.0	
Missing	NAP	776	54.4		
	Total	1,426	100.0		

*This table has been slightly edited to improve readability.

ABORTION IF PREGNANCY IS THE RESULT OF RAPE (*abrape*)*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	YES	490	34.4	79.4	79.4
	NO	127	8.9	20.6	100.0
	Total	617	43.3	100.0	
Missing	NAP	776	54.4		
	Total	1,426	100.0		

*This table has been slightly edited to improve readability.

SUMMARY SCALE (*abscale*)*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	2.00	268	18.8	44.2	44.2
	3.00	219	15.4	36.1	80.2
	4.00	120	8.4	19.8	100.0
	Total	607	42.6	100.0	
Missing	System	819	57.4		
	Total	1,426	100.0		

*This table has been slightly edited to improve readability.

Missing Cases

Note that only 630 and 617 of the 1,426 total respondents to the 2006 GSS answered the two specific abortion items. Remember that no respondent is given the entire GSS, and the vast majority of the missing cases received a form of the GSS that did not include these two items. Now look at *abscale* and note that even fewer cases (607) are included in the summary scale than in either of the two original items. When SPSS executes a **Compute** statement, it automatically eliminates any cases that are missing scores on any of the constituent items. If these cases were not eliminated, a variety of errors and misclassifications could result. For example, if cases with missing scores were included, a person who scored a 2 (anti-abortion) on *abany* and then failed to respond to *abrape* would have a total score of 2 on *abscale.* Thus, this case would be treated as pro-abortion when the only information we have indicates that this respondent is anti-abortion. To eliminate this kind of error, cases with missing scores on any of the constituent variables are deleted from calculations.

STEP 2: Interpreting the Results

Compare the distributions of these three variables with each other and write a report in which you answer the following questions.

1. How does the level of approval for abortion differ from one specific situation to another?
2. What percentage of the respondents approved or disapproved of abortion in both circumstances? What percentage approved of abortion in one situation, but not the other? (Use the distribution of *abscale* to answer this question.)
3. What do these patterns reveal about value consensus in American society? Are Americans generally in agreement about the issue of abortion?

6 The Normal Curve

LEARNING OBJECTIVES

By the end of this chapter, you will be able to:

1. Define and explain the concept of the normal curve.
2. Convert empirical scores to *Z* scores and use *Z* scores and the normal curve table (Appendix A) to find areas above, below, and between points on the curve.
3. Express areas under the curve in terms of probabilities.

6.1 INTRODUCTION

The **normal curve** is a concept of great importance in statistics. In combination with the mean and standard deviation, the normal curve can be used to construct precise descriptive statements about empirical distributions. In addition, as we shall see in Part II, it is also central to the theory that underlies inferential statistics. Thus, this chapter will conclude our treatment of descriptive statistics in Part I and lay important groundwork for Part II.

The normal curve is a theoretical model, a kind of frequency polygon or line chart that is unimodal (i.e., it has a single mode or peak), and perfectly smooth and symmetrical (unskewed) so that its mean, median, and mode are all exactly the same value. It is bell shaped and its tails extend infinitely to the left and to the right. Of course, no empirical distribution has a shape that perfectly matches this ideal model, but many variables (e.g., test results from large classes, standardized test scores, people's height and weight) are close enough to permit the assumption of normality. In turn, this assumption makes possible one of the most important uses of the normal curve—the description of empirical distributions based on our knowledge of the theoretical normal curve.

The crucial point about the normal curve is that distances along the horizontal axis of the distribution, when measured in standard deviations from the mean, always encompass the same proportion of the total area under the curve. In other words, the distance from any given point to the mean (when measured in standard deviations) will mark off exactly the same proportion of the curve's area.

To illustrate, Figures 6.1 and 6.2 present two hypothetical distributions of IQ scores, one for a group of males and one for a group of females, both normally distributed (or nearly so). The basic statistical characteristics of the two samples are as follows:

Males	Females
$\bar{X} = 100$	$\bar{X} = 100$
$s = 20$	$s = 10$
$N = 1{,}000$	$N = 1{,}000$

Figures 6.1 and 6.2 are drawn with two scales on the horizontal axis of the graph. The upper scale is stated in IQ units and the lower scale in standard

FIGURE 6.1 IQ SCORES FOR A GROUP OF MALES

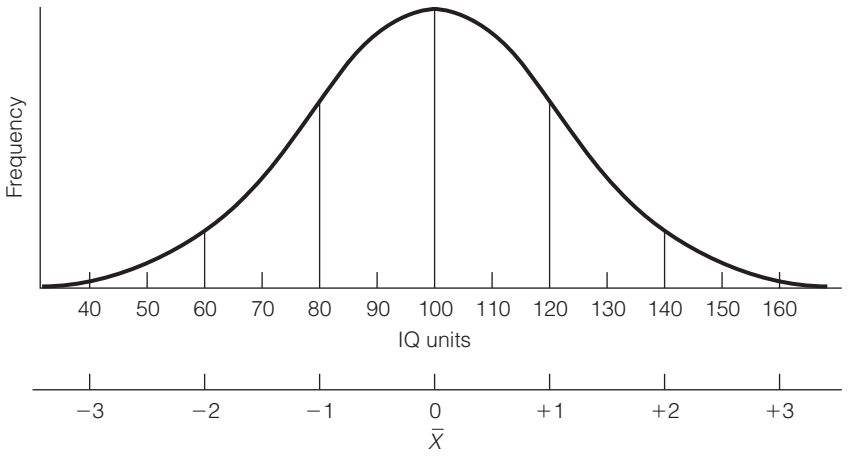
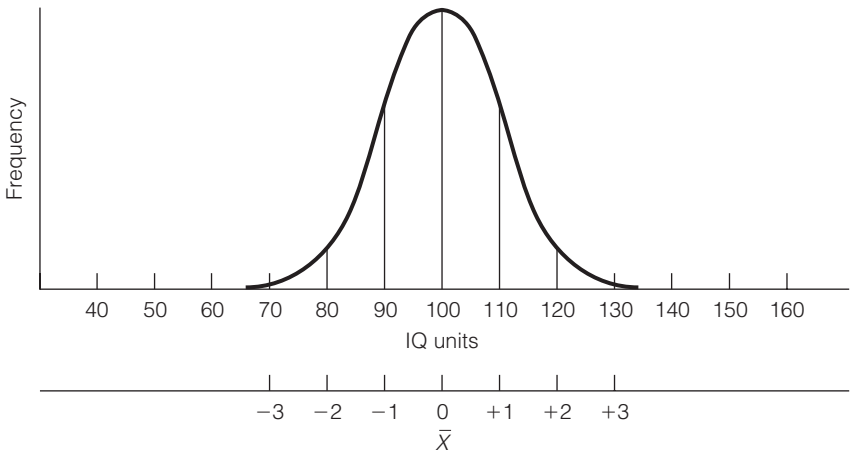


FIGURE 6.2 IQ SCORES FOR A GROUP OF FEMALES



deviations from the mean. These scales are interchangeable, and we can easily shift from one to the other. For example, for the males, an IQ score of 120 is one standard deviation (remember that $s = 20$ for the male group) above (to the right of) the mean, and an IQ of 140 is two standard deviations above (to the right of) the mean. Scores below or to the left of the mean are marked as negative values because they are less than the mean. For the males, an IQ of 80 is one standard deviation below the mean, an IQ score of 60 is two standard deviations below the mean, and so forth. Figure 6.2 is marked in a similar way, except the markings occur at different points since its standard deviation is a different value ($s = 10$ for the female group). For the female sample, one standard deviation above the mean is an IQ of 110, one standard deviation below the mean is an IQ of 90, and so forth.

Recall that, for the normal curve, the crucial point is that we always encompass exactly the same proportion of the total area under the curve when we measure distances along the horizontal axis in standard deviations. Specifically, the distance between one standard deviation above (to the right of) the mean and one standard deviation below (to the left of) the mean (or ±1 standard deviation) encompasses exactly 68.26% of the total area under the curve. This means that in Figure 6.1, 68.26% of the total area lies between the score of 80 (−1 standard deviation) and 120 (+1 standard deviation). The standard deviation for females is 10, so the same percentage of the area (68.26%) lies between the scores of 90 and 110. As long as an empirical distribution is normal, 68.26% of the total area will always lie between ±1 standard deviation, regardless of the trait being measured and the number values of the mean and standard deviation.

It will be useful to familiarize yourself with the following relationships between distances from the mean and areas under the curve.

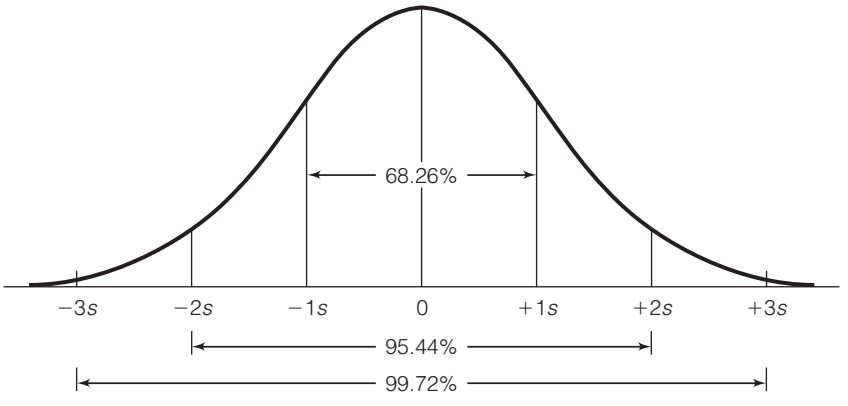
Between	Lies
±1 standard deviation	68.26% of the area
±2 standard deviations	95.44% of the area
±3 standard deviations	99.72% of the area

These relationships are displayed graphically in Figure 6.3.

The relationship between distance from the mean and area allows us to describe empirical distributions that are at least approximately normal. The position of individual scores can be described with respect to the mean, the distribution as a whole, or any other score in the distribution.

The areas between scores can also be expressed, if desired, in numbers of cases rather than percentage of total area. For example, a normal distribution of 1,000 cases will contain about 683 cases (68.26% of 1,000 cases) between ±1 standard deviation of the mean, about 954 between ±2 standard deviations, and about 997 between ±3 standard deviations. Thus, for any normal distribution, only a few cases will be farther away from the mean than ±3 standard deviations.

FIGURE 6.3 AREAS UNDER THE THEORETICAL NORMAL CURVE



6.2 COMPUTING *Z* SCORES

To find the percentage of the total area (or number of cases) above, below, or between scores in an empirical distribution, the original scores must first be expressed in units of the standard deviation or converted into ***Z* scores.** The original scores could be in any unit of measurement (inches, IQ, dollars), but *Z* scores always have the same values for their mean (0) and standard deviation (1).

Think of converting the original scores into *Z* scores as a process of changing value scales, similar to changing from meters to yards, kilometers to miles, or gallons to liters. These units are different but equally valid ways of expressing distance, length, or volume. For example, a mile is equal to 1.61 kilometers, so two towns that are 10 miles apart are also 16.1 kilometers apart and a 5-K race covers about 3.10 miles. Although you may be more familiar with miles than kilometers, either unit works perfectly well as a way of expressing distance.

In the same way, the original (or *raw*) scores and *Z* scores are two equally valid but different ways of measuring distances under the normal curve. In Figure 6.1, for example, we could describe a particular score in terms of IQ units ("John's score was 120") or standard deviations ("John scored one standard deviation above the mean").

When we compute *Z* scores, we convert the original units of measurement (IQ scores, inches, dollars, etc.) to *Z* scores and, thus, "standardize" the normal curve to a distribution that has a mean of 0 and a standard deviation of 1. The mean of the empirical normal distribution will be converted to 0, its standard deviation to 1, and all values will be expressed in *Z*-score form. The formula for computing *Z* scores is as follows.

FORMULA 6.1

$$Z = \frac{X_i - \bar{X}}{s}$$

This formula will convert any score (X_i) from an empirical normal distribution into the equivalent *Z* score. To illustrate with the men's IQ data (Figure 6.1), the *Z*-score equivalent of a raw score of 120 would be

$$Z = \frac{120 - 100}{20} = +1.00$$

The *Z* score of +1.00 means that the empirical score (IQ = 120) lies one standard deviation unit above (to the right of) the mean. A negative score would fall below (to the left of) the mean. *(For practice in computing Z scores, see any of the problems at the end of this chapter.)*

ONE STEP AT A TIME **Computing *Z* Scores**

Step	*Operation*
1.	Subtract the value of the mean ($\bar{X}$) from the value of the score (X_i).
2.	Divide the quantity found in Step 1 by the value of the standard deviation (s).

TABLE 6.1 AN ILLUSTRATION OF HOW TO FIND AREAS UNDER THE NORMAL CURVE USING APPENDIX A

(a) *Z*	(b) Area Between Mean and *Z*	(c) Area Beyond *Z*
0.00	0.0000	0.5000
0.01	0.0040	0.4960
0.02	0.0080	0.4920
0.03	0.0120	0.4880
.	.	.
.	.	.
1.00	0.3413	0.1587
1.01	0.3438	0.1562
1.02	0.3461	0.1539
1.03	0.3485	0.1515
.	.	.
.	.	.
1.50	0.4332	0.0668
1.51	0.4345	0.0655
1.52	0.4357	0.0643
1.53	0.4370	0.0630

6.3 THE NORMAL CURVE TABLE

The theoretical normal curve has been very thoroughly analyzed and described by statisticians. The areas related to any *Z* score have been precisely determined and organized into a table format. This **normal curve table** or *Z*-score table is presented as Appendix A in this book, and a small portion of it is reproduced here as Table 6.1 for purposes of illustration.

The normal curve table consists of three columns, with *Z* scores in the left-hand column (a), areas between the *Z* score and the mean of the curve in the middle column (b), and areas beyond the *Z* score in the right-hand column (c). To find the area between any *Z* score and the mean, go down the column labeled *Z* until you find the *Z* score. For example, go down column a either in Appendix A or in Table 6.1 until you find a *Z* score of 1.00. The entry in column b (Area Between Mean and *Z*) is 0.3413. The table presents all areas in the form of proportions, but we can easily translate these into percentages by multiplying them by 100 (see Chapter 2). We could say either that a proportion of 0.3413 of the total area under the curve lies between a *Z* score of 1.00 and the mean, or that 34.13% of the total area lies between a score of 1.00 and the mean.

To illustrate further, find the *Z* score of 1.50 either in column a of Appendix A or the abbreviated table presented in Table 6.1. This score is 1.5 standard deviations to the right of the mean and corresponds to an IQ of 130 for the men's IQ data. The area in column b for this score is 0.4332. This means that a proportion of 0.4332, or a percentage of 43.32%, of all the area under the curve lies between this score and the mean.

The third column in the table presents "Areas Beyond *Z*." These are areas above (to the right of) positive scores or below (to the left of) negative scores. This column will be used when we want to find an area above or below certain *Z* scores, an application that will be explained in Section 6.4.

To conserve space, the normal curve table in Appendix A includes only positive *Z* scores. Since the normal curve is perfectly symmetrical, however, the

area between a negative score and the mean (column b) will be exactly the same as that for a positive score of the same numerical value. For example, the area between a Z score of −1.00 and the mean will be 34.13%, exactly the same as the area we found previously for a score of +1.00. As will be repeatedly demonstrated below, however, the sign of the Z score is extremely important and should be carefully noted.

For practice in using Appendix A to describe areas under an empirical normal curve, verify that the Z scores and areas given below are correct for the men's IQ distribution. For each IQ score, the equivalent Z score is computed using Formula 6.1 and then Appendix A is used to find areas between the score and the mean. ($\bar{X} = 100$, $s = 20$ throughout.)

IQ Score	*Z* Score	Area Between *Z* and the Mean
110	+0.50	19.15%
125	+1.25	39.44%
133	+1.65	45.05%
138	+1.90	47.13%

The same procedures apply when the Z-score equivalent of an actual score happens to be a minus value (that is, when the raw score lies below the mean).

IQ Score	*Z* Score	Area Between *Z* and the Mean
93	−0.35	13.68%
85	−0.75	27.34%
67	−1.65	45.05%
62	−1.90	47.13%

Remember that the areas in Appendix A will be the same for Z scores of the same numerical value regardless of sign. The area between the score of 138 (+1.90) and the mean is the same as the area between 62 (−1.90) and the mean. *(For practice in using the normal curve table, see any of the problems at the end of this chapter.)*

6.4 FINDING TOTAL AREA ABOVE AND BELOW A SCORE

To this point, we have seen how the normal curve table (Appendix A) can be used to find areas between a Z score and the mean. The table can also be used to find other kinds of areas in empirical distributions that are at least approximately normal in shape. For example, suppose you need to determine the total area below the scores of two male subjects in the distribution described in Figure 6.1. The first subject has a score of 117 ($X_1 = 117$), which is equivalent to a Z score of +0.85:

$$Z_1 = \frac{X_i - \bar{X}}{s} = \frac{117 - 100}{20} = \frac{17}{20} = +0.85$$

The plus sign of the Z score indicates that the score should be placed above (to the right of) the mean. To find the area below a positive Z score, the area between the score and the mean (given in column b) must be added to the area below the mean. As we noted earlier, the normal curve is symmetrical

FIGURE 6.4 FINDING THE AREA BELOW A POSITIVE *Z* SCORE

(unskewed), and its mean will be equal to its median. Therefore, the area below the mean (because it is equal to the median) will be 50%. Study Figure 6.4 carefully. We are interested in the shaded area.

By consulting the normal curve table, we find that the area between the score and the mean (see column b) is 30.23% of the total area. The area below a *Z* score of +0.85 is therefore 80.23% (50.00% + 30.23%). This subject scored higher than 80.23% of the persons tested.

The second subject has an IQ score of 73 ($X_2 = 73$), which is equivalent to a *Z* score of −1.35:

$$Z_2 = \frac{X_i - \overline{X}}{s} = \frac{73 - 100}{20} = -\frac{27}{20} = -1.35$$

To find the area below a negative score, we use the column labeled "Area Beyond *Z*." The area of interest is depicted in Figure 6.5, and we must determine the size of the shaded area. The area beyond a score of −1.35 is given as 0.0885, which we can express as 8.85%. The second subject ($X_2 = 73$) scored higher than 8.85% of the tested group.

In the examples above, we found the area *below* a score. The same procedures are used to find the area *above* a score. If we need to determine the area above an IQ score of 108, for example, we would first convert to a *Z* score,

$$Z = \frac{X_i - \overline{X}}{s} = \frac{108 - 100}{20} = \frac{8}{20} = +0.40$$

FIGURE 6.5 FINDING THE AREA BELOW A NEGATIVE *Z* SCORE

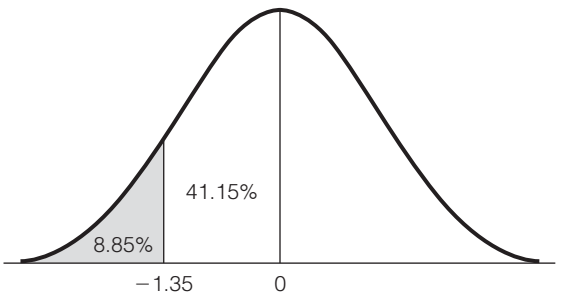
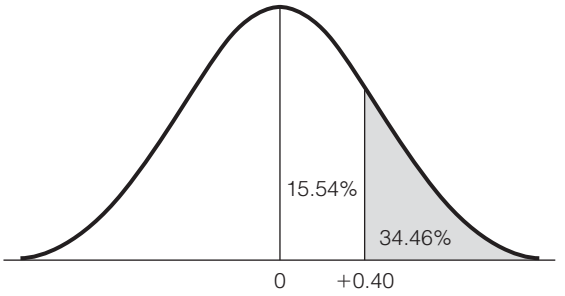


FIGURE 6.6 FINDING THE AREA ABOVE A POSITIVE *Z* SCORE



and then proceed to Appendix A. The shaded area in Figure 6.6 represents the area in which we are interested. The area above a positive score is found in the "Area Beyond *Z*" column, and in this case, the area is 0.3446, or 34.46%.

These procedures are summarized in the One Step at a Time box. To find the total area above a positive *Z* score or below a negative *Z* score, go down the "*Z*" column of Appendix A until you find the score. The area you are seeking will be in the Area Beyond *Z* column (column c). The value in column c can be left as a proportion or changed to a percentage (by multiplying be 100).

To find the total area below a positive *Z* score or above a negative score, locate the *Z* score in column a and then add the area in the "Area Between Mean and *Z*" (column b) to either 0.5000 or change to percentages and add it to 50.00%. These techniques might be confusing at first, and you will find it helpful to draw the curve and shade in the areas in which you are interested. *(For practice in finding areas above or below Z scores, see Problems 6.1–6.7.)*

ONE STEP AT A TIME **Finding Areas Above and Below Positive and Negative *Z* Scores**

Step	***Operation***
1.	Compute the Z score. Note whether the score is positive or negative.
2.	Find the Z score in column a of the normal curve table (Appendix A).
To find the total area below a positive Z score:	
3.	Add the column b area for this score to 0.5000 or change to percentages and add it to 50.00%.
To find the total area above a positive Z score:	
3.	Look in column c for this score for the area expressed as a proportion. Multiply the column c area by 100 to express the area as a percentage.
To find the total area below a negative Z score:	
3.	Look in column c for this score for the area expressed as a proportion. Multiply the column c area by 100 to express the area as a percentage.
To find the total area above a negative Z score:	
3.	Add the column b area for this score to 0.5000 or change to percentages and add it to 50.00%.

Application 6.1

You have just received your score on a test of intelligence. If your score was 78 and you know that the mean score on the test was 67, with a standard deviation of 5, how does your score compare with the distribution of all test scores?

If you can assume that the test scores are normally distributed, you can compute a Z score and find the area below or above your score. The Z-score equivalent of your raw score would be

$$Z = \frac{X_i - \overline{X}}{s} = \frac{78 - 67}{5} = \frac{11}{5} = +2.20$$

Turning to Appendix A, we find that the area between mean and Z for a Z score of 2.20 is 0.4861, which could also be expressed as 48.61%. Since this is a positive Z score, we need to add this area to 50.00% to find the total area below. Your score is higher than 48.61 + 50.00, or 98.61%, of all the test scores. You did pretty well!

6.5 FINDING AREAS BETWEEN TWO SCORES

On occasion, you will need to determine the area between two scores rather than the total area above or below one score. In the case where the scores are on opposite sides of the mean, the area between the scores can be found by adding the areas between each score and the mean. Using the men's IQ data as an example, if we wished to know the area between the IQ scores of 93 and 112, we would convert both scores to Z scores, find the area between each score and the mean from Appendix A, and add these two areas together. The first IQ score of 93 converts to a Z score of −0.35:

$$Z_1 = \frac{X_i - \overline{X}}{s} = \frac{93 - 100}{20} = -\frac{7}{20} = -0.35$$

The second IQ score (112) converts to +0.60:

$$Z_2 = \frac{X_i - \overline{X}}{s} = \frac{112 - 100}{20} = \frac{12}{20} = 0.60$$

Both scores are placed on Figure 6.7. We are interested in the total shaded area. The total area between these two scores is 13.68% + 22.57%, or 36.25%. Therefore, 36.25% of the total area (or about 363 of the 1,000 cases) lies between the IQ scores of 93 and 112.

FIGURE 6.7 FINDING THE AREA BETWEEN TWO SCORES

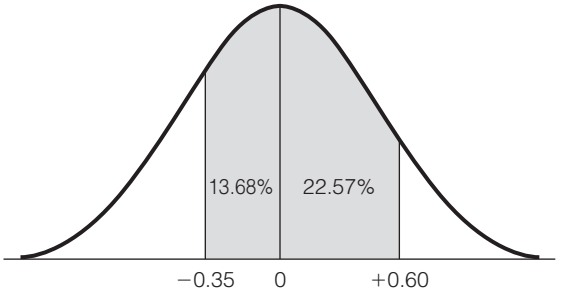
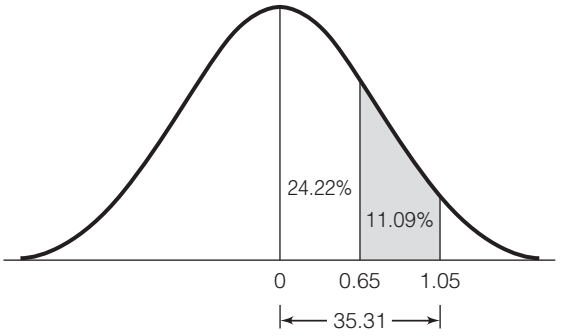


FIGURE 6.8 FINDING THE AREA BETWEEN TWO SCORES



When the scores of interest are on the same side of the mean, a different procedure must be followed to determine the area between them. For example, if we were interested in the area between the scores of 113 and 121, we would begin by converting these scores into *Z* scores:

$$Z_2 = \frac{X_i - \bar{X}}{s} = \frac{113 - 100}{20} = \frac{13}{20} = +0.65$$

$$Z_2 = \frac{X_i - \bar{X}}{s} = \frac{121 - 100}{20} = \frac{21}{20} = +1.05$$

The scores are noted in Figure 6.8; we are interested in the shadowed area. To find the area between two scores on the same side of the mean, find the area between each score and the mean (given in column b of Appendix A) and then subtract the smaller area from the larger. Between the *Z* score of +0.65 and the mean lies 24.22% of the total area. Between +1.05 and the mean lies 35.31% of the total area. Therefore, the area between these two scores is 35.31% − 24.22%, or 11.09% of the total area. The same technique would be followed if both scores had been below the mean. The procedures for finding areas between two scores are summarized in the One Step at a Time box. *(For practice in finding areas between two scores, see Problems 6.3, 6.4, 6.6–6.9.)*

ONE STEP AT A TIME **Finding Areas Between *Z* scores**

Step	***Operation***
1.	Compute the *Z* scores for both raw scores. Note whether the scores are positive or negative.
2.	Find areas between each score and the mean in column b.

If the scores are on the same *side of the mean:*

3.	Subtract the smaller area from the larger area. Multiply this value by 100 to express it as a percentage.

If the scores are on opposite *sides of the mean:*

3.	Add the two areas together to get the total area between the scores. Multiply this value by 100 to express it as a percentage.

Application 6.2

Students in all Biology 101 classes at a large university were given the same final exam. Test scores were distributed normally, with a mean of 72 and a standard deviation of 8. What percentage of students scored between 60 and 69 (a grade of D) and what percentage scored between 70 and 79 (a grade of C)? The first two scores are both below the mean. Using the One Step at a Time box as a guide, we must first compute Z scores, find areas between each score and the mean, and then subtract the smaller area from the larger.

$$Z_1 = \frac{X_i - \bar{X}}{s} = \frac{60 - 72}{8} = -\frac{12}{8} = -1.50$$

$$Z_2 = \frac{X_i - \bar{X}}{s} = \frac{69 - 72}{8} = -\frac{3}{8} = -0.38$$

Using column b, we see that the area between $Z = -1.50$ and the mean is 0.4332 and the area between $Z = -0.38$ and the mean is .1480. Subtracting the smaller from the larger (0.4332 − 0.1480) gives 0.2852. Changing to percentage format, we can say that 28.52% of the students earned a D on the test.

To find the percentage of students who earned a C, we must add column b areas together since the scores (70 and 79) are on opposite sides of the mean.

$$Z_1 = \frac{X_i - \bar{X}}{s} = \frac{70 - 72}{8} = -\frac{2}{8} = -0.25$$

$$Z_2 = \frac{X_i - \bar{X}}{s} = \frac{79 - 72}{8} = \frac{7}{8} = 0.88$$

Using column b, we see that the area between $Z = -0.25$ and the mean is 0.0987 and the area between $Z = 0.88$ and the mean is 0.3133. Therefore, the total area between these two scores is (0.0987 + 0.3133) or 0.4120. Translating to percentages again, we can say that 41.20% of the students earned a C on this test.

6.6 USING THE NORMAL CURVE TO ESTIMATE PROBABILITIES

To this point, we have thought of the theoretical normal curve as a way of describing the percentage of total area above, below, and between scores in an empirical distribution. We have also seen that these areas can be converted into the number of cases above, below, and between scores. In this section, we introduce the idea that the theoretical normal curve may also be thought of as a distribution of probabilities. Specifically, we may use the properties of the theoretical normal curve (Appendix A) to estimate the **probability** that a case randomly selected from an empirical normal distribution will have a score that falls in a certain range. In terms of techniques, these probabilities will be found in exactly the same way as areas were found. Before we consider these mechanics, however, let us examine what is meant by the concept of probability.

Although we are rarely systematic or rigorous about it, we all attempt to deal with probabilities every day, and, indeed, we base our behavior on our estimates of the likelihood that certain events will occur. We constantly ask (and answer) questions such as What is the probability of rain? Of drawing to an inside straight in poker? Of the worn-out tires on my car going flat? Of passing a test if I don't study?

To estimate the probability of an event, we must first be able to define what would constitute a "success." The examples above contain several different definitions of a success (that is, rain, drawing a certain card, flat tires, and passing grades). To determine a probability, a fraction must be established, with the numerator equaling the number of events that would constitute a success and the denominator equaling the total number of possible events where a success could theoretically occur:

$$\text{Probability} = \frac{\text{Number of successes}}{\text{Number of events}}$$

To illustrate, assume that we wish to know the probability of selecting a specific card—say, the king of hearts—in one draw from a well-shuffled deck of cards. Our definition of a success is quite specific (drawing the king of hearts), and with the information given, we can establish a fraction. Only one card satisfies our definition of success, so the number of events that would constitute a success is 1; this value will be the numerator of the fraction. There are 52 possible events (that is, 52 cards in the deck), so the denominator will be 52. The fraction is thus 1/52, which represents the probability of selecting the king of hearts on one draw from a well-shuffled deck of cards. Our probability of success is 1 out of 52.

We can leave the fraction established above as it is, or we can express it in several other ways. For example, we can express it as an odds ratio by inverting the fraction, showing that the odds of selecting the king of hearts on a single draw are 52:1 (or 52 to 1). We can express the fraction as a proportion by dividing the numerator by the denominator. For our example above, the corresponding proportion is 0.0192, which is the proportion of all possible events that would satisfy our definition of a success. In the social sciences, probabilities are usually expressed as proportions, and we will follow this convention throughout the remainder of this section. Using p to represent *probability*, the probability of drawing the king of hearts (or any specific card) can be expressed as

$$p \text{ (king of hearts)} = \frac{\text{Number of successes}}{\text{Number of events}} = \frac{1}{52} = 0.0192$$

As conceptualized here, probabilities have an exact meaning: over the long run, the events that we define as successes will bear a certain proportional relationship to the total number of events. The probability of 0.0192 for selecting the king of hearts in a single draw really means that, over thousands of selections of 1 card at a time from a full deck of 52 cards, the proportion of successful draws would be 0.0192. Or, for every 10,000 draws, 192 would be the king of hearts and the remaining 9,808 selections would be other cards. Thus, when we say that the probability of drawing the king of hearts in one draw is 0.0192, we are essentially applying our knowledge of what would happen over thousands of draws to a single draw.

Like proportions, probabilities range from 0.00 (meaning that the event has absolutely no chance of occurrence) to 1.00 (a certainty). As the value of the probability increases, the likelihood that the defined event will occur also increases. A probability of 0.0192 is close to zero, and this means that the event (drawing the king of hearts) is unlikely or improbable.

These techniques can be used to establish simple probabilities in any situation in which we can specify the number of successes and the total number of events. For example, a single die has six sides or faces, each with a different value ranging from 1 to 6. The probability of getting any specific number (say, a 4) in a single roll of a die is therefore

$$p \text{ (rolling a four)} = \frac{1}{6} = 0.1667$$

Combining this way of thinking about probability with our knowledge of the theoretical normal curve allows us to estimate the likelihood of selecting a case that has a score within a certain range. For example, suppose we wished

to estimate the probability that a randomly chosen subject from the distribution of men's IQ scores would have an IQ score between 95 and the mean score of 100. Our definition of a success here would be the selection of any subject with a score in the specified range. Normally, we would next establish a fraction with the numerator equal to the number of subjects with scores in the defined range and the denominator equal to the total number of subjects. However, if the empirical distribution is normal in form, we can skip this step since the probabilities, in proportion form, are already stated in Appendix A. That is, the areas in Appendix A can be interpreted as probabilities.

To determine the probability that a randomly selected case will have a score between 95 and the mean, we would convert the original score to a Z score:

$$Z = \frac{X_i - \bar{X}}{s} = \frac{95 - 100}{20} = \frac{-5}{20} = -0.25$$

Using Appendix A, we see that the area between this score and the mean is 0.0987. This is the probability we are seeking. The probability that a randomly selected case will have a score between 95 and 100 is 0.0987 (or, rounded off, 0.1, or 1 out of 10). In the same fashion, the probability of selecting a subject from any range of scores can be estimated. Note that the techniques for estimating probabilities are exactly the same as those for finding areas. The only new information introduced in this section is the idea that the areas in the normal curve table can also be thought of as probabilities.

To consider an additional example, what is the probability that a randomly selected male will have an IQ less than 123? We will find probabilities the same way we found areas. The score (X_i) is above the mean, and we will find the probability we are seeking by adding the area in column b to 0.5000. First, we find the Z score:

$$Z = \frac{X_i - \bar{X}}{s} = \frac{123 - 100}{20} = \frac{23}{20} = +1.15$$

Next, look in column b of Appendix A to find the area between this score and the mean. Then, add the area (0.3749) to 0.5000. The probability of selecting a male with an IQ of less than 123 is 0.3749 + 0.5000 or 0.8749. Rounding this value to .88, we can say that the odds are .88 (very high) that we will select a male with an IQ score in this range. Remember that technically this probability expresses what would happen over the long run: for every 100 males selected from this group over an infinite number of trials, 88 would have IQ scores less than 123 and 12 would not.

ONE STEP AT A TIME **Finding Probabilities**

Step	*Operation*
1.	Compute the Z score (or scores). Note whether the score is positive or negative.
2.	Find the Z score (or scores) in column a of the normal curve table (Appendix A).
3.	Find the area above or below the score (or between the scores) as you would normally (see the two previous One Step at a Time boxes in this chapter), and express the result as a proportion. Typically, probabilities are expressed as a value between 0.00 and 1.00, rounded to two digits beyond the decimal point.

BECOMING A CRITICAL CONSUMER: Applying the Laws of Probability

As mentioned in the chapter, we all work with probabilities all the time, but we are rarely systematic or rigorous in our thinking. One reason for this is that, in everyday life, it's difficult—usually impossible—to figure out the numbers that would go in the fraction used to calculate a probability. For example, if you are trying to figure the odds of passing a test without studying, you would have to know how many people with your characteristics have earned a passing grade on this test under these conditions. Needless to say, these numbers are simply not available (and there's only one you!).

On the other hand, games of chance—lotteries, card games, board games, sports betting, etc.—are a big part of daily life. For many people, the attraction of gambling comes from the possibility of winning really huge prizes, like the millions of dollars at stake in a state lottery. Others might be drawn by games that combine rational analysis with random chance—the challenge of trying to figure the odds and maximize the chances for victory. Many people enjoy the tension and drama generated by the element of chance, the uncertainty of the outcome, and the possibility that a skilled expert could be ignominiously defeated or that some naive greenhorn might stumble into victory.

People who enjoy competing against the odds engage in an extraordinary array of behaviors to try to control their fate, including lucky charms or mantras and prayers. There are also more rational and scientific ways of increasing your chances of walking away a winner, and a careful study of the role of probability in your preferred game can dramatically improve your chances for victory. (Of course, there are no guarantees—that's why they call it gambling!)

One example of using rigorous logic to understand and tame the odds is reported in Ben Mizrich's 2003 best seller *Bringing Down the House* (New York: Free Press), later made into a movie called *21*. A group of MIT college students, along with some advisors and financial backers, applied the mathematics of probability to the game of blackjack (or twenty-one).[1] They studied the nature of the game, using computers and advanced math, and, based on their research, they developed a way to beat the game over the long run. In other words, their system virtually guaranteed the group a profit from gambling. They took their system to casinos in Las Vegas, Atlantic City, and elsewhere and won—literally—millions of dollars.

Their use of probability theory goes far beyond the material presented in this chapter, but it is very much in the tradition of statistics. The laws of probability were first established and demonstrated several centuries ago by scientists and mathematicians trying to understand the outcomes of various gambling games and improve their chances of winning. The MIT students simply extended a long tradition of mathematical research.

What was their system? The book does not reveal all of their secrets, but basically, they counted cards (i.e., they kept track of which cards had been dealt), played in carefully orchestrated teams, and practiced their strategies religiously. There was nothing illegal about what they did, but the establishments they played in had the right to refuse service to anyone that was abusing the hospitality of the casino. So, in addition to the math and strategizing, the MIT team had to develop elaborate disguises, secret code words, and other routines to obscure what they were doing from casino security. While they had a long run of fabulous success, security did eventually catch up with them and bring their career to a grinding (and sometimes bloody) stop.

Still, the story is fascinating and illustrates how a study of the odds can improve your chances of winning in every game from Old Maid to roulette. (Of course, the book also illustrates that, despite the most brilliant ploys, the house will still find a way to win at the end of the day.)

[1]If you are not familiar with this game, here's a brief summary of the rules. Players are dealt two cards from an ordinary deck and may request additional cards. The winner of a hand is the player with the highest total less than or equal to 21. In calculating total value of a hand, cards 2 through 10 count as their face value, all picture cards (Jack, Queen, and King) count as 10, and the ace counts as 1 or 11, whichever helps the most. If the total value of the hand exceeds 21, the player loses; getting exactly 21 is called *blackjack*.

Let us close by stressing a very important point about probabilities and the normal curve. The probability is very high that any case randomly selected from a normal distribution will have a score close in value to that of the mean. The normal curve reaches its highest point at the mean (the same as the median and the mode), and thus, there are more cases at this point than at any other (remember that the curve is a frequency polygon with scores arrayed along the horizontal axis and number of cases along the vertical axis). For any normal curve, cases are clustered around the mean and decline in frequency as we move farther away—either to the right or to the left—from the mean value. In fact, given what we know about the normal curve, the probability that a randomly selected case will have a score within ±1 standard deviations of the mean is 0.6826. Rounding off, we can say that 68 out of 100 cases—or about two-thirds of all cases—selected over the long run will have a score between ±1 standard deviations or *Z* scores of the mean. The probability is high that any randomly selected case will have a score close in value to the mean.

In contrast, the probability of the case having a score beyond three standard deviations from the mean is very small. Look in column c ("Area Beyond *Z*") for a *Z* score of 3.00, and you will find the value 0.0014. Adding the areas in the upper tail (beyond +3.00) to the area in the lower tail (beyond −3.00) gives us 0.0014 + 0.0014, for a total of 0.0028. The probability of selecting a case with a very high score or a very low score is 0.0028. If we randomly select cases from a normally distributed variable, we would select cases with *Z* scores beyond ±3.00 only 28 times out of every 10,000 trials.

The general point to remember is that cases with scores close to the mean are common and cases with scores far above or below the mean are rare. This relationship is central for an understanding of inferential statistics, which we discuss in Part II. *(For practice in using the normal curve table to find probabilities, see Problems 6.8–6.10 and 6.13.)*

SUMMARY

1. The normal curve, in combination with the mean and standard deviation, can be used to construct precise descriptive statements about empirical distributions that are normally distributed. This chapter also lays some important groundwork for Part II.
2. To work with the theoretical normal curve, raw scores must be transformed into their equivalent *Z* scores. *Z* scores allow us to find areas under the theoretical normal curve (Appendix A).
3. We considered three uses of the theoretical normal curve: finding total areas above and below a score, finding areas between two scores, and expressing these areas as probabilities. This last use of the normal curve is especially germane because inferential statistics are centrally concerned with estimating the probabilities of defined events in a fashion very similar to the process introduced in Section 6.6.

SUMMARY OF FORMULAS

FORMULA 6.1 *Z* scores:

$$Z = \frac{X_i - \bar{X}}{s}$$

GLOSSARY

Normal curve. A theoretical distribution of scores that is symmetrical, unimodal, and bell shaped. The standard normal curve always has a mean of 0 and a standard deviation of one.

Normal curve table. A detailed description of the area between a *Z* score and the mean of any standardized normal distribution. See Appendix A.

Probability. The likelihood that a defined event will occur.

***Z* scores.** Standard scores; the way scores are expressed after they have been standardized to the theoretical normal curve.

PROBLEMS

(Problems are labeled with the social science discipline from which they are drawn: SOC for sociology, SW for social work, PS for political science, CJ for criminal justice, PA for public administration, and GER for gerontology.)

6.1 Scores on a quiz were normally distributed and had a mean of 10 and a standard deviation of 3. For each score below, find the *Z* score and the percentage of area above and below the score.

X_i	*Z* Score	% Area Above	% Area Below
5			
6			
7			
8			
9			
11			
12			
14			
15			
16			
18			

6.2 Assume that the distribution of a college entrance exam is normal, with a mean of 500 and a standard deviation of 100. For each score below, find the equivalent *Z* score, the percentage of the area above the score, and the percentage of the area below the score.

X_i	*Z* Score	% Area Above	% Area Below
650			
400			
375			
586			
437			
526			
621			
498			
517			
398			

6.3 The senior class has been given a comprehensive examination to assess their educational experience. The mean on the test was 74, and the standard deviation was 10. What percentage of the students had scores

a. between 75 and 85?
b. between 80 and 85?
c. above 80?
d. above 83?
e. between 80 and 70?
f. between 75 and 70?
g. below 75?
h. below 77?
i. below 80?
j. below 85?

6.4 For a normal distribution where the mean is 50 and the standard deviation is 10, what percentage of the area is

a. between the scores of 40 and 47?
b. above a score of 47?
c. below a score of 53?
d. between the scores of 35 and 65?
e. above a score of 72?
f. below a score of 31 and above a score of 69?
g. between the scores of 55 and 62?
h. between the scores of 32 and 47?

6.5 SOC At St. Algebra College, the 200 freshmen enrolled in Introductory Biology took a final exam on which their mean score was 72 and their standard deviation was 6. The table below presents the grades of 10 students. Convert each into a *Z* score and determine the number of people who scored higher or lower than each of the 10 students. (*HINT: Multiply the appropriate proportion by* N *and round the result.*)

X_i	Z Score	Number of Students Above	Number of Students Below
60			
57			
55			
67			
70			
72			
78			
82			
90			
95			

6.6 If a distribution of test scores is normal, with a mean of 78 and a standard deviation of 11, what percentage of the area lies

a. below 60?
b. below 70?
c. below 80?
d. below 90?
e. between 60 and 65?
f. between 65 and 79?
g. between 70 and 95?
h. between 80 and 90?
i. above 99?
j. above 89?
k. above 75?
l. above 65?

6.7 SOC A scale measuring prejudice has been administered to a large sample of respondents. The distribution of scores is approximately normal, with a mean of 31 and a standard deviation of 5. What percentage of the sample had scores

a. below 20?
b. below 40?
c. between 30 and 40?
d. between 35 and 45?
e. above 25?
f. above 35?

6.8 CJ The average burglary rate for a jurisdiction has been 311 per year, with a standard deviation of 50. What is the probability that next year the number of burglaries will be

a. less than 250?
b. less than 300?
c. more than 350?
d. more than 400?
e. between 250 and 350?
f. between 300 and 350?
g. between 350 and 375?

6.9 SOC For a math test on which the mean was 59 and the standard deviation was 4, what is the probability that a student randomly selected from this class will have a score

a. between 55 and 65?
b. between 60 and 65?
c. above 65?
d. between 60 and 50?
e. between 55 and 50?
f. below 55?

6.10 SOC On the scale mentioned in Problem 6.7, if a score of 40 or more is considered "highly prejudiced," what is the probability that a person selected at random will have a score in that range?

6.11 CJ The local police force gives all applicants an entrance exam and accepts only those applicants who score in the top 15% on this test. If the mean score this year is 87 and the standard deviation is 8, would an individual with a score of 110 be accepted?

6.12 SW After taking the state merit examinations for the positions of social worker and employment counselor, you receive the following information on the tests and on your performance. On which of the tests did you do better?

Social Worker	Employment Counselor
$\bar{X} = 118$	$\bar{X} = 27$
$s = 17$	$s = 3$
Your score = 127	Your score = 29

6.13 In a distribution of scores with a mean of 35 and a standard deviation of 4, which event is more likely: that a randomly selected score will be between 29 and 31 or that a randomly selected score will be between 40 and 42?

6.14 To be accepted into an honor society, students must have GPAs in the top 10% of the school. If the mean GPA is 2.78 and the standard deviation is 0.33, which of the following GPAs would qualify?

a. 3.20
b. 3.21
c. 3.25
d. 3.30
e. 3.35

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Part II Inferential Statistics

The chapters in Part II cover the techniques and concepts of inferential or inductive statistics. Generally speaking, these applications allow us to learn about large groups (populations) from small, carefully selected subgroups (samples). These statistical techniques are powerful and extremely useful and they are used to assess public opinion, research the potential market for new products, project the winners of elections, test the effects of new drugs, and in hundreds of other ways both inside and outside the social sciences.

Chapter 7 includes a brief description of sampling, but the most important part of this chapter concerns the sampling distribution, the single most important concept in inferential statistics. The sampling distribution is normal in shape, and it is the key link between populations and samples. The chapter also covers estimation, the first of the two main applications of inferential statistics. In this section, you will learn how to use statistical information from a sample (e.g., a mean or a proportion) to estimate the characteristics of a population. This technique is most commonly used in public opinion polling and election projection.

Chapters 8–11 cover a second application of inferential statistics: hypothesis testing. Most of the relevant concepts for this material are introduced in Chapter 8, and each Chapter covers a different situation in which hypothesis testing is done. For example, Chapter 9 presents the techniques that are used when we are comparing information from two different samples or groups (e.g., men vs. women) and Chapter 10 covers applications involving more than two groups or samples (e.g., Republicans vs. Democrats vs. Independents).

Hypothesis testing is one of the more challenging applications of statistics for beginning students, and I have included an abundance of learning aids to ease the chore of assimilating this material. Hypothesis testing is also one of the most common and important statistical applications to be found in social science research, and mastery of this material is essential for developing the ability to read the professional literature.

7

Introduction to Inferential Statistics, the Sampling Distribution, and Estimation

LEARNING OBJECTIVES

By the end of this chapter, you will be able to:

1. Explain the purpose of inferential statistics in terms of generalizing from a sample to a population.
2. Explain the principle of random sampling and the terms *population, sample, parameter, statistic, representative, EPSEM.*
3. Differentiate between the sampling distribution, the sample, and the population.
4. Explain the two theorems presented.
5. Explain the logic of estimation and the role of the sample, sampling distribution, and population.
6. Define and explain the concepts of bias and efficiency.
7. Construct and interpret confidence intervals for sample means and sample proportions.

7.1 INTRODUCTION

One of the goals of social science research is to test theories and hypotheses using many different people, groups, societies, and historical eras. Obviously, we have the greatest confidence in theories that withstand testing against the greatest variety of cases and social settings. A major problem in social science research, however, is that the populations in which we are interested may be too large to test. For example, a theory concerning political party preference among U.S. citizens would be best tested using the entire electorate, but it is impossible to interview every member of this population (over 120 million people). Indeed, even for theories that could be reasonably tested with smaller populations—such as a local community or the student body at a university—the logistics of gathering data from every single case (entire populations) are staggering to contemplate.

If it is too difficult or expensive to do research with entire populations, how can we reasonably test our theories? To deal with this problem, social scientists select samples, or subsets of cases, from the populations of interest. Our goal in inferential statistics is to learn about the characteristics (or **parameters**) of a population based on what we can learn from our samples. Two applications of inferential statistics are covered in this text. In *estimation procedures,* covered in this chapter, a "guess" of the population parameter is made based on what is known about the sample. In *hypothesis testing*, covered in Chapters 8 through 11, the validity of a hypothesis about the population is tested against sample outcomes. Before we can address these applications, however, we need to consider sampling (the techniques for selecting cases for a sample) and a key concept in inferential statistics, the *sampling distribution*.

7.2 PROBABILITY SAMPLING

In this section, we will review the basic procedure for selecting probability samples, the only type of sample that fully supports the use of inferential statistical techniques to generalize to populations. These types of samples are often described as *random,* and you may be more familiar with this term. Because of its greater familiarity, I will often use the phrase *random sample* in the following chapters. The term *probability sample* is preferred, however, because in everyday language, random is often used to mean "by coincidence" or to give a connotation of unpredictability. To the contrary, probability samples are selected by techniques that are careful and methodical and leave no room for haphazardness. Interviewing the people you happen to meet in a mall one afternoon may be random in some sense, but this technique will not result in a sample that would support inferential statistics.

Before considering probability sampling, let me point out that social scientists often use nonprobability samples. For example, researchers who are studying small-group dynamics or the structure of attitudes or personal values might use the students enrolled in their classes as subjects. Such "convenience" samples are very useful for a number of purposes (e.g., exploring ideas or pretesting survey forms before embarking on a more ambitious project) and are typically less costly and easier to assemble. The major limitation of these samples is that results cannot be generalized beyond the group being tested. If a theory of prejudice, for example, has been tested only on the students who happen to have been enrolled in a particular section of Introductory Sociology at a particular university in a particular year, then the researcher cannot generalize the findings to other types of people in other social locations or at other times. Even when the evidence is very strong, we cannot place a lot of confidence in the generalizability of theories tested on nonprobability samples only.

The goal of probability sampling is to select cases so that the final sample is **representative** of the population from which it was drawn. A sample is representative if it reproduces the important characteristics of the population. For example, if the population consists of 60% females and 40% males, the sample should contain the same proportions. In other words, a representative sample is very much like the population, only smaller. It is crucial for inferential statistics that samples be representative: if they are not, generalizing to the population becomes, at best, extremely hazardous.

How can we assure ourselves that our samples are representative? Unfortunately, it is not possible to guarantee that samples are representative. However, we can maximize the chances of drawing a representative sample by following the principle of **EPSEM** (the **e**qual **p**robability of **se**lection **m**ethod). To use this method, we select the sample so that every case in the population has an equal probability of being selected for the sample. Our goal is to select a representative sample, and the technique we use to maximize the chance of achieving that goal is to follow the rule of EPSEM.

The most basic EPSEM sampling technique produces a **simple random sample.** There are numerous variations on and refinements of this technique, but in this text, we will consider only the most straightforward application. To draw a simple random sample, we need a list of all cases in the population and a system for selecting cases that ensures every case has an equal chance of being selected for the sample. The selection process could be based on a number of different kinds of operations (for example, drawing cards from a well-shuffled deck, flipping coins, throwing dice, drawing numbers from a hat, and so on).

Cases are often selected by using a list of random numbers that have been generated by a computer program or that are presented in a table of random numbers. In either case, the numbers are random in the sense that they have no order or pattern: each number in the list is just as likely as any other number. An example of a table of random numbers is available at the Web site for this text.

To use random numbers to select a simple random sample, first assign each case on the population list a unique identification number. Then, select cases for the sample by matching their identification number to the number chosen from the table. This procedure will produce an EPSEM sample because the numbers in the table are in random order and any number is just as likely as any other number. Stop selecting cases when you have reached your desired sample size, and if an identification number is selected more than once, ignore the repeats.[1]

Remember that the EPSEM selection technique and the representativeness of the final sample are two different things. In other words, the fact that a sample is selected according to EPSEM does not guarantee that it will be an exact representation or microcosm of the population. The probability is very high that an EPSEM sample will be representative, but just as a perfectly honest coin will sometimes show 10 heads in a row when flipped, an EPSEM sample will occasionally present an inaccurate picture of the population. One of the great strengths of inferential statistics is that they allow the researcher to estimate the probability of this type of error and interpret results accordingly.

In this section we learned that the purpose of inferential statistics is to acquire knowledge about populations based on the information derived from samples of that population. Each of the applications of inferential statistics to be presented in this book requires that samples be selected according to EPSEM. While even the most painstaking and sophisticated sampling techniques will not guarantee representativeness, the probability is high that EPSEM samples will be representative of the populations from which they are selected.

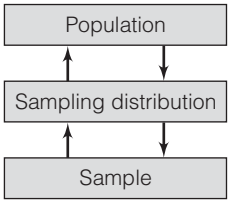
7.3 THE SAMPLING DISTRIBUTION

Once we have selected a probability sample, what do we know? Remember that our goal is to learn more about the population from which the sample was selected. We can gather information from the cases in the sample and then use that information to help us learn more about the population: sample information is important primarily insofar as it allows us to generalize to the population.

When we use inferential statistics, we generally measure some variable (e.g., age, political party preference, or opinions about abortion) in the sample and then use the information from the sample to learn more about that variable in the population. In Part I of this text, you learned that three types of information are generally necessary to adequately characterize a variable: (1) the shape of its distribution, (2) some measure of central tendency, and (3) some measure of dispersion. Clearly, we can gather all three types of information about the cases in the sample. Just as clearly, none of the information is available for the population. The means and standard deviations of variables in the population, as well as their shapes, are unknown: if we had this information for the population, inferential statistics would be unnecessary.

[1]Ignoring identification numbers when they are repeated is called *sampling without replacement*. Technically, this practice compromises the randomness of the selection process. However, if the sample is a small fraction of the total population, we will be unlikely to select the same case twice, and ignoring repeats will not bias our conclusions.

FIGURE 7.1 THE RELATIONSHIPS BETWEEN THE SAMPLE, SAMPLING DISTRIBUTION, AND POPULATION



In statistics, we link information from the sample to the population with a device known as the **sampling distribution,** which is the theoretical, probabilistic distribution of a statistic for all possible samples of a certain sample size (N). That is, the sampling distribution includes statistics that represent every conceivable combination of cases from the population. A crucial point about the sampling distribution is that its characteristics are based on the laws of probability, not on empirical information, and are very well known. In fact, the sampling distribution is the central concept in inferential statistics, and a prolonged examination of its characteristics is certainly in order.

As illustrated by Figure 7.1, we move between the sample and population by means of the sampling distribution. Thus, three separate and distinct distributions are involved in every application of inferential statistics.

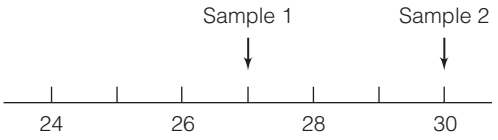
1. The sample distribution, which is empirical (i.e., it exists in reality) and known in the sense that the shape, central tendency, and dispersion of any variable can be ascertained for the sample. Remember that the information from the sample is important primarily insofar as it allows the researcher to learn about the population.
2. The population distribution, which, while empirical, is unknown. Amassing information about or making inferences to the population is the sole purpose of inferential statistics.
3. The sampling distribution, which is nonempirical or theoretical. Because of the laws of probability, a great deal is known about this distribution. Specifically, the shape, central tendency, and dispersion of the distribution can be deduced, and therefore, the distribution can be adequately characterized.

The utility of the sampling distribution is implied by its definition. Because it encompasses all possible sample outcomes, the sampling distribution enables us to estimate the probability of any particular sample outcome, a process that will occupy our attention for the remainder of this chapter and the next four chapters to come.

The sampling distribution is theoretical, which means that it is never obtained in reality by the researcher. However, to understand better the structure and function of the distribution, let's consider an example of how it might be constructed. Suppose that we wanted to gather some information about the age of a particular community of 10,000 individuals. We draw an EPSEM sample of 100 residents, ask all 100 respondents their age, and use those individual scores to compute a mean age of 27. This score is noted on the graph in Figure 7.2. Note that this sample is one of countless possible combinations of 100 people taken from this population of 10,000 and the mean of 27 is one of millions of possible sample outcomes.

Now, replace the 100 respondents in the first sample, draw another sample of the same size ($N = 100$), and again compute the average age. Assume

FIGURE 7.2 CONSTRUCTING A SAMPLE DISTRIBUTION

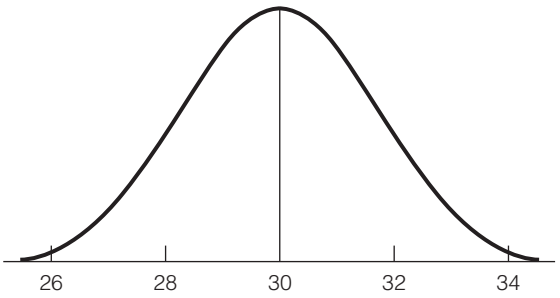


that the mean for the second sample is 30 and note this sample outcome on Figure 7.2. This second sample is another of the countless possible combinations of 100 people taken from this population of 10,000, and the sample mean of 30 is another of the millions of possible sample outcomes. Replace the respondents from the second sample and draw still another sample, calculate and note the mean, replace this third sample, and draw a fourth sample, continuing these operations an infinite number of times, calculating and noting the mean of each sample. Now, try to imagine what Figure 7.2 would look like after tens of thousands of individual samples had been collected and the mean had been computed for each sample. What shape, mean, and standard deviation would this distribution of sample means have after we had collected all possible combinations of 100 respondents from the population of 10,000?

For one thing, we know that each sample will be at least slightly different from every other sample, since it is very unlikely that we will sample exactly the same 100 people twice. Since each sample will almost certainly be a unique combination of individuals, each sample mean will be at least slightly different in value. We also know that even though the samples are chosen according to EPSEM, they will not be representative of the population in every single case. For example, if we continue taking samples of 100 people long enough, we will eventually choose a sample that includes only the very youngest residents. Such a sample would have a mean much lower than the true population mean. Likewise, by random chance alone, some of our samples will include only senior citizens and will have means that are much higher than the population mean. Common sense suggests, however, that such nonrepresentative samples will be rare and that most sample means will cluster around the true population value.

To illustrate further, assume that we somehow come to know that the true mean age of the population is 30. As we have seen, if the population mean is 30, most of the sample means will also be approximately 30 and the sampling distribution of these sample means should peak at 30. Some of the sample means will be much too low or much too high, but the frequency of such misses should decline as we get farther away from 30. That is, the distribution should slope to the base as we get farther away from the population value–sample means of 29 or 31 should be common, and means of 20 or 40 should be rare. Since the samples are random, the means should miss an equal number of times on either side of the population value, and the distribution itself should therefore be roughly symmetrical. In other words, the sampling distribution of all possible sample means should be approximately normal and will resemble the distribution presented in Figure 7.3. Recall from Chapter 6 that, on any normal curve, cases close

FIGURE 7.3 A SAMPLING DISTRIBUTION OF SAMPLE MEANS



to the mean (say, within ±1 standard deviation) are common and cases far away from the mean (say, beyond ±3 standard deviations) are rare.

These commonsense notions about the shape of the sampling distribution and other very important information about central tendency and dispersion are stated in two theorems. The first of these theorems states

> **If repeated random samples of size N are drawn from a normal population with mean μ and standard deviation σ, then the sampling distribution of sample means will be normal with a mean μ and a standard deviation of $\sigma/\sqrt{N}$.**

To translate: if we begin with a trait that is normally distributed across a population (IQ, height, or weight, for example) and take an infinite number of equally sized random samples from that population, then the sampling distribution of sample means will be normal. If it is known that the variable is distributed normally in the population, it can be assumed that the sampling distribution will be normal.

The theorem tells us more than the shape of the sampling distribution of all possible sample means, however. It also defines its mean and standard deviation. In fact, it says that the mean of the sampling distribution will be exactly the same value as the mean of the population (μ). That is, if we know that the mean IQ of the entire population is 100, then we know that the mean of any sampling distribution of sample mean IQs will also be 100. Exactly why this should be so is not a matter that can be fully explained at this level. Recall, however, that most sample means will cluster around the population value over the long run. Thus, the fact that these two values are equal should have intuitive appeal. As for dispersion, the theorem says that the standard deviation of the sampling distribution, also called the **standard error of the mean,** will be equal to the standard deviation of the population divided by the square root of N (symbolically: $\sigma/\sqrt{N}$).

If the mean and standard deviation of a normally distributed population are known, the theorem allows us to compute the mean and standard deviation of the sampling distribution.[2] Thus, we will know exactly as much about the sampling distribution (shape, central tendency, and dispersion) as we ever knew about any empirical distribution.

The first theorem requires a normal population distribution. What happens when the distribution of the variable in question is unknown or is known not to be normal in shape (such as income, which always has a positive skew)? These eventualities (very common, in fact) are covered by a second theorem, called the **central limit theorem:**

> **If repeated random samples of size N are drawn from any population, with mean μ and standard deviation σ, then, as N becomes large, the sampling distribution of sample means will approach normality, with mean μ and standard deviation $\sigma/\sqrt{N}$.**

[2]In the typical research situation, the values of the population mean and standard deviation are, of course, unknown. However, these values can be estimated from sample statistics, as we shall see in the chapters that follow.

To translate: For *any* trait or variable, even those that are not normally distributed in the population, as sample size grows larger, the sampling distribution of sample means will become normal in shape. When N is large, the mean of the sampling distribution will equal the population mean and its standard deviation (or the standard error of the mean) will be equal to $\sigma/\sqrt{N}$.

The importance of the central limit theorem is that it removes the constraint of normality in the population. Whenever sample size is large, we can assume that the sampling distribution is normal, with a mean equal to the population mean and a standard deviation equal to $\sigma/\sqrt{N}$ regardless of the shape of the variable in the population. Thus, even if we are working with a variable that is known to have a skewed distribution (such as income), we can still assume a normal sampling distribution.

The issue remaining, of course, is to define what is meant by a large sample. A good rule of thumb is that if sample size (N) is 100 or more, the central limit theorem applies, and you may assume that the sampling distribution is normal in shape. When N is less than 100, you must have good evidence of a normal population distribution before you may assume that the sampling distribution is normal. Thus, a normal sampling distribution can be ensured by the expedient of using fairly large samples.

7.4 THE SAMPLING DISTRIBUTION: AN ADDITIONAL EXAMPLE

Developing an understanding of the sampling distribution—what it is and why it is important—is often one of the more challenging tasks for beginning students of statistics. It may be helpful to briefly list the most important points about the sampling distribution.

1. *Its definition:* The sampling distribution is the distribution of a statistic (such as a mean or a proportion) for all possible sample outcomes of a certain size.
2. *Its shape:* The shape is normal (see Chapter 6 and Appendix A).
3. *Its central tendency and dispersion:* The mean of the sampling distribution is the same value as the mean of the population. The standard deviation of the sampling distribution—or the standard error—is equal to the population standard deviation divided by the square root of N (see the two theorems above).
4. *Its role in inferential statistics:* It links the sample with the population (see Figure 7.1).

To reinforce these points, let's consider an additional example of how the sampling distribution works together with the sample and the population. Consider the General Social Survey (GSS), the database used for computer exercises in this book. The GSS has been administered to randomly selected samples of adult Americans since 1972 and explores a broad range of characteristics and issues, including confidence in the Supreme Court, attitudes about assisted suicide, number of siblings, and level of education. The GSS has its limits, of course, but it has proven to be a very valuable resource for testing theory and learning more about American society. Focusing on this survey, let's review the roles played by the population, sample, and sampling distribution when we use this database.

We'll start with the population or the group we are actually interested in and want to learn more about. In the case of the GSS, the population consists

of all adult (older than 18) Americans, which includes about 225 million people. Clearly, we can never interview all of these people and learn what they are like or what they think about abortion, capital punishment, gun control, affirmative action, sex education in the public schools, or any number of other issues. We should also note that this information is worth having. It could help inform public debates, provide some basis in fact for the discussion of many controversial issues (e.g., the polls show consistently that the majority of Americans favor some form of gun control), and assist people in clarifying their personal beliefs. If the information is valuable, what can be done to learn more about this huge population?

This brings us to the sample, a carefully chosen subset of the population. The GSS has been given to samples ranging in size from about 1,500 to almost 4,000 people. Each respondent is chosen by a sophisticated technology based on the principle of EPSEM. A key point to remember is that samples chosen by this method are very likely to be representative of the populations from which they were selected. In other words, whatever is true of the sample will also be true of the population (with some limits and qualifications, of course).

The respondents are contacted at home and asked for background information (religion, gender, years of education, etc.) as well as their opinions and attitudes. When all of this information is collated, the GSS database includes information (shape, central tendency, dispersion) on hundreds of variables (age, level of prejudice, marital status) for the people in the sample. So, we have a lot of information about the variables for the sample (the people who actually responded to the survey), but no information about these variables for the population (the 225 million adult Americans). How do we get from the known characteristics of the sample to the unknown population? This is the central question of inferential statistics, and the answer, as we hope you realize by now, is by using the sampling distribution.

Remember that, unlike the sample and the population, the sampling distribution is theoretical. We can work with the sampling distribution because its shape, central tendency, and dispersion are defined by the theorems presented earlier in this chapter. For any variable from the GSS, we know that the sampling distribution will be normal in shape because the sample is large (N is much greater than 100). Secondly, the theorems tell us that the mean of the sampling distribution will be the same value as the mean of the population. If *all* adult Americans have completed an average of 13.5 years of schooling ($\mu = 13.5$), the mean of the sampling distribution will also be 13.5.

Thirdly, the theorems tell us that the standard deviation (or standard error) of the sampling distribution is equal to the population standard deviation (σ) divided by the square root of N. Thus, the theorems tell us the statistical characteristics of the sampling distribution (shape, central tendency, and dispersion) and this information allows us to link the sample to the population.

How does the sampling distribution connect the sample to the population? The fact that the sampling distribution is normal in shape is crucial. Remember that the sampling distribution includes *all possible* sample outcomes. As on any normal curve, the sampling distribution includes more than two-thirds (about 68%) of all samples within ± 1 Z of the mean (which is the same value as the population mean), about 95% within ± 2 Z scores, and so forth. We do not (and cannot) know the actual value of the mean of the sampling distribution, but we do know that the odds are excellent that any statistic computed from our

sample will be approximately equal to the parameter. Likewise, the theorems give us crucial information about the mean and standard error of the sampling distribution that we can use, as you will see, to link information from the sample to the population.

To summarize, our goal is to infer information about the population (in the case of the GSS, all adult Americans). When populations are too large to test (and contacting 225 million adult Americans is far beyond the capacity of even the most energetic pollster), we use information from randomly selected samples, carefully drawn from the population of interest, to estimate the characteristics of the population. In the case of the GSS, the full sample consists of thousands of adult Americans who have responded to the questions on the survey. The sampling distribution, the theoretical distribution whose characteristics are defined by the theorems, links the known sample to the unknown population.

7.5 SYMBOLS AND TERMINOLOGY

In inferential statistics, we work with three entirely different distributions (sample, population, and sampling distribution). Furthermore, we will be concerned with several different kinds of sampling distributions—including the sampling distribution of sample means and the sampling distribution of sample proportions.

To distinguish clearly among these various distributions, we will often use symbols. The symbols used for the means and standard deviations of samples and populations have already been introduced in Chapters 4 and 5. Table 7.1 introduces some of the symbols that will be used for the sampling distribution in summary form for quick reference. Basically, the sampling distribution is denoted with Greek letter symbols that are subscripted according to the sample statistic of interest.

Note that the mean and standard deviation of a sample are denoted with English letters ($\overline{X}$ and s) and the mean and standard deviation of a population are denoted with the Greek letter equivalents (μ and σ). Proportions calculated on samples are symbolized as $\boldsymbol{P_s}$ (read P sub s, with s for sample) and population proportions are denoted as $\boldsymbol{P_u}$ (P sub u, with u for universe or population). The symbols for the sampling distribution are Greek letters with English letter subscripts. The mean and standard deviation of a sampling distribution of sample means are $\mu_{\overline{X}}$ (mu sub x bar) and $\sigma_{\overline{X}}$ sigma sub x bar). The mean and standard deviation of a sampling distribution of sample proportions are $\boldsymbol{\mu_P}$ (mu sub p) and σ_P (sigma sub p).

TABLE 7.1 SYMBOLS FOR MEANS AND STANDARD DEVIATIONS OF THREE DISTRIBUTIONS

	Mean	Standard Deviation	Proportion
1. Samples	$\overline{X}$	s	P_s
2. Populations	μ	σ	P_u
3. Sampling distributions			
Of means	$\mu_{\overline{X}}$	$\sigma_{\overline{X}}$	
Of proportions	μ_P	σ_P	

7.6 INTRODUCTION TO ESTIMATION

The object of this branch of inferential statistics is to estimate population values or parameters from statistics computed from samples. Although these techniques may be new to you, you are certainly familiar with their most common applications: public opinion polls and election projections. Polls and surveys on every conceivable issue—from the sublime to the trivial—have become a staple of the mass media and popular culture. The techniques you will learn in this chapter are essentially the same as those used by the most reputable, sophisticated, and scientific pollsters.

The standard procedure for estimating population values is to construct **confidence intervals,** a mathematical statement that says that the parameter lies within a certain range of values or interval. For example, a confidence interval estimate might be stated as "between 71% and 77% of Americans approve of capital punishment." The interval places the population value (the percentage of *all* Americans who support capital punishment) between 71% and 77%, but does not specify an exact value.

7.7 BIAS AND EFFICIENCY

Estimation procedures are based on sample statistics. Which of the many available sample statistics should be used? Estimators can be selected according to two criteria: **bias** and **efficiency.** Estimates should be based on sample statistics that are unbiased and relatively efficient. We cover each of these criteria separately.

Bias. An estimator is unbiased if the mean of its sampling distribution is equal to the population value of interest. We know from the theorems presented earlier in this chapter that sample means conform to this criterion. The mean of the sampling distribution of sample means (which we will note symbolically as $\mu_{\bar{X}}$) is the same as the population mean (μ).

Sample proportions (P_s) are also unbiased. That is, if we calculate sample proportions from repeated random samples of size N and then array them in a line chart or frequency polygon, the sampling distribution of sample proportions will have a mean (μ_p) equal to the population proportion (P_u). Thus, if we are concerned with coin flips and sample honest coins 10 at a time ($N = 10$), the sampling distribution will have a mean equal to 0.5, which is the probability that an honest coin will be heads (or tails) when flipped. All statistics other than sample means and sample proportions are biased (that is, have sampling distributions with means not equal to the population value).[3]

Knowing that sample means and proportions are unbiased is crucial because it allows us to determine the probability that they lie within a given distance of the population values we are trying to estimate. To illustrate, consider a specific problem. Assume that we wish to estimate the average income of a community. A random sample of 500 households is taken ($N = 500$), and a sample mean

[3]In particular, the sample standard deviation (s) is a biased estimator of the population standard deviation (σ). As you might expect, there is less dispersion in a sample than in a population, and as a consequence, s will underestimate σ. As we shall see, however, sample standard deviation can be corrected for this bias and still serve as an estimate of the population standard deviation for large samples.

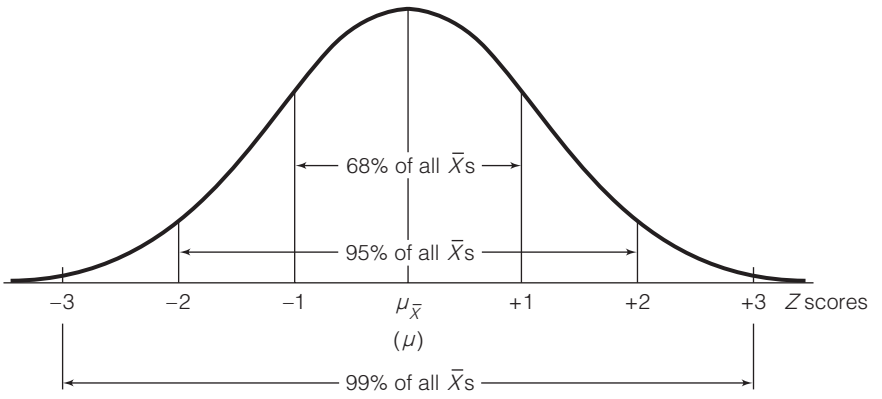
of $45,000 is computed. In this example, the population mean is the average income of *all* households in the community and the sample mean is the average income for the 500 households that happened to be selected for our sample. Note that we do not know the value of the population mean (μ)—if we did, we wouldn't need the sample—but it is μ that we are interested in. The sample mean of $45,000 is important and interesting primarily because it can give us information about the population mean.

The two theorems presented earlier in this chapter give us a great deal of information about the sampling distribution of all possible sample means. Because N is large, we know that the sampling distribution is normal and that its mean is equal to the population mean. We also know that all normal curves contain about 68% of the cases (the cases here are sample means) within ± 1 Z, 95% of the cases within ± 2 Zs, and more than 99% of the cases within ± 3 Zs of the mean. Remember that we are discussing the sampling distribution here—the distribution of all possible sample outcomes or, in this instance, sample means. Thus, the probabilities are very good (approximately 68 out of 100 chances) that our sample mean of $45,000 is within ± 1 Z, excellent (95 out of 100) that it is within ± 2 Zs, and overwhelming (99 out of 100) that it is within ± 3 Zs of the mean of the sampling distribution (which is the same value as the population mean). These relationships are graphically depicted in Figure 7.4.

If an estimator is unbiased, it is almost certainly an accurate estimate of the population parameter (μ in this case). However, in less than 1% of the cases, a sample mean will be more than ± 3 Zs away from the mean of the sampling distribution (very inaccurate) by random chance alone. We literally have no idea if our particular sample mean of $45,000 is in this small minority. We do know, however, that the odds are high that our sample mean is considerably closer than ± 3 Zs to the mean of the sampling distribution and, thus, to the population mean.

Efficiency. The second desirable characteristic of an estimator is efficiency, which is the extent to which the sampling distribution is clustered about its mean. Efficiency, or clustering, is essentially a matter of dispersion, as we saw

FIGURE 7.4 AREAS UNDER THE SAMPLING DISTRIBUTION OF SAMPLE MEANS



in Chapter 5 (see Figure 5.1). The smaller the standard deviation of a sampling distribution, the greater the clustering and the higher the efficiency. Remember that the standard deviation of the sampling distribution of sample means, or the standard error of the mean, is equal to the population standard deviation divided by the square root of N. Therefore, the standard deviation of the sampling distribution is an inverse function of N ($\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}}$). As sample size increases, $\sigma_{\bar{X}}$ will decrease. We can improve the efficiency (or decrease the standard deviation of the sampling distribution) for any estimator by increasing sample size.

An example should make this clearer. Consider two samples of different sizes.

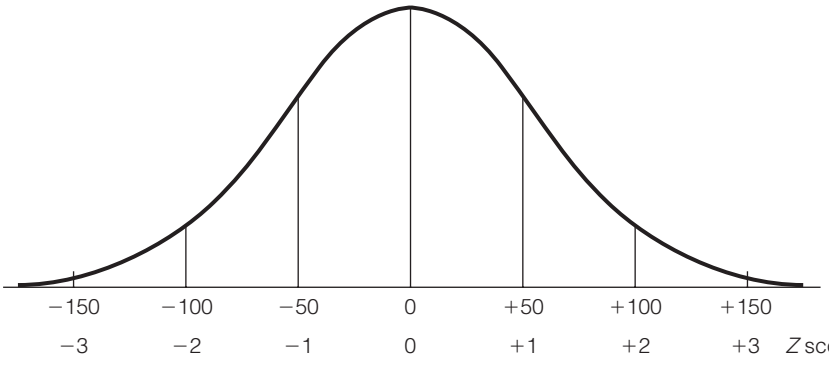
Sample 1	Sample 2
$\bar{X}$ = \$45,000	$\bar{X}$ = \$45,000
N_1 = 100	N_2 = 1,000

Both sample means are unbiased, but which is the more efficient estimator? Consider sample 1 and assume, for the sake of illustration, that the population standard deviation (σ) is \$500.[4] In this case, the standard deviation of the sampling distribution of all possible sample means with an N of 100 would be $\sigma/\sqrt{N}$ or $500/\sqrt{100}$ or \$50.00. For sample 2, the standard deviation of all possible sample means with an N of 1,000 would be much smaller. Specifically, it would be equal to $500/\sqrt{1{,}000}$ or \$15.81.

Sampling distribution 2 is much more clustered than sampling distribution 1. In fact, distribution 2 contains 68% of all possible sample means within ±15.81 of μ and distribution 1 requires a much broader interval of ±50.00 to do the same. The estimate based on a sample with 1,000 cases is much more likely to be close in value to the population parameter than is an estimate based on a sample of 100 cases. Figures 7.5 and 7.6 illustrate these relationships graphically.

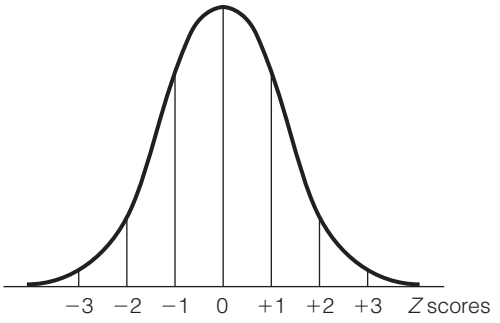
To summarize, the standard deviation of all sampling distributions is an inverse function of N. The larger the sample, the greater the clustering and the

FIGURE 7.5 A SAMPLING DISTRIBUTION WITH N = 100 AND = \$50.00



[4]In reality, of course, the value of σ would be unknown.

FIGURE 7.6 A SAMPLING DISTRIBUTION WITH $N = 1{,}000$ AND $\sigma_{\bar{X}} = \$15.81$



higher the efficiency. In part, these relationships between sample size and the standard deviation of the sampling distribution do nothing more than underscore our commonsense notion that much more confidence can be placed in large samples than in small (as long as both have been randomly selected).

7.8 ESTIMATION PROCEDURES: INTRODUCTION

The first step in constructing an interval estimate is to decide on how much risk of being wrong you are willing to take. An interval estimate is wrong if it does not include the population parameter. This probability of error is called **alpha** (symbolized as α). The exact value of alpha will depend on the nature of the research situation, but a 0.05 probability is commonly used. Setting alpha equal to 0.05, also called using the 95% **confidence level,** means that over the long run, the researcher is willing to be wrong only 5% of the time. Or, to put it another way, if an infinite number of intervals were constructed at this alpha level (and with all other things being equal), 95% of them would contain the population value and 5% would not. In reality, of course, only one interval is constructed, and by setting the probability of error very low, we are setting the odds in our favor that the interval will include the population value.

The second step is to picture the sampling distribution, divide the probability of error equally into the upper and lower tails of the distribution, and then find the corresponding Z score. For example, if we decided to set alpha equal to 0.05, we would place half (0.025) of this probability in the lower tail and half in the upper tail of the distribution. The sampling distribution would thus be divided as illustrated in Figure 7.7.

FIGURE 7.7 THE SAMPLING DISTRIBUTION WITH ALPHA (α) EQUAL TO 0.05

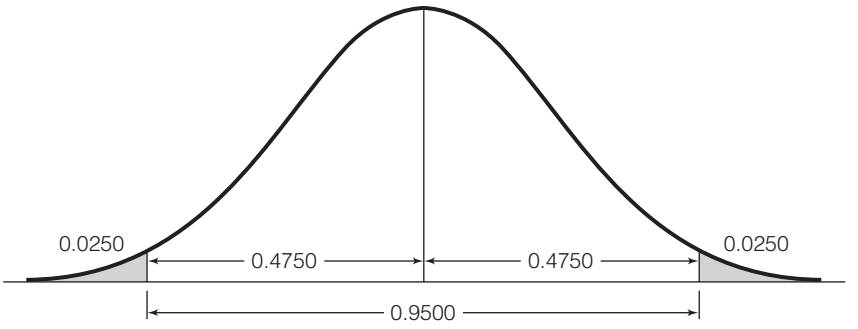
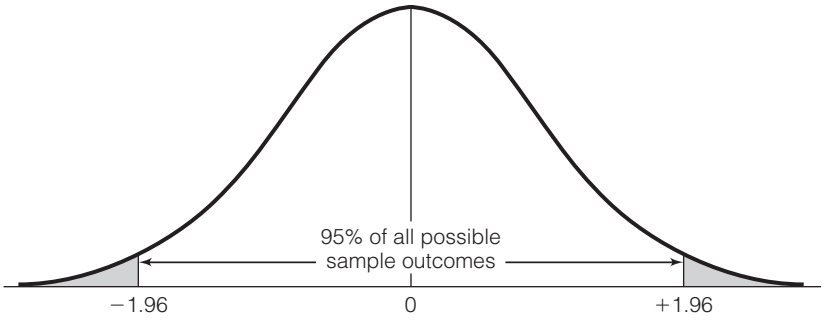


FIGURE 7.8 FINDING THE *Z* SCORE THAT CORRESPONDS TO AN ALPHA (α) OF 0.05



We need to find the *Z* score that marks the beginnings of the shaded areas in Figure 7.7. In Chapter 6, we learned how to first calculate a *Z* score and then find an area under the normal curve. Here, we will reverse that process. We need to find the *Z* score beyond which lies a proportion of 0.0250 of the total area. To do this, go down column c of Appendix A until you find this proportional value (0.0250). The associated *Z* score is 1.96. Since the curve is symmetrical and we are interested in both the upper and lower tails, we designate the *Z* score that corresponds to an alpha of 0.05 as ±1.96 (see Figure 7.8).

We now know that 95% of all possible sample outcomes fall within ±1.96 *Z*-score units of the population value. In reality, of course, there is only one sample outcome, but if we construct an interval estimate based on ±1.96 *Z*s, the probabilities are that 95% of all such intervals will trap the population value. Thus, we can be 95% confident that our interval contains the population value.

Besides the 95% level, there are three other commonly used confidence levels: the 90% level ($\alpha = 0.10$), the 99% level ($\alpha = 0.01$), and the 99.9% level ($\alpha = 0.001$). To find the corresponding *Z* scores for these levels, follow the procedures outlined above for an alpha of 0.05. Table 7.2 summarizes all the information you will need.

You should turn to Appendix A and confirm for yourself that the *Z* scores in Table 7.2 do indeed correspond to these alpha levels. As you do, note that in the cases where alpha is set at 0.10 and 0.01, the precise areas we seek do not appear in the table. For example, with an alpha of 0.10, we would look in column c ("Area Beyond") for the area 0.0500. Instead, we find an area of 0.0505 ($Z = \pm 1.64$) and an area of 0.0495 ($Z = \pm 1.65$). The *Z* score we are seeking is somewhere between these two other scores. When this condition occurs, take the larger of the two scores as *Z*. This will make the interval as wide as possible under the circumstances and is thus the most conservative course of action. In

TABLE 7.2 *Z* SCORES FOR VARIOUS LEVELS OF ALPHA (α)

Confidence Level	Alpha	$\alpha/2$	*Z* Score
90%	0.10	0.0500	±1.65
95%	0.05	0.0250	±1.96
99%	0.01	0.0050	±2.58
99.9%	0.001	0.0005	±3.29

the case of an alpha of 0.01, we encounter the same problem (the exact area 0.0050 is not in the table), resolve it the same way, and take the larger score as *Z*. Finally, note that in the case where alpha is set at 0.001, we can choose from several *Z* scores. Although our table is not detailed enough to show it, the closest *Z* score to the exact area we want is ±3.291, which we can round off to ±3.29. *(For practice in finding Z scores for various levels of confidence, see Problem 7.3)*

The third step is to actually construct the confidence interval. In the sections that follow, we illustrate how to construct an interval estimate first with sample means and then with sample proportions.

7.9 INTERVAL ESTIMATION PROCEDURES FOR SAMPLE MEANS (LARGE SAMPLES)

The formula for constructing a confidence interval based on sample means is given in Formula 7.1:

FORMULA 7.1

$$c.i. = \bar{X} \pm Z\left(\frac{\sigma}{\sqrt{N}}\right)$$

Where: $c.i.$ = confidence interval
$\bar{X}$ = the sample mean
Z = the Z score as determined by the alpha level
$\frac{\sigma}{\sqrt{N}}$ = the standard deviation of the sampling distribution or the standard error of the mean

For example, suppose you wanted to estimate the average IQ of a community and had randomly selected a sample of 200 residents, with a sample mean IQ of 105. Assume that the population standard deviation for IQ scores is about 15, so we can set σ equal to 15. If we are willing to run a 5% chance of being wrong and set alpha at 0.05, the corresponding *Z* score will be 1.96. These values can be directly substituted into Formula 7.1, and an interval can be constructed:

$$c.i. = \bar{X} \pm Z\left(\frac{\sigma}{\sqrt{N}}\right)$$

$$c.i. = 105 \pm 1.96\left(\frac{15}{\sqrt{200}}\right)$$

$$c.i. = 105 \pm 1.96\left(\frac{15}{14.14}\right)$$

$$c.i. = 105 \pm (1.96)(1.06)$$

$$c.i. = 105 \pm 2.08$$

That is, our estimate is that the average IQ for the population in question is somewhere between 102.92 (105 − 2.08) and 107.08 (105 + 2.08). Since 95% of all possible sample means are within ±1.96 *Z*s (or 2.08 IQ units in this case) of the mean of the sampling distribution, the odds are very high that our interval will contain the population mean. In fact, even if the sample mean is as far off as ±1.96 *Z*s (which is unlikely), our interval will still contain $\mu_{\bar{X}}$ and, thus, μ. Only if our sample mean is one of the few that is more than ±1.96 *Z*s from the mean of the sampling distribution will we have failed to include the population mean.

Note that in the example above, the value of the population standard deviation was supplied; it is unusual to have such information about a population. In the great majority of cases, we will have no knowledge of σ. In such cases,

however, we can estimate σ with s, the sample standard deviation. Unfortunately, s is a biased estimator of σ, and the formula must be changed slightly to correct for the bias. For larger samples, the bias of s will not affect the interval very much. The revised formula for cases in which σ is unknown is

FORMULA 7.2

$$c.i. = \bar{X} \pm Z\left(\frac{S}{\sqrt{N-1}}\right)$$

In comparing this formula with Formula 7.1, note that there are two changes. First, σ is replaced by s, and second, the denominator of the last term is the square root of $N - 1$ rather than the square root of N. The latter change is the correction for the fact that s is biased.

Let me stress here that the substitution of s for σ is permitted only for large samples (that is, samples with 100 or more cases). For smaller samples, when the value of the population standard deviation is unknown, the standardized normal distribution summarized in Appendix A cannot be used in the estimation process. To construct confidence intervals from sample means with samples smaller than 100, we must use a different theoretical distribution, called the Student's t distribution, to find areas under the sampling distribution. We will defer the presentation of the t distribution until Chapter 8 and confine our attention here to estimation procedures for large samples only.

Let us close this section by working through a sample problem with Formula 7.2. Average income for a random sample of a particular community is \$35,000, with a standard deviation of \$200. What is the 95% interval estimate of the population mean, μ? Given that

$$\bar{X} = \$35{,}000$$
$$s = \$200$$
$$N = 500$$

and using an alpha of 0.05, the interval can be constructed as follows:

$$c.i. = \bar{X} \pm Z\left(\frac{S}{\sqrt{N-1}}\right)$$

$$c.i. = 35{,}000 \pm 1.96\left(\frac{200}{\sqrt{499}}\right)$$

$$c.i. = 35{,}000 \pm 17.55$$

The average income for the community as a whole is between \$34,982.45 (35,000 − 17.55) and \$35,017.55 (35,000 + 17.55). Remember that this interval has only a 5% chance of being wrong (that is, of not containing the population mean).

In social science research, we need to complete one more step after the confidence interval has been constructed. The fourth and final step in the process is to express our results in a way that can be easily understood and that identifies each of the following elements: the sample mean, the upper and lower limits of the interval, the alpha level, and the sample size (N). The results for the example used in this section might be expressed as follows: "Average income for this community is \$35,000 ± \$17.55. This estimate is based on a sample of 500 respondents and we can be 95% confident that the interval estimate is correct." *(For practice in constructing and expressing confidence intervals for sample means, see Problems 7.1, 7.4–7.7, 7.18a–7.18c.)*

ONE STEP AT A TIME — Constructing Confidence Intervals for Sample Means

Step ***Operation***

1. Select an alpha level. Commonly used alpha levels are 0.10, 0.05, 0.01, 0.001, and the 0.05 level is particularly common.
2. Divide the value of alpha in half. For example, if alpha is 0.05, half of alpha would be 0.025. Find this value in column c of Appendix A to find the Z score that corresponds to your selected alpha level. If alpha = 0.05, $Z = \pm 1.96$.

   (*Note:* If you are using the conventional alpha level of 0.05, the Z score will always be ± 1.96 and you may omit the first two steps. For other commonly used alpha levels, see Table 7.2.)
3. Substitute the sample values into the proper formula. If the value of the population standard deviation (σ) is known, use Formula 7.1. If the value of the population standard deviation (σ) is not known, use Formula 7.2.

   Solving Formula 7.1:
4. Find the square root of N.
5. Divide this value into sigma (σ).
6. Multiply this value by Z.
7. The value you found in Step 6 is the width of the confidence interval. Insert this value into Formula 7.1 following the $\pm$ sign.
8. State the confidence interval in a sentence or two that identifies the
   - **a.** sample mean.
   - **b.** upper and lower limits of the interval.
   - **c.** alpha level (e.g., 0.05) or confidence level (e.g., 95%).
   - **d.** sample size (N).

   Solving Formula 7.2:
4. Find the square root of $N - 1$.
5. Divide this value into s.
6. Multiply this value by Z.
7. The value you found in Step 6 is the width of the confidence interval. Insert this value into Formula 7.2 following the $\pm$ sign.
8. State the confidence interval in a sentence or two that identifies the
   - **a.** sample mean.
   - **b.** upper and lower limits of the interval.
   - **c.** alpha level (e.g., 0.05) or confidence level (e.g., 95%).
   - **d.** sample size (N).

Application 7.1

A study of the leisure activities of Americans was conducted on a sample of 1,000 households. The respondents identified television viewing as a major form of recreation. If the sample reported an average of 6.2 hours of television viewing a day, what is the estimate of the population mean? The information from the sample is

$$\overline{X} = 6.2$$
$$s = 0.7$$
$$N = 1{,}000$$

(continued next page)

Application 7.1 *(continued)*

If we set alpha at 0.05, the corresponding Z score will be ±1.96 and the 95% confidence interval will be

$$c.i. = \bar{X} \pm Z\left(\frac{s}{\sqrt{N-1}}\right)$$

$$c.i. = 6.2 \pm 1.96\left(\frac{0.7}{\sqrt{1{,}000-1}}\right)$$

$$c.i. = 6.2 \pm 1.96\left(\frac{0.7}{31.61}\right)$$

$$c.i. = 6.2 \pm (1.96)(0.02)$$

$$c.i. = 6.2 \pm 0.04$$

Based on this result, we would estimate that the population spends an average of 6.2 ± 0.04 hours per day viewing television. The lower limit of our interval estimate (6.2 − 0.04) is 6.16, and the upper limit (6.2 + 0.04) is 6.24. Thus, another way to state the interval would be

$$6.16 \leq \mu \leq 6.24$$

The population mean is greater than or equal to 6.16 and less than or equal to 6.24. Because alpha was set at the 0.05 level, this estimate has a 5% chance of being wrong (that is, of not containing the population mean).

7.10 INTERVAL ESTIMATION PROCEDURES FOR SAMPLE PROPORTIONS (LARGE SAMPLES)

Estimation procedures for sample proportions are essentially the same as those for sample means. The major difference is that, since proportions are different statistics, we must use a different sampling distribution. In fact, again based on the central limit theorem, we know that sample proportions have sampling distributions that are normal in shape with means (μ_p) equal to the population value (P_u) and standard deviations (σ_p) equal to $\sqrt{P_u(1-P_u)/N}$. The formula for constructing confidence intervals based on sample proportions is

FORMULA 7.3

$$c.i. = P_s \pm Z\sqrt{\frac{P_u(1-P_u)}{N}}$$

The values for P_s and N come directly from the sample, and the value of Z is determined by the confidence level, as was the case with sample means. This leaves one unknown in the formula, P_u—the same value we are trying to estimate. This dilemma can be resolved by setting the value of P_u at 0.5. Since the second term in the numerator under the radical $(1 - P_u)$ is the reciprocal of Pu, the entire expression will always have a value of 0.5 × 0.5, or 0.25, which is the maximum value this expression can attain. That is, if we set P_u at any value other than 0.5, the expression $P_u(1 - P_u)$ will decrease in value. If we set P_u at 0.4, for example, the second term $(1 - P_u)$ would be 0.6, and the value of the entire expression would decrease to 0.24. Setting P_u at 0.5 ensures that the expression $P_u(1 - P_u)$ will be at its maximum possible value and, consequently, the interval will be at maximum width. This is the most conservative solution possible to the dilemma posed by having to assign a value to P_u in the estimation equation.

To illustrate these procedures, assume that you wish to estimate the proportion of students at your university who missed at least one day of classes because of illness last semester. Out of a random sample of 200 students, 60 reported that they had been sick enough to miss classes at least once during the previous semester. The sample proportion upon which we will base our estimate is thus 60/200, or 0.30. At the 95% level, the interval estimate will be

$$c.i. = P_s \pm Z\sqrt{\frac{P_u(1-P_u)}{N}}$$

$$c.i. = 0.30 \pm 1.96\sqrt{\frac{(0.5)(0.5)}{200}}$$

$$c.i. = 0.30 \pm 1.96\sqrt{\frac{0.25}{200}}$$

$$c.i. = 0.30 \pm 1.96\sqrt{0.00125}$$

$$c.i. = 0.30 \pm (1.96)(0.035)$$

$$c.i. = 0.30 \pm 0.07$$

Based on this sample proportion of 0.30, you would estimate the proportion of students who missed at least one day of classes because of illness to be between 0.23 and 0.37. The estimate could, of course, also be phrased in percentages by reporting that between 23% and 37% of the student body was affected by illness at least once during the past semester.

As was the case with sample means, the final step in the process is to express the confidence interval in a way that is easy to understand and that includes the value of the sample proportion, the upper and lower limits of the interval, the alpha level, and the sample size (*N*). The results for the example used in this section might be expressed as follows: "The percentage of students sick enough to miss at least one day of class is 30% ± 7%. This estimate is based on a sample of 200 respondents, and we can be 95% confident that the interval estimate is correct." *(For practice with confidence intervals for sample proportions, see Problems 7.2, 7.8–7.12, 7.16, 7.17, and 7.18d–g.)*

ONE STEP AT A TIME **Constructing Confidence Intervals for Sample Proportions**

Step	*Operation*
1.	Select an alpha level. Commonly used alpha levels are 0.10, 0.05, 0.01, 0.001, and the 0.05 level is particularly common.
2.	Divide the value of alpha in half. For example, if alpha is 0.05, half of alpha would be 0.025. Find this value in column c of Appendix A to find the *Z* score that corresponds to your selected alpha level. If alpha = 0.05, *Z* = 1.96. (*Note:* If you are using the conventional alpha level of 0.05, the *Z* score will always be ±1.96 and you can omit the first two steps. For other commonly used alpha levels, see Table 7.2.)
3.	Substitute the sample values into Formula 7.3.
4.	Substitute a value of 0.5 for P_u. This will make the numerator of the fraction under the square root sign 0.25.
5.	Divide *N* into 0.25.
6.	Find the square root of this value.
7	Multiply this value by the value of *Z*.
8	State the confidence interval in a sentence or two that identifies the **a.** sample proportion. **b.** upper and lower limits of the interval. **c.** alpha level (e.g., 0.05) or confidence level (e.g., 95%). **d.** sample size (*N*).

BECOMING A CRITICAL CONSUMER: Public Opinion Polls, Election Projections, and Surveys

Public opinion polls have become a part of everyday life in the United States and in many other societies, and statements such as those below are routinely found in the press, on the Internet, and in mass media:

> 55% of drivers have changed their driving habits as a result of the high cost of fuel.
>
> 40% of voters are likely to vote for Candidate X.
>
> The president's approval rating stands at 62%.
>
> 17% of Americans have watched an X-rated movie in the past 6 months.

Can reports like these be trusted? How much credence should be accorded to statements such as these? How many times have you said (or heard someone else say), "Where do they get these numbers? They aren't talking to anyone I know." How can you evaluate these claims?

Three Cautions

First of all, whenever you encounter an attempt to characterize how "the public" feels or thinks, you need to examine the source of the statement. Generally, you can place more trust in reports that come from reputable polling firms (e.g., Gallup) or a national news source (CBS News, *USA Today*) and very little (if any) in polls commissioned for partisan purposes (that is, by organizations that represent a particular point of view, such as political parties or advocacy groups).

Second, you should examine how the information is reported. Professional polling firms use interval estimates, and responsible reporting by the media will usually emphasize the estimate itself (for example, "In a survey of the American public, 47% approved of gay marriage.") but also will report the width of the interval ("This estimate is accurate to within ±3%," or "Figures from this poll are subject to a sampling error of ±3%."); the alpha level (usually as the confidence level of 95%); and the size of the sample ("1,458 households were surveyed."). You should be suspicious if any of this information is missing.

Third, you should examine the sample, not only for adequate size, but also for representativeness. In particular, you should greatly discount reports based on the folksy, "man in the street" approach used in many news programs and "comments from our readers or viewers" sometimes found in mass media outlets. These may be interesting and even useful, but because they are not based on EPSEM samples, the results *cannot* be generalized or used to characterize the opinions of anyone other than the actual respondents.

Election Projections and Presidential Approval Ratings

In politics, the very same estimation techniques presented in this chapter are used to track public sentiment, measure how citizens perceive the performance of our leaders, and project the likely winners of upcoming elections.

We should note that these applications are controversial. Many people wonder if the easy availability of polls makes our political leaders too sensitive to the whims of public sentiment. Also, there is concern that election projections could work against people's willingness to participate in the political process and cast their votes on Election Day. These are serious concerns, but we can do little more than acknowledge them here and hope that you will have the opportunity to pursue them fully in other contexts.

Here we'll examine the accuracy of election projections for the 2004 and 2000 presidential election, and we'll also examine the polls that have measured the approval ratings of incumbent U.S. presidents since the middle of the 20th century. Both kinds of polls use the same formulas introduced in this chapter to construct confidence intervals (although the random samples were assembled according to a complex and sophisticated technology that is beyond the scope of this book).

"Too Close to Call"

Pollsters have become very accurate in predicting the outcomes of presidential elections, but remember that 95% confidence intervals are accurate only to ±3 percentage points. This means that polls cannot identify the likely winner in very close races. Both the 2004 and 2000 elections were very close, and the polls indicated a statistical dead heat right up to the end of the campaigns. The table

BECOMING A CRITICAL CONSUMER: *(continued)*

below shows CNN's "poll of polls"—or averages of polls from various sources—for the final weeks of the 2004 campaign and the final breakdown of votes for the two major candidates. The polls included by CNN were based on sample sizes of about 1,000.

POLLING RESULTS AND ACTUAL VOTE, 2004 PRESIDENTIAL ELECTION

	Actual Vote	
	BUSH	KERRY
	51%	48%
	Election Projections	
	Percentage of Sample Estimated to Vote for:	
Date of Poll	Bush	Kerry
November 1	48	46
October 25	49	46
October 18	50	45

The final polls in October and November show that the race was very close and that the difference between the candidates was so small that a winner could not be projected. For example, in the November 1 poll, Bush's support could have been as low as 45% (48% − 3%) and Kerry's could have been as high as 49% (46% + 3%). When the confidence intervals overlap, the race is said to be "too close to call" and "a statistical dead heat."

The 2000 presidential election was even closer. When the ballots were finally counted (and recounted), the candidates were in a virtual tie for the popular vote, separated by only a few thousand votes out of more than 100 million votes cast. The Democratic candidate, Senator Al Gore of Tennessee, received 48.3% of the popular vote and Republican George Bush garnered 48.1%. After a lengthy and intense battle over the electoral votes of Florida, the election was decided in favor of Bush by the Supreme Court. The table below shows the actual vote and the final election projections made by the Gallup polls. These estimates are based on sample sizes ranging between 700 and 2,300.

POLLING RESULTS AND ACTUAL VOTE, 2000 PRESIDENTIAL ELECTION

	Actual Vote	
	BUSH	GORE
	48%	48%
	Election Projections	
	Percentage of Sample Estimated to Vote for:	
Date of Poll	Bush	Gore
November 6	48	46
November 1	47	43
October 22	46	44

Although these races were too close for the pollsters to identify a likely winner, note that in both 2000 and 2004 the polls were within the ±3% margin of error that is associated with interval estimates based on the 95% confidence level and using 1,000 to 2,000 respondents.

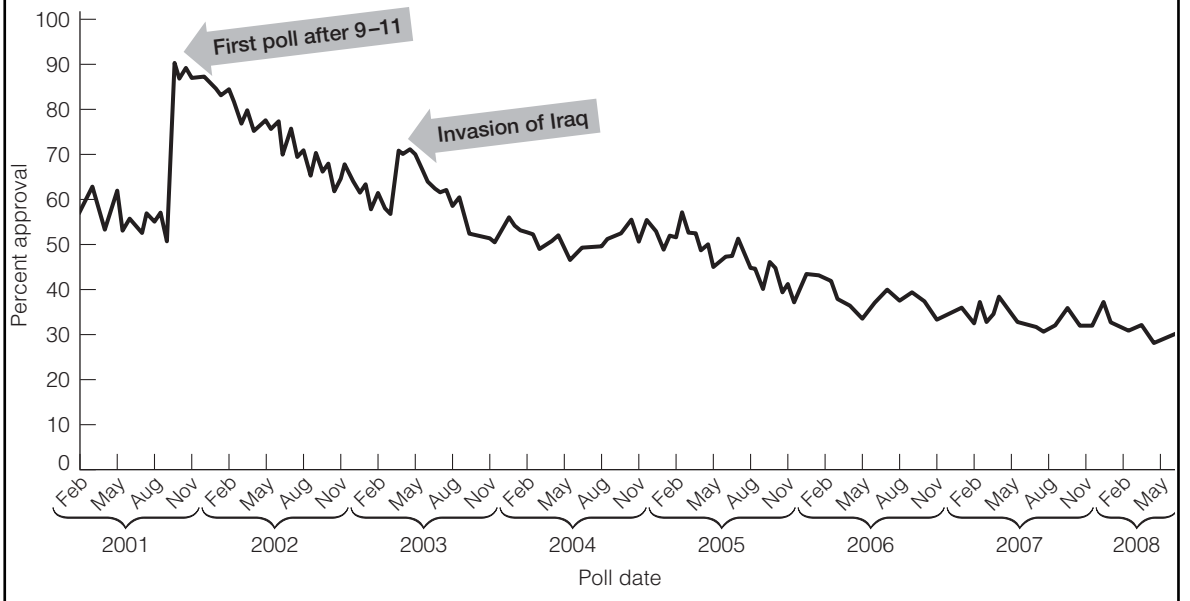
The Ups and Downs of Presidential Popularity

Once you get to be president, you are not free of polls and confidence intervals. Since the middle of the 20th century, pollsters have tracked the president's popularity by asking randomly selected samples of adult Americans if they approve or disapprove of the way the president is handling his job. President Bush's approval ratings (the percent of the sample that approves) are presented below. For purposes of clarity, only the sample proportions are used for this graph, but you should remember that the confidence interval estimate would range about ±3% (at the 95% confidence level) around these points.

The single most dramatic feature of the graph is the huge increase in approval that followed the 9/11 terrorist attacks on the World Trade Center and the Pentagon in 2001. This burst of support reflects the increased solidarity, strong emotions, and high levels of patriotism with which Americans responded to the attacks. The president's approval rating reached an astounding 90% shortly after the attacks, but then, inevitably, began to trend down

BECOMING A CRITICAL CONSUMER: *(continued)*

APPROVAL OF PRESIDENT BUSH, FEBRUARY 2001 TO JULY 2008



as the society (and politics) gradually returned to the daily routine of everyday business.

By the spring of 2003, the president's approval rating had fallen back into the high 50s, much lower than the peaks of fall 2001, but still higher than the historical average. President Bush's approval received another boost with the invasion of Iraq in March 2003, an echo of the "rally" effect that followed 9/11, but then began to sink to prewar levels and below, reflecting the substantial reluctance of many Americans to support this war effort. In 2008, his approval ratings sank to less than 30%, among the very lowest scores ever accorded a modern president.

Surveys in the Professional Literature

For the social sciences, probably the single most important consequence of the growth in opinion polling is that many nationally representative databases are now available for research purposes. These high-quality databases are often available for a nominal fee, and they make it possible to conduct state-of-the-art research without the expense and difficulty of collecting data yourself. This is an important development because we can now test our theories against very high-quality data, and our conclusions will therefore have a stronger empirical basis. Our research efforts will have greater credibility with our colleagues, with policy makers, and with the public at large.

One of the more important and widely used databases of this sort is called the General Social Survey, or the GSS. Since 1972, the National Opinion Research Council has questioned a nationally representative sample of Americans about a wide variety of issues and concerns. Since many of the questions are asked every year, the GSS offers a more than three-decade-long longitudinal record of American sentiment and opinion about a large variety of topics. Each year, new topics of current concern are added and explored, and the variety of information available continues to expand. Like other nationally representative samples, the GSS sample is chosen by a complex probability design. Sample size varies from 1,400 to over 4,000, and estimates based on samples this large will be accurate to within about $\pm 3\%$ (see Table 7.5 and Section 7.7). The computer exercises in this text are based on the 2006 GSS, and this database is described more fully in Appendix F and Appendix G.

Application 7.2

If 45% of a random sample of 1,000 Americans reports that walking is their major physical activity, what is the estimate of the population value? The sample information is

$$P_s = 0.45$$
$$N = 1{,}000$$

Note that the percentage of walkers has been stated as a proportion. If we set alpha at 0.05, the corresponding Z score will be ± 1.96, and the interval estimate of the population proportion will be

$$c.i. = P_s \pm Z\sqrt{\frac{P_u(1 - P_u)}{N}}$$
$$c.i. = 0.45 \pm 1.96\sqrt{\frac{(0.5)(0.5)}{1{,}000}}$$
$$c.i. = 0.45 \pm 1.96\sqrt{0.00025}$$
$$c.i. = 0.45 \pm (1.96)(0.016)$$
$$c.i. = 0.45 \pm 0.03$$

We can now estimate that the proportion of the population for which walking is the major form of physical exercise is between 0.42 and 0.48. That is, the lower limit of the interval estimate is $(0.45 - 0.03)$ or 0.42, and the upper limit is $(0.45 + 0.03)$ or 0.48. We may also express this result in percentages and say that between 42% and 48% of the population walk as their major form of physical exercise. This interval has a 5% chance of not containing the population value.

Application 7.3

A total of 1,609 adult Canadians were randomly selected to participate in a study of attitudes toward homosexuality and same-sex marriages. Some results are reported below. What is the level of support in the population? The sample information expressed in terms of the proportion agreeing, is as follows.

"Gays and lesbians should have the same rights as heterosexuals."

$$P_s = 0.72$$
$$N = 1{,}609$$

"Marriage should be expanded to include same-sex unions."

$$P_s = 0.60$$
$$N = 1{,}609$$

For the first item, the confidence interval estimate to the population at the 95% confidence level is

$$c.i = P_s \pm Z\sqrt{\frac{P_u(1 - P_u)}{N}}$$
$$c.i = 0.72 \pm 1.96\sqrt{\frac{(0.5)(0.5)}{1{,}609}}$$
$$c.i = 0.72 \pm 1.96\sqrt{0.00016}$$
$$c.i = 0.72 \pm (1.96)(0.013)$$
$$c.i = 0.72 \pm 0.03$$

Expressing these results in terms of percentages, we can conclude that at the 95% confidence level, between 69% and 75% of adult Canadians support equal rights for gays and lesbians.

For the second survey item, the confidence interval estimate for the population at the 95% confidence level is

$$c.i = P_s \pm Z\sqrt{\frac{P_u(1 - P_u)}{N}}$$
$$c.i = 0.60 \pm 1.96\sqrt{\frac{(0.5)(0.5)}{1{,}609}}$$
$$c.i = 0.60 \pm 1.96\sqrt{0.00016}$$
$$c.i = 0.60 \pm (1.96)(0.013)$$
$$c.i = 0.60 \pm 0.03$$

Again expressing results in terms of percentages, we can conclude that at the 95% confidence level, between 57% and 63% of adult Canadians support same-sex marriages. (*Note*: The width of the second confidence interval is exactly the same as the first. This is because we are using the same values for Z score and sample size in both estimates.)

TABLE 7.3 CHOOSING FORMULAS FOR CONFIDENCE INTERVALS

If the sample statistic is a	and		Use formula
mean	the population standard deviation is known	7.1	$c.i. = \bar{X} \pm Z\left(\frac{\sigma}{\sqrt{N}}\right)$
mean	the population standard deviation is unknown	7.2	$c.i. = \bar{X} \pm Z\left(\frac{s}{\sqrt{N-1}}\right)$
proportion		7.3	$c.i. = P_s \pm Z\sqrt{\frac{P_u(1-P_u)}{N}}$

7.11 A SUMMARY OF THE COMPUTATION OF CONFIDENCE INTERVALS

To this point, we have covered the construction of confidence intervals for sample means and sample proportions. In both cases, the procedures assume large samples (N greater than 100). The procedures for constructing confidence intervals for small samples are not covered in this text. Table 7.3 presents the three formulas for confidence intervals organized by the situations in which they are used. For sample means, when the population standard deviation is known, use Formula 7.1. When the population standard deviation is unknown (which is the usual case), use Formula 7.2. For sample proportions, always use Formula 7.3.

7.12 CONTROLLING THE WIDTH OF INTERVAL ESTIMATES

The width of a confidence interval for either sample means or sample proportions can be partly controlled by manipulating two terms in the equation. First, the confidence level can be raised or lowered, and second, the interval can be widened or narrowed by gathering samples of different size. The researcher alone determines how much risk of being wrong he or she is willing to take (that is, of not including the population value in the interval estimate). The exact confidence level (or alpha level) will depend, in part, on the purpose of the research. For example, if potentially harmful drugs were being tested, the researcher would naturally demand very high levels of confidence (99.99% or even 99.999%). On the other hand, if intervals are being constructed only for loose "guesstimates," then much lower confidence levels can be tolerated (such as 90%).

The relationship between interval size and confidence level is that intervals widen as confidence levels increase. This relationship should make intuitive sense. Wider intervals are more likely to trap the population value; hence, more confidence can be placed in them.

To illustrate this relationship, let us return to the example where we estimated the average income for a community. In this problem, we were working with a sample of 500 residents, and the average income for this sample was \$35,000, with a standard deviation of \$200. We constructed the 95% confidence interval and found that it extended 17.55 around the sample mean (that is, the interval was \$35,000 ± 17.55).

If we had constructed the 90% confidence interval for these sample data (a lower confidence level), the Z score in the formula would have decreased to ±1.65, and the interval would have been narrower:

$$c.i. = \bar{X} \pm Z\left(\frac{s}{\sqrt{N-1}}\right)$$

$$c.i. = 35{,}000 \pm 1.65\left(\frac{200}{\sqrt{499}}\right)$$
$$c.i. = 35{,}000 \pm 1.65(8.95)$$
$$c.i. = 35{,}000 \pm 14.77$$

On the other hand, if we had constructed the 99% confidence interval, the Z score would have increased to ± 2.58, and the interval would have been wider:

$$c.i. = \bar{X} \pm Z\left(\frac{s}{\sqrt{N-1}}\right)$$
$$c.i. = 35{,}000 \pm 2.58\left(\frac{200}{\sqrt{499}}\right)$$
$$c.i. = 35{,}000 \pm 2.58(8.95)$$
$$c.i. = 35{,}000 \pm 23.09$$

At the 99.9% confidence level, the Z score would be ± 3.29, and the interval would be wider still

$$c.i. = \bar{X} \pm Z\left(\frac{s}{\sqrt{N-1}}\right)$$
$$c.i. = 35{,}000 \pm 3.29\left(\frac{200}{\sqrt{499}}\right)$$
$$c.i. = 35{,}000 \pm 3.29(8.95)$$
$$c.i. = 35{,}000 \pm 29.45$$

These four intervals are grouped together in Table 7.4, and the increase in interval size can be readily observed. Although sample means have been used to illustrate the relationship between interval width and confidence level, exactly the same relationships apply to sample proportions. *(To further explore the relationship between alpha and interval width, see Problem 7.13.)*

Sample size has the opposite relationship to interval width. As sample size increases, interval width decreases. Larger samples give more precise (narrower) estimates. Again, an example should make this clearer. In Table 7.5, confidence intervals for four samples of various sizes are constructed and then grouped together for purposes of comparison. The sample data are the same as in Table 7.4, and the confidence level is 95% throughout. The relationships illustrated in Table 7.5 also hold true, of course, for sample proportions. *(To further explore the relationship between sample size and interval width, see Problem 7.14.)*

TABLE 7.4 INTERVAL ESTIMATES FOR FOUR CONFIDENCE LEVELS ($\bar{X}$ = \$35,000, s = \$200, N = 500 throughout)

Alpha	Confidence Level	Interval	Interval Width
0.10	90%	\$35,000 ± 14.77	\$29.54
0.05	95%	\$35,000 ± 17.55	\$35.10
0.01	99%	\$35,000 ± 23.09	\$46.18
0.001	99.9%	\$35,000 ± 29.45	\$58.90

TABLE 7.5 INTERVAL ESTIMATES FOR FOUR DIFFERENT SAMPLES ($\bar{X}$ = \$35,000, s = \$200, alpha = 0.05 throughout)

Sample 1 (N = 100)	Sample 2 (N = 500)
$c.i. = 35{,}000 \pm 1.96\left(\frac{200}{\sqrt{99}}\right)$	$c.i. = 35{,}000 \pm 1.96\left(\frac{200}{\sqrt{499}}\right)$
$c.i. = 35{,}000 \pm 39.40$	$c.i. = 35{,}000 \pm 17.55$
Sample 3 (N = 1,000)	**Sample 4 (N = 10,000)**
$c.i. = 35{,}000 \pm 1.96\left(\frac{200}{\sqrt{999}}\right)$	$c.i. = 35{,}000 \pm 1.96\left(\frac{200}{\sqrt{9{,}999}}\right)$
$c.i. = 35{,}000 \pm 12.40$	$c.i. = 35{,}000 \pm 3.92$

Sample	N	Interval Width
1	100	\$78.80
2	500	\$35.10
3	1,000	\$24.80
4	10,000	\$7.84

Notice that the decrease in interval width (or, increase in precision) does not bear a constant or linear relationship with sample size. For example, sample 2 is five times larger than sample 1, but the interval constructed with the larger sample size is not five times as narrow. This is an important relationship because it means that N might have to be increased many times over to appreciably improve the accuracy of an estimate. Since the cost of a research project is directly related to sample size, this relationship implies a point of diminishing returns in estimation procedures. A sample of 10,000 will cost about twice as much as a sample of 5,000, but estimates based on the larger sample will not be twice as precise.

SUMMARY

1. Since populations are almost always too large to test, a fundamental strategy of social science research is to select a sample from the defined population and then use information from the sample to generalize to the population. This is done either by estimation or by hypothesis testing.
2. Simple random samples are created by selecting cases from a list of the population following the rule of EPSEM (each case has an equal probability of being selected). Samples selected by the rule of EPSEM have a very high probability of being representative.
3. The sampling distribution, the central concept in inferential statistics, is a theoretical distribution of all possible sample outcomes. Since its overall shape, mean, and standard deviation are known (under the conditions specified in the two theorems), the sampling distribution can be adequately characterized and used by researchers.
4. The two theorems that were introduced in this chapter state that when the variable of interest is normally distributed in the population or when sample size is large, the sampling distribution will be normal in shape, its mean will be equal to the population mean, and its standard deviation (or standard error) will be equal to the population standard deviation divided by the square root of N.
5. Population values can be estimated with sample values. With confidence intervals, which can be based on either proportions or means, we estimate

that the population value falls within a certain range of values. The width of the interval is a function of how much risk of being wrong we are willing to take (the alpha level) and the sample size. The interval widens as our probability of being wrong decreases and as sample size decreases.

6. Estimates based on sample statistics must be unbiased and relatively efficient. Of all the sample statistics, only means and proportions are unbiased. The means of the sampling distributions of these statistics are equal to the respective population values. Efficiency is largely a matter of sample size. The greater the sample size, the lower the value of the standard deviation of the sampling distribution, the more tightly clustered the sample outcomes will be around the mean of the sampling distribution, and the more efficient the estimate.

SUMMARY OF FORMULAS

FORMULA 7.1 Confidence interval for a sample mean, large samples, population standard deviation known:

$$c.i. = \overline{X} \pm Z\left(\frac{\sigma}{\sqrt{N}}\right)$$

FORMULA 7.2 Confidence interval for a sample mean, large samples, population standard deviation unknown:

$$c.i. = \overline{X} \pm Z\left(\frac{s}{\sqrt{N-1}}\right)$$

FORMULA 7.3 Confidence interval for a sample proportion, large samples:

$$c.i. = P_s \pm Z\sqrt{\frac{P_u(1-P_u)}{N}}$$

GLOSSARY

Alpha (α). The probability of error or the probability that a confidence interval does not contain the population value. Alpha levels are usually set at 0.10, 0.05, 0.01, or 0.001.

Bias. A criterion used to select sample statistics as estimators. A statistic is unbiased if the mean of its sampling distribution is equal to the population value of interest.

Central limit theorem. A theorem that specifies the mean, standard deviation, and shape of the sampling distribution, given that the sample is large.

Confidence interval. An estimate of a population value in which a range of values is specified.

Confidence level. A frequently used alternate way of expressing alpha, the probability that an interval estimate will not contain the population value. Confidence levels of 90%, 95%, 99%, and 99.9% correspond to alphas of 0.10, 0.05, 0.01, and 0.001, respectively.

Efficiency. The extent to which the sample outcomes are clustered around the mean of the sampling distribution.

EPSEM. Equal probability of selection method for selecting samples. Every element or case in the population must have an equal probability of selection for the sample.

μ. The mean of a population.

$\mu_{\overline{X}}$. (Read mu bar *X*.) The mean of a sampling distribution of sample means.

μ_p. (Read mu sub *p*.) The mean of a sampling distribution of sample proportions.

Parameter. A characteristic of a population.

P_s. (Read *P* sub *s*.) Any sample proportion.

P_u. (Read *P* sub *u*.) Any population proportion.

Representative. The quality a sample is said to have if it reproduces the major characteristics of the population from which it was drawn.

Sampling distribution. The distribution of a statistic for all possible sample outcomes of a certain size. Under conditions specified in two theorems, the sampling distribution will be normal in shape, with a mean equal to the population value and a standard deviation equal to the population standard deviation divided by the square root of *N*.

Simple random sample. A method for choosing cases from a population by which every case and every combination of cases has an equal chance of being included.

Standard error of the mean. The standard deviation of a sampling distribution of sample means.

PROBLEMS

(Problems are labeled with the social science discipline from which they are drawn: SOC for sociology, SW for social work, PS for political science, CJ for criminal justice, PA for public administration, and GER for gerontology.)

7.1 For each set of sample outcomes below, construct the 95% confidence interval for estimating μ, the population mean.

a. $\overline{X} = 5.2$, $s = 0.7$, $N = 157$
b. $\overline{X} = 100$, $s = 9$, $N = 620$
c. $\overline{X} = 20$, $s = 3$, $N = 220$
d. $\overline{X} = 1{,}020$, $s = 50$, $N = 329$
e. $\overline{X} = 7.3$, $s = 1.2$, $N = 105$
f. $\overline{X} = 33$, $s = 6$, $N = 220$

7.2 For each set of sample outcomes below, construct the 99% confidence interval for estimating P_u.

a. $P_s = 0.14$, $N = 100$
b. $P_s = 0.37$, $N = 522$
c. $P_s = 0.79$, $N = 121$
d. $P_s = 0.43$, $N = 1{,}049$
e. $P_s = 0.40$, $N = 548$
f. $P_s = 0.63$, $N = 300$

7.3 For each confidence level below, determine the corresponding *Z* score.

Confidence Level	Alpha	Area Beyond *Z*	*Z* Score
95%	0.05	0.0250	±1.96
94%			
92%			
97%			
98%			
99.9%			

7.4 SW You have developed a series of questions to measure burnout in social workers. A random sample of 100 social workers working in greater metropolitan Shinbone, Kansas, has an average score of 10.3, with a standard deviation of 2.7. What is your estimate of the average burnout score for the population as a whole? Use the 95% confidence level.

7.5 SOC A researcher has gathered information from a random sample of 178 households. For each variable below, construct confidence intervals to estimate the population mean. Use the 90% level.

a. An average of 2.3 people resides in each household. Standard deviation is 0.35.
b. There was an average of 2.1 television sets ($s = 0.10$) and 0.78 telephones ($s = 0.55$) per household.
c. The households averaged 6.0 hours of television viewing per day ($s = 3.0$).

7.6 SOC A random sample of 100 television programs contained an average of 2.37 acts of physical violence per program. At the 99% level, what is your estimate of the population value?

$\overline{X} = 2.37$
$s = 0.30$
$N = 100$

7.7 SOC A random sample of 429 college students was interviewed about a number of matters.

a. They reported that they had spent an average of $378.23 on textbooks during the previous semester. If the sample standard deviation for these data is $15.78, construct an estimate of the population mean at the 99% level.
b. They also reported that they had visited the health clinic an average of 1.5 times a semester. If the sample standard deviation is 0.3, construct an estimate of the population mean at the 99% level.
c. On average, the sample had missed 2.8 days of classes per semester because of illness. If the sample standard deviation is 1.0, construct an estimate of the population mean at the 99% level.
d. On average, the sample had missed 3.5 days of classes per semester for reasons other than illness. If the sample standard deviation is 1.5, construct an estimate of the population mean at the 99% level.

7.8 CJ A random sample of 500 residents of Shinbone, Kansas, shows that exactly 50 of the respondents had been the victims of violent crime over the past year. Estimate the proportion of victims for the population as a whole, using the 90% confidence level. (*HINT: Calculate the sample proportion P_s before using Formula 7.3. Remember that proportions are equal to frequency divided by N.*)

7.9 SOC The survey mentioned in Problem 7.5 found that 25 of the 178 households consisted of unmarried couples who were living together. What is your estimate of the population proportion? Use the 95% level.

7.10 PA A random sample of 324 residents of a community revealed that 30% were very satisfied with the quality of trash collection. At the 99% level, what is your estimate of the population value?

7.11 SOC A random sample of 1,496 respondents of a major metropolitan area was questioned about a number of issues. Construct estimates to the population at the 90% level for each of the results reported below. Express the final confidence interval in percentages (e.g., between 40 and 45% agreed that premarital sex was always wrong).

a. When asked to agree or disagree with the statement "Explicit sexual books and magazines lead to rape and other sex crimes," 823 agreed.
b. When asked to agree or disagree with the statement "Hand guns should be outlawed," 650 agreed.
c. 375 of the sample agreed that marijuana should be legalized.
d. 1023 of the sample said that they had attended church or synagogue at least once within the past month.
e. 800 agreed that public elementary schools should have sex education programs starting in the fifth grade.

7.12 SW A random sample of 100 patients who had been treated in a program for alcoholism and drug dependency over the past 10 years was selected. It was determined that 53 of the patients had been readmitted to the program at least once. At the 95% level, construct an estimate to the population proportion.

7.13 For the sample data below, construct four different interval estimates of the population mean, one each for the 90%, 95%, 99%, and 99.9% level. What happens to the interval width as confidence level increases? Why?

$$\overline{X} = 100$$
$$s = 10$$
$$N = 500$$

7.14 For each of the three sample sizes below, construct the 95% confidence interval. Use a sample proportion of 0.40 throughout. What happens to interval width as sample size increases? Why?

$$P_s = 0.40$$
Sample A: $N = 100$
Sample B: $N = 1{,}000$
Sample C: $N = 10{,}000$

7.15 PS Two individuals are running for mayor of Shinbone. You conduct an election survey a week before the election and find that 51% of the respondents prefer candidate A. Can you predict a winner? Use the 99% level. (*HINT: In a two-candidate race, what percentage of the vote would the winner need? Does the confidence interval indicate that candidate A has a sure margin of victory? Remember that while the population parameter is probably ($\alpha = 0.01$) in the confidence interval, it may be anywhere in the interval.*)

$$P_s = 0.51$$
$$N = 578$$

7.16 SOC The World Values Survey (http://www.worldvaluessurvey.org/) is administered periodically to random samples from societies around the globe. Listed below are the number of respondents in each nation who said that they are "very happy." Compute sample proportions and construct confidence interval estimates for each nation at the 95% level.

Nation	Year	Number "Very Happy"	Sample Size	Confidence Interval
France	1999	505	1,615	_____
Japan	2000	378	1,362	_____
Chili	2000	432	1,200	_____
Nigeria	2000	1,351	2,000	_____
China	2001	115	1,000	_____

7.17 SOC The fraternities and sororities at St. Algebra College have been plagued by declining membership over the past several years and want to know if the incoming freshman class will be a fertile recruiting ground. Not having enough money to survey all 1,600 freshmen, they commission you to survey the interests of a random sample. You find that 35 of your 150 respondents are extremely interested in social clubs. At the 95% level, what is your estimate of the number of freshmen who would be extremely interested? (*HINT: The high and low values of your final confidence interval are proportions. How can proportions also be expressed as numbers?*)

7.18 SOC The results listed below are from a survey given to a random sample of the American public. For each sample statistic, construct a confidence

interval estimate of the population parameter at the 95% confidence level. Sample size (N) is 2,987 throughout.

a. The average occupational prestige score was 43.87, with a standard deviation of 13.52.
b. The respondents reported watching an average of 2.86 hours of TV per day with a standard deviation of 2.20.
c. The average number of children was 1.81, with a standard deviation of 1.67.
d. Of the 2,987 respondents, 876 identified themselves as Catholic.
e. Of the 2,987 respondents, 535 said that they had never married.
f. The proportion of respondents who said they voted for Bush in the 2004 presidential election was 0.43.
g. When asked about capital punishment, 2,425 of the respondents said that they favored the death penalty for murder.

YOU ARE THE RESEARCHER: Estimating the Characteristics of the Typical American

SPSS does not provide a program specifically for constructing confidence intervals, although some of the procedures we'll cover in future chapters do include confidence intervals as part of the output. Rather than make use of these programs, we will use SPSS to produce sample statistics from the 2006 GSS, which you can then use as the basis for interval estimates to the population (U.S. society as of 2006).

STEP 1: Choosing the Variables

As a framework for this exercise, we will return to the task of describing the "typical American" begun in Chapter 4 and continued in Chapter 5. Select four of the variables you used in the earlier exercises and add four more that you did *not* use earlier. Make sure that you have at least one variable from the nominal level and at least one measured at the ordinal or interval-ratio level. List the variables, along with level of measurement, here.

Variable	SPSS Name	Explain Exactly What This Variable Measures	Level of Measurement
1			
2			
3			
4			
5			
6			
7			
8			

STEP 2: Getting Sample Statistics

For interval-ratio variables and ordinal level variables with four or more scores, use **Descriptives** to get sample means, standard deviations, and sample sizes. For nominal level variables and for ordinal level variables with three or fewer scores, use **Frequencies** to produce frequency distributions. For either procedure, click

Analyze → Descriptive Statistics and then select either **Frequencies** or **Descriptives.** Select your variables from the box on the left, and click the arrow to move the variable name to the window on the right. SPSS will process all variables listed at the same time.

STEP 3: Constructing Confidence Intervals

Once you have the SPSS output, use the results to construct 95% confidence intervals around the sample statistics. For nominal level variables and ordinal level variables with three or fewer scores, select one category (e.g., female for *sex* or Catholic for *relig*) and look in the **Valid Percent** column of the frequency distribution to get the percentage of cases in the sample in that category. Change this value to a proportion (divide by 100). This value is P_s and can be substituted directly into Formula 7.3. Remember to estimate P_u at 0.5. After you have found the confidence interval, change your proportion back to a percentage and record your results in the table below.

For ordinal level variables with four or more scores and for interval ratio variables, find the sample mean, standard deviation, and sample size in the output of the **Descriptives** procedure. Substitute these values into Formula 7.2 and record your results in the table below.

STEP 4: Recording Results

Use the table below to summarize your confidence intervals.

Variable	SPSS Name	Sample Statistic ($\bar{X}$ or P_s)	95% Confidence Interval	*N*
1				
2				
3				
4				
5				
6				
7				
8				

STEP 5: Reporting Results

For each statistic, express the confidence interval in words, as if you were reporting results in a newspaper story. Be sure to include each of the following: the sample statistic, sample size, the upper and lower limits of the confidence interval, the confidence level (95%), and the identity of the population. A sample sentence might read as follows: I estimate that 45% of all Americans support candidate X for president. This estimate is accurate to within ±3% and has a 95% chance of being accurate, and it is based on a sample of 1,362 adult Americans.

8

Hypothesis Testing I
The One-Sample Case

LEARNING OBJECTIVES

By the end of this chapter, you will be able to:

1. Explain the logic of hypothesis testing.
2. Define and explain the conceptual elements involved in hypothesis testing, especially the null hypothesis, the sampling distribution, the alpha level, and the test statistic.
3. Explain what it means to reject the null hypothesis or fail to reject the null hypothesis.
4. Identify and cite examples of situations in which one-sample tests of hypotheses are appropriate.
5. Test the significance of single-sample means and proportions using the five-step model and correctly interpret the results.
6. Explain the difference between one- and two-tailed tests and specify when each is appropriate.
7. Define and explain Type I and Type II errors and relate each to the selection of an alpha level.

8.1 INTRODUCTION

Chapter 7 introduced the techniques for estimating population parameters from sample statistics. In Chapters 8 through 11, we will investigate a second application of inferential statistics called **hypothesis testing** or **significance testing.** In this chapter, the techniques for hypothesis testing in the one-sample case will be introduced. These procedures could be used in situations such as the following.

1. A researcher has selected a sample of 789 older citizens who live in a particular state and has information on the percentage of the entire population of the state that was victimized by crime during the past year. Are older citizens, as represented by this sample, more or less likely to be victimized than the population in general?
2. Are the GPAs of college athletes different from the GPAs of the student body as a whole? To investigate, the academic records of a random sample of 235 student athletes from a large state university are compared with the overall GPA of all students.
3. A sociologist has been hired to assess the effectiveness of a rehabilitation program for alcoholics in her city. The program serves a large area, and she does not have the resources to test every single client. Instead, she draws a random sample of 127 people from the list of all clients and questions them on a variety of issues. She notices that, on the average, the people in her sample miss fewer days of work each year than the city as a whole. Are alcoholics treated by the program more reliable than workers in general?

In each of these situations, we have randomly selected samples (of senior citizens, athletes, or treated alcoholics) that we want to compare to a population (the entire state, student body, or city). Note that we are not interested in the sample per se, but in the larger group from which it was selected (*all* senior citizens in the state, *all* athletes on this campus, or *all* people who have completed the treatment program). Specifically, we want to know if the groups represented by the samples are different from the populations on a specific trait or variable (victimization rates, GPAs, or absenteeism).

Of course, it would be better if we could include all senior citizens, athletes, or treated alcoholics rather than these smaller samples. However, as we have seen, researchers usually do not have the resources necessary to test everyone in a large group and must use random samples instead. In these situations, conclusions will be based on a comparison of a single sample (which represents a larger group) and the population. For example, if we found that the rate of victimization for the sample of senior citizens was higher than the state rate, we might conclude that senior citizens are significantly more likely to be crime victims. The word *significantly* is a key word: it means that the difference between the sample's victimization rate and the population's rate is very unlikely to have been caused by random chance alone. In other words, it is very likely that *all* senior citizens (not just the 789 people in the sample) have a higher victimization rate than the state as a whole. On the other hand, if we found no significant difference between the GPAs of the sample of athletes and the student body as a whole, we might conclude that athletes (*all* athletes on this campus) are essentially the same as other students in terms of academic achievement.

Thus, we can use samples to represent larger groups (senior citizens, athletes, or treated alcoholics), compare and contrast the characteristics of the sample with the population, and be extremely confident in our conclusions. Remember, however, that the equal probability of selection method (EPSEM) procedure does not guarantee that samples will be representative; thus, there will always be a small amount of uncertainty in our conclusions. One of the great advantages of inferential statistics, however, is that we will be able to estimate the probability of error and evaluate our decisions accordingly.

8.2 AN OVERVIEW OF HYPOTHESIS TESTING

We'll begin with a general overview of hypothesis testing, using our third research situation listed above as an example, and introduce the more technical considerations and proper terminology throughout the remainder of the chapter. Let's examine this situation in some detail. First of all, the main question is, Are people treated in this program more reliable workers than people in the community in general? In other words, what the researcher would really like to do is compare information gathered from *all* clients (the *population* of alcoholics treated in the program) with information about the entire metropolitan area. If she had information for both of these groups (all clients and the entire metro population), she could answer the question easily, completely, and finally.

The problem is that the researcher does not have the time or money to gather information on the thousands of people who have been treated by the program. Instead, following the rule of EPSEM, she has drawn a random

sample of 127 clients from agency records. The absentee rates for the sample and the community follow.

Community	Sample of Treated Alcoholics
$\mu = 7.2$ days per year	$\bar{X} = 6.8$ days per year
$\sigma = 1.43$	$N = 127$

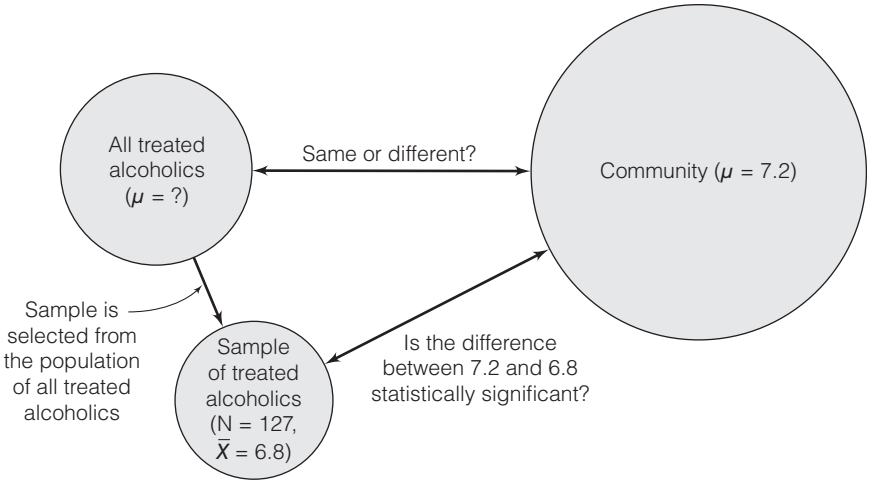
We can see that there is a difference in rates of absenteeism and that the average rate of absenteeism for the sample is lower than the rate for the community. Although it's tempting, we can't make any conclusions yet because we are working with a random sample of the population we are interested in, not the population itself (all people treated in the program).

Figure 8.1 should clarify these relationships. The community is symbolized by the largest circle because it is the largest group. The population of all treated alcoholics is also symbolized by a large circle because it is a sizable group, although only a small fraction of the community as a whole. The random sample of 127, the smallest of the three groups, is symbolized by the smallest circle.

The labels on the arrows connecting the circles summarize the major questions and connections in this research situation. As we noted earlier, the main question the researcher wants to answer is, Are all treated alcoholics the same as or different from the community in terms of absenteeism? The population of treated alcoholics, too large to test, is represented by the randomly selected sample of 127. The main question related to the sample concerns the cause of the observed difference between its mean of 6.8 and the community mean of 7.2. There are two possible explanations for this difference, and we will consider them one at a time.

The first explanation, which we will call explanation A, is that the difference between the community mean of 7.2 days and the sample mean of

FIGURE 8.1 A TEST OF HYPOTHESIS FOR SINGLE SAMPLE MEANS



6.8 days reflects a real difference in absentee rates between the population of all treated alcoholics and the community. The difference is statistically significant in the sense that it is very unlikely to have occurred by random chance alone. If explanation A is true, the population of all treated alcoholics is different from the community and the sample did *not* come from a population with a mean absentee rate of 7.2 days.

The second explanation, or explanation B, is that the observed difference between sample and community means was caused by mere random chance. In other words, there is no important difference between treated alcoholics and the community as a whole, and the difference between the sample mean of 6.8 days and the mean of 7.2 days of absenteeism for the community is trivial and due to random chance. If explanation B is true, the population of treated alcoholics is just like everyone else and has a mean absentee rate of 7.2 days.

Which explanation is correct? As long as we are working with a sample rather than the entire group, we cannot be absolutely (100%) sure about the answer to this question. However, we can set up a decision-making procedure so conservative that one of the two explanations can be chosen knowing that the probability of choosing the incorrect explanation is very low.

This decision-making process begins with the assumption that explanation B is correct. Symbolically, the assumption that the mean absentee rate for all treated alcoholics is the same as the rate for the community as a whole can be stated as

$$\mu = 7.2 \text{ days per year}$$

Remember that this μ refers to the mean for *all* treated alcoholics, not just the 127 in the sample. This assumption, $\mu = 7.2$, can be tested statistically.

If explanation B (the population of treated alcoholics is not different from the community as a whole and has a μ of 7.2) is true, then the probability of getting the observed sample outcome ($\bar{X} = 6.8$) can be found. Let us add an objective decision rule in advance. If the odds of getting the observed difference are less than 0.05 (5 out of 100, or 1 in 20), we will reject explanation B. If this explanation were true, a difference of this size (7.2 days vs. 6.8 days) would be a very rare event, and in hypothesis testing we always bet *against* rare events.

How can we estimate the probability of the observed sample outcome ($\bar{X} = 6.8$) if explanation B is correct? This value can be determined by using our knowledge of the sampling distribution of all possible sample outcomes. Looking back at the information we have and applying the central limit theorem (see Chapter 7), we can assume that the sampling distribution is normal in shape, has a mean of 7.2 (because $\mu_{\bar{X}} = \mu$) and a standard deviation of $1.43/\sqrt{127}$ because $\sigma_{\bar{X}} = \sigma/\sqrt{N}$. We also know that the standard normal distribution can be interpreted as a distribution of probabilities (see Chapter 6) and that the particular sample outcome noted above ($\bar{X} = 6.8$) is one of thousands of possible sample outcomes. The sampling distribution, with the sample outcome noted, is depicted in Figure 8.2.

Using our knowledge of the standardized normal distribution, we can add further useful information to this sampling distribution of sample means. Specifically, with Z scores, we can depict the decision rule stated previously: any sample outcome with probability less than 0.05 (assuming that

FIGURE 8.2 THE SAMPLING DISTRIBUTION OF ALL POSSIBLE SAMPLE MEANS

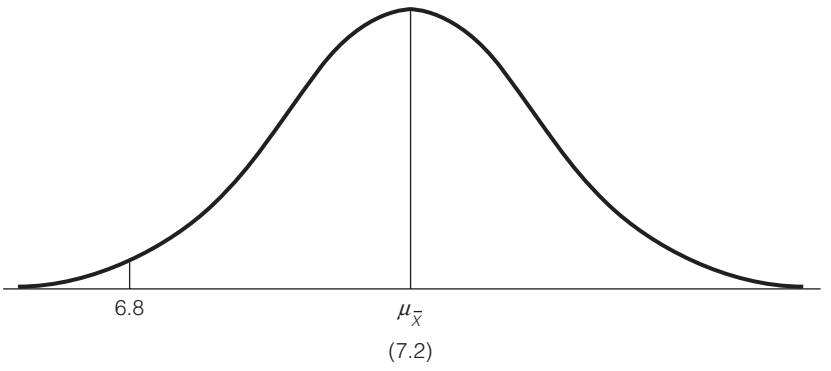
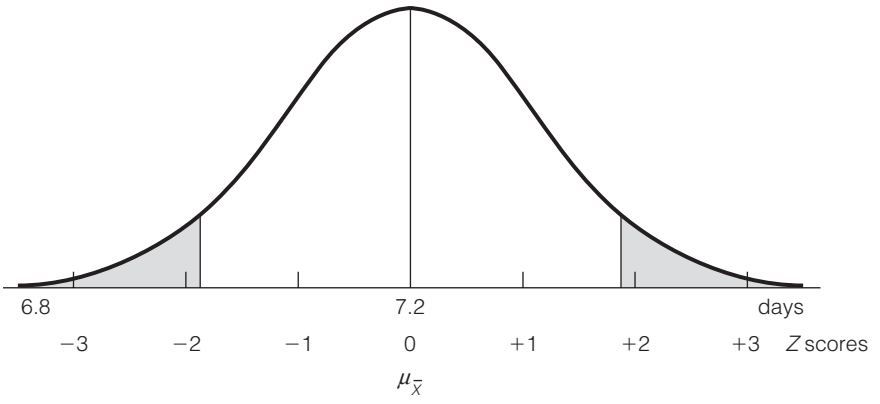


FIGURE 8.3 THE SAMPLING DISTRIBUTION OF ALL POSSIBLE SAMPLE MEANS



explanation B is true) will cause us to reject explanation B. The probability of 0.05 can be translated into an area and divided equally into the upper and lower tails of the sampling distribution. Using Appendix A, we find that the Z-score equivalent of this area is ± 1.96. The areas and Z scores are depicted in Figure 8.3.

The decision rule can now be rephrased. Any sample outcome falling in the shaded areas depicted in Figure 8.3 by definition has a probability of occurrence of less than 0.05. Such an outcome would be a rare event and would cause us to reject explanation B.

All that remains is to translate our sample outcome into a Z score so we can see where it falls on the curve. To do this, we use the standard formula for locating any particular raw score under a normal distribution. When we use known or empirical distributions, this formula is expressed as

$$Z = \frac{X_i - \bar{X}}{s}$$

Or, to find the equivalent Z score for any raw score, subtract the mean of the distribution from the raw score and divide by the standard deviation of the distribution. Since we are now concerned with the sampling distribution of all

sample means rather than an empirical distribution, the symbols in the formula will change, but the form remains exactly the same:

FORMULA 8.1

$$Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{N}}$$

Or, to find the equivalent Z score for any sample mean, subtract the mean of the sampling distribution, which is equal to the population mean or μ, from the sample mean and divide by the standard deviation of the sampling distribution.

Recalling the data given on this problem, we can now find the Z-score equivalent of the sample mean.

$$Z = \frac{6.8 - 7.2}{1.43/\sqrt{127}}$$

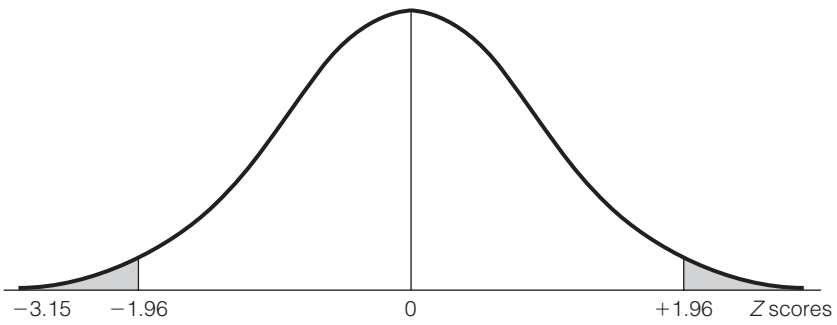
$$Z = \frac{-0.40}{0.127}$$

$$Z = -3.15$$

In Figure 8.4, this Z score of -3.15 is noted on the distribution of all possible sample means, and we see that the sample outcome does fall in the shaded area. If explanation B is true, this particular sample outcome has a probability of occurrence of less than 0.05 (how much less than 0.05 is irrelevant, since the decision rule was established in advance). The sample outcome ($\overline{X} = 6.8$ or $Z = -3.15$) would therefore be rare if explanation B were true, and the researcher may reject explanation B. If explanation B were true, this sample outcome would be extremely unlikely. The sample of 127 treated alcoholics comes from a population that is significantly different from the community on the trait of absenteeism. Or, to put it another way, the sample does not come from a population that has a mean of 7.2 days of absences.

Keep in mind that our decisions in significance testing are based on information gathered from random samples. On rare occasions, a sample may not be representative of the population from which it was selected. The decision-making process outlined above has a very high probability of resulting in correct decisions, but as long as we must work with samples rather than populations, we face an element of risk. That is, the decision to reject explanation B might be incorrect if this sample happens to be one of the few that is unrepresentative of

FIGURE 8.4 THE SAMPLING DISTRIBUTION OF SAMPLE MEANS WITH THE SAMPLE OUTCOME ($\overline{X} = 6.8$) NOTED IN Z SCORES



the population of alcoholics treated in this program. One important strength of hypothesis testing is that the probability of making an incorrect decision can be estimated. In the example at hand, explanation B was rejected, and the probability of this decision being incorrect is 0.05—the decision rule established at the beginning of the process. To say that the probability of rejecting explanation B incorrectly is 0.05 means that if we repeated this same test an infinite number of times, we would incorrectly reject explanation B only 5 times out of every 100.

8.3 THE FIVE-STEP MODEL FOR HYPOTHESIS TESTING

All the formal elements and concepts used in hypothesis testing were introduced in the preceding discussion. This section presents their proper names and introduces a **five-step model** for organizing all hypothesis testing.

Step 1. Making assumptions and meeting test requirements

Step 2. Stating the null hypothesis

Step 3. Selecting the sampling distribution and establishing the critical region

Step 4. Computing the test statistic

Step 5. Making a decision and interpreting the results of the test

We will look at each step individually, using the problem from Section 8.2 as an example throughout.

Step 1. Making Assumptions and Meeting Test Requirements. Any application of statistics requires that certain assumptions be made. In other words, in order for the test to be valid, the elements of the test and the variables involved have to have certain characteristics. Specifically, three assumptions have to be satisfied when conducting a test of an hypothesis with a single sample mean. First, we must be sure that we are working with a random sample, one that has been selected according to the rules of EPSEM (see Chapter 7). Second, to justify computation of a mean, we must assume that the variable being tested is interval-ratio in level of measurement. Finally, we must assume that the sampling distribution of all possible sample means is normal in shape so that we may use the standardized normal distribution to find areas under the sampling distribution. We can be sure that this assumption is satisfied by using large samples (see the central limit theorem in Chapter 7).

Usually, we will state these assumptions in abbreviated form as a mathematical model for the test. For example,

Model: Random sampling
Level of measurement is interval-ratio
Sampling distribution is normal

Step 2. Stating the Null Hypothesis (H_0). The **null hypothesis** is the formal name for explanation B and is always a statement of "no difference." The exact form of the null hypothesis will vary depending on the test being conducted. In the single-sample case, the null hypothesis states that the sample comes from a population with a certain characteristic. In our example, the null is that the population of treated alcoholics are "no different" from the community

as a whole, that their average days of absenteeism are also 7.2, and that the difference between 7.2 and the sample mean of 6.8 is caused by random chance. Symbolically, the null would be stated as

$$H_0: \mu = 7.2$$

where μ refers to the mean of the population of treated alcoholics. The null hypothesis is the central element in any test of hypothesis because the entire process is aimed at rejecting or failing to reject the H_0.

Usually, consistent with explanation A, the researcher believes that there is a significant difference and desires to reject the null hypothesis. At this point in the five-step model, the researcher's belief is stated in a **research hypothesis (H_1),** a statement that directly contradicts the null hypothesis. Thus, the researcher's goal in hypothesis testing is often to gather evidence for the research hypothesis by rejecting the null hypothesis.

The research hypothesis can be stated in several ways. One form would simply assert that the population from which the sample was selected did not have a certain characteristic or, in terms of our example, had a mean that was not equal to a specific value:

$$(H_1: \mu \neq 7.2)$$
where $\neq$ means "not equal to"

Symbolically, this statement asserts that the sample does not come from a population with a mean of 7.2 or that the population of all treated alcoholics is different from the community as a whole. The research hypothesis is enclosed in parentheses to emphasize that it has no formal standing or role in the hypothesis-testing process (except, as we shall see in the next section, in choosing between one-tailed and two-tailed tests). It serves as a reminder of what the researcher believes to be the truth.

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region. The sampling distribution is, as always, the probabilistic yardstick against which a particular sample outcome is measured. By assuming that the null hypothesis is true (and *only* by this assumption), we can attach values to the mean and standard deviation of the sampling distribution and thus measure the probability of any specific sample outcome. There are several different sampling distributions, but for now we will confine our attention to the sampling distribution described by the standard normal curve as summarized in Appendix A.

The **critical region** consists of the areas under the sampling distribution that include unlikely sample outcomes. Prior to the test of the hypothesis, we must define what we mean by *unlikely.* That is, we must specify in advance those sample outcomes so unlikely that they will lead us to reject the H_0. This decision rule will establish the critical region or region of rejection. The word *region* is used because, essentially, we are describing those areas under the sampling distribution that contain unlikely sample outcomes. In the example above, this area corresponded to a Z score of ± 1.96, called ***Z*(critical),** that was graphically displayed in Figure 8.3. The shaded area is the critical region. Any sample outcome for which the Z-score equivalent fell in this area (that is, below -1.96 or above $+1.96$) would have caused us to reject the null hypothesis.

By convention, the size of the critical region is reported as alpha (α), the proportion of all of the area included in the critical region. In the example above, our **alpha level** was 0.05. Other commonly used alphas are 0.10, 0.01, and 0.001.

In abbreviated form, all the decisions made in this step are noted below. The critical region is noted by the Z scores that mark its beginnings.

$$\begin{aligned} \text{Sampling distribution} &= Z \text{ distribution} \\ \alpha &= 0.05 \\ Z(\text{critical}) &= \pm 1.96 \end{aligned}$$

[For practice in finding Z(critical) scores, see Problem 8.1a.]

Step 4. Computing the Test Statistic. To evaluate the probability of any given sample outcome, the sample value must be converted into a Z score. Solving the equation for Z-score equivalents is called computing the **test statistic,** and the resultant value will be referred to as ***Z*(obtained)** in order to differentiate the test statistic from the critical region. In our example above, we found a Z(obtained) of -3.15. *(For practice in computing obtained Z scores for means, see Problems 8.1c, 8.2–8.7, and 8.15e and f.)*

Step 5. Making a Decision and Interpreting the Results of the Test. As the last step in the hypothesis-testing process, the test statistic is compared with the critical region. If the test statistic falls into the critical region, our decision will be to reject the null hypothesis. If the test statistic does not fall into the critical region, we fail to reject the null hypothesis. In our example, the two values were

$$\begin{aligned} Z(\text{critical}) &= \pm 1.96 \\ Z(\text{obtained}) &= -3.15 \end{aligned}$$

and we saw that the Z(obtained) fell in the critical region (see Figure 8.4). Our decision was to reject the null hypothesis, a statement that treated alcoholics have a mean absentee rate of 7.2 days, or in other words, that there is no difference between treated alcoholics and the community. When we reject this statement, we are saying that treated alcoholics do *not* have a mean absentee rate of 7.2 days and that there *is* a difference between them and the community. We can also conclude that the difference between the sample mean of 6.8 and the community mean of 7.2 is statistically significant, or unlikely to be caused by random chance alone. On the trait of absenteeism, treated alcoholics are different from the community as a whole.

Note that in order to complete Step 5, you have to do two things. First you make a decision about the null hypothesis: if the test statistic falls in the critical region, you reject H_0. If the test statistic does not fall in the critical region, you fail to reject H_0.

Second, you need to say what that decision means. In this case, the null hypothesis was rejected, and that means that there is a significant difference between the mean of the sample and the mean of the community; therefore, treated alcoholics are different from the community as a whole.

This five-step model will serve us as a framework for decision-making throughout the hypothesis-testing chapters. The exact nature and method of expression for our decisions will be different for different situations. However, familiarity with the five-step model will assist you in mastering this material by providing a common frame of reference for all significance testing.

ONE STEP AT A TIME

Testing the Significance of the Difference Between a Sample Mean and a Population Mean: Computing *Z*(obtained) and Interpreting Results

Use these procedures if the population standard deviation (σ) is known or sample size (N) is greater than 100. See Section 8.6 for procedures when σ is unknown and N is less than 100.)

Step 4: Computing *Z*(obtained). Use Formula 8.1 to compute the test statistic.

Step	*Operation*
1.	Find the square root of *N*.
2.	Divide the square root of *N* into the population standard deviation (σ).
3.	Subtract the population mean (μ) from the sample mean ($\bar{X}$).
4.	Divide the quantity you found in Step 3 by the quantity you found in Step 2. This value is *Z*(obtained).

Step 5: Making a Decision and Interpreting the Test Result.

5. Compare the *Z*(obtained) you computed in Step 4 to your *Z*(critical). If *Z*(obtained) is *in* the critical region, *reject* the null hypothesis. If *Z*(obtained) is *not in* the critical region, *fail to reject* the null hypothesis.
6. Interpret the decision to reject or fail to reject the null hypothesis in the terms of the original question. For example, our conclusion for the example problem used in Section 8.3 was "Treated alcoholics miss significantly fewer days of work than the community as a whole."

8.4 ONE-TAILED AND TWO-TAILED TESTS OF HYPOTHESIS

The five-step model for hypothesis testing is fairly rigid, and the researcher has little room for making choices. Follow the steps one by one as specified above, and you cannot make a mistake. Nonetheless, the researcher must still make two crucial choices. First, he or she must decide between a one-tailed and a two-tailed test. Second, an alpha level must be selected. In this section, we will discuss the former decision and, in Section 8.5, the latter.

Choosing a One- or Two-Tailed Test. The choice between a one-tailed test and two-tailed test is based on the researcher's expectations about the population from which the sample was selected. These expectations are reflected in the research hypothesis (H_1), which is contradictory to the null hypothesis and usually states what the researcher believes to be "the truth." In most situations, the researcher will wish to support the research hypothesis by rejecting the null hypothesis.

The format for the research hypothesis may take either of two forms, depending on the relationship between what the null hypothesis states and what the researcher believes to be the truth. The null hypothesis states that the population has a specific characteristic. In the example that has served us throughout this chapter, the null stated, in symbols, "All treated alcoholics have the *same* absentee rate (7.2 days) as the community." The researcher might believe that the population of treated alcoholics actually has *less* absenteeism (their population mean is *lower than* the value stated in the null hypothesis), *more* absenteeism (their population mean is *greater than* the value stated in the null hypothesis), or he or she might be unsure about the direction of the difference.

If the researcher is unsure about the direction, the research hypothesis would state only that the population mean is "not equal" to the value stated in the null hypothesis. The research hypothesis stated in Section 8.3 ($\mu \neq 7.2$) was in this format. This is called a **two-tailed test** of significance because it means that the researcher will be equally concerned with the possibility that the true population value is greater than the value specified in the null hypothesis and the possibility that the true population value is less than the value specified in the null hypothesis.

In other situations, the researcher might be concerned only with differences in a specific direction. If the direction of the difference can be predicted, or if the researcher is concerned only with differences in one direction, a **one-tailed test** can be used. A one-tailed test may take one of two forms, depending on the researcher's expectations about the direction of the difference. If the researcher believes that the true population value is greater than the value specified in the null, the research hypothesis would reflect that belief. In our example, if we had predicted that treated alcoholics had *higher* absentee rates than the community (or, averaged *more* than 7.2 days of absenteeism), our research hypothesis would have been

$$(H_1: \mu > 7.2)$$
where $>$ signifies "greater than"

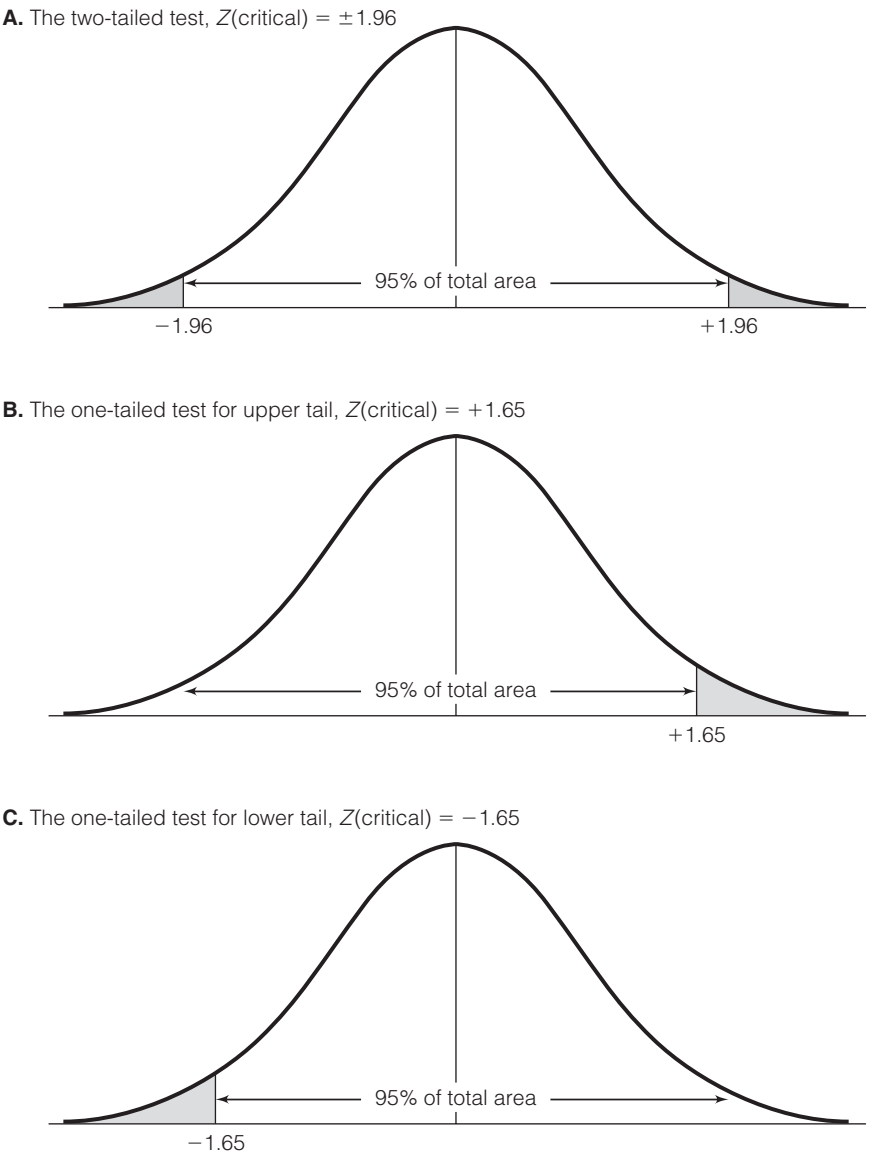
If we predicted that treated alcoholics had lower absentee rates than the community (or, averaged *fewer* than 7.2 days of absenteeism), our research hypothesis would have been

$$(H_1: \mu < 7.2)$$
where $<$ signifies "less than"

One-tailed tests are often appropriate when programs designed to solve a problem or improve a situation are being evaluated. For example, if the program for treating alcoholics made them *less* reliable workers with *higher* absentee rates, the program would be considered a failure, at least on that one criterion. In a situation like this, researchers may focus only on outcomes that would indicate that the program is a success (i.e., when treated alcoholics have lower rates) and conduct a one-tailed test with a research hypothesis in the form $H_1: \mu < 7.2$. Or, consider the evaluation of a program designed to reduce unemployment. The evaluators would be concerned only with outcomes that show a decrease in the unemployment rate. If the rate shows no change, or if unemployment increases, the program is a failure, and both of these outcomes might be considered equally negative by the researchers. Thus, the researchers could legitimately use a one-tailed test that stated that unemployment rates for graduates of the program would be less than ($<$) rates in the community.

Placing the Critical Region in One-Tailed Tests. In terms of the five-step model, the choice of a one-tailed or two-tailed test determines where we place the critical region under the sampling distribution in Step 3. As you recall, in a two-tailed test, we split the critical region equally into the upper and lower tails of the sampling distribution. In a one-tailed test, we place the entire critical area in one tail of the sampling distribution. If we believe that the population characteristic is greater than the value stated in the null hypothesis (if the H_1 includes the $>$ symbol), we place the entire critical region in the upper tail. If we believe

FIGURE 8.5 ESTABLISHING THE CRITICAL REGION, ONE-TAILED TESTS VERSUS TWO-TAILED TESTS (alpha = 0.05)



that the characteristic is less than the value stated in the null hypothesis (if the H_1 includes the $<$ symbol), the entire critical region goes in the lower tail.

For example, in a two-tailed test with alpha equal to 0.05, the critical region begins at $Z(\text{critical}) = \pm 1.96$. In a one-tailed test at the same alpha level, the Z(critical) is $+1.65$ if the upper tail is specified and -1.65 if the lower tail is specified. Table 8.1 summarizes the procedures to follow in terms of the nature of the research hypothesis. The difference in placing the critical region is graphically summarized in Figure 8.5, and the critical Z scores for the most common alpha levels are given in Table 8.2 for both one- and two-tailed tests.

TABLE 8.1 ONE-TAILED VS. TWO-TAILED TESTS, $\alpha = 0.05$

If the Research Hypothesis Uses	The Test Is	And Concern Is with	Z(critical) =
$\neq$	Two-tailed	Both tails	± 1.96
$>$	One-tailed	Upper tail	$+1.65$
$<$	One-tailed	Lower tail	-1.65

TABLE 8.2 FINDING CRITICAL Z SCORES FOR ONE-TAILED TESTS (single sample means)

Alpha	Two-Tailed Value	One-Tailed Values	
		Upper Tail	Lower Tail
0.10	± 1.65	$+1.29$	-1.29
0.05	± 1.96	$+1.65$	-1.65
0.01	± 2.58	$+2.33$	-2.33
0.001	± 3.29	$+3.10$	-3.10

Note that the critical Z values for one-tailed tests at all values of alpha are smaller in value and closer to the mean of the sampling distribution. Thus, a one-tailed test is more likely to reject the H_0 without changing the alpha level (assuming that we have specified the correct tail). One-tailed tests are a way of statistically both having and eating your cake and should be used whenever (1) the direction of the difference can be confidently predicted or (2) the researcher is concerned only with differences in one tail of the sampling distribution.

A Test of Hypothesis Using a One-Tailed Test. After many years of work, a sociologist has noted that sociology majors seem more sophisticated, charming, and cosmopolitan than the rest of the student body. A Sophistication Scale test has been administered to the entire student body and to a random sample of 100 sociology majors, and these results have been obtained

Student Body	Sociology Majors
$\mu = 17.3$	$\bar{X} = 19.2$
$\sigma = 7.4$	$N = 100$

We will use the five-step model to test the H_0 of no difference between sociology majors and the general student body.

Step 1. Making Assumptions and Meeting Test Requirements. Since we are using a mean to summarize the sample outcome, we must assume that the Sophistication Scale generates interval-ratio-level data. With a sample size of 100, the central limit theorem applies, and we can assume that the sampling distribution is normal in shape.

Model: Random sampling
Level of measurement is interval-ratio
Sampling distribution is normal

Step 2. Stating the Null Hypothesis (H_0). The null hypothesis states that there is no difference between sociology majors and the general student body. The research hypothesis (H_1) will also be stated at this point. The researcher has predicted a direction for the difference (sociology majors are *more* sophisticated), so a one-tailed test is justified. The two hypotheses may be stated as

$$H_0: \mu = 17.3$$
$$(H_1: \mu > 17.3)$$

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region. We will use the standardized normal distribution (Appendix A) to find areas under the sampling distribution. If alpha is set at 0.05, the critical region will begin at the Z score +1.65. That is, the researcher has predicted that sociology majors are *more* sophisticated and that this sample comes from a population that has a mean *greater than* 17.3, so he will be concerned only with sample outcomes in the upper tail of the sampling distribution. If sociology majors are *the same as* other students in terms of sophistication (if the H_0 is true), or if they are *less* sophisticated (and come from a population with a mean less than 17.3), the theory is disproved. These decisions may be summarized as

$$\text{Sampling distribution} = Z \text{ distribution}$$
$$\alpha = 0.05$$
$$Z(\text{critical}) = +1.65$$

Step 4. Computing the Test Statistic.

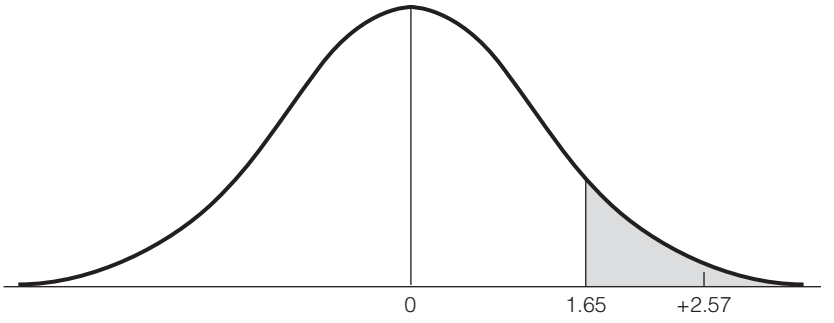
$$Z(\text{obtained}) = \frac{\bar{X} - \mu}{\sigma/\sqrt{N}}$$
$$Z(\text{obtained}) = \frac{19.2 - 17.3}{7.4/\sqrt{100}}$$
$$Z(\text{obtained}) = +2.57$$

Step 5. Making a Decision and Interpreting Test Results. Comparing the Z(obtained) with the Z(critical):

$$Z(\text{critical}) = +1.65$$
$$Z(\text{obtained}) = +2.57$$

We see that the test statistic falls into the critical region. This outcome is depicted graphically in Figure 8.6. We will reject the null hypothesis because, if

FIGURE 8.6 Z(OBTAINED) VERSUS Z(CRITICAL) (alpha = 0.05, one-tailed test)



the H_0 were true, a difference of this size would be very unlikely. There is a significant difference between sociology majors and the general student body in terms of sophistication. Since the null hypothesis has been rejected, the research hypothesis (sociology majors are more sophisticated) is supported. *(For practice in dealing with tests of significance for means that may call for one-tailed tests, see Problems 8.2, 8.3, 8.6, 8.8, and 8.17).*

8.5 SELECTING AN ALPHA LEVEL

In addition to deciding between one-tailed and two-tailed tests, the researcher must also select an alpha level. We have seen that the alpha level plays a crucial role in hypothesis testing. When we assign a value to alpha, we define what we mean by an "unlikely" sample outcome. If the probability of the observed sample outcome is lower than the alpha level (if the test statistic falls into the critical region), we reject the null hypothesis as untrue. Thus, the alpha level will have important consequences for our decision in Step 5.

How can reasonable decisions be made with respect to the value of alpha? Recall that in addition to defining what will be meant by *unlikely,* the alpha level is also the probability that the decision to reject the null hypothesis, if the test statistic falls into the critical region, will be incorrect. In hypothesis testing, the error of incorrectly rejecting the null hypothesis or rejecting a null hypothesis that is actually true is called **Type I error,** or **alpha error.** To minimize this type of error, use very small values for alpha.

To elaborate, when an alpha level is specified, the sampling distribution is divided into two sets of possible sample outcomes. The critical region includes all unlikely or rare sample outcomes. Outcomes in this region will cause us to reject the null hypothesis. The remainder of the area consists of all sample outcomes that are "non-rare." The lower the level of alpha, the smaller the critical region and the greater the distance between the mean of the sampling distribution and the beginnings of the critical region. Compare, for the sake of illustration, the following alpha levels and values for Z (critical) for two-tailed tests. As you may recall, this information was also presented in Table 7.2.

If Alpha Equals	The Two-Tailed Critical Region Will Begin at Z(critical) Equal to
0.10	±1.65
0.05	±1.96
0.01	±2.58
0.001	±3.29

As alpha goes down, the critical region becomes smaller and moves farther away from the mean of the sampling distribution. The lower the alpha level, the harder it will be to reject the null hypothesis and, since a Type I error can be made only if our decision in Step 5 is to reject the null hypothesis, the lower the probability of Type I error. To minimize the probability of rejecting a null hypothesis that is in fact true, use very low alpha levels.

However, there is a complication. As the critical region decreases in size (as alpha levels decrease), the noncritical region—the area between the two Z(critical) scores in a two-tailed test—must become larger. All other things being equal, the lower the alpha level, the less likely that the sample outcome will fall into the critical region. This raises the possibility of a second type of incorrect

TABLE 8.3 DECISION MAKING AND THE NULL HYPOTHESIS

	Decision	
The H_0 Is Actually:	Reject	Fail to Reject
True	Type I or α error	OK
False	OK	Type II or β error

decision, called **Type II error,** or **beta error:** failing to reject a null that is, in fact, false. The probability of Type I error decreases as alpha level decreases, but the probability of Type II error increases. Thus, the two types of error are inversely related, and it is not possible to minimize both in the same test. As the probability of one type of error decreases, the other increases, and vice versa.

It may be helpful to clarify the relationships between decision making and errors in a table format. Table 8.3 lists the two decisions we can make in Step 5 of the five-step model: we either reject or fail to reject the null hypothesis. The other dimension of Table 8.3 lists the two possible conditions of the null hypothesis: it is either actually true or actually false. The table combines these possibilities into a total of four possible combinations, two of which are desirable (OK) and two of which indicate that an error has been made.

The two desirable outcomes are rejecting null hypotheses that are actually false and failing to reject null hypotheses that are actually true. The goal of any scientific investigation is to verify true statements and reject false statements. The remaining two combinations are errors or situations that, naturally, we wish to avoid. If we reject a null hypothesis that is in fact true, we are saying that a true statement is false. Likewise, if we fail to reject a null hypothesis that is in fact false, we are saying that a false statement is true. Obviously, we would prefer to always wind up in one of the areas labeled "OK" in Table 8.3—to always reject false statements and accept the truth when we find it. Remember, however, that hypothesis testing always carries an element of risk and that it is not possible to minimize the chances of both Type I and Type II error simultaneously.

What all of this means, finally, is that you must think of selecting an alpha level as an attempt to balance the two types of error. Higher alpha levels will minimize the probability of Type II error (saying that false statements are true), and lower alpha levels will minimize the probability of Type I error (saying that true statements are false). Normally, in social science research we want to minimize Type I error, and thus lower alpha levels (0.05, 0.01, 0.001 or lower) will be used. The 0.05 level in particular has emerged as a generally recognized indicator of a significant result. However, the widespread use of the 0.05 level is simply a convention, and there is no reason that alpha cannot be set at virtually any sensible level (such as 0.04, 0.027, 0.083). The researcher has the responsibility of selecting the alpha level that seems most reasonable in terms of the goals of the research project.

8.6 THE STUDENT'S *t* DISTRIBUTION

To this point, we have considered only one type of hypothesis test. Specifically, we have focused on situations involving single sample means where the value of the population standard deviation (σ) was known. Obviously, in most research situations the value of σ will not be known. However, a value for σ is required in order to compute the standard error of the mean (σ/N), convert

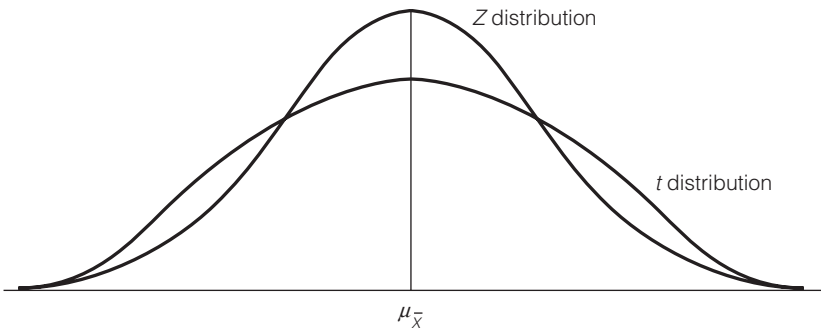
our sample outcome into a Z score, and place the Z(obtained) on the sampling distribution (Step 4). How can a value for the population standard deviation reasonably be obtained?

It might seem sensible to estimate σ with s, the sample standard deviation. As we noted in Chapter 5, s is a biased estimator of σ, but the degree of bias decreases as sample size increases. For large samples (that is, samples with 100 or more cases), the sample standard deviation yields an adequate estimate of σ. Thus, for large samples, we simply substitute s for σ in the formula for Z(obtained) in Step 4 and continue to use the standard normal curve to find areas under the sampling distribution.[1]

For smaller samples, however, when σ is unknown, an alternative distribution called the **Student's *t* distribution** must be used to find areas under the sampling distribution and establish the critical region. The shape of the t distribution varies as a function of sample size. The relative shapes of the t and Z distributions are depicted in Figure 8.7. For small samples, the t distribution is much flatter than the Z distribution, but, as sample size increases, the t distribution comes to resemble the Z distribution more and more until the two are essentially identical when sample size is greater than 120. As N increases, the sample standard deviation (s) becomes a more and more adequate estimator of the population standard deviation (σ), and the t distribution becomes more and more like the Z distribution.

The Distribution of Student's *t*: Using Appendix B. The t distribution is summarized in Appendix B. The t table differs from the Z table in several ways. First, there is a column at the left of the table labeled *df* for "degrees of freedom."[2] As mentioned above, the exact shape of the t distribution and thus the exact location of the critical region for any alpha level varies as a function of sample size.

FIGURE 8.7 THE *t* DISTRIBUTION AND THE *Z* DISTRIBUTION



[1]Even though its effect will be minor and will decrease with sample size, we will always correct for the bias in s by using the term $N - 1$ rather than N in the computation for the standard deviation of the sampling distribution when σ is unknown.

[2]Degrees of freedom refer to the number of values in a distribution that are free to vary. For a sample mean, a distribution has $N - 1$ degrees of freedom. This means that for a specific value of a mean, $N - 1$ scores are free to vary. For example, if the mean is 3 and $N = 5$, the distribution of five scores would have $N - 1$, or four degrees of freedom. When the values of four of the scores are known, the value of the fifth is fixed. If four scores are 1, 2, 3, and 4, the fifth must be 5 and no other value.

Degrees of freedom, which are equal to $N - 1$ in the case of a single-sample mean, must first be computed before the critical region for any alpha can be located. Second, alpha levels are arrayed across the top of Appendix B in two rows, one row for the one-tailed tests and one for two-tailed tests. To use the table, begin by locating the selected alpha level in the appropriate row.

The third difference is that the entries in the table are the actual scores, called ***t*(critical),** that mark the beginnings of the critical regions and not areas under the sampling distribution. To illustrate how to use this table with single-sample means, find the critical region for alpha equal to 0.05, two-tailed test, for $N = 30$. The degrees of freedom will be $N - 1$, or 29; reading down the proper column, you should find a value of 2.045. Thus, the critical region for this test will begin at $t(\text{critical}) = \pm 2.045$.

Take a moment to notice some additional features of the t distribution. First, note that the t(critical) we found above is larger in value than the comparable Z(critical), which for a two-tailed test at an alpha of 0.05 would be ± 1.96. This relationship reflects the fact that the t distribution is flatter than the Z distribution (see Figure 8.6). When you use the t distribution, the critical regions will begin farther away from the mean of the sampling distribution and, therefore, the null hypothesis will be harder to reject. Furthermore, the smaller the sample size (the lower the degrees of freedom), the larger the value of t(obtained) necessary for a rejection of the H_0.

Second, scan the column for an alpha of 0.05, two-tailed test. Note that, for one degree of freedom, the t(critical) is ± 12.706 and that the value of t(critical) decreases as degrees of freedom increase. For degrees of freedom greater than 120, the value of t(critical) is the same as the comparable value of Z(critical), or ± 1.96. As sample size increases, the t distribution comes to resemble the Z distribution more and more until, with sample sizes greater than 120, the two distributions are essentially identical.[3]

A Test of Hypothesis Using Student's *t*. To demonstrate the uses of the t distribution in more detail, we will work through an example problem. Note that, in terms of the five-step model, the changes required by using t scores occur mostly in Steps 3 and 4. In Step 3, the sampling distribution will be the t distribution, and degrees of freedom (df) must be computed before locating the critical region as marked by t(critical). In Step 4, a slightly different formula for computing the test statistic, ***t*(obtained),** will be used. As compared with the formula for Z(obtained), s will replace σ and $N - 1$ will replace N.

Specifically,

FORMULA 8.2

$$t(\text{obtained}) = \frac{\bar{X} - \mu}{s/\sqrt{N-1}}$$

A researcher wonders if commuter students are different from the general student body in terms of academic achievement. She has gathered a

[3]Appendix B abbreviates the t distribution by presenting a limited number of critical t scores for degrees of freedom between 31 and 120. If the degrees of freedom for a specific problem equal 77 and alpha equals 0.05, two-tailed, we have a choice between a t(critical) of ± 2.000 ($df = 60$) and a t(critical) of ± 1.980 ($df = 120$). In situations such as these, take the larger table value as t(critical). This will make rejection of the H_0 less likely and is therefore the more conservative course of action.

random sample of 30 commuter students and has learned from the registrar that the mean grade-point average for all students is 2.50 ($\mu = 2.50$), but the standard deviation of the population (σ) has never been computed. Sample data are reported below. Is the sample from a population that has a mean of 2.50?

Student Body	Commuter Students
$\mu = 2.50\ (= \mu_{\bar{X}})$	$\bar{X} = 2.78$
$\sigma = ?$	$s = 1.23$
	$N = 30$

Step 1. Making Assumptions and Meeting Test Requirements.

Model: Random sampling
Level of measurement is interval-ratio
Sampling distribution is normal

Step 2. Stating the Null Hypothesis.

$$H_0: \mu = 2.50$$
$$(H_1: \mu \neq 2.50)$$

You can see from the research hypothesis that the researcher has not predicted a direction for the difference. This will be a two-tailed test.

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region. Since σ is unknown and sample size is small, the t distribution will be used to find the critical region. Alpha will be set at 0.01.

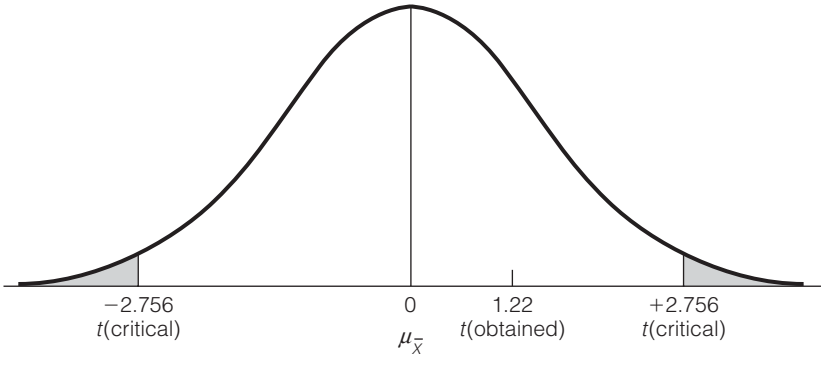
$$\text{Sampling distribution} = t \text{ distribution}$$
$$\alpha = 0.01, \text{ two-tailed test}$$
$$df = (N - 1) = 29$$
$$t(\text{critical}) = \pm 2.756$$

Step 4. Computing the Test Statistic.

$$t(\text{obtained}) = \frac{\bar{X} - \mu}{s/\sqrt{N}}$$
$$t(\text{obtained}) = \frac{2.78 - 2.50}{1.23/\sqrt{29}}$$
$$t(\text{obtained}) = \frac{0.28}{0.23}$$
$$t(\text{obtained}) = +1.22$$

Step 5. Making a Decision and Interpreting Test Results. The test statistic does not fall into the critical region. Therefore, the researcher fails to reject the H_0. The difference between the sample mean (2.78) and the population mean (2.50) is no greater than what would be expected if only random chance were operating. The test statistic and critical regions are displayed in Figure 8.8.

FIGURE 8.8 SAMPLING DISTRIBUTION SHOWING *t*(OBTAINED) VERSUS *t*(CRITICAL) ($\alpha = 0.05$, two-tailed test, $df = 29$)



To summarize, when testing single-sample means, we must make a choice regarding the theoretical distribution we will use to establish the critical region. The choice is straightforward. If the population standard deviation (σ) is known or sample size is large, the Z distribution (summarized in Appendix A) will be used. If σ is unknown and the sample is small, the t distribution (summarized in Appendix B) will be used. *(For practice in using the* t *distribution in a test of hypothesis, see Problems 8.8–8.10 and 8.17.)*

ONE STEP AT A TIME

Testing the Significance of the Difference Between a Sample Mean and a Population Mean Using the Student's *t* distribution: Computing *t*(obtained) and Interpreting Results

Use these procedures if the population standard deviation (σ) is unknown and sample size (N) is less than 100. See Section 8.3 for procedures when σ is known or N is more than 100.

Step 4: Computing *t*(obtained). Use Formula 8.2 to compute the test statistic.

Step	*Operation*
1.	Take the square root of $N - 1$.
2.	Divide the quantity you found in Step 1 into the sample standard deviation (s).
3.	Subtract the population mean (μ) from the sample mean ($\overline{X}$).
4.	Divide the quantity you found in Step 3 by the quantity you found in Step 2.

Step 5: Making a Decision and Interpreting the Test Result.

5. Compare the *t*(obtained) you computed in Step 4 to your *t*(critical). If *t*(obtained) is *in* the critical region, *reject* the null hypothesis. If *t*(obtained) is *not in* the critical region, *fail to reject* the null hypothesis.
6. Interpret the decision to reject or fail to reject the null hypothesis in the terms of the original question. For example, our conclusion for the example problem used in Section 8.6 was "There is no significant difference between the average GPA of commuter students and the general student body."

Application 8.1

For a random sample of 152 felony cases tried in a local court, the average prison sentence was 27.3 months. Is this significantly different from the average prison term for felons nationally? We will use the five-step model to organize the decision-making process.

Step 1. Making Assumptions and Meeting Test Requirements.

Model: Random Sampling
Level of measurement is interval-ratio
Sampling distribution is normal

From the information given (this is a large sample with $N > 100$ and length of sentence is an interval-ratio variable), we can conclude that the model assumptions are satisfied.

Step 2. Stating the Null Hypothesis (H_0). The null hypothesis would say that the average sentence locally (for *all* felony cases) is equal to the national average. In symbols:

$$H_0: \mu = 28.7$$

The question does not specify a direction: it only asks if the local sentences are "different from" (nor higher or lower than) national averages. This seems to suggest a two-tailed test.

$$(H_1: \mu \neq 28.7)$$

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region.

$$\text{Sampling distribution} = Z \text{ distribution}$$
$$\alpha = 0.05$$
$$Z(\text{critical}) = \pm 1.96$$

Step 4. Computing the Test Statistic. The necessary information for conducting a test of the null hypothesis is

$$\bar{X} = 27.3 \qquad \mu = 28.7$$
$$s = 3.7$$
$$N = 152$$

The test statistic, Z(obtained), would be

$$Z(\text{obtained}) = \frac{\bar{X} - \mu}{s/\sqrt{N-1}}$$

$$Z(\text{obtained}) = \frac{27.3 - 28.7}{3.7/\sqrt{152-1}}$$

$$Z(\text{obtained}) = \frac{-1.40}{3.7/\sqrt{151}}$$

$$Z(\text{obtained}) = \frac{-1.40}{0.30}$$

$$Z(\text{obtained}) = -4.67$$

Step 5. Making a Decision and Interpreting Test Results. With alpha set at 0.05, the critical region would begin at Z(critical) $= \pm 1.96$. With an obtained Z score of -4.67, the null would be rejected. This means that the difference between the prison sentences of felons convicted in the local court and felons convicted nationally is statistically significant. The difference is so large that we may conclude that it did not occur by random chance. The decision to reject the null hypothesis has a 0.05 probability of being wrong.

8.7 TESTS OF HYPOTHESES FOR SINGLE-SAMPLE PROPORTIONS (LARGE SAMPLES)

In many cases, the variables in which we are interested will not be measured in a way that justifies the assumption of interval-ratio level of measurement. One alternative in this situation would be to use a sample proportion (P_s) rather than a sample mean as the test statistic. As we shall see below, the overall procedures for testing single-sample proportions are the same as those for testing means. The central question is still, Does the population from which the sample was drawn have a certain characteristic? We still conduct the test based on the assumption that the null hypothesis is true, and we still evaluate the probability of the obtained sample outcome against a sampling distribution of all possible sample outcomes. Our decision at the end of the test is also the same. If the obtained test statistic falls into

the critical region (is unlikely, given the assumption that the H_0 is true), we reject the H_0.

Having stressed the continuity in procedures and logic, I must hastily point out the important differences as well. These differences are best related in terms of the five-step model for hypothesis testing. In Step 1, when working with sample proportions, we assume that the variable is measured at the nominal level of measurement. In Step 2, the symbols used to state the null hypothesis are different, even though the null is still a statement of "no difference."

In Step 3, we will use only the standardized normal curve (the Z distribution) to find areas under the sampling distribution and locate the critical region. This will be appropriate as long as sample size is large. We will not consider small-sample tests of hypothesis for proportions in this text.

In Step 4, computing the test statistic, the form of the formula remains the same. That is, the test statistic, Z(obtained), equals the sample statistic minus the mean of the sampling distribution, divided by the standard deviation of the sampling distribution. However, the symbols will change because we are basing the tests on sample proportions. The formula can be stated as

FORMULA 8.3

$$Z(\text{obtained}) = \frac{P_s - P_u}{\sqrt{P_u(1 - P_u)/N}}$$

Step 5 is exactly the same as before. If the test statistic, Z(obtained), falls into the critical region, as marked by Z(critical), reject the H_0.

A Test of Hypothesis Using Sample Proportions. An example should clarify these procedures. A random sample of 122 households in a low-income neighborhood revealed that 53 (or a proportion of 0.43) of the households were headed by females. In the city as a whole, the proportion of female-headed households is 0.39. Are households in the lower-income neighborhood significantly different from the city as a whole in terms of this characteristic?

Step 1. Making Assumptions and Meeting Test Requirements.

Model: Random sampling
Level of measurement is nominal
Sampling distribution is normal in shape

Step 2. Stating the Null Hypothesis. The research question, as stated above, asks only if the sample proportion is different from the population proportion. Since no direction is predicted for the difference, a two-tailed test will be used.

$$H_0: P_u = 0.39$$
$$(H_1: P_u \neq 0.39)$$

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region.

$$\begin{aligned} \text{Sampling distribution} &= Z \text{ distribution} \\ \alpha &= 0.10, \text{ two-tailed test} \\ Z(\text{critical}) &= \pm 1.65 \end{aligned}$$

Step 4. Computing the Test Statistic.

$$Z(\text{obtained}) = \frac{P_s - P_u}{\sqrt{P_u(1 - P_u)/N}}$$

$$Z(\text{obtained}) = \frac{0.43 - 0.39}{\sqrt{0.39(0.61)/122}}$$

$$Z(\text{obtained}) = +0.91$$

Step 5. Making a Decision and Interpreting Test Results. The test statistic, Z(obtained), does not fall into the critical region. Therefore, we fail to reject the H_0. There is no statistically significant difference between the low-income community and the city as a whole in terms of the proportion of households headed by females. Figure 8.9 displays the sampling distribution, the critical region, and the Z(obtained). *(For practice in tests of significance using sample proportions, see Problems 8.1c, 8.11–8.14, 8.15a–d, and 8.16.)*

ONE STEP AT A TIME

Testing the Significance of the Difference Between a Sample Proportion and a Population Proportion: Computing *Z*(obtained) and Interpreting Results

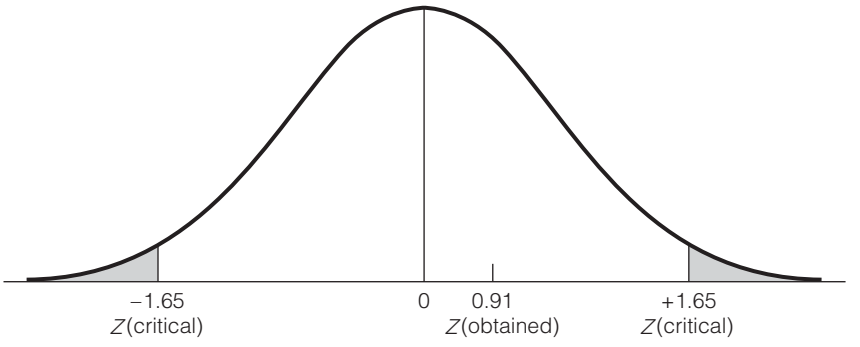
Step 4: Computing *Z*(obtained). Use Formula 8.3 to compute the test statistic.

Step	*Operation*
1.	Start with the denominator of Formula 8.3 and substitute in the value of P_u. This value will be given in the statement of the problem.
2.	Find $(1 - P_u)$ by subtracting P_u from 1.
3.	Multiply the value you found in Step 2 by the value you found in Step 1.
4.	Divide the quantity you found in Step 3 by N.
5.	Take the square root of the quantity you found in Step 4.
6.	Subtract the value of P_u from P_s.
7.	Divide the quantity you found in Step 6 by the quantity you found in Step 5.

Step 5: Making a Decision and Interpreting the Test Result.

5. Compare the Z(obtained) you computed in Step 7 to your Z(critical). If Z(obtained) is *in* the critical region, *reject* the null hypothesis. If Z(obtained) is *not in* the critical region, *fail to reject* the null hypothesis.
6. Interpret the decision to reject or fail to reject the null hypothesis in the terms of the original question. For example, our conclusion for the example problem used in Section 8.7 was "There is no significant difference between the low income community and the city as a whole in the proportion of households that are headed by females."

FIGURE 8.9 SAMPLING DISTRIBUTION SHOWING *Z*(OBTAINED) VERSUS *Z*(CRITICAL) ($\alpha = 0.10$, two-tailed test)



Application 8.2

In a random sample drawn from the most affluent neighborhood in a community, 76% of the respondents reported that they had voted Republican in the most recent presidential election. For the community as a whole, 66% of the electorate voted Republican. Was the affluent neighborhood significantly more likely to have voted Republican?

Step 1. Making Assumptions and Meeting Test Requirements.

Model: Random sampling
Level of measurement is nominal
Sampling distribution is normal

This is a large sample, so we may assume a normal sampling distribution. The variable, percent Republican, is only nominal in level of measurement.

Step 2. Stating the Null Hypothesis (H_0). The null hypothesis says that the affluent neighborhood is not different from the community as a whole.

$$H_0: P_u = 0.66$$

The original question ("was the affluent neighborhood *more* likely to vote Republican") suggests a one-tailed research hypothesis:

$$(H_1: P_u > 0.66)$$

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region.

$$\text{Sampling distribution} = Z \text{ distribution}$$
$$\alpha = 0.05$$
$$Z(\text{critical}) = +1.65$$

The research hypothesis says that we will be concerned only with outcomes in which the neighborhood is more likely to vote Republican or with sample outcomes in the upper tail of the sampling distribution.

Step 4. Computing the Test Statistic. The information necessary for a test of the null hypothesis, expressed in the form of proportions, is as follows.

Neighborhood	Community
$P_s = 0.76$	$P_u = 0.66$
$N = 103$	

The test statistic, *Z*(obtained), would be

$$Z(\text{obtained}) = \frac{P_s - P_u}{\sqrt{P_u(1 - P_u)/N}}$$

$$Z(\text{obtained}) = \frac{0.76 - 0.66}{\sqrt{(0.66)(1 - 0.66)/103}}$$

$$Z(\text{obtained}) = \frac{0.10}{\sqrt{(0.2244)/103}}$$

$$Z(\text{obtained}) = \frac{0.100}{0.047}$$

$$Z(\text{obtained}) = 2.13$$

Step 5. Making a Decision and Interpreting Test Results. With alpha set at 0.05, one-tailed, the critical region would begin at *Z*(critical) = +1.65. With an obtained *Z* score of 2.13, the null hypothesis is rejected. The difference between the affluent neighborhood and the community as a whole is statistically significant and in the predicted direction. Residents of the affluent neighborhood were significantly more likely to have voted Republican in the last presidential election.

SUMMARY

1. All the basic concepts and techniques for testing hypotheses were presented in this chapter. We saw how to test the null hypothesis of "no difference" for single sample means and proportions. In both cases, the central question is whether the population represented by the sample has a certain characteristic.
2. All tests of a hypothesis involve finding the probability of the observed sample outcome, given that the null hypothesis is true. If the outcomes have low probabilities, we reject the null hypothesis. In the usual research situation, we will wish to reject the null hypothesis and thereby support the research hypothesis.
3. The five-step model will be our framework for decision making throughout the hypothesis-testing chapters. We will always (1) make assumptions; (2) state the null hypothesis; (3) select a sampling distribution, specify alpha, and find the critical region; (4) compute a test statistic; and (5) make a decision. What we do during each step, however, will vary, depending on the specific test being conducted.
4. If we can predict a direction for the difference in stating the research hypothesis, a one-tailed test is called for. If no direction can be predicted, a two-tailed test is appropriate. There are two kinds of errors in hypothesis testing. Type I, or alpha, error is rejecting a true null; Type II, or beta, error is failing to reject a false null. The probabilities of committing these two types of error are inversely related and cannot be simultaneously minimized in the same test. By selecting an alpha level, we try to balance the probability of these two kinds of error.
5. When testing sample means, the *t* distribution must be used to find the critical region when the population standard deviation is unknown and sample size is small.
6. Sample proportions can also be tested for significance. Tests are conducted using the five-step model. Compared to the test for the sample mean, the major differences lie in the level-of-measurement assumption (Step 1), the statement of the null (Step 2), and the computation of the test statistic (Step 4).
7. If you are still confused about the uses of inferential statistics described in this chapter, don't be alarmed or discouraged. A sizable volume of rather complex material has been presented and only rarely will a beginning student fully comprehend the unique logic of hypothesis testing on the first exposure. After all, it is not every day that you learn how to test a statement you don't believe (the null hypothesis) against a distribution that doesn't exist (the sampling distribution)!

SUMMARY OF FORMULAS

FORMULA 8.1 Single-sample means, large samples: $Z(\text{obtained}) = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{N}}$

FORMULA 8.2 Single-sample means when samples are small and population standard deviation is unknown: $t(\text{obtained}) = \dfrac{\bar{X} - \mu}{s/\sqrt{N-1}}$.

FORMULA 8.3 Single-sample proportions, large samples: $Z(\text{obtained}) = \dfrac{P_s - P_u}{\sqrt{P_u(1 - P_u)/N}}$

GLOSSARY

Alpha level (α). The proportion of area under the sampling distribution that contains unlikely sample outcomes, given that the null hypothesis is true. Also, the probability of Type I error.

Critical region (region of rejection). The area under the sampling distribution that, in advance of the test itself, is defined as including unlikely sample outcomes, given that the null hypothesis is true.

Five-step model. A step-by-step guideline for conducting tests of hypotheses. A framework that organizes decisions and computations for all tests of significance.

Hypothesis testing. Statistical tests that estimate the probability of sample outcomes if assumptions about the population (the null hypothesis) are true.

Null hypothesis (H_0). A statement of "no difference." In the context of single-sample tests of significance, the population from which the sample was drawn is assumed to have a certain characteristic or value.

One-tailed test. A type of hypothesis test used when (1) the direction of the difference can be predicted or (2) concern focuses on outcomes in only one tail of the sampling distribution.

Research hypothesis (H_1). A statement that contradicts the null hypothesis. In the context of single-sample tests of significance, the research hypothesis says that the population from which the sample was drawn does not have a certain characteristic or value.

Significance testing. See Hypothesis testing.

Student's *t* distribution. A distribution used to find the critical region for tests of sample means when σ is unknown and sample size is small.

***t*(critical).** The *t* score that marks the beginning of the critical region of a *t* distribution.

***t*(obtained).** The test statistic computed in Step 4 of the five-step model. The sample outcome expressed as a *t* score.

Test statistic. The value computed in Step 4 of the five-step model that converts the sample outcome into either a *t* score or a *Z* score.

Two-tailed test. A type of hypothesis test used when (1) the direction of the difference cannot be predicted or (2) concern focuses on outcomes in both tails of the sampling distribution.

Type I error (alpha error). The probability of rejecting a null hypothesis that is, in fact, true.

Type II error (beta error). The probability of failing to reject a null hypothesis that is, in fact, false.

***Z*(critical).** The *Z* score that marks the beginnings of the critical region on a *Z* distribution.

***Z*(obtained).** The test statistic computed in Step 4 of the five-step model. The sample outcomes expressed as a *Z* score.

PROBLEMS

(Problems are labeled with the social science discipline from which they are drawn: SOC for sociology, SW for social work, PS for political science, CJ for criminal justice, PA for public administration, and GER for gerontology.)

8.1 a. For each situation, find *Z*(critical).

Alpha	Form	*Z*(Critical)
0.05	One-tailed	
0.10	Two-tailed	
0.06	Two-tailed	
0.01	One-tailed	
0.02	Two-tailed	

b. For each situation, find the critical *t* score.

Alpha	Form	*N*	*t*(Critical)
0.10	Two-tailed	31	
0.02	Two-tailed	24	
0.01	Two-tailed	121	
0.01	One-tailed	31	
0.05	One-tailed	61	

c. Compute the appropriate test statistic (*Z* or *t*) for each situation:

1. $\mu = 2.40$, $\sigma = 0.75$, $\overline{X} = 2.20$, $N = 200$

2. $\mu = 17.1$, $\overline{X} = 16.8$, $s = 0.9$, $N = 45$

3. $\mu = 10.2$, $\overline{X} = 9.4$, $s = 1.7$, $N = 150$

4. $P_u = .57$, $P_s = 0.60$, $N = 117$

5. $P_u = 0.32$, $P_s = 0.30$, $N = 322$

8.2 SOC **a.** The student body at St. Algebra College attends an average of 3.3 parties per month. A random sample of 117 sociology majors averages 3.8 parties per month with a standard deviation of 0.53. Are sociology majors significantly different from the student body as a whole? [*HINT: The wording of the research question suggests a two-tailed test. This means that the alternative or research hypothesis in Step 2 will be stated as* H_1: $\mu \neq 3.3$ *and that the critical region will be split between the upper and lower tails of the sampling distribution. [See Table 7.2 for values of* Z*(critical) for various alpha levels.*]

b. What if the research question were changed to "Do sociology majors attend a significantly *greater* number of parties"? How would the test conducted in 8.2a change? [*HINT: This wording implies a one-tailed test of significance. How would the research hypothesis change? For the alpha you used in Problem 8.2a, what would the value of Z(critical) be?*]

8.3 SW **a.** Nationally, social workers average 10.2 years of experience. In a random sample, 203 social workers in greater metropolitan Shinbone average only 8.7 years with a standard deviation of 0.52. Are social workers in Shinbone significantly less experienced? (Note the wording of the research hypotheses. These situations may justify one-tailed tests of significance. If you chose a one-tailed test, what form would the research hypothesis take, and where would the critical region begin?)

b. The same sample of social workers reports an average annual salary of $25,782 with a standard deviation of $622. Is this figure significantly higher than the national average of $24,509? (The wording of the research hypotheses suggests a one-tailed test. What form would the research hypothesis take, and where would the critical region begin?)

8.4 SOC Nationally, the average score on the college entrance exams (verbal test) is 453 with a standard deviation of 95. A random sample of 152 freshmen entering St. Algebra College shows a mean score of 502. Is there a significant difference?

8.5 SOC A random sample of 423 Chinese Americans has finished an average of 12.7 years of formal education with a standard deviation of 1.7. Is this significantly different from the national average of 12.2 years?

8.6 SOC A sample of 105 workers in the Overkill Division of the Machismo Toy Factory earns an average of $24,375 per year. The average salary for all workers is $24,230 with a standard deviation of $523. Are workers in the Overkill Division overpaid? Conduct both one- and two-tailed tests.

8.7 GER **a.** Nationally, the population as a whole watches 6.2 hours of TV per day. A random sample of 1,017 senior citizens report watching an average of 5.9 hours per day with a standard deviation of 0.7. Is the difference significant?

b. The same sample of senior citizens reports that they belong to an average of 2.1 volunteer organizations and clubs with a standard deviation of 0.5. Nationally, the average is 1.7. Is the difference significant?

8.8 SOC A school system has assigned several hundred "chronic and severe underachievers" to an alternative educational experience. To assess the program, a random sample of 35 has been selected for comparison with all students in the system.

a. In terms of GPA, did the program work?

Systemwide GPA	Program GPA
$\mu = 2.47$	$\bar{X} = 2.55$ $s = 0.70$ $N = 35$

b. In terms of absenteeism (number of days missed per year), what can be said about the success of the program?

Systemwide	Program
$\mu = 6.137$	$\bar{X} = 4.78$ $s = 1.11$ $N = 35$

c. In terms of standardized test scores in math and reading, was the program a success?

Math Test—Systemwide	Math Test—Program
$\mu = 103$	$\bar{X} = 106$ $s = 2.0$ $N = 35$

Reading Test—Systemwide	Reading Test—Program
$\mu = 110$	$\bar{X} = 113$ $s = 2.0$ $N = 35$

(*HINT: Note the wording of the research questions. Is a one-tailed test justified? Is the program a success if the students in the program are no different from students systemwide? What if the program students were performing at lower levels? If a one-tailed test is used, what form should the research hypothesis take? Where will the critical region begin?*)

8.9 SOC A random sample of 26 local sociology graduates scored an average of 458 on the Graduate

Record Examination (GRE) advanced sociology test with a standard deviation of 20. Is this significantly different from the national average ($\mu = 440$)?

8.10 [PA] Nationally, the per capita property tax is $130. A random sample of 36 southeastern cities average $98 with a standard deviation of $5. Is the difference significant? Summarize your conclusions in a sentence or two.

8.11 [GER/CJ] A survey shows that 10% of the population is victimized by property crime each year. A random sample of 527 older citizens (65 years or more of age) shows a victimization rate of 14%. Are older people more likely to be victimized? Conduct both one- and two-tailed tests of significance.

8.12 [CJ] A random sample of 113 convicted rapists in a state prison system completed a program designed to change their attitudes toward women, sex, and violence before being released on parole. Fifty-eight eventually became repeat sex offenders. Is this recidivism rate significantly different from the rate for all offenders (57%) in that state? Summarize your conclusions in a sentence or two. (*HINT: You must use the information given in the problem to compute a sample proportion. Remember to convert the population percentage to a proportion.*)

8.13 [PS] In a recent statewide election, 55% of the voters rejected a proposal to institute a state lottery. In a random sample of 150 urban precincts, 49% of the voters rejected the proposal. Is the difference significant? Summarize your conclusions in a sentence or two.

8.14 [CJ] Statewide, the police clear by arrest 35% of the robberies and 42% of the aggravated assaults reported to them. A researcher takes a random sample of all the robberies ($N = 207$) and aggravated assaults ($N = 178$) reported to a metropolitan police department in one year and finds that 83 of the robberies and 80 of the assaults were cleared by arrest. Are the local arrest rates significantly different from the statewide rate? Write a sentence or two interpreting your decision.

8.15 [SOC/SW] A researcher has compiled a file of information on a random sample of 317 families in a city that has chronic, long-term patterns of child abuse. Below are reported some of the characteristics of the sample along with values for the city as a whole. For each trait, test the null hypothesis of "no difference" and summarize your findings.

a. Mothers' educational level (proportion completing high school):

City	Sample
$P_u = 0.63$	$P_s = 0.61$

b. Family size (proportion of families with four or more children):

City	Sample
$P_u = 0.21$	$P_s = 0.26$

c. Mothers' work status (proportion of mothers with jobs outside the home):

City	Sample
$P_u = 0.51$	$P_s = 0.27$

d. Relations with kin (proportion of families that have contact with kin at least once a week):

City	Sample
$P_u = 0.82$	$P_s = 0.43$

e. Fathers' educational achievement (average years of formal schooling):

City	Sample
$\mu = 12.3$	$\bar{X} = 12.5$ $s = 1.7$

f. Fathers' occupational stability (average years in present job):

City	Sample
$\mu = 5.2$	$\bar{X} = 3.7$ $s = 0.5$

8.16 [SW] You are the head of an agency seeking funding for a program to reduce unemployment among teenage males. Nationally, the unemployment rate for this group is 18%. A random sample of 323 teenage males in your area reveals an unemployment rate of 21.7%. Is the difference significant? Can you demonstrate a need for the program? Should you use a one-tailed test in this situation? Why? Explain the result of your test of significance as you would to a funding agency.

8.17 PA The city manager of Shinbone has received a complaint from the local union of firefighters to the effect that they are underpaid. Not having much time, the city manager gathers the records of a random sample of 27 firefighters and finds that their average salary is $38,073 with a standard deviation of $575. If she knows that the average salary nationally is $38,202, how can she respond to the complaint? Should she use a one-tailed test in this situation? Why? What would she say in a memo to the union that would respond to the complaint?

8.18 The following essay questions review the basic principles and concepts of inferential statistics. The order of the questions roughly follows the five-step model.

a. Hypothesis testing or significance testing can be conducted only with a random sample. Why?

b. Under what specific conditions can it be assumed that the sampling distribution is normal in shape?

c. Explain the role of the sampling distribution in a test of hypothesis.

d. The null hypothesis is an assumption about reality that makes it possible to test sample outcomes for their significance. Explain.

e. What is the critical region? How is the size of the critical region determined?

f. Describe a research situation in which a one-tailed test of hypothesis would be appropriate.

g. Thinking about the shape of the sampling distribution, why does use of the *t* distribution (as opposed to the *Z* distribution) make it more difficult to reject the null hypothesis?

h. What exactly can be concluded in the one-sample case when the test statistic falls into the critical region?

9

Hypothesis Testing II
The Two-Sample Case

LEARNING OBJECTIVES

By the end of this chapter, you will be able to:

1. Identify and cite examples of situations in which the two-sample test of hypothesis is appropriate.
2. Explain the logic of hypothesis testing as applied to the two-sample case.
3. Explain what an independent random sample is.
4. Perform a test of hypothesis for two-sample means or two-sample proportions following the five-step model and correctly interpret the results.
5. List and explain each of the factors (especially sample size) that affect the probability of rejecting the null hypothesis. Explain the differences between statistical significance and importance.

9.1 INTRODUCTION

In Chapter 8, we dealt with hypothesis testing in the one-sample case. In that situation, our concern was with the significance of the difference between a sample value and a population value. In this chapter, we will consider research situations in which we are concerned with the significance of the difference between two separate populations. For example, do men and women in the United States vary in their support for gun control? Obviously, we cannot ask every male and female for their opinions on this issue. Instead, we must draw random samples of both groups and use the information gathered from these samples to infer population patterns.

The central question asked in hypothesis testing in the two-sample case is: Is the difference between the samples large enough to allow us to conclude (with a known probability of error) that the populations represented by the samples are different? Thus, if we find a large enough difference in support for gun control between random samples of men and women, we can argue that the difference between the samples did not occur by simple random chance, but rather represents a real difference between men and women in the population.

In this chapter, we will consider tests for the significance of the difference between sample means and sample proportions. In both tests, the five-step model will serve as a framework for organizing our decision making. The general flow of the hypothesis-testing process is very similar to that followed in the one-sample case, but we will also need to consider some important differences.

9.2 HYPOTHESIS TESTING WITH SAMPLE MEANS (LARGE SAMPLES)

Two-Sample Versus One-Sample Tests. There are several important differences between the two-sample tests covered in this chapter and the one-sample tests covered in Chapter 8, the first of which occurs in Step 1 of the five-step model. The one-sample case requires that the sample be selected following the

principle of EPSEM (each case in the population must have an equal chance of being selected for the sample). The two-sample situation requires that the samples be selected independently as well as randomly. This requirement is met when the selection of a case for one sample has no effect on the probability that any particular case will be included in the other sample. In our example, this would mean that the selection of a specific male for the sample would have no effect on the probability of selecting any particular female. This new requirement will be stated as **independent random sampling** in Step 1.

The requirement of independent random sampling can be satisfied by drawing EPSEM samples from separate lists (for example, one for females and one for males). It is usually more convenient, however, to draw a single EPSEM sample from a single list of the population and then subdivide the cases into separate groups (males and females, for example). As long as the original sample is selected randomly, any subsamples created by the researcher will meet the assumption of independent random samples.

The second important difference in the five-step model for the two-sample case is in the form of the null hypothesis. The null is still a statement of "no difference." Now, however, instead of saying that the population from which the sample is drawn has a certain characteristic, it will say that the two populations are not different. ("There is no significant difference between men and women in their support of gun control.") If the test statistic falls in the critical region, the null hypothesis of no difference between the populations can be rejected, and the argument that the populations are different on the trait of interest will be supported.

A third important new element concerns the sampling distribution: the distribution of all possible sample outcomes. In Chapter 8, the sample outcome was either a mean or a proportion. Now we are dealing with two samples (e.g., samples of men and women), and the sample outcome is the *difference between* the sample statistics. In terms of our example, the sampling distribution would include all possible differences in sample means for support of gun control between men and women. If the null hypothesis is true and men and women do *not* have different views about gun control, the difference between the population means would be zero, the mean of the sampling distribution will be zero, and the huge majority of differences between sample means would be zero (or, at any rate, very small in value). The greater the differences between the sample means, the further the sample outcome (the *difference* between the two sample means) will be from the mean of the sampling distribution (zero) and the more likely it will be that the difference reflects a real difference between the populations represented by the samples.

A Test of Hypothesis for Two Sample Means. To illustrate the procedure for testing sample means, assume that a researcher has access to a nationally representative random sample and that the individuals in the sample have responded to a scale that measures attitudes toward gun control. The sample is divided by sex, and sample statistics are computed for males and females separately. Assuming that the scale yields interval-ratio level data, a test for the significance of the difference in sample means can be conducted.

As long as sample size is large (that is, as long as the combined number of cases in the two samples exceeds 100), the sampling distribution of the differences in sample means will be normal, and the normal curve (Appendix A) can be used to establish the critical regions. The test statistic, Z(obtained), will be computed by

the usual formula: sample outcome (the difference between the sample means) minus the mean of the sampling distribution, divided by the standard deviation of the sampling distribution. The formula is presented as Formula 9.1. Note that numerical subscripts are used to identify the samples and the two populations they represent. The subscript attached to σ ($\sigma_{\bar{X}-\bar{X}}$) indicates that we are dealing with the sampling distribution of the *differences* in sample means.

FORMULA 9.1

$$Z(\text{obtained}) = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{X}-\bar{X}}}$$

Where: $(\bar{X}_1 - \bar{X}_2)$ = the difference in the sample means
$(\mu_1 - \mu_2)$ = the difference in the population means
$\sigma_{\bar{X}-\bar{X}}$ = the standard deviation of the sampling distribution of the differences in sample means

The second term in the numerator, $(\mu_1 - \mu_2)$, reduces to zero because we assume that the null hypothesis (which will be stated as H_0: $\mu_1 = \mu_2$) is true. Recall that tests of significance are always based on the assumption that the null hypothesis is true. If the means of the two populations are equal, then the term $(\mu_1 - \mu_2)$ will be zero and can be dropped from the equation. In effect, then, the formula we will actually use to compute the test statistic in step 4 will be

FORMULA 9.2

$$Z(\text{obtained}) = \frac{(\bar{X}_1 - \bar{X}_2)}{\sigma_{\bar{X}-\bar{X}}}$$

For large samples, the standard deviation of the sampling distribution of the difference in sample means is defined as

FORMULA 9.3

$$\sigma_{\bar{X}-\bar{X}} = \sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}}$$

Since we will rarely, if ever, know the values of the population standard deviations (σ_1 and σ_2), we must use the sample standard deviations, suitably corrected for bias, to estimate them. Formula 9.4 displays the equation used to estimate the standard deviation of the sampling distribution in this situation. This is called a **pooled estimate** because it combines information from both samples.

FORMULA 9.4

$$\sigma_{\bar{X}-\bar{X}} = \sqrt{\frac{s_1^2}{N_1 - 1} + \frac{s_2^2}{N_2 - 1}}$$

The sample outcomes for support of gun control are reported below, and a test for the significance of the difference can now be conducted.

Sample 1 (Men)	Sample 2 (Women)
$\bar{X}_1 = 6.2$	$\bar{X}_2 = 6.5$
$s_1 = 1.3$	$s_2 = 1.4$
$N_1 = 324$	$N_2 = 317$

We see from the sample statistics that men have a lower average score on the support for gun control scale and are thus less supportive of gun control. The test of the hypothesis will tell us if this difference is large enough to justify the conclusion that it did not occur by random chance alone but rather reflects an actual difference between the populations of men and women on this issue.

Step 1. Making Assumptions and Meeting Test Requirements. Note that although we now assume that the random samples are independent, the rest of the model is the same as in the one-sample case.

Model: Independent random samples
Level of measurement is interval-ratio
Sampling distribution is normal

Step 2. Stating the Null Hypothesis. The null hypothesis states that the *populations* represented by the samples are not different on this variable. Since no direction for the difference has been predicted, a two-tailed test is called for, as reflected in the research hypothesis.

$$H_0: \mu_1 = \mu_2$$
$$(H_1: \mu_1 \neq \mu_2)$$

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region. For large samples, the Z distribution can be used to find areas under the sampling distribution and establish the critical region. Alpha will be set at 0.05.

Sampling distribution = Z distribution
Alpha = 0.05
Z(critical) = ±1.96

Step 4. Computing the Test Statistic. Since the population standard deviations are unknown, Formula 9.4 will be used to estimate the standard deviation of the sampling distribution. This value will then be substituted into Formula 9.2 and Z(obtained) will be computed:

$$\sigma_{\bar{X}-\bar{X}} = \sqrt{\frac{s_1^2}{N_1 - 1} + \frac{s_2^2}{N_2 - 1}} = \sqrt{\frac{(1.3)^2}{324 - 1} + \frac{(1.4)^2}{317 - 1}} = \sqrt{(0.0052) + (0.0062)}$$

$$= \sqrt{0.0114} = 0.107$$

$$Z(\text{obtained}) = \frac{(\bar{X}_1 - \bar{X}_2)}{\sigma_{\bar{X}-\bar{X}}} = \frac{6.2 - 6.5}{0.107} = \frac{-0.300}{0.107} = -2.80$$

Step 5. Making a Decision and Interpreting the Results of the Test. Comparing the test statistic with the critical region,

Z(obtained) = −2.80
Z(critical) = ±1.96

We see that the Z score clearly falls into the critical region. This outcome indicates that a difference as large as −0.300 (6.2 − 6.5) between the sample means is unlikely if the null hypothesis is true. The null hypothesis of no difference can be rejected, and the notion that men and women are different in terms of their support of gun control is supported. The decision to reject the null hypothesis has only a 0.05 probability (the alpha level) of being incorrect.

Note that the value for Z(obtained) is negative, indicating that men have significantly lower scores than women for support for gun control. The sign of the test statistics reflects our arbitrary decision to label men sample 1 and women sample 2. If we had reversed the labels and called women sample 1

ONE STEP AT A TIME — Testing the Difference in Sample Means for Significance (Large Samples): Computing *Z*(obtained) and Interpreting Results

Use these procedures when the sample size is large ($N_1 + N_2 > 100$).

Step 4: Computing *Z*(obtained). Solve Formula 9.4 before computing the test statistic.

Step	*Operation*
1.	Subtract 1 from N_1.
2.	Square the value of the standard deviation for the first sample (s_1^2).
3.	Divide the quantity you found in Step 2 by the quantity you found in Step 1.
4.	Subtract 1 from N_2.
5.	Square the value of the standard deviation for the second sample (s_2^2).
6.	Divide the quantity you found in Step 5 by the quantity you found in Step 4.
7.	Add the quantity you found in Step 6 to the quantity you found in Step 3.
8.	Take the square root of the quantity you found in Step 7.

Solving Formula 9.2:

9.	Subtract $\overline{X}_2$ from $\overline{X}_1$.
10.	Divide the quantity you found in Step 9 by the quantity you found in Step 8 ($\sigma_{\overline{X}_2 - \overline{X}_1}$).

Step 5: Making a Decision and Interpreting the Results of the Test

11.	Compare the *Z*(obtained) you computed in Step 10 to *Z*(critical). If *Z*(obtained) is *in* the critical region, *reject* the null hypothesis. If *Z*(obtained) is *not* in the critical region, *fail to reject* the null hypothesis.
12.	Interpret the decision to reject or fail to reject the null hypothesis in terms of the original question. For example, our conclusion for the example problem used in Section 9.2 was "There is a significant difference between men and women in their support for gun control."

and men sample 2, the sign of the *Z*(obtained) would have been positive, but its value (2.80) would have been exactly the same, as would our decision in Step 5. *(For practice in testing the significance of the difference between sample means for large samples, see Problems 9.1–9.6 and 9.15d–f.)*

Application 9.1

An attitude scale measuring satisfaction with family life has been administered to a sample of married respondents. On this scale, higher scores indicate greater satisfaction. The sample has been divided into respondents with no children and respondents with at least one child, and means and standard deviations have been computed for both groups. Is there a significant difference in satisfaction with family life between these two groups? The sample information is as follows:

Sample 1 (No Children)	Sample 2 (at Least One Child)
$\overline{X}_1 = 11.3$	$\overline{X}_2 = 10.8$
$s_1 = 0.6$	$s_2 = 0.5$
$N_1 = 78$	$N_2 = 93$

We can see from the sample results that respondents with no children are happier. The significance of this difference will be tested following the five-step model.

(continued next page)

Application 9.1 *(continued)*

Step 1. Making Assumptions and Meeting Test Requirements.

Model: Independent random samples
Level of measurement is interval-ratio
Sampling distribution is normal

Step 2. Stating the Null Hypothesis.

$$H_0: \mu_1 = \mu_2$$
$$(H_1: \mu_1 \neq \mu_2)$$

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region.

Sampling distribution = Z distribution
Alpha = 0.05, two-tailed
Z(critical) = ±1.96

Step 4. Computing the Test Statistic.

$$\sigma_{\bar{X}-\bar{X}} = \sqrt{\frac{s_1^2}{N_1 - 1} + \frac{s_2^2}{N_2 - 1}} = \sqrt{\frac{(0.6)^2}{78 - 1} + \frac{(0.5)^2}{93 - 1}}$$
$$= \sqrt{0.008} = 0.09$$

$$Z(\text{obtained}) = \frac{(\bar{X}_1 - \bar{X}_2)}{\sigma_{\bar{X}-\bar{X}}} = \frac{11.3 - 10.8}{0.09}$$
$$= \frac{0.50}{0.09} = 5.56$$

Step 5. Making a Decision. Comparing the test statistic with the critical region,

Z(obtained) = 5.56
Z(critical) = ±1.96

We would reject the null hypothesis. This test supports the conclusion that parents and childless couples are different with respect to satisfaction with family life. Given the direction of the difference, we also note that childless couples are significantly happier.

9.3 HYPOTHESIS TESTING WITH SAMPLE MEANS (SMALL SAMPLES)

As with single-sample means, when the population standard deviation is unknown and sample size is small (combined sample sizes of less than 100), the Z distribution can no longer be used to find areas under the sampling distribution. Instead, we will use the t distribution to find the critical region and thus to identify unlikely sample outcomes. To use the t distribution for testing two sample means, we need to perform one additional calculation and make one additional assumption. The calculation is for degrees of freedom, a quantity required for proper use of the t table (Appendix B). In the two-sample case, degrees of freedom are equal to $N_1 + N_2 - 2$.

The additional assumption is a more complex matter. When samples are small, we must assume that the variances of the populations of interest are equal in order to justify the assumption of a normal sampling distribution and to form a pooled estimate of the standard deviation of the sampling distribution. The assumption of equal variance in the population can be tested, but for our purposes here, we will simply assume equal population variances without formal testing. This assumption is safe as long as sample sizes are approximately equal.

A Test of Hypothesis for Two Sample Means: Small Samples. To illustrate this procedure, assume that a researcher believes that center-city families are significantly larger than suburban families, as measured by number of children. Random samples from both areas are gathered and sample statistics are computed.

Sample 1 (Suburban)	Sample 2 (Center City)
$\bar{X}_1 = 2.37$	$\bar{X}_2 = 2.78$
$s_1 = 0.63$	$s_2 = 0.95$
$N_1 = 42$	$N_2 = 37$

The sample data reveal a difference in the predicted direction. The significance of this observed difference can be tested with the five-step model.

Step 1. Making Assumptions and Meeting Test Requirements. Sample size is small, and the population standard deviation is unknown. Hence, we must assume equal population variances in the model.

Model: Independent random samples
Level of measurement is interval-ratio
Population variances are equal ($\sigma_1^2 = \sigma_2^2$)
Sampling distribution is normal

Step 2. Stating the Null Hypothesis. Since a direction has been predicted (center-city families are larger), a one-tailed test will be used, and the research hypothesis is stated in accordance with this decision.

$$H_0: \mu_1 = \mu_2$$
$$(H_1: \mu_1 < \mu_2)$$

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region. With small samples, the t distribution is used to establish the critical region. Alpha will be set at 0.05, and a one-tailed test will be used.

$$\text{Sampling distribution} = t \text{ distribution}$$
$$\text{Alpha} = 0.05, \text{ one-tailed}$$
$$\text{Degrees of freedom} = N_1 + N_2 - 2 = 42 + 37 - 2 = 77$$
$$t(\text{critical}) = -1.671$$

Note that the critical region is placed in the lower tail of the sampling distribution in accordance with the direction specified in H_1.

Step 4. Computing the Test Statistic. With small samples, a different formula (Formula 9.5) is used for the pooled estimate of the standard deviation of the sampling distribution. This value is then substituted directly into the denominator of the formula for t(obtained) given in Formula 9.6.

FORMULA 9.5

$$\sigma_{\bar{X}-\bar{X}} = \sqrt{\frac{N_1 s_1^2 + N_2 s_2^2}{N_1 + N_2 - 2}} \sqrt{\frac{N_1 + N_2}{N_1 N_2}}$$

$$\sigma_{\bar{X}-\bar{X}} = \sqrt{\frac{(42)(0.63)^2 + (37)(0.95)^2}{42 + 37 - 2}} \sqrt{\frac{42 + 37}{(42)(37)}}$$

$$\sigma_{\bar{X}-\bar{X}} = \sqrt{\frac{50.06}{77}} \sqrt{\frac{79}{1554}}$$

$$\sigma_{\bar{X}-\bar{X}} = (0.81)(0.23)$$

$$\sigma_{\bar{X}-\bar{X}} = 0.19$$

FORMULA 9.6

$$t(\text{obtained}) = \frac{(\bar{X}_1 - \bar{X}_2)}{\sigma_{\bar{X}-\bar{X}}}$$

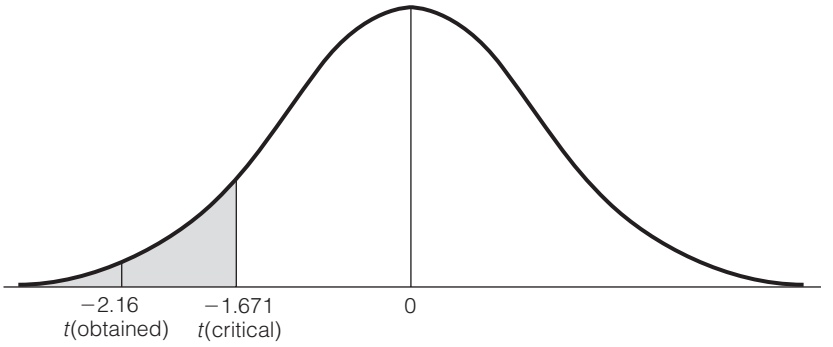
$$t(\text{obtained}) = \frac{2.37 - 2.78}{0.19} = \frac{-0.41}{0.19} = -2.16$$

Step 5. Making a Decision and Interpreting Test Results. Comparing the test statistic with the critical region,

$$t(\text{obtained}) = -2.16$$
$$t(\text{critical}) = -1.671$$

we can see that the test statistic falls into the critical region. If the null ($\mu_1 = \mu_2$) were true, this would be a very unlikely outcome, so the null can be rejected. There is a statistically significant difference (a difference so large that it is unlikely to be due

FIGURE 9.1 THE SAMPLING DISTRIBUTION WITH CRITICAL REGION AND TEST STATISTIC DISPLAYED



to random chance) in the sizes of center-city and suburban families. Furthermore, center-city families are significantly larger in size. The test statistic and sampling distribution are depicted in Figure 9.1. *(For practice in testing the significance of the difference between sample means for small samples, see Problems 9.7 and 9.8.)*

ONE STEP AT A TIME — Testing the Difference in Sample Means for Significance (Small Samples): Computing *t*(obtained) and Interpreting Results

Use these procedures when the sample size is large ($N_1 + N_2 < 100$).

Step 4: Computing *Z*(obtained).

Step ***Operation***

Solving Formula 9.5:

1. Add N_1 and N_2 and then subtract 2 from this total.
2. Square the standard deviation for the first sample (s_1^2).
3. Multiply the quantity you found in Step 2 by N_1.
4. Square the standard deviation for the second sample (s_2^2).
5. Multiply the quantity you found in Step 4 by N_2.
6. Add the quantities you found in Step 3 and Step 5.
7. Divide the quantity you found in Step 6 by the quantity you found in Step 1.
8. Take the square root of the quantity you found in Step 7.
9. Multiply N_1 and N_2.
10. Add N_1 and N_2.
11. Divide the quantity you found in Step 10 by the quantity you found in Step 9.
12. Take the square root of the quantity you found in Step 11.
13. Multiply the quantity you found in Step 12 by the quantity you found in Step 8.

Solving Formula 9.6:

14. Subtract $\overline{X}_2$ from $\overline{X}_1$.
15. Divide the quantity you found in Step 14 by the quantity you found in Step 13.

Step 5: Making a Decision and Interpreting the Results of the Test.

16. Compare the *t*(obtained) you computed in Step 15 to *t*(critical). If *t*(obtained) is *in* the critical region, *reject* the null hypothesis. If *t*(obtained) is *not* in the critical region, *fail to reject* the null hypothesis.
17. Interpret the decision to reject or fail to reject the null hypothesis in terms of the original question. For example, our conclusion for the example problem used in Section 9.3 was "There is a significant difference between the average size of center-city and suburban families."

9.4 HYPOTHESIS TESTING WITH SAMPLE PROPORTIONS (LARGE SAMPLES)

Testing for the significance of the difference between two sample proportions is analogous to testing sample means. The null hypothesis states that no difference exists between the populations from which the samples are drawn for the trait being tested. The sample proportions form the basis of the test statistic computed in Step 4, which is then compared with the critical region. When sample sizes are large (combined sample sizes of more than 100), the Z distribution may be used to find the critical region. We will not consider tests of significance for proportions based on small samples in this text.

In order to find the value of the test statistics, several preliminary equations must be solved. Formula 9.7 uses the values of the two sample proportions (P_s) to give us an estimate of the population proportion (P_u), the proportion of cases in the population that have the trait under consideration assuming the null hypothesis is true.

FORMULA 9.7

$$P_u = \frac{N_1 P_{s1} + N_2 P_{s2}}{N_1 + N_2}$$

The estimated value of P_u is then used to determine a value for the standard deviation of the sampling distribution of the difference in sample proportions in Formula 9.8:

FORMULA 9.8

$$\sigma_{p-p} = \sqrt{P_u(1 - P_u)}\sqrt{\frac{N_1 + N_2}{N_1 N_2}}$$

This value is then substituted into the formula for computing the test statistic, presented as Formula 9.9:

FORMULA 9.9

$$Z(\text{obtained}) = \frac{(P_{s1} - P_{s2}) - (P_{u1} - P_{u2})}{\sigma_{p-p}}$$

Where: $(P_{s1} - P_{s2})$ = the difference between the sample proportions
$(P_{u1} - P_{u2})$ = the difference between the population proportions
σ_{p-p} = the standard deviation of the sampling distribution of the difference between sample proportions

As was the case with sample means, the second term in the numerator is assumed to be zero by the null hypothesis. Therefore, the formula reduces to

FORMULA 9.10

$$Z(\text{obtained}) = \frac{(P_{s1} - P_{s2})}{\sigma_{p-p}}$$

Remember to solve these equations in order, starting with Formula 9.7 (and skipping Formula 9.9).

A Test of Hypothesis for Two Sample Proportions. An example will clarify these procedures. Assume that random samples of black and white senior citizens have been selected, and each respondent has been classified as high or low in terms of the number of memberships he or she holds in voluntary associations. Is there a statistically significant difference in the participation patterns of black and white elderly? The proportion of each

group classified as "high" in participation and sample size for both groups is reported below.

Sample 1 (Black Senior Citizens)	Sample 2 (White Senior Citizens)
$P_{s1} = 0.34$	$P_{s2} = 0.25$
$N_1 = 83$	$N_2 = 103$

Step 1. Making Assumptions and Meeting Test Requirements.

Model: Independent random samples
Level of measurement is nominal
Sampling distribution is normal

Step 2. Stating the Null Hypothesis. Since no direction has been predicted, this will be a two-tailed test.

$$H_0: P_{u1} = P_{u2}$$
$$(H_1: P_{u1} \neq P_{u2})$$

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region. Since sample size is large, the Z distribution will be used to establish the critical region. Setting alpha at 0.05, we have

Sampling distribution = Z distribution
Alpha = 0.05, two-tailed
Z(critical) = ± 1.96

Step 4. Computing the Test Statistic. Begin with the formula for estimating P_u (Formula 9.7), substitute the resultant value into Formula 9.8, and then solve for Z(obtained) with Formula 9.10.

$$P_u = \frac{N_1 P_{s1} + N_2 P_{s2}}{N_1 + N_2} = \frac{(83)(0.34) + (103)(0.25)}{83 + 103} = 0.29$$

$$\sigma_{p-p} = \sqrt{P_u(1 - P_u)}\sqrt{\frac{N_1 + N_2}{N_1 N_2}} = \sqrt{(0.29)(0.71)}\sqrt{\frac{83 + 103}{(83)(103)}}$$

$$= (0.45)(0.15) = 0.07$$

$$Z\text{(obtained)} = \frac{(P_{s1} - P_{s2})}{\sigma_{p-p}} = \frac{0.34 - 0.25}{0.07} = 1.29$$

Step 5. Making a Decision and Interpreting the Results of the Test. Since the test statistic, Z(obtained) = 1.29, does not fall into the critical region as marked by the Z(critical) of ± 1.96, we fail to reject the null hypothesis. The difference between the sample proportions is no greater than that which would be expected if the null hypothesis were true and only random chance were operating. Black and white senior citizens are not significantly different in terms of participation patterns as measured in this test. *(For practice in testing the significance of the difference between sample proportions, see Problems 9.10–9.14 and 9.15a–c.)*

ONE STEP AT A TIME

Testing the Difference in Sample Proportions for Significance (Large Samples): Computing *Z*(obtained) and Interpreting Results Step-by-Step

Step 4: Computing *Z*(obtained).

Step ***Operation***

Solving Formula 9.7:

1. Add N_1 and N_2.
2. Multiply P_{s1} by N_1.
3. Multiply P_{s2} by N_2.
4. Add the quantity you found in Step 3 to the quantity you found in Step 2.
5. Divide the quantity you found in Step 4 by the quantity you found in Step 1.

Solving Formula 9.8:

6. Multiply P_u (see Step 5) by $(1 - P_u)$.
7. Take the square root of the quantity you found in Step 6.
8. Multiply N_1 and N_2.
9. Add N_1 and N_2 (see Step 1).
10. Divide the quantity you found in Step 9 by the quantity you found in Step 8.
11. Take the square root of the quantity you found in Step 10.
12. Multiply the quantity you found in Step 11 by the quantity you found in Step 7.

Solving Formula 9.10

13. Subtract P_{s2} from P_{s1}.
14. Divide the quantity you found in Step 13 by the quantity you found in Step 12.

Step 5: Making a Decision and Interpreting the Results of the Test.

15. Compare *Z*(obtained) to *Z*(critical). If *Z*(obtained) is *in* the critical region, *reject* the null hypothesis. If *Z*(obtained) is *not* in the critical region, *fail to reject* the null hypothesis.
16. Interpret the decision to reject or fail to reject the null hypothesis in terms of the original question. For example, our conclusion for the example problem used in Section 9.4 was "There is no significant difference between the participation patterns of black and white senior citizens."

Application 9.2

Do attitudes toward sex vary by gender? The respondents in a national survey have been asked if they think that premarital sex is "always wrong" or only "sometimes wrong." The proportion of each sex that feels that premarital sex is always wrong is as follows.

Females	Males
$P_{s1} = 0.35$	$P_{s2} = 0.32$
$N_1 = 450$	$N_2 = 417$

This is all the information we will need to conduct a test of the null hypothesis following the familiar five-step model with alpha set at 0.05, using a two-tailed test.

Step 1. Making Assumptions and Meeting Test Requirements.

Model: Independent random samples
Level of measurement is nominal
Sampling distribution is normal

(continued next page)

Application 9.2 *(continued)*

Step 2. Stating the Null Hypothesis.

$$H_0: P_{u1} = P_{u2}$$
$$(H_1: P_{u1} \neq P_{u2})$$

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region

Sampling distribution = Z distribution
Alpha = 0.05, two-tailed
Z(critical) = ±1.96

Step 4. Computing the Test Statistic. Remember to start with Formula 9.7, substitute the value for P_u into Formula 9.8, and then substitute that value into Formula 9.10 to solve for Z(obtained).

$$P_u = \frac{N_1 P_{s1} + N_2 P_{s2}}{N_1 + N_2} = \frac{(450)(0.35) + (417)(0.32)}{450 + 417}$$

$$= \frac{290.94}{867} = 0.34$$

$$\sigma_{p-p} = \sqrt{P_u(1 - P_u)} \sqrt{\frac{N_1 + N_2}{N_1 N_2}}$$

$$= \sqrt{(0.34)(0.66)} \sqrt{\frac{450 + 417}{(450)(417)}}$$

$$= \sqrt{0.2244} \sqrt{0.0046} = (0.47)(0.068) = 0.032$$

$$Z(\text{obtained}) = \frac{(P_{s1} - P_{s2})}{\sigma_{p-p}} = \frac{0.35 - 0.32}{0.032}$$

$$= \frac{0.030}{0.032} = 0.94$$

Step 5. Making a Decision. With an obtained Z score of 0.94, we would fail to reject the null hypothesis. There is no statistically significant difference between males and females on attitudes toward premarital sex.

9.5 THE LIMITATIONS OF HYPOTHESIS TESTING: SIGNIFICANCE VERSUS IMPORTANCE

Given that we are usually interested in rejecting the null hypothesis, we should take a moment to consider systematically the factors that affect our decision in Step 5. Generally speaking, the probability of rejecting the null hypothesis is a function of four independent factors:

1. The size of the observed difference(s)
2. The alpha level
3. The use of one- or two-tailed tests
4. The size of the sample

Only the first of these four is not under the direct control of the researcher. The size of the difference (either between the sample outcome and the population value or between two sample outcomes) is partly a function of the testing procedures (that is, how variables are measured), but should generally reflect the underlying realities we are trying to probe.

The relationship between alpha level and the probability of rejection is straightforward. The higher the alpha level, the larger the critical region, the higher the percentage of all possible sample outcomes that fall in the critical region, and the greater the probability of rejection. Thus, it is easier to reject the H_0 at the 0.05 level than at the 0.01 level, and easier still at the 0.10 level. The danger here, of course, is that higher alpha levels will lead to more frequent Type I errors, and we might find ourselves declaring small differences to be statistically significant. In similar fashion, using a one-tailed test will increase the probability of rejection (assuming that the proper direction has been predicted).

The final factor is sample size: with all other factors constant, the probability of rejecting H_0 increases with sample size. In other words, the larger the sample, the more likely we are to reject the null hypothesis, and with very

TABLE 9.1 TEST STATISTICS FOR SINGLE-SAMPLE MEANS COMPUTED FROM SAMPLES OF VARIOUS SIZES ($\bar{X} = 80$, $\mu = 77$, $s = 5$ throughout)

Sample Size	Test Statistic, *Z*(obtained)
100	1.99
200	2.82
500	4.47

large samples (say, samples with thousands of cases), we may declare small, unimportant differences to be statistically significant.

This relationship may appear to be surprising, but the reasons for it can be appreciated with a brief consideration of the formulas used to compute test statistics in step 4. In all these formulas, for all tests of significance, sample size (N) is in the "denominator of the denominator." Algebraically, this is equivalent to being in the numerator of the formula and means that the value of the test statistic is directly proportional to N and that the two will increase together. To illustrate, consider Table 9.1, which shows the value of the test statistic for single sample means from samples of various sizes. The value of the test statistic, Z(obtained), increases as N increases, even though none of the other terms in the formula changes. This pattern of higher probabilities for rejecting H_0 with larger samples holds for all tests of significance.

On one hand, the relationship between sample size and the probability of rejecting the null should not alarm us unduly. Larger samples are, after all, better approximations of the populations they represent. Thus, decisions based on larger samples can be trusted more than decisions based on smaller samples.

On the other hand, this relationship clearly underlines what is perhaps the most significant limitation of hypothesis testing. Simply because a difference is statistically significant does not guarantee that it is important in any other sense. Particularly with very large samples, relatively small differences may be statistically significant. Even with small samples, of course, differences that are otherwise trivial or uninteresting may be statistically significant. The crucial point is that statistical significance and theoretical or practical importance can be two very different things. Statistical significance is a necessary but not sufficient condition for theoretical or practical importance. A difference that is not statistically significant is almost certainly unimportant. However, significance by itself does not guarantee importance. Even when it is clear that the research results were not produced by random chance, the researcher must still assess their importance. Do they firmly support a theory or hypothesis? Are they clearly consistent with a prediction or analysis? Do they strongly indicate a line of action in solving some problem? These are the kinds of questions a researcher must ask when assessing the importance of the results of a statistical test.

Also, we should note that researchers have access to some very powerful ways of analyzing the importance (vs. the statistical significance) of research results. These statistics, including bivariate measures of association and multivariate statistical techniques, will be introduced in Parts III and IV of this text.

BECOMING A CRITICAL CONSUMER: When Is a Difference a Difference?

How big does a difference have to be in order to be considered a difference? The question may sound whimsical or even silly, but this is a serious issue because it relates to our ability to identify the truth when we see it. Using the gender income gap as an example, how big a difference must there be between average incomes for men and women before the gap becomes a problem or evidence of gender inequality? Would we be concerned if U.S. men averaged $55,000 and women averaged $54,500? Across millions of cases, a difference of $500—about 1% of the average incomes—seems small and unimportant. How about if the difference was $1,000? $5,000? $10,000? At what point do we declare the difference to be important?

Very large or very small differences are easy to deal with. Small differences—relative to the scale of the variable (e.g., a difference of a few dollars in average incomes)—can be dismissed as trivial. At the other extreme, large differences, again thinking in terms of the scale of the variable (e.g., differences of more than $10,000 dollars in average income), are almost certainly worthy of attention. But what about differences between these two extremes? How big is big?

There are, of course, no absolute rules that would always enable us to identify important differences. However, we can discuss some guidelines that will help us know when a difference is consequential. We'll do this in general terms first and then relate this discussion to significance testing, the subject of Chapters 8–11.

Differences in General

First, as I suggested above, think about the difference in terms of the scale of the variable. A drop of a cent or two in the cost of a gallon of gas when the average is $4.00 probably won't make a difference in the budgets of anyone except very high-mileage drivers. In very general (and arbitrary) terms, a change of 10% or more (if gas costs $4.00 a gallon, a 10% change would be a rise or fall of 40 cents) probably signals an important difference. This rule of thumb works for many social indicators: population growth, crime rates, and birth rates, for example.

Second, you need to look at the raw frequencies to judge the importance of a change. Most people would be alarmed by a headline that reported a doubling of the number of teen pregnancies in a locality. However, consider two scenarios: one in which the number doubled from 10 to 20 in a town of 100,000 and another when, in a city of the same size, the numbers went from 2,500 to 5,000. The raw frequencies can add a valuable context to the perception of a change (which is one reason they are always reported in the professional research literature).

Third, and another way to add some context to a change, is to look at a broader time period. A report that voter turnout declined by 20% in a locality between 2000 and 2002 would naturally result in a good deal of alarm. However, turnout often declines between years featuring presidential elections (2000) and those that do not (2002). It would be more meaningful to compare 2000 with 2004 and, perhaps, even more revealing to get the data for earlier years.

Differences in Social Research

In social research, the problem of identifying important differences is additionally complicated by the vagaries of random chance when we work with samples rather than populations. That is, the size of a difference between sample statistics may be the result of random chance rather than (or in addition to) actual differences in the population. One of the great strengths of hypothesis testing is that it provides a system for identifying important differences. When we say that a difference is statistically significant, we reject the argument that random chance alone is responsible (with a known probability of error—the alpha level or *p* value) and support the idea that the difference in the sample statistics reflects a difference that also exists in the population. Small differences (e.g., a difference of only a few hundred dollars in average income between the genders) are unlikely to be significant at the 0.05 level. The larger the difference between the sample statistics, the more likely it is to be significant. The larger the difference and the lower the alpha level, the more confidence we can have

(continued next page)

BECOMING A CRITICAL CONSUMER *(continued)*

that the difference reflects actual patterns in the population.

Obviously, this form of decision making or system for identifying important differences is not infallible. We must remember that there is a chance of making an incorrect decision: we could declare a trivial difference to be important, or we could conclude that an important difference is trivial.

Reading Social Research

When reporting the results of tests of significance, professional researchers use a vocabulary that is much terser than ours. This is partly because of space limitations in scientific journals and partly because professional researchers can assume a certain level of statistical literacy in their audiences. Thus, they omit many of the elements—such as the null hypothesis or the critical region—that we have been so careful to state.

Instead, researchers report only the sample values (for example, means or proportions), the value of the test statistic (for example, a Z or t score), the alpha level, the degrees of freedom (if applicable), and sample size. The results of the example problem in Section 9.3 might be reported in the professional literature as "the difference between the sample means of 2.37 (suburban families) and 2.78 (center-city families) was tested and found to be significant ($t = -2.16$, $df = 77$, $p < 0.05$)." Note that the alpha level is reported as $p < 0.05$. This is shorthand for "the probability of a difference of this magnitude occurring by chance alone, if the null hypothesis of no difference is true, is less than 0.05" and is a good illustration of how researchers can convey a great deal of information in just a few symbols. In a similar fashion, our somewhat long-winded phrase "the test statistic falls in the critical region and, therefore, the null hypothesis is rejected" is rendered tersely and simply: "the difference . . . was . . . found to be significant."

When researchers need to report the results of many tests of significance, they will often use a summary table to report the sample information and whether the difference is significant at a certain alpha level. If you read the researcher's description and analysis of such tables, you should have little difficulty interpreting and understanding them. These comments about how significance tests are reported in the literature apply to all of the tests of hypotheses covered in Part II of this book.

To illustrate the reporting style you would find in the professional research literature, we can use a study authored by sociologists Dana Haynie and Scott Smith. They used a representative national sample (the National Longitudinal Study of Adolescent Health) to study the relationship between residential mobility and violence for teenagers. The researchers examined variables that might affect the relationship between

	Movers (N = 1,479)		Stayers (N = 6,559)		
Variables	$\bar{X}$ or %	s	$\bar{X}$ or %	s	Significant at $p < 0.05$?
Dependent					
Violence	0.67	1.15	0.47	1.00	Yes
Background					
Female	55.05%		52.09%		Yes
Two-parent family	62.98%		75.92%		Yes
Parent education	6.10	2.03	6.31	2.03	Yes
Distress					
Depression index	11.62	7.83	10.62	7.83	Yes
Network Behavior					
Peer deviance	3.12	2.62	2.88	2.62	Yes

Source: Dana Haynie and Scott Smith. 2005. "Residential Mobility and Adolescent Violence." *Social Forces:* (84)1: 361–374.

(continued next page)

BECOMING A CRITICAL CONSUMER *(continued)*

mobility and violence, including race, family type, psychological depression, and the characteristics of the adolescent's friendship networks. The table on page 220 presents some of their findings, using both means and percentages. All of the differences included in the table were statistically significant.

Violence, the dependent variable, was measured by asking the respondents about their involvement in six violent activities, including fighting and using a weapon to threaten someone. The scale measuring violence ranged from 0 to 3, so the means reported in the table indicate that violence was uncommon—or at least closer to 0 than the maximum score of 3—for both groups. Still, residentially mobile adolescents were significantly more violent than "stayers." There were also significant differences between the groups on a number of other variables that might impact the relationship between mobility and violence: gender, family characteristics, level of depression (movers were, on the average, significantly more depressed than stayers), and the behavior of the network of friends to which they belonged (movers had significantly more deviant peers than stayers).

How did all these factors affect the relationship between mobility and violence for teens? You can follow up by consulting the actual article for yourself: the complete citation is given under the table.

SUMMARY

1. A common research situation is to test for the significance of the difference between two populations. Sample statistics are calculated for random samples of each population, and then we test for the significance of the difference between the samples as a way of inferring differences between the specified populations.
2. When sample information is summarized in the form of sample means, and N is large, the Z distribution is used to find the critical region. When N is small, the t distribution is used to establish the critical region. In the latter circumstance, we must also assume equal population variances before forming a pooled estimate of the standard deviation of the sampling distribution.
3. Differences in sample proportions may also be tested for significance. For large samples, the Z distribution is used to find the critical region.
4. In all tests of hypothesis, a number of factors affect the probability of rejecting the null: the size of the difference, the alpha level, the use of one-tailed versus two-tailed tests, and sample size. Statistical significance is not the same thing as theoretical or practical importance. Even after a difference is found to be statistically significant, the researcher must still demonstrate the relevance or importance of his or her findings. The statistics presented in Parts III and IV of this text will give us the tools we need to deal directly with issues beyond statistical significance.

SUMMARY OF FORMULAS

FORMULA 9.1 Test statistic for two sample means, large samples:

$$Z(\text{obtained}) = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{X} - \bar{X}}}$$

FORMULA 9.2 Test statistic for two sample means, large samples (simplified formula):

$$Z(\text{obtained}) = \frac{(\bar{X}_1 - \bar{X}_2)}{\sigma_{\bar{X} - \bar{X}}}$$

FORMULA 9.3 Standard deviation of the sampling distribution of the difference in sample means, large samples:

$$\sigma_{\bar{X} - \bar{X}} = \sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}}$$

FORMULA 9.4 Pooled estimate of the standard deviation of the sampling distribution of the difference in sample means, large samples: $\sigma_{\bar{X}-\bar{X}} = \sqrt{\frac{s_1^2}{N_1 - 1} + \frac{s_2^2}{N_2 - 1}}$

FORMULA 9.5 Pooled estimate of the standard deviation of the sampling distribution of the difference in sample means, small samples: $\sigma_{\bar{X}-\bar{X}} = \sqrt{\frac{N_1 s_1^2 + N_2 s_2^2}{N_1 + N_2 - 2}} \sqrt{\frac{N_1 + N_2}{N_1 N_2}}$

FORMULA 9.6 Test statistic for two sample means, small samples: $t(\text{obtained}) = \frac{(\bar{X}_1 - \bar{X}_2)}{\sigma_{\bar{X}-\bar{X}}}$

FORMULA 9.7 Pooled estimate of population proportion, large samples: $P_u = \frac{N_1 P_{s1} + N_2 P_{s2}}{N_1 + N_2}$

FORMULA 9.8 Standard deviation of the sampling distribution of the difference in sample proportions, large samples: $\sigma_{p-p} = \sqrt{P_u(1 - P_u)} \sqrt{\frac{N_1 + N_2}{N_1 N_2}}$

FORMULA 9.9 Test statistic for two sample proportions, large samples: $Z(\text{obtained}) = \frac{(P_{s1} - P_{s2}) - (P_{u1} - P_{u2})}{\sigma_{p-p}}$

FORMULA 9.10 Test statistic for two sample proportions, large samples (simplified formula): $Z(\text{obtained}) = \frac{(P_{s1} - P_{s2})}{\sigma_{p-p}}$

GLOSSARY

Independent random samples. Random samples gathered in such a way that the selection of a particular case for one sample has no effect on the probability that any other particular case will be selected for the other samples.

Pooled estimate. An estimate of the standard deviation of the sampling distribution of the difference in sample means based on the standard deviations of both samples.

σ_{p-p}. symbol for the standard deviation of the sampling distribution of the differences in sample proportions.

$\sigma_{\bar{X}-\bar{X}}$. Symbol for the standard deviation of the sampling distribution of the differences in sample means.

PROBLEMS

(Problems are labeled with the social science discipline from which they are drawn: SOC for sociology, SW for social work, PS for political science, CJ for criminal justice, PA for public administration, and GER for gerontology.)

9.1 For each problem below, test for the significance of the difference in sample statistics using the five-step model. (*HINT: Remember to solve Formula 9.4 for before attempting to solve Formula 9.2. Also, in Formula 9.4, perform the mathematical operations in the proper sequence. First square each sample standard deviation, then divide by N − 1, add the resultant values, and then find the square root of the sum.*)

a.

Sample 1	Sample 2
$\bar{X}_1 = 72.5$	$\bar{X}_2 = 76.0$
$s_1 = 14.3$	$s_2 = 10.2$
$N_1 = 136$	$N_2 = 257$

b.

Sample 1	Sample 2
$\bar{X}_1 = 107$	$\bar{X}_2 = 103$
$s_1 = 14$	$s_2 = 17$
$N_1 = 175$	$N_2 = 200$

9.2 SOC Gessner and Healey administered questionnaires to samples of undergraduates. Among other things, the questionnaires contained a scale that measured attitudes toward interpersonal violence (higher scores indicate greater approval of interpersonal violence). Test the results as reported below for sexual, racial, and social-class differences.

a.

Sample 1 (Males)	Sample 2 (Females)
$\bar{X}_1 = 2.99$	$\bar{X}_2 = 2.29$
$s_1 = 0.88$	$s_2 = 0.91$
$N_1 = 122$	$N_2 = 251$

b.

Sample 1 (Blacks)	Sample 2 (Whites)
$\bar{X}_1 = 2.76$	$\bar{X}_2 = 2.49$
$s_1 = 0.68$	$s_2 = 0.91$
$N_1 = 43$	$N_2 = 304$

c.

Sample 1 (White Collar)	Sample 2 (Blue Collar)
$\bar{X}_1 = 2.46$	$\bar{X}_2 = 2.67$
$s_1 = 0.91$	$s_2 = 0.87$
$N_1 = 249$	$N_2 = 97$

d. Summarize your results in terms of the significance and the direction of the differences. Which of these three factors seems to make the biggest difference in attitudes toward interpersonal violence?

9.3 SOC Do athletes in different sports vary in terms of intelligence? Below are reported College Board scores of random samples of college basketball and football players. Is there a significant difference? Write a sentence or two explaining the difference.

a.

Sample 1 (Basketball Players)	Sample 2 (Football Players)
$\bar{X}_1 = 460$	$\bar{X}_2 = 442$
$s_1 = 92$	$s_2 = 57$
$N_1 = 102$	$N_2 = 117$

b. What about male and female college athletes?

Sample 1 (Males)	Sample 2 (Females)
$\bar{X}_1 = 452$	$\bar{X}_2 = 480$
$s_1 = 88$	$s_2 = 75$
$N_1 = 107$	$N_2 = 105$

9.4 PA A number of years ago, the fire department in Shinbone, Kansas, began recruiting minority group members through an affirmative action program. In terms of efficiency ratings as compiled by their superiors, how do the affirmative action employees rate? The ratings of random samples of both groups were collected, and the results are reported below (higher ratings indicate greater efficiency).

Sample 1 (Affirmative Action)	Sample 2 (Regular)
$\bar{X}_1 = 15.2$	$\bar{X}_2 = 15.5$
$s_1 = 3.9$	$s_2 = 2.0$
$N_1 = 97$	$N_2 = 100$

Write a sentence or two of interpretation.

9.5 SOC Are middle-class families more likely than working-class families to maintain contact with kin? Write a paragraph summarizing the results of these tests.

a. A sample of middle-class families reported an average of 8.3 visits per year with close kin while a sample of working-class families averaged 8.2 visits. Is the difference significant?

Visits

Sample 1 (Middle Class)	Sample 2 (Working Class)
$\bar{X}_1 = 7.3$	$\bar{X}_2 = 8.2$
$s_1 = 0.3$	$s_2 = 0.5$
$N_1 = 89$	$N_2 = 55$

b. The middle-class families averaged 2.3 phone calls and 8.7 e-mail messages per month with close kin. The working-class families averaged 2.7 calls and 5.7 e-mail messages per month. Are these differences significant?

Phone Calls

Sample 1 (Middle Class)	Sample 2 (Working Class)
$\bar{X}_1 = 2.3$	$\bar{X}_2 = 2.7$
$s_1 = 0.5$	$s_2 = 0.8$
$N_1 = 89$	$N_2 = 55$

E-Mail Messages

Sample 1 (Middle Class)	Sample 2 (Working Class)
$\bar{X}_1 = 8.7$	$\bar{X}_2 = 5.7$
$s_1 = 0.3$	$s_2 = 1.1$
$N_1 = 89$	$N_2 = 55$

9.6 SOC Are college students who live in dormitories significantly more involved in campus life than students who commute to campus? The data below report the average number of hours per week students devote to extracurricular activities. Is the difference between these randomly selected samples of commuter and residential students significant?

Sample 1 (Residential)	Sample 2 (Commuter)
$\bar{X}_1 = 12.4$	$\bar{X}_2 = 10.2$
$s_1 = 2.0$	$s_2 = 1.9$
$N_1 = 158$	$N_2 = 173$

9.7 SOC Are senior citizens who live in retirement communities more socially active than those who live in age-integrated communities? Write a sentence or two explaining the results of these tests. (*HINT: Remember to use the proper formulas for small sample sizes.*)

a. A random sample of senior citizens living in a retirement village reported that they had an average of 1.42 face-to-face interactions per day with their neighbors. A random sample of those living in age-integrated communities reported 1.58 interactions. Is the difference significant?

Sample 1 (Retirement Community)	Sample 2 (Age-integrated Neighborhood)
$\bar{X}_1 = 1.42$	$\bar{X}_2 = 1.58$
$s_1 = 0.10$	$s_2 = 0.78$
$N_1 = 43$	$N_2 = 37$

b. Senior citizens living in the retirement village reported that they had 7.43 telephone calls with friends and relatives each week whereas those in the age-integrated communities reported 5.50 calls. Is the difference significant?

Sample 1 (Retirement Community)	Sample 2 (Age-integrated Neighborhood)
$\bar{X}_1 = 7.43$	$\bar{X}_2 = 5.50$
$s_1 = 0.75$	$s_2 = 0.25$
$N_1 = 43$	$N_2 = 37$

9.8 SW As the director of the local Boys Club, you have claimed for years that membership in your club reduces juvenile delinquency. Now, a cynical member of your funding agency has demanded proof of your claim. Fortunately, your local sociology department is on your side and springs to your aid with student assistants, computers, and hand calculators at the ready. Random samples of members and nonmembers are gathered and interviewed with respect to their involvement in delinquent activities. Each respondent is asked to enumerate the number of delinquent acts he has engaged in over the past year. The results are in and reported below (the average number of admitted acts of delinquency). What can you tell the funding agency?

Sample 1 (Members)	Sample 2 (Nonmembers)
$\bar{X}_1 = 10.3$	$\bar{X}_2 = 12.3$
$s_1 = 0.27$	$s_2 = 4.2$
$N_1 = 40$	$N_2 = 55$

9.9 SOC A survey has been administered to random samples of respondents in each of five nations. For each nation, are men and women significantly different in terms of their reported levels of satisfaction? Respondents were asked, "How satisfied are you with your life as a whole?" Responses varied from 1 (very dissatisfied) to 10 (very satisfied). Conduct a test for the significance of the difference in mean scores for each nation.

France

Males	Females
$\bar{X}_1 = 7.4$	$\bar{X}_2 = 7.7$
$s_1 = 0.20$	$s_2 = 0.25$
$N_1 = 1{,}005$	$N_2 = 1{,}234$

Nigeria

Males	Females
$\bar{X}_1 = 6.7$	$\bar{X}_2 = 7.8$
$s_1 = 0.16$	$s_2 = 0.23$
$N_1 = 1{,}825$	$N_2 = 1{,}256$

China

Males	Females
$\bar{X}_1 = 7.6$	$\bar{X}_2 = 7.1$
$s_1 = 0.21$	$s_2 = 0.11$
$N_1 = 1{,}400$	$N_2 = 1{,}200$

Mexico	
Males	Females
$\bar{X}_1 = 8.3$	$\bar{X}_2 = 9.1$
$s_1 = 0.29$	$s_2 = 0.30$
$N_1 = 1{,}645$	$N_2 = 1{,}432$

Japan	
Males	Females
$\bar{X}_1 = 8.8$	$\bar{X}_2 = 9.3$
$s_1 = 0.34$	$s_2 = 0.32$
$N_1 = 1{,}621$	$N_2 = 1{,}683$

9.10 For each problem, test the sample statistics for the significance of the difference. (*HINT: In testing proportions, remember to begin with Formula 9.7, then solve Formulas 9.8 and 9.9.*)

a.

Sample 1	Sample 2
$P_{s1} = 0.17$	$P_{s2} = 0.20$
$N_1 = 101$	$N_2 = 114$

b.

Sample 1	Sample 2
$P_{s1} = 0.62$	$P_{s2} = 0.60$
$N_1 = 532$	$N_2 = 478$

9.11 CJ About half of the police officers in Shinbone, Kansas, have completed a special course in investigative procedures. Has the course increased their efficiency in clearing crimes by arrest? The proportions of cases cleared by arrest for samples of trained and untrained officers are reported below.

Sample 1 (Trained)	Sample 2 (Untrained)
$P_{s1} = 0.47$	$P_{s2} = 0.43$
$N_1 = 157$	$N_2 = 113$

9.12 SW A large counseling center needs to evaluate several experimental programs. Write a paragraph summarizing the results of these tests. Did the new programs work?

a. One program is designed for divorce counseling; the key feature of the program is its counselors, who are married couples working in teams. About half of all clients have been randomly assigned to this special program and half to the regular program, and the proportion of cases that eventually ended in divorce was recorded for both. The results for random samples of couples from both programs are reported below. In terms of preventing divorce, did the new program work?

Sample 1 (Special Program)	Sample 2 (Regular Program)
$P_{s1} = 0.53$	$P_{s2} = 0.59$
$N_1 = 78$	$N_2 = 82$

b. The agency is also experimenting with peer counseling for depressed children. About half of all clients were randomly assigned to peer counseling. After the program ran for a year, a random sample of children from the new program were compared with a random sample of children who did not receive peer counseling. In terms of the percentage who were judged to be much improved, did the new program work?

Sample 1 (Peer Counseling)	Sample 2 (No Peer Counseling)
$P_{s1} = 0.10$	$\bar{X}_{s2} = 0.15$
$N_1 = 52$	$N_2 = 56$

9.13 SOC At St. Algebra College, the sociology and psychology departments have been feuding for years about the respective quality of their programs. In an attempt to resolve the dispute, you have gathered data about the graduate school experience of random samples of both groups of majors. The results are presented below: the proportion of majors who applied to graduate schools, the proportion of majors accepted into their preferred programs, and the proportion of these who completed their programs. As measured by these data, is there a significant difference in program quality?

a. Proportion of majors who applied to graduate school are as follows.

Sample 1 (Sociology)	Sample 2 (Psychology)
$P_{s1} = 0.53$	$P_{s2} = 0.40$
$N_1 = 150$	$N_2 = 175$

b. Proportion accepted by program of first choice are as follows:

Sample 1 (Sociology)	Sample 2 (Psychology)
$P_{s1} = 0.75$	$P_{s2} = 0.85$
$N_1 = 80$	$N_2 = 70$

c. Proportion completing the programs are as follows:

Sample 1 (Sociology)	Sample 2 (Psychology)
$P_{s1} = 0.75$	$P_{s2} = 0.69$
$N_1 = 60$	$N_2 = 60$

9.14 CJ The local police chief started a "crimeline" program some years ago and wonders if it's really working. The program publicizes unsolved violent crimes in the local media and offers cash rewards for information leading to arrests. Are "featured" crimes more likely to be cleared by arrest than other violent crimes? Results from random samples of both types of crimes are reported as follows:

Sample 1 (Crimeline Crimes Cleared by Arrest)	Sample 2 (Non-crimeline Crimes Cleared by Arrest)
$P_{s1} = 0.35$	$P_{s2} = 0.25$
$N_1 = 178$	$N_2 = 212$

9.15 SOC Some results from a survey administered to a nationally representative sample are reported below in terms of differences by sex. Which of these differences, if any, are significant? Write a sentence or two of interpretation for each test.

a. Proportion favoring the legalization of marijuana are as follows:

Sample 1 (Males)	Sample 2 (Females)
$P_{s1} = 0.37$	$P_{s2} = 0.31$
$N_1 = 202$	$N_2 = 246$

b. Proportion strongly agreeing that "kids are life's greatest joy" are as follows:

Sample 1 (Males)	Sample 2 (Females)
$P_{s1} = 0.47$	$P_{s2} = 0.51$
$N_1 = 251$	$N_2 = 351$

c. Proportion voting for President Bush in 2004 are as follows:

Sample 1 (Males)	Sample 2 (Females)
$P_{s1} = 0.59$	$P_{s2} = 0.47$
$N_1 = 399$	$N_2 = 509$

d. Average hours spent with e-mail each week are as follows:

Sample 1 (Males)	Sample 2 (Females)
$\overline{X}_1 = 4.18$	$\overline{X}_2 = 3.38$
$s_1 = 7.21$	$s_2 = 5.92$
$N_1 = 431$	$N_2 = 535$

e. Average rate of church attendance (number of times per year) is as follows:

Sample 1 (Males)	Sample 2 (Females)
$\overline{X}_1 = 3.19$	$\overline{X}_2 = 3.99$
$s_1 = 2.60$	$s_2 = 2.72$
$N_1 = 641$	$N_2 = 808$

f.. Number of children are as follows:

Sample 1 (Males)	Sample 2 (Females)
$\overline{X}_1 = 1.49$	$\overline{X}_2 = 1.93$
$s_1 = 1.50$	$s_2 = 1.50$
$N_1 = 635$	$N_2 = 803$

YOU ARE THE RESEARCHER: Gender Gaps and Support for Traditional Gender Roles

There are two projects presented below. The first uses *t* tests to test for significant differences between men and women on four variables of your own choosing. The second uses the **Compute** command to explore attitudes toward abortion or traditional gender roles. You are urged to complete both projects.

PROJECT 1: Exploring the Gender Gap with *t* Tests

In this enlightened age, with its heavy stress on gender equality, how many important differences persist between the sexes? In this section, you will use SPSS to conduct *t* tests with sex as the independent variable. You will select four dependent variables and test to see if significant differences remain between the sexes in the areas measured by your variables.

STEP 1: Choosing Dependent Variables

Select four variables from the 2006 GSS to serve as dependent variables. Choose *only* interval-ratio variables or ordinal variables with three or more scores or categories. As you select variables, you might keep in mind the issues at the forefront of the debate over gender equality: income, education, and other measures of equality. Or you might choose variables that relate to lifestyle choices and patterns of everyday life: religiosity, TV viewing habits, desired family size, political ideas, or use of the Internet.

List your four dependent variables in the table below.

Variable	SPSS Name	What Exactly Does This Variable Measure?
1		
2		
3		
4		

STEP 2: Stating Hypotheses

For each dependent variable, state a hypothesis about the difference you expect to find between men and women. For example, you might hypothesize that men will be more liberal or women will be more educated. You can base your hypotheses on your own experiences or on the information about gender differences that you have acquired in your courses or from other sources.

Hypotheses:

1.

2.

3.

4.

STEP 3: Getting the Output

SPSS for Windows includes several tests for the significance of the difference between means. In this demonstration, we'll use the **Independent-Samples T Test,** the test we covered in Section 9.2, to test for the significance of the difference between men and women. If there are statistically significant differences between the sample means for men and women, we can conclude that there are differences between all U.S. men and U.S. women on this variable.

Start *SPSS for Windows* and load the 2006 GSS database. From the main menu bar, click **Analyze,** then **Compare Means,** and then **Independent-Samples T Test.** The **Independent-Samples T Test** dialog box will open with the usual list of variables on the left. Find and move the cursor over the names of the dependent variables you selected in Step 1. Click the top arrow in the middle of the window to move the variable names to the **Test Variable(s)** box.

Next, find and highlight *sex* and click the bottom arrow in the middle of the window to move *sex* to the **Grouping Variable** box. Two question marks will appear in the **Grouping Variable** box, and the **Define Groups** button will become active. SPSS needs to know which cases go in which groups, and, in the case at hand, the instructions we need to supply are straightforward. Males (indicated by a score of 1 on *sex*) go into group 1 and females (a score of 2) will go into group 2.

Click the **Define Groups** button, and the **Define Groups** window will appear. The cursor will be blinking in the box beside Group 1—SPSS is asking for the score that will determine which cases go into this group. Type a *1* in this box (for males) and then click the box next to Group *2* and type a *2* (for females). Click **Continue** to return to the **Independent-Samples T Test** window, click **OK,** and your output will be produced.

STEP 4: Reading the Output

To illustrate the appearance and interpretation of SPSS *t* test output, I will present a test for the significance of the gender difference in average age (note that age is not a good choice as a dependent variable: it works better as a cause than as an effect). Here's what the output looks like. (*Note:* Several columns of the output have been deleted to conserve space and improve clarity.)

Group Statistics

RESPONDENTS	SEX	*N*	Mean	Std. Deviation	Std. Error Mean
AGE	MALE	641	46.50	16.328	.645
	FEMALE	776	47.20	17.692	.635

Independent Samples T Test

	Levene's Test for Equality of Variances		T test for Equality of Means		
	F	Sig.	*t*	df	Sig. (2-tailed)
Equal variances assumed	5.853	0.016	−0.766	1415	0.444
Equal variances not assumed			−0.772	1397.703	0.440

The first block of output (Group Statistics) presents descriptive statistics. There were 641 males in the sample, and their average age was 46.50 with a standard deviation of 16.328. The 776 females averaged 47.20 years of age with a standard deviation of 17.692. We can see from this output that the sample means are different and that, on the average, females are a little older. Is the difference between the sample means significant?

The results of the test for significance are reported in the next block of output. *SPSS for Windows* does a separate test for each assumption about the population variance (see Sections 9.2 and 9.3), but we will look only at the "Equal variances assumed" reported in the top row. This is basically the same model used in Section 9.2.

Skip over the first columns of the output block (which reports the results of a test for equality of the population variances). In the top row, *SPSS for Windows* reports a *t* value (−0.776), the degrees of freedom (*df* = 1415), and a Sig. (2-tailed) of 0.444. This last piece of information is an alpha level, except it is the *exact* probability of getting the observed difference in sample means if only chance is operating. Thus, there is no need to look up the test statistic in a *t* or *Z* table. This value is much greater than 0.05, our usual indicator of significance. We will fail to reject the null hypothesis and conclude that the difference is not statistically significant. There is no difference in average years of age between men and women in the population.

STEP 5: Recording Your Results

Run *t* tests for your dependent variables and gender and record your results in the table below. A column has been provided for each piece of information. Write the SPSS variable name in the first column and then record the descriptive statistics (mean, standard deviation, and *N*). Next, record the results of the test of significance, using the top row (Equal variance assumed) of the Independent Samples output box. Record the *t* score, the degrees of freedom (*df*), and whether the difference is significant at the 0.05 level. If the value of Sig (2-tailed) is less than 0.05, reject the null hypothesis and write *yes* in this column. If the value of Sig (2-tailed) is more than 0.05, fail to reject the null hypothesis and write *no* in the column.

Dependent Variable		Mean	*s*	*N*	*t* score	*df*	Sig (2-tailed) < 0.05?
	Men						
	Women						
	Men						
	Women						
	Men						
	Women						
	Men						
	Women						

STEP 6: Interpreting Your Results

Summarize your findings. For each dependent variable, include the following.

1. At least one sentence summarizing the test in which you identify the variables being tested, the sample means for each group, *N,* the *t* score, and the significance level. In the professional research literature, you might find the results

reported as "For a sample of 1,417 respondents, there was no significant difference between the average age of men (46.50) and the average age of women (47.20) ($t = -0.77$, $df = 1{,}415$. $p > 0.05$)."

2. A sentence relating to your hypotheses. Were they supported? How?

PROJECT 2: Using the Compute Command to Explore Gender Differences

In this project, you will use the **Compute** command, which was introduced in Chapter 5, to construct a summary scale for either support for legal abortion or support for traditional gender roles. Do these attitudes vary significantly by gender? You will also choose a second independent variable other than gender to test for significant differences.

STEP 1: Creating Summary Scales

To refresh your memory, I used the **Compute** command in Chapter 5 to create a summary scale (*abscale*) for attitudes toward abortion by adding the scores on the two constituent items (*abhlth* and *abany*). Remember that, once created, a computed variable is added to the active file and can be used like any of the variables previously recorded in the file. If you did not save the data file with *abscale* included, you can quickly recreate the variable by following the instructions in Chapter 5.

The GSS data set supplied with this text also includes two variables that measure support for traditional gender roles. One of these (*fefam*) states "It is much better for everyone involved if the man is the achiever outside the home and the woman takes care of the home and family." There are four possible responses to this item ranging from "Strongly Agree" (1) to "Strongly Disagree" (4). The second item (*fepresch*) states "A preschool child is likely to suffer if his or her mother works." This item also has four possible responses, labeled the same as *fefam*. Since the two items have exactly the same number of scores and the same direction (for both, agreement indicates support for traditional gender roles), we can create a summary scale by simply adding the two variables together. Follow the commands in Chapter 5 to create the scale, which we will call *fescale*. The computed variable will have a total of seven possible scores, with lower scores indicating more support for traditional gender roles and higher scores indicating less support.

STEP 2: Stating Hypotheses

State your hypotheses about what differences you expect to find between men and women on these scales. Which gender will be more supportive (have a lower average score on *abscale*) of legal abortion? Why? Will men or women be more supportive (have a lower average score on *fescale*) of traditional gender roles? Why?

Hyphotheses:

1.

2.

STEP 3: Getting and Interpreting the Output

Run the **Independent Samples T Test** as before with the computed scales as the **Test Variable** and *sex* as the **Grouping Variable.** See the instructions for Project 1 above.

STEP 4: Interpreting Your Results

Summarize your results as in Project 1, Step 6. Was your hypothesis confirmed? How?

STEP 5: Extending the Test by Selecting an Additional Independent Variable

What other independent variable besides gender might be related to attitudes toward abortion or traditional gender roles? Select another independent variable besides *sex* and conduct an additional *t* test with *abscale* or *fescale* as the dependent variable. Remember that the *t* test requires the independent variable to have *only* two categories. For variables with more than two categories (*relig* or *racecen1,* for example), you can meet this requirement by using the **Define Groups** button in the **Grouping Variables** box to select specific categories of a variable. You could, for example, compare Protestants and Catholics on *relig* by choosing scores of 1 (Protestants) and 2 (Catholics).

STEP 6: Stating Hypotheses

State a hypothesis about what differences you expect to find between the categories of your independent variable. Which category will be more supportive of legal abortion or more supportive of traditional gender roles? Why?

STEP 7: Getting and Interpreting the Output

Run the **Independent Samples T Test** as before with the scale you selected as the **Test Variable** and your independent variable as the **Grouping Variable.**

STEP 8: Interpreting Your Results

Summarize your results as in Project 1, step 6. Was your hypothesis confirmed? How?

10

Hypothesis Testing III
The Analysis of Variance

LEARNING OBJECTIVES

By the end of this chapter, you will be able to:

1. Identify and cite examples of situations in which ANOVA is appropriate.
2. Explain the logic of hypothesis testing as applied to ANOVA.
3. Perform the ANOVA test, using the five-step model as a guide, and correctly interpret the results.
4. Define and explain the concepts of population variance, total sum of squares, the sum of squares between, and the sum of squares within, and mean square estimates.
5. Explain the difference between the statistical significance and the importance of relationships between variables.

10.1 INTRODUCTION

In this chapter, we will examine a very flexible and widely used test of significance called the **analysis of variance** (often abbreviated as **ANOVA**). This test is designed to be used with interval-ratio level dependent variables and is a powerful tool for analyzing the most sophisticated and precise measurements you are likely to encounter.

It is perhaps easiest to think of ANOVA as an extension of the *t* test for the significance of the difference between two sample means, which was presented in Chapter 9. The *t* test can be used only in situations in which our independent variable has exactly two categories (e.g., Protestants and Catholics). The analysis of variance, on the other hand, is appropriate for independent variables with more than two categories (e.g., Protestants, Catholics, Jews, people with no religious affiliation, and so forth).

To illustrate, suppose we were interested in examining the social basis of support for capital punishment. Why does support for the death penalty vary from person to person? Could there be a relationship between religion (the independent variable) and support for capital punishment (the dependent variable)? Opinion about the death penalty has an obvious moral dimension and may well be affected by a person's religious background.

Suppose that we administered a scale that measures support for capital punishment at the interval-ratio level to a randomly selected sample that includes Protestants, Catholics, Jews, people with no religious affiliation (None), and people from other religions (Other). We will have five categories of subjects, and we want to see if support for the death penalty varies significantly by religious affiliation. We will also want to answer other questions: Which religion shows the least or most support for capital punishment? Are Protestants significantly more supportive than Catholics or Jews? How do people with no religious affiliation compare to people in the other categories? The analysis of variance provides a very useful statistical context in which the questions can be addressed.

10.2 THE LOGIC OF THE ANALYSIS OF VARIANCE

For ANOVA, the null hypothesis is that the populations from which the samples are drawn are equal on the characteristic of interest. As applied to our problem, the null hypothesis could be phrased as "People from different religious denominations do not vary in their support for the death penalty," or symbolically as $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k$. (Note that this is an extended version of the null hypothesis for the two-sample *t* test). As usual, the researcher will normally be interested in rejecting the null and, in this case, showing that support is related to religion.

If the null hypothesis of "no difference" between the various religious populations (*all* Catholics, *all* Protestants, and so forth) is true, then any means calculated from randomly selected samples should be roughly equal in value. If the populations are truly the same, the average score for the Protestant sample should be about the same as the average score for the Catholic sample, the Jewish sample, and so forth. Note that the averages are unlikely to be exactly the same value even if the null hypothesis really is true, since we will always encounter some error or chance fluctuations in the measurement process. We are *not* asking "are there differences between the samples or categories of the independent variable (or, in our example, the religions)?" Rather, we are asking "are the differences between the samples large enough to reject the null hypothesis and justify the conclusion that the populations represented by the samples are different?"

Now, consider what kinds of outcomes we might encounter if we actually administered a Support of Capital Punishment Scale and organized the scores by religion. Of the infinite variety of possibilities, let's focus on two extreme outcomes as exemplified by Tables 10.1 and 10.2. In the first set of hypothetical results (Table 10.1), we see that the means and standard deviations of the groups are quite similar. The average scores are about the same for every religious group, and all five groups exhibit about the same dispersion. These results would be quite consistent with the null hypothesis of no difference. Neither the average score nor the dispersion of the scores changes in any important way by religion.

Now consider another set of fictitious results as displayed in Table 10.2. Here we see substantial differences in average score from category to category, with Jews showing the lowest support and Protestants showing the highest. Also, the standard deviations are low and similar from category to category, indicating that there is not much variation within the religions. Table 10.2 shows marked differences *between* religions combined with homogeneity *within* religions, as indicated by the low values of the standard deviations. These results would contradict the null hypothesis and support the notion that support for the death penalty does vary by religion.

The ANOVA test is based on the kinds of comparisons outlined above. The test compares the amount of variation between categories (for example,

TABLE 10.1 SUPPORT FOR CAPITAL PUNISHMENT BY RELIGION (fictitious data)

	Protestant	Catholic	Jew	None	Other
Mean	10.3	11.0	10.1	9.9	10.5
Standard deviation	2.4	1.9	2.2	1.7	2.0

TABLE 10.2 SUPPORT FOR CAPITAL PUNISHMENT BY RELIGION (fictitious data)

	Protestant	Catholic	Jew	None	Other
Mean	14.7	11.3	5.7	8.3	7.1
Standard deviation	2.4	1.9	2.2	1.7	2.0

from Protestants to Catholics to Jews to None to Other) with the amount of variation within categories (among Protestants, among Catholics, and so forth). The greater the differences *between* categories, relative to the differences *within* categories, the more likely that the null hypothesis of no difference is false and can be rejected. If support for capital punishment truly varies by religion, then the sample mean for each religion should be quite different from the others and dispersion within the categories should be relatively low.

10.3 THE COMPUTATION OF ANOVA

Even though we have been thinking of ANOVA as a test for the significance of the difference between sample means, the computational routine actually involves developing two separate estimates of the population variance, σ^2 (hence the name *analysis of variance*). Recall from Chapter 5 that the variance and standard deviation both measure dispersion and that the variance is simply the standard deviation squared. One estimate of the population variance is based on the amount of variation *within* each of the categories of the independent variable, and the other is based on the amount of variation *between* categories.

Before constructing these estimates, we need to introduce some new concepts and statistics. The first new concept is the total variation of the scores, which is measured by a quantity called the **total sum of squares,** or ***SST***

FORMULA 10.1

$$SST = \Sigma X^2 - N\bar{X}^2$$

To solve this formula, first find the sum of the squared scores (in other words, square each score and then add up the squared scores). Next, square the mean of all scores, multiply that value by the total number of cases in the sample (N), and subtract that quantity from the sum of the squared scores.

Formula 10.1 may seem vaguely familiar. A similar expression, $\Sigma(X_i - \bar{X}^2)$, appears in the formula for the standard deviation and variance (see Chapter 5). All three statistics incorporate information about the variation of the scores (or, in the case of *SST*, the squared scores) around the mean (or, in the case of *SST*, the square of the mean multiplied by N). In other words, all three statistics are measures of the variation or dispersion of the scores.

To construct the two separate estimates of the population variance, the total variation (*SST*) is divided into two components. One of these reflects the pattern of variation within the categories and is called the **sum of squares within (*SSW*).** In our example problem, *SSW* would measure the amount of variety in support for the death penalty within each of the religions.

The other component is based on the variation *between* categories and is called the **sum of squares between (*SSB*).** Again using our example to illustrate, *SSB* measures the size of the difference from religion to religion in support

for capital punishment. *SSW* and *SSB* are components of *SST*, as reflected in Formula 10.2:

FORMULA 10.2

$$SST = SSB + SSW$$

Let's start with the computation of *SSB*, our measure of the variation in scores between categories. We use the category means as summary statistics to determine the size of the difference from category to category. In other words, we compare the average support for the death penalty for each religion with the average support for all other religions to determine *SSB*. The formula for the sum of squares between (*SSB*) is

FORMULA 10.3

$$SSB = \Sigma N_k(\bar{X}_k - \bar{X})^2$$

Where: SSB = the sum of squares between the categories
N_k = the number of cases in a category
$\bar{X}_k$ = the mean of a category

To find *SSB*, subtract the overall mean of all scores ($\bar{X}$) from each category mean ($\bar{X}_k$), square the difference, multiply by the number of cases in the category, and add the results across all the categories.

The second estimate of the population variance (*SSW*) is based on the amount of variation within the categories. Look at Formula 10.2 again and you will see that the total sum of squares (*SST*) is equal to the addition of *SSW* and *SSB*. This relationship provides an easy method for finding *SSW* by simple subtraction. Formula 10.4 rearranges the symbols in Formula 10.2.

FORMULA 10.4

$$SSW = SST - SSB$$

Let's pause for a second to remember what we are after here. If the null hypothesis is *true*, then there should not be much variation from category to category (see Table 10.1) relative to the variation within categories, and the two estimates to the population variance based on *SSW* and *SSB* should be roughly equal. If the null hypothesis is *not true,* there will be large differences between categories (see Table 10.2) relative to the differences within categories, and *SSB* should be much larger than *SSW*. *SSB* will increase as the differences *between* category means increase, especially when there is not much variation *within* the categories (*SSW*). The larger *SSB* is as compared to *SSW*, the more likely it is that we will reject the null hypothesis.

The next step in the computational routine is to construct the estimates of the population variance. To do this, we will divide each sum of squares by its respective degrees of freedom. To find the degrees of freedom associated with *SSW*, subtract the number of categories (k) from the number of cases (N). The degrees of freedom associated with *SSB* are the number of categories minus one. In summary,

FORMULA 10.5

$$dfw = N - k$$

Where: dfw = degrees of freedom associated with *SSW*
N = total number of cases
k = number of categories

FORMULA 10.6

$$dfb = k - 1$$

Where: dfb = degrees of freedom associated with *SSB*
k = number of categories

The actual estimates of the population variance, called the **mean square estimates,** are calculated by dividing each sum of squares by its respective degrees of freedom:

FORMULA 10.7

$$\text{Mean square within} = \frac{SSW}{dfw}$$

FORMULA 10.8

$$\text{Mean square between} = \frac{SSB}{dfb}$$

The test statistic calculated in Step 4 of the five-step model is called the ***F* ratio** and its value is determined by the following formula:

FORMULA 10.9

$$F = \text{Mean square between/Mean square within}$$

As you can see, the value of the *F* ratio will be a function of the amount of variation between categories (based on *SSB*) to the amount of variation within the categories (based on *SSW*). The greater the variation between the categories relative to the variation within, the higher the value of the *F* ratio and the more likely we will reject the null hypothesis. These procedures are summarized in the One Step at a Time box and illustrated in the next section.

ONE STEP AT A TIME — Computing ANOVA

It is highly recommended that you use a computing table such as Table 10.3 to organize these computations.

Step	*Operation*
1.	To find *SST* by Formula 10.1:
	a. Find Σ^2 by squaring each score and adding the squared scores together.
	b. Find $N\bar{X}^2$ by squaring the value of the mean of all scores and then multiplying the result by *N*.
	c. Subtract the quantity you found in Step b from the quantity you found in Step a.
2.	To find *SSB* by Formula 10.3:
	a. Subtract the mean of all scores ($\bar{X}$) from the mean of each category ($\bar{X}_k$) and then square each difference.
	b. Multiply each of the squared differences you found in Step a by the number of cases in the category (N_k).
	c. Add the quantities you found in Step b together.
3.	To find *SSW* by Formula 10.4: Subtract the value of *SSB* from the value of *SST*.
4.	Calculate degrees of freedom.
	a. For *dfw,* use Formula 10.5. Subtract the number of categories (k) from the number of cases (N).
	b. For *dfb,* use Formula 10.6. Subtract 1 from the number of categories (k).
5.	Construct the two mean square estimates to the population variance.
	a. To find *MSW,* divide *SSW* (see Step 3) by *dfw* (see Step 4a).
	b. To find *MSB,* divide *SSB* (see Step 2) by *dfb* (see Step 4b).
6.	Find the obtained *F* ratio by Formula 10.9. Divide the mean square between estimate (*MSB*; see Step 5b) by the mean square within estimate (*MSW*; see Step 5a).

10.4 A COMPUTATIONAL EXAMPLE

Assume that we have administered our support for capital punishment scale to a sample of 20 individuals who are equally divided into the five religions. (Obviously, this sample is much too small for any serious research and is intended solely for purposes of illustration.) All scores are reported in Table 10.3 along with the squared scores, the category means, and the overall mean.

TABLE 10.3 SUPPORT FOR CAPITAL PUNISHMENT BY RELIGION FOR 16 SUBJECTS (fictitious data)

Protestant		Catholic		Jew		None		Other	
X	X^2	X	X^2	X	X^2	X	X^2	X	X^2
8	64	12	144	12	144	15	225	10	100
12	144	20	400	13	169	16	256	18	324
13	169	25	625	18	324	23	529	12	144
17	289	27	729	21	441	28	784	12	144
50	666	84	1,898	64	1,078	82	1,794	52	712
$\bar{X}_k = 12.5$		$\bar{X}_k = 21.0$		$\bar{X}_k = 16.0$		$\bar{X}_k = 20.5$		$\bar{X}_k = 13.0$	
				$\bar{X} = 16.6$					

To organize our computations, we'll follow the routine summarized in the One Step at a Time box at the end of Section 10.3. These steps are presented in Table 10.4.

TABLE 10.4 COMPUTING ANOVA

Step	Quantity	Formula	Solution
1.	*SST*	10.1 $SST = \Sigma X^2 - N\bar{X}^2$	$SST = 6{,}148 - (20)(16.6)^2 =$ ***636.8***
2.	*SSB*	10.3 $SSB = \Sigma N_k(\bar{X}_k - \bar{X})^2$	$SSB = 4(12.5 - 16.6)^2 + 4(21.0 - 16.6)^2 + 4(16.0 - 16.6)^2 + 4(20.5 - 16.6)^2 + 4(13.0 - 16.6)^2 =$ $67.24 + 77.44 + 1.44 + 60.84 + 51.84 =$ ***258.80***
3.	*SSW*	10.4 $SSW = SST - SSB$	$SSW = 636.8 - 258.8 =$ ***378.00***
4.	*dfw*	10.5 $dfw = N - k$	$dfw = 20 - 5 =$ ***15***
	dfb	10.6 $dfb = k - 1$	$dfb = 5 - 1 =$ ***4***
5.	*MSW*	10.7 $MSW = SSW/dfw$	$MSW = 378.00/15 =$ ***25.20***
	MSB	10.8 $MSB = SSB/dfb$	$MSB = 258.80/4 =$ ***64.70***
6.	*F* ratio	10.9 $F = MSW/MSB$	$F = 64.70/25.20 =$ ***2.57***

The *F* ratio computed in Step 6 must still be evaluated for its significance. *(Solve any of the end-of-chapter problems to practice computing these quantities and solving these formulas.)*

10.5 A TEST OF SIGNIFICANCE FOR ANOVA

In this section, we will see how to test an *F* ratio for significance, and we will also take a look at some of the assumptions underlying the ANOVA test. As usual, we will follow the five-step model as a convenient way of organizing the decision-making process.

Step 1. Making Assumptions and Meeting Test Requirements.

Model: Independent random samples
Level of measurement is interval-ratio
Populations are normally distributed
Population variances are equal

The model assumptions are quite stringent and underscore the fact that ANOVA should be used only with dependent variables that have been carefully and precisely measured. However, as long as sample sizes are equal (or nearly so), ANOVA can tolerate some violation of the model assumptions. In situations where you are uncertain or have samples of very different size, it is probably advisable to use an alternative test. (Chi square in Chapter 11 is one option.)

Step 2. Stating the Null Hypothesis. For ANOVA, the null hypothesis always states that the means of the populations from which the samples were drawn are equal. For our example problem, we are concerned with five different populations or categories, so our null hypothesis would be

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

where μ_1 represents the mean for Protestants, μ_2 the mean for Catholics, and so forth.

The alternative hypothesis states simply that at least one of the population means is different. The wording here is important. If we reject the null, ANOVA does not identify which mean or means are significantly different, but we can usually identify the most important differences by inspection of the sample means.

(H_1: At least one of the population means is different.)

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region. The sampling distribution for ANOVA is the *F* distribution, which is summarized in Appendix D. Note that there are separate tables for alphas of 0.05 and 0.01, respectively. As with the *t* table, the value of the critical *F* score will vary by degrees of freedom. For ANOVA, there are two separate degrees of freedom, one for each estimate of the population variance. The numbers across the top of the table are the degrees of freedom associated with the between estimate (*dfb*), and the numbers down the side of the table are those associated with the within estimate (*dfw*). In our example, *dfb* is ($k - 1$), or 4, and *dfw* is ($N - k$), or 15 (see Formulas 10.5 and 10.6). So, if we set alpha at 0.05, our critical *F* score will be 3.06.

Summarizing these considerations:

Sampling distribution = *F* distribution
Alpha = 0.05
Degrees of freedom (within) = $(N - k) = 15$
Degrees of freedom (between) = $(k - 1) = 4$
F(critical) = 3.06

Taking a moment to inspect the two *F* tables, you will notice that all the values are greater than 1.00. This is because ANOVA is a one-tailed test, and we are concerned only with outcomes in which there is more variance between categories than within categories. *F* values of less than 1.00 would indicate that

the between estimate was lower in value than the within estimate, and since we would always fail to reject the null in such cases, we simply ignore this class of outcomes.

Step 4. Computing the Test Statistic. This was done in the previous section, where we found an obtained F ratio of 2.57.

Step 5. Making a Decision and Interpreting the Results of the Test. Compare the test statistic with the critical value:

$$F(\text{critical}) = 3.06$$
$$F(\text{obtained}) = 2.57$$

Since the test statistic does not fall into the critical region, our decision would be to fail to reject the null. Support for capital punishment does not differ significantly by religion, and the variation we observed in the sample means is unimportant.

10.6 AN ADDITIONAL EXAMPLE FOR COMPUTING AND TESTING THE ANALYSIS OF VARIANCE

In this section, we will work through an additional example of the computation and interpretation of the ANOVA test. We will first review matters of computation, find the obtained F ratio, and then test the statistic for its significance. In the computational section, we will follow the step-by-step guidelines presented at the end of Section 10.3.

A researcher has been asked to evaluate the efficiency with which each of three social service agencies is administering a particular program. One area of concern is the speed of the agencies in processing paperwork and determining the eligibility of potential clients. The researcher has gathered information on the number of days required for processing a random sample of 10 cases in each agency. Is there a significant difference? The data are reported in Table 10.5, which also includes some additional information we will need to complete our calculations.

TABLE 10.5 NUMBER OF DAYS REQUIRED TO PROCESS CASES FOR THREE AGENCIES (fictitious data)

	Agency A		Agency B		Agency C	
Client	X	X^2	X	X^2	X	X^2
1	5	25	12	144	9	81
2	7	49	10	100	8	64
3	8	64	19	361	12	144
4	10	100	20	400	15	225
5	4	16	12	144	20	400
6	9	81	11	121	21	441
7	6	36	13	169	20	400
8	9	81	14	196	19	361
9	6	36	10	100	15	225
10	6	36	9	81	11	121
	$\Sigma X = 70$	$\Sigma X^2 = 524$	$\Sigma X = 130$	$\Sigma X^2 = 1{,}816$	$\Sigma X = 150$	$\Sigma X^2 = 2{,}462$
		$\bar{X}_k = 7.0$		$\bar{X}_k = 13.0$		$\bar{X}_k = 15.0$
				$\bar{X} = 350/30 = 11.67$		

TABLE 10.6 COMPUTING ANOVA

Step	Quantity	Formula	Solution
1.	*SST*	10.1 $SST = \Sigma X^2 - N\bar{X}^2$	$SST = (524 + 1{,}816 + 2{,}462) - 30(11.67)^2 = 4{,}802 - 4{,}085.7 =$ ***716.30***
2.	*SSB*	10.3 $SSB = \Sigma N_k (\bar{X}_k - \bar{X})^2$	$SSB = (10)(7.0 - 11.67)^2 + (10)(13.0 - 11.67)^2 + (10)(15.0 - 11.67)^2 = (10)(21.81) + (10)(1.77) + (10)(11.09) = 218.10 + 17.70 + 110.90 =$ ***346.70***
3.	*SSW*	10.4 $SSW = SST - SSB$	$SSW = 716.30 - 346.70 =$ ***369.60***
4.	*dfw*	10.5 $dfw = N - k$	$dfw = 30 - 3 = 27$
	dfb	10.6 $dfb = k - 1$	$dfb = 3 - 1 = 2$
5.	*MSW*	10.7 $MSW = SSW/dfw$	$MSW = 369.60/27 =$ ***13.69***
	MSB	10.8 $MSB = SSB/dfb$	$MSB = 346.7/2 =$ ***173.35***
6.	*F ratio*	10.9 $F = MSW/MSB$	$F = 173.35/13.69 =$ ***12.66***

The actual computations for ANOVA are presented in Table 10.6, following the computational routine introduced in Table 10.4.

We can now test the *F ratio* for its significance.

Step 1. Making Assumptions and Meeting Test Requirements.

Model: Independent random samples
Level of measurement is interval-ratio
Populations are normally distributed
Population variances are equal

The researcher will always be in a position to judge the adequacy of the first two assumptions in the model. The second two assumptions are more problematical, but remember that ANOVA will tolerate some deviation from its assumptions as long as sample sizes are roughly equal.

Step 2. Stating the Null Hypothesis.

$$H_0: \mu_1 = \mu_2 = \mu_3$$

(H_1: At least one of the population means is different.)

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region.

Sampling distribution = *F* distribution
Alpha = 0.05
Degrees of freedom (within) = $(N - k) = (30 - 3) = 27$
Degrees of freedom (between) = $(k - 1) = (3 - 1) = 2$
F(critical) = 3.35

Step 4. Computing the Test Statistic. We found an obtained *F* ratio of 12.66.

Step 5. Making a Decision and Interpreting the Results of the Test. Compare the test statistic with the critical value:

F(critical) = 3.35
F(obtained) = 12.66

The test statistic is in the critical region, and we would reject the null of no difference. The differences between the three agencies are very unlikely to have occurred by chance alone. The agencies are significantly different in the speed with which they process paperwork and determine eligibility. *(For practice in*

Application 10.1

An experiment in teaching introductory biology was recently conducted at a large university. One section was taught by the traditional lecture-lab method, a second was taught by an all-lab/demonstration approach with no lectures, and a third was taught entirely by a series of videotaped lectures and demonstrations that the students were free to view at any time and as often as they wanted. Students were randomly assigned to each of the three sections and, at the end of the semester, random samples of final exam scores were collected from each section. Is there a significant difference in student performance by teaching method?

FINAL EXAM SCORES BY TEACHING METHOD

Lecture		Demonstration		Videotape	
X	X^2	X	X^2	X	X^2
55	3,025	56	3,136	50	2,500
57	3,249	60	3,600	52	2,704
60	3,600	62	3,844	60	3,600
63	3,969	67	4,489	61	3,721
72	5,184	70	4,900	63	3,969
73	5,329	71	5,041	69	4,761
79	6,241	82	6,724	71	5,041
85	7,225	88	7,744	80	6,400
92	8,464	95	9,025	82	6,724
$X = 636$	$X^2 = 46{,}286$	$X = 651$	$X^2 = 48{,}503$	$X = 588$	$X^2 = 39{,}420$
$\bar{X}_k = 70.67$		$\bar{X}_k = 72.33$		$\bar{X}_k = 65.33$	
		$\bar{X} = 1{,}875/27 = 69.44$			

We can see by inspection that the "Videotape" group had the lowest average score and that the "Demonstration" group had the highest average score. The ANOVA test will tell us if these differences are large enough to justify the conclusion that they did not occur by chance alone. The table below follows the computational routine presented in Table 10.4.

COMPUTING ANOVA

Step	Quantity	Formula	Solution
1.	*SST*	9.10 $SST = \Sigma X^2 - N\bar{X}^2$	$SST = (46{,}286 + 48{,}503 + 39{,}420) - 27(69.44)^2 =$ ***4,017.33***
2.	*SSB*	9.4 $SSB = \Sigma N_k (\bar{X}_k - \bar{X})^2$	$SSB = (9)(70.67 - 69.44)^2 + (9)(72.33 - 69.44)^2 + (9)(65.33 - 69.44)^2 = 13.62 + 75.17 + 152.03 =$ ***240.82***
3.	*SSW*	9.11 $SSW = SST - SSB$	$SSW = 4{,}017.33 - 240.82 =$ ***3,776.51***
4.	*dfw*	9.5 $dfw = N - k$	$dfw = 27 - 3 =$ ***24***
	dfb	9.6 $dfb = k - 1$	$dfb = 3 - 1 =$ ***2***
5.	*MSW*	9.7 $MSW = SSW/dfw$	$MSW = 3776.51/24 =$ ***157.36***
	MSB	9.8 $MSB = SSB/dfb$	$MSB = 240.82/2 =$ ***120.41***
6.	*F* ratio	9.9 $F = MSW/MSB$	$F = 120.41/157.36 =$ **0.77**

(continued next page)

Application 10.1 *(continued)*

We can now conduct the test of significance.

Step 1. Making Assumptions and Meeting Test Requirements.

Model: Independent random samples
Level of measurement is interval-ratio
Populations are normally distributed
Population variances are equal

Step 2. Stating the Null Hypothesis.

H_0: $\mu_1 = \mu_2 = \mu_3$
(H_1: At least one of the population means is different.)

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region.

Sampling distribution = F distribution
Alpha = 0.05
Degrees of freedom (within) = $(N - k)$
$= (27 - 3) = 24$
Degrees of freedom (between) = $(k - 1)$
$= (3 - 1) = 2$
F(critical) = 3.40

Step 4. Computing the Test Statistic. We found an obtained F ratio of 0.77.

Step 5. Making a Decision and Interpreting the Results of the Test. Compare the test statistic with the critical value:

F(critical) = 3.40
F(obtained) = 0.77

We would clearly fail to reject the null hypothesis (the population means are equal) and would conclude that the observed differences among the category means were the results of random chance. Student performance in this course does not vary significantly by teaching method.

conducting the ANOVA test, see Problems 9.2–9.8. Begin with the lower-numbered problems because they have smaller data sets, fewer categories, and, therefore, the simplest calculations.)

10.7 THE LIMITATIONS OF THE TEST

ANOVA is appropriate whenever you want to test differences between the means of an interval-ratio level variable across three or more categories of an independent variable. This application is called **one-way analysis of variance,** because it involves the effect of a single variable (for example, religion) on another (for example, support for capital punishment). This is the simplest application of ANOVA, and you should be aware that the technique has numerous more advanced and complex forms. For example, you may encounter research projects in which the effects of two separate variables (for example, religion and gender) on some third variable were observed.

One important limitation of ANOVA is that it requires interval-ratio measurement of the dependent variable and roughly equal numbers of cases in each of the categories of the independent variable. The former condition may be difficult to meet with complete confidence for many variables of interest to the social sciences. The latter condition may create problems when the research hypothesis calls for comparisons between groups that are, by their nature, unequal in numbers (for example, white versus black Americans) and may call for some unusual sampling schemes in the data-gathering phase of a research project. Neither of these limitations should be particularly crippling since ANOVA can tolerate some deviation from its model assumptions, but you should be aware of these limitations in planning your own research as well as in judging the adequacy of research conducted by others.

A second limitation of ANOVA actually applies to all forms of significance testing and was introduced in Section 9.5. These tests are designed to detect

nonrandom differences: differences so large that they are very unlikely to be produced by random chance alone. The problem is that differences that are statistically significant are not necessarily important in any other sense. Statistical techniques that can assess the importance of results directly are presented in Parts III and IV of this text.

A final limitation of ANOVA relates to the research hypothesis. As you recall, when the null hypothesis is rejected, the alternative hypothesis is supported. The limitation is that the alternative hypothesis is not specific: it simply asserts that at least one of the population means is different from the others. Obviously, we would like to know which differences are significant. We can sometimes make this determination by simple inspection. In our problem involving social service agencies, for example, it is pretty clear from Table 10.5 that Agency A is the source of most of the differences. This informal, "eyeball" method can be misleading, however, and you should exercise caution in making conclusions about which means are significantly different.

BECOMING A CRITICAL CONSUMER: Reading the Professional Literature

It is extremely unlikely that you would encounter a report using ANOVA in everyday life or in the popular media, thus I will confine this section to the professional research literature. As I have pointed out previously, reports about tests of significance in social science research journals will be short on detail, but you will still be able to locate all of the essential information needed to understand the results of the test.

We can use a recent article to illustrate how to read ANOVA results in the professional literature. Researchers Choi, Meininger, and Roberts administered a battery of standardized tests that measured stress and mental health to over 300 middle school students in the Houston area. In addition, they measured self-esteem and family cohesion, resources the students could use to combat stress and mental health problems. Since the researchers were concerned with a variety of dependent variables, they reported their results in a summary table, a version of which is presented here. Note that the table lists the independent variable (racial or ethnic group) in the columns and five of the dependent variables used in the study in the rows. The means for each test are noted in the body of the table, and you should inspect these statistics to look for patterns in the differences between groups.

The right-hand columns of the table list the *F* ratios—*F*(obtained) in our terminology—for each test and the exact probability (*p*) that the differences occurred by random chance alone. The *p* values are another way to represent what we call the alpha level, and values in this column that are less than 0.05 are statistically significant (we would say that the "null hypothesis has been rejected").

In two of the five tests reported here, the differences are significant at the 0.05 level. For the test measuring stress, European American children had the lowest average score, and for the test of self-esteem, they had higher scores than two of the other three groups. For two of the other tests (family cohesion and suicidal ideation), the differences in sample means are clearly not significant. In the fifth test (depression), the differences approach significance and, again, European American children show the most favorable score.

What do these tests show? Looking over the full array of their evidence—not just the partial results reported here—the authors conclude that minority group adolescents are more vulnerable to stress and to some kinds of mental health problems. They also have access to resources (e.g., cohesive families) that can be used to deal

(continued next page)

BECOMING A CRITICAL CONSUMER *(continued)*

with these problems, but, overall, minority group children require more attention from health-care providers. Want to learn more? The citation is given below.

Stress, Resources, and Mental Distress by Group

	Group					
	European Americans	African Americans	Hispanic Americans	Asian Americans		
Measure	Mean	Mean	Mean	Mean	*F*	*p*
General social stress	24.93	28.15	34.84	32.26	9.42	0.000
Self-esteem	32.29	35.29	29.78	26.28	6.30	0.000
Family Cohesion	6.75	6.65	6.44	5.72	1.17	0.321
Depression	40.52	42.91	45.26	43.25	2.21	0.087
Suicidal ideation	4.07	4.25	4.47	3.80	0.85	0.470

Source: Heeseung Choi, Janet Meininger, and Robert Roberts. 2006. "Ethnic Differences in Adolescents' Mental Distress, Social Stress, and Resources" *Adolescence* 41: 263–283. Based on Table 2, p. 243.

SUMMARY

1. One-way analysis of variance is a powerful test of significance that is commonly used when comparisons across more than two categories or samples are of interest. It is perhaps easiest to conceptualize ANOVA as an extension of the test for the difference in sample means.
2. ANOVA compares the amount of variation within the categories to the amount of variation between categories. If the null of no difference is false, there should be relatively great variation between categories and relatively little variation within categories. The greater the differences from category to category relative to the differences within the categories, the more likely we will be able to reject the null.
3. The computational routine for even simple applications of ANOVA can quickly become quite complex. The basic process is to construct separate estimates to the population variance based on the variation within the categories and the variation between the categories. The test statistic is the *F* ratio, which is based on a comparison of these two estimates. The basic computational routine is summarized in Table 10.4. This is probably an appropriate time to mention the widespread availability of statistical packages such as SPSS, the purpose of which is to perform complex calculations such as these accurately and quickly. If you haven't yet learned how to use such programs, ANOVA may provide you with the necessary incentive.
4. The ANOVA test can be organized into the familiar five-step model for testing the significance of sample outcomes. Although the model assumptions (Step 1) require high-quality data, the test can tolerate some deviation as long as sample sizes are roughly equal. The null takes the familiar form of stating that there is no difference of any importance among the population values, while the alternative hypothesis asserts that at least one population mean is different. The sampling distribution is the *F* distribution, and the test is always one-tailed. The decision to reject or to fail to reject the null is based on a comparison of the obtained *F* ratio with the critical *F* ratio as determined for a given alpha level and degrees of freedom. The decision to reject the null indicates only that one or more of the population means is different from the others. We can often determine which sample mean(s) account for the difference by inspecting the sample data, but this informal method should be used with caution.

SUMMARY OF FORMULAS

FORMULA 10.1 Total sum of squares: $SST = \Sigma X^2 - N\overline{X}^2$

FORMULA 10.2 The two components of the total sum of squares: $SST = SSB + SSW$

FORMULA 10.3 Sum of squares within: $SSB = \Sigma N_k \left(\overline{X}_k - \overline{X}\right)^2$

FORMULA 10.4 Sum of squares between: $SSW = SST - SSB$

FORMULA 10.5 Degrees of freedom for *SSW:* $dfw = N - k$

FORMULA 10.6 Degrees of freedom for *SSB:* $dfb = k - 1$

FORMULA 10.7 Mean square within: Mean square within = SSW/dfw

FORMULA 10.8 Mean square between: Mean square between = SSB/dfb

FORMULA 10.9 *F* ratio: F = Mean square between/Mean square within

GLOSSARY

Analysis of variance. A test of significance appropriate for situations in which we are concerned with the differences among more than two sample means.

ANOVA. See Analysis of variance.

***F* ratio.** The test statistic computed in Step 4 of the ANOVA test.

Mean square estimate. An estimate of the variance calculated by dividing the sum of squares within (*SSW*) or the sum of squares between (*SSB*) by the proper degrees of freedom.

One-way analysis of variance. Applications of ANOVA in which the effect of a single independent variable on a dependent variable is observed.

Sum of squares between (*SSB*). The sum of the squared deviations of the sample means from the overall mean, weighted by sample size.

Sum of squares within (*SSW*). The sum of the squared deviations of scores from the category means.

Total sum of squares (*SST*). The sum of the squared deviations of the scores from the overall mean.

PROBLEMS

(Problems are labeled with the social science discipline from which they are drawn: SOC for sociology, SW for social work, PS for political science, CJ for criminal justice, PA for public administration, and GER for gerontology.)

(*NOTE:* The number of cases in these problems is very low—a fraction of the sample size necessary for any serious research—in order to simplify computations.)

10.1 Conduct the ANOVA test for each set of scores below. (*HINT: Keep track of all sums and means by constructing computational tables like Table 10.3 or 10.4.*)

a.

Category		
A	B	C
5	10	12
7	12	16
8	14	18
9	15	20

b.

Category		
A	B	C
1	2	3
10	12	10
9	2	7
20	3	14
8	1	1

c.

A	B	C	D
13	45	23	10
15	40	78	20
10	47	80	25
11	50	34	27
10	45	30	20

10.2 SOC What type of person is most involved in the neighborhood and community? Who is more likely to volunteer for organizations such as PTA, scouts, or Little League? A random sample of 15 people have been asked for their number of memberships in community voluntary organizations and some other information. Which differences are significant?

a.

Membership by Education

Less than High School	High School	College
0	1	0
1	3	3
2	3	4
3	4	4
4	5	4

b.

Membership by Length of Residence in Present Community

Less than 2 Years	2–5 Years	More than 5 Years
0	0	1
1	2	3
3	3	3
4	4	4
4	5	4

c.

Membership by Extent of Television Watching

Little or None	Moderate	High
0	3	4
0	3	4
1	3	4
1	3	4
2	4	5

d.

Membership by Number of Children

None	One Child	More than One Child
0	2	0
1	3	3
1	4	4
3	4	4
3	4	5

10.3 SOC In a local community, a random sample of 18 couples has been assessed on a scale that measures the extent to which power and decision making are shared (lower scores) or monopolized by one party (higher scores) and on marital happiness (lower scores indicate lower levels of unhappiness). The couples were also classified by type of relationship: traditional (only the husband works outside the home), dual career (both parties work), and cohabitational (parties living together but not legally married, regardless of work patterns). Does decision making or happiness vary significantly by type of relationship?

a.

Decision Making

Traditional	Dual Career	Cohabitational
7	8	2
8	5	1
2	4	3
5	4	4
7	5	1
6	5	2

b.

Happiness

Traditional	Dual Career	Cohabitational
10	12	12
14	12	14
20	12	15
22	14	17
23	15	18
24	20	22

10.4 CJ Two separate crime-reduction programs have been implemented in the city of Shinbone. One involves a neighborhood watch program with citizens actively involved in crime prevention. The second involves officers patrolling the neighborhoods on foot rather than in patrol cars. In terms of the percentage reduction in crimes reported to the police over a one-year period, were the programs successful? The results are for random

samples of 18 neighborhoods drawn from the entire city.

Neighborhood Watch	Foot Patrol	No Program
−10	−21	+30
−20	−15	−10
+10	−80	+14
+20	−10	+80
+70	−50	+50
+10	−10	−20

10.5 Are sexually active teenagers any better informed about AIDS and other potential health problems related to sex than teenagers who are sexually inactive? A 15-item test of general knowledge about sex and health was administered to random samples of teens who are sexually inactive, teens who are sexually active but with only a single partner ("going steady"), and teens who are sexually active with more than one partner. Is there any significant difference in the test scores?

Inactive	Active—One Partner	Active—More than One Partner
10	11	12
12	11	12
8	6	10
10	5	4
8	15	3
5	10	15

10.6 SOC Does the rate of voter turnout vary significantly by the type of election? A random sample of voting precincts displays the following pattern of voter turnout by election type. Assess the results for significance.

Local Only	State	National
33	35	42
78	56	40
32	35	52
28	40	66
10	45	78
12	42	62
61	65	57
28	62	75
29	25	72
45	47	51
44	52	69
41	55	59

10.7 GER Do older citizens lose interest in politics and current affairs? A brief quiz on recent headline stories was administered to random samples of respondents from each of four different age groups. Is there a significant difference? The data below represent numbers of correct responses.

High School (15–18)	Young Adult (21–30)	Middle-Aged (30–55)	Retired (65+)
0	0	2	5
1	0	3	6
1	2	3	6
2	2	4	6
2	4	4	7
2	4	5	7
3	4	6	8
5	6	7	10
5	7	7	10
7	7	8	10
7	7	8	10
9	10	10	10

10.8 SOC A small random sample of respondents has been selected from the General Social Survey database. Each respondent has been classified as either a city dweller, a suburbanite, or a rural dweller. Are there statistically significant differences by place of residence for any of the variables listed below?

a.

Occupational Prestige

Urban	Suburban	Rural
32	40	30
45	48	40
42	50	40
47	55	45
48	55	45
50	60	50
51	65	52
55	70	55
60	75	55
65	75	60

b.

Number of Children

Urban	Suburban	Rural
1	0	1
1	1	4
0	0	2
2	0	3
1	2	3
0	2	2
2	3	5
2	2	0
1	2	4
0	1	6

c.

Family income

Urban	Suburban	Rural
5	6	5
7	8	5
8	11	11
11	12	10
8	12	9
9	11	6
8	11	10
3	9	7
9	10	9
10	12	8

d.

Church Attendance

Urban	Suburban	Rural
0	0	1
7	0	5
0	2	4
4	5	4
5	8	0
8	5	4
7	8	8
5	7	8
7	2	8
4	6	5

e.

Hours of TV Watching per Day

Urban	Suburban	Rural
5	5	3
3	7	7
12	10	5
2	2	0
0	3	1
2	0	8
3	1	5
4	3	10
5	4	3
9	1	1

10.9 SOC Does support for suicide (death with dignity) vary by social class? Is this relationship different in different nations? Small samples in three nations were asked if it is ever justified for a person with an incurable disease to take his or her own life. Respondents answered in terms of a 10-point scale on which 10 was "always justified" (the strongest support for death with dignity) and 1 was "never justified" (the lowest level of support). Results are reported below.

MEXICO

Lower Class	Working Class	Middle Class	Upper Class
5	2	1	2
2	2	1	4
4	1	3	5
5	1	4	7
4	6	1	8
2	5	2	10
3	7	1	10
1	2	5	9
1	3	1	8
3	1	1	8

CANADA

Lower Class	Working Class	Middle Class	Upper Class
7	5	1	5
7	6	3	7
6	7	4	8
4	8	5	9
7	8	7	10
8	9	8	10
9	5	8	8
9	6	9	5
6	7	9	8
5	8	5	9

UNITED STATES

Lower Class	Working Class	Middle Class	Upper Class
4	4	4	1
5	5	6	5
6	1	7	8
1	4	5	9
3	3	8	9
3	3	9	9
3	4	9	8
5	2	8	6
3	1	7	9
6	1	2	9

YOU ARE THE RESEARCHER: Why Are Some People Liberal (or Conservative)? Why Are Some People More Sexually Active?

Complete the two projects below to practice your computer skills and apply your knowledge of ANOVA. The first project investigates political ideology using *polviews*—a seven-point scale that measures how liberal or conservative a person is—as the dependent variable. The second project looks at a matter of nearly universal fascination (sex, of course) and uses *sexfreq,* a measure of how sexually active the respondent is. Both variables are ordinal in level of measurement, but will be treated as interval-ratio for purposes of this test.

Before starting the projects, I will demonstrate ANOVA with SPSS and also introduce you to a new SPSS command called **recode,** which enables us to collapse scores on a variable. This is extremely useful because it allows us to change the nature of a variable to fit a particular task. Here I will demonstrate how we can take a variable such as *age,* which has a wide range of scores (respondents in the 2006 GSS ranged in age from 18 to 89), and transform it into a variable with just a few scores that we can use as an independent variable in ANOVA.

Recoding Variables

We will use the **Recode** command to create a new version of *age* that has three categories. When we are finished, we will have two versions of the same variable in the data set: the original interval-ratio version with age measured in years and a new ordinal-level version with collapsed categories. If we wish, the new version of age can be added to the permanent data file and used in the future.

The decision to use three categories for *age* is arbitrary, and we could easily have decided on four, five, or even six categories for the new, recoded independent variable. If we find that we are unhappy with the three-category version of the variable, we can always return to these procedures and develop a more elaborate version of the variable.

We will collapse the values of *age* into three broad categories with an roughly equal number of cases in each category. How can we define these new categories? Begin by running the **Frequencies** command for *age* to inspect the distribution of the scores. I used the cumulative percent column of the frequency distribution to find ages that divided the variable into thirds. I found that 32% of the 2006 GSS sample were younger than 36 and that 65.9% were younger than 53. I decided to use these ages as the dividing point for the new categories, as summarized below:

Ages	Percent of Sample
18–36	32.0%
37–53	33.9%
54–89	34.1%

To recode *age* into these categories, follow these steps:

1. In the **SPSS Data Editor** window, click **Transform** from the menu bar and then click **Recode**. A window will open that gives us two choices: **into same variable** or **into different variable**. If we choose **into same variable**, the new version of the variable will replace the old version—the original version of *age* (with actual years) would disappear. We definitely do *not* want this to

happen, so we will choose (click on) **into different variable.** This option will allow us to keep both the old and new versions of the variable.

2. The **Recode into Different Variable** window will open. A box containing an alphabetical list of variables will appear on the left. Use the cursor to highlight *age* and then click on the arrow button to move the variable to the **Input Variable → Output Variable** box. The input variable is the old version of age, and the output variable is the new, recoded version we will soon create.
3. In the **Output Variable** box on the right, click in the **Name** box and type a name for the new (output) variable. I suggest *ager* (age recoded) for the new variable, but you can assign any name as long as it does not duplicate the name of some other variable in the data set and is no longer than eight characters. Click the **Change** button and the expression *age → ager* will appear in the **Input Variable → Output Variable** box.
4. Click on the **Old and New Values** button in the middle of the screen, and a new dialog box will open. Read down the left-hand column until you find the **Range** button. Click on the button, and the cursor will move to the small box that is immediately below. In these boxes we will specify the low and high points of each interval of the new variable *ager*.
5. Type 18 (the youngest age in the sample) into the left-hand **Range** dialog box and then click on the right-hand box and type 36. In the **New Value** box in the upper-right-hand corner of the screen, click the **Value** button. Type 1 in the **Value** dialog box and then click the **Add** button directly below. The expression 18 – 36 → 1 will appear in the **Old → New** dialog box.
6. Continue recoding by returning to the **Range** dialog boxes on the left. Type 37 in the left-hand box and 53 in the right-hand box and then click the **Value** button in the **New Values** box. Type 2 in the **Value** dialog box and then click the **Add** button. The expression 37 – 53 → 2 appears in the **Old → New** dialog box.
7. Finish the recoding by returning to the **Range** dialog box and entering the value 54 in the left-hand box and 89 in the right-hand box. Click the **Value** button in the **New Values** box. Type 3 in the **Value** dialog box and then click the **Add** button. The expression 54 – 89 → 3 appears in the **Old → New** dialog box.
8. Click the **Continue** button at the bottom of the screen and you will return to the **Recode into Different Variable** dialog box. Click **OK** and SPSS will execute the transformation.

You now have a data set with one more variable named *ager* (or whatever name you gave the recoded variable). SPSS adds the new variable to the data set, and you can find it in the last column at the right in the data window. You can make the new variable a permanent part of the data set by saving the data file at the end of the session. If you do not wish to save the new, expanded data file, click **No** when you are asked if you want to save the data file. If you are using the student version of SPSS for Windows, remember that you are limited to a maximum of 50 variables, and you may not be able to save the new variable.

Using ANOVA to Analyze the Effect of Age on Number of Sex Partners (*partnrs5)*

To demonstrate how to run ANOVA with SPSS, I will conduct a test with recoded age as the independent variable and *partnrs5,* a measure of sexual activity that asks how many different sexual partners the respondent has had over the past five years, as the dependent variable. Note the coding scheme for *partnrs5*

(see Appendix G or click **Utilities** → **Variables** on the SPSS menu bar). The first five scores (0 partners to 4 partners) are actual numbers, but the higher scores represent broad categories (e.g., "5" means the respondent had between 5 and 10 different partners). This variable is a combination of interval-ratio and ordinal scoring, and we will have to treat the means with some caution.

SPSS provides several different ways of conducting the ANOVA test. The procedure summarized below is the most accessible of these, but it still incorporates options and capabilities that we have not covered in this chapter. If you wish to explore these possibilities, please use the online **Help** facility.

To use the ANOVA procedure, click **Analyze, Compare Means,** and then **One-way ANOVA.** The **One-way ANOVA** window appears. Find *partnrs5* in the variable list on the left and click the arrow to move the variable name into the **Dependent List** box. Note that you can request more than one dependent variable at a time. Next, find the name of the recoded *age* variable (perhaps called *ager?*) and click the arrow to move the variable name into the **Factor** box.

Click **Options** and then click the box next to **Descriptive** in the **Statistics** box to request means and standard deviations along with the analysis of variance. Click **Continue** and then click **OK,** and the following output will be produced.

Descriptives

	N	Mean	Std. Deviation	Std. Error
1.00	240	2.50	1.899	.123
2.00	243	1.51	1.271	.082
3.00	253	.99	1.146	.072
Total	736	1.65	1.595	.059

Note: This table has been edited and will not look exactly like the raw SPSS output.

ANOVA
SEX OF THE PARTNER LAST 5 YEARS

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	287.272	2	143.636	66.481	.000
Within Groups	1583.685	733	2.161		
Total	1870.957	735			

The output box labeled **Descriptives** presents the category means and shows, not surprisingly, that the youngest respondents (category 1 or people 18 to 36 years old) had the highest average number of different partners. The oldest respondents (category 3 or people 54 to 89) had the lowest average number of sex partners, and the overall mean was 1.65 (see the Total row).

The output box labeled **ANOVA** includes the various degrees of freedom, all of the sums of squares, the mean square estimates, the *F* ratio (66.481), and, at the far right, the exact probability ("Sig.") of getting these results if the null hypothesis is true. This is reported as 0.000, much lower than our usual alpha level of 0.05. The differences in *partnrs5* for the various age groups are statistically significant.

Your turn.

PROJECT 1: Political Ideology (*polviews*)

STEP 1: Choosing Independent Variables

Select three variables from the 2006 GSS to serve as independent variables. What factors might help to explain why some people are more liberal and some are more conservative? Choose *only* independent variables with three to six scores or categories. Among other possibilities, you might consider education (use *degree*), religious denomination, age (use the recoded version), or social class. Use the **recode** command to collapse independent variables with more than six categories into three or four categories. In general, variables that measure characteristics or traits (like gender or race) will work better than those that measure attitude or opinion (like *cappun* or *gunlaw*), which are more likely to be manifestations of political ideology, not causes.

List your independent variables in the table below:

Variable	SPSS Name	What Exactly Does This Variable Measure?
1		
2		
3		

STEP 2: Stating Hypotheses

For each independent variable, state a hypothesis about its relationship with *polviews*. For example, you might hypothesize that people with greater education will be more liberal or that the more religious will be more conservative. You can base your hypotheses on your own experiences or on information you have acquired in your courses.

Hypotheses:

1.

2.

3.

STEP 3: Getting and Reading the Output

We discussed these tasks in the demonstration above.

STEP 4: Recording Results

Record the results of your ANOVA tests in the table below, using as many rows for each independent variable as necessary. Write the SPSS variable name in the first column and then write the names of the categories of that independent variable in the next column. In the next columns, record the descriptive statistics (mean, standard deviation, and *N*). Write the value of the *F* ratio and, in the far right-hand column, indicate whether or not the results are significant at the 0.05 level. If the value in the "Sig." column of the ANOVA output is less than 0.05, write *yes* in this column. If the value in the "Sig." column of the ANOVA output is more than 0.05, write *no* in this column. Finally, for each independent variable, record the overall mean, standard deviation, and sample size in the row labeled "Totals =."

Independent Variables:	Categories	Mean	Std. Dev.	*N*	*F* ratio	Sig. at 0.05 Level?
1. _____	1.				_____	____
	2.					
	3.					
	4.					
	5.					
	6.					
	Totals =					
2. _____	1.				_____	_____
	2.					
	3.					
	4.					
	5.					
	6.					
	Totals =					
3. _____	1.				_____	_____
	2.					
	3.					
	4.					
	5.					
	6.					
	Totals =					

STEP 6: Interpreting Your Results

Summarize your findings. For each test, write the following.

1. At least one sentence summarizing the test in which you identify the variables being tested, the sample means for each group, *N,* the *F* ratio, and the significance level. In the professional research literature, you might find the results reported as follows: "For a sample of 1,417 respondents, there was no significant difference between the average age of Southerners (46.50), Northerners (44.20), Midwesterners (47.80), and Westerners (47.20) ($F = 1.77$, $p > 0.05$)."
2. A sentence relating to your hypotheses. Were they supported? How?

PROJECT 2: Sexual Activity (*sexfreq*)

STEP 1: Choosing Independent Variables

Select three variables from the 2006 GSS to serve as independent variables. What factors might help to explain why some people are more sexually active than others? Choose *only* independent variables with three to six scores or categories. Among other possibilities, you might consider education, marital status, age (use the recoded version), or social class. Use the **recode** command to collapse independent variables with more than six categories into three or four categories.

List your independent variables in the table below.

Variable	SPSS Name	What Exactly Does This Variable Measure?
1		
2		
3		

STEP 2: Stating Hypotheses

For each independent variable, state a hypothesis about its relationship with *sexfreq*. For example, you might hypothesize that married people would have higher rates of sexual activity than single people (or would it be the other way around?).

Hypotheses:

1.

2.

3.

STEP 3: Getting and Reading the Output

We discussed these tasks in the demonstration above.

STEP 4: Recording Results

Record the results of your ANOVA tests in the table below, using as many rows for each independent variable as necessary. Write the SPSS variable name in the first column and then write the names of the categories of that independent variable in the next column. In the next columns, record the descriptive statistics (mean, standard deviation, and *N*). Write the value of the *F* ratio and, in the far right-hand column, indicate whether or not the results are significant at the 0.05 level. If the value in the "Sig." column of the ANOVA output is less than 0.05, write *yes* in this column. If the value in the "Sig." column of the ANOVA output is more than 0.05, write *no* in this column. Finally, for each independent variable, record the mean, standard deviation, and sample size in the row labeled "Totals =."

Independent Variables:	Categories	Mean	Std. Dev.	*N*	*F* ratio	Sig. at 0.05 Level?
1. _____	1.				_____	____
	2.					
	3.					
	4.					
	5.					
	6.					
	Totals =					
2. _____	1.				_____	_____
	2.					
	3.					
	4.					
	5.					
	6.					
	Totals =					

(continued next page)

Independent Variables:	Categories	Mean	Std. Dev.	*N*	*F* ratio	Sig. at 0.05 Level?
3. _____	1.				_____	_____
	2.					
	3.					
	4.					
	5.					
	6.					
	Totals =					

STEP 6: Interpreting Your Results

Summarize your findings. For each test, write the following.

1. At least one sentence summarizing the test in which you identify the variable being tested, the sample means for each group, *N,* the *F* ratio, and the significance level. In the professional research literature, you might find the results reported as follows: "For a sample of 1,417 respondents, there was no significant difference between the average age of Southerners (46.50), Northerners (44.20), Midwesterners (47.80), and Westerners (47.20) ($F = 1.77$, $p > 0.05$)."
2. A sentence relating to your hypotheses. Were they supported? How?

11 Hypothesis Testing IV
Chi Square

LEARNING OBJECTIVES

By the end of this chapter, you will be able to:

1. Identify and cite examples of situations in which the chi square test is appropriate.
2. Explain the structure of a bivariate table and the concept of independence as applied to expected and observed frequencies in a bivariate table.
3. Explain the logic of hypothesis testing as applied to a bivariate table.
4. Perform the chi square test using the five-step model and correctly interpret the results.
5. Explain the limitations of the chi square test and, especially, the difference between statistical significance and importance.

11.1 INTRODUCTION

The **chi square (χ^2) test** has probably been the most frequently used test of hypothesis in the social sciences, a popularity that is due largely to the fact that the assumptions and requirements in Step 1 of the five-step model are easy to satisfy. Specifically, the test can be conducted with variables measured at the nominal level (the lowest level of measurement). Thus, the chi square test has no restrictions in terms of level of measurement. Also, the test is **nonparametric** or "distribution-free," which means that it requires no assumption at all about the shape of the population or sampling distribution. (See www.cengage.com/sociology/healey for other nonparametric tests of significance.)

These easily satisfied assumptions are an advantage because the decision to reject the null hypothesis (Step 5) is not specific: it means only that one statement in the model (Step 1) *or* the null hypothesis (Step 2) is wrong. Usually, of course, we single out the null hypothesis for rejection. The more certain we are of the model, the greater our confidence that the null hypothesis is the faulty assumption. A "weak" or easily satisfied model means that our decision to reject the null hypothesis can be made with even greater certainty.

Chi square has also been popular for its flexibility. Not only can the test be used with variables at any level of measurement (unlike the ANOVA test covered in Chapter 10), it can also be used with variables that have many values or scores. For example, in Chapter 9 we tested the significance of the difference in the proportions of black and white citizens who were "highly participatory" in volunteer associations. What if the researcher wished to expand the test to include Americans of Hispanic and Asian descent? The two-sample test would no longer be applicable, but chi square handles the more complex variable easily.

11.2 BIVARIATE TABLES

Chi square is computed from **bivariate tables,** so called because they display the scores of cases on two different variables at the same time. Bivariate tables are used to ascertain if there is a significant relationship between the

two variables as well as for other purposes that we will investigate in later chapters. In fact, these tables are very commonly used in research, and a detailed examination of them is in order.

First of all, bivariate tables have (of course) two dimensions. The horizontal (across) dimension is referred to as **rows,** and the vertical dimension (up and down) is referred to as **columns.** Each column or row represents a score on a variable, and the intersections of the row and columns **(cells)** represent the various combined scores on both variables.

Let's use an example to clarify. Suppose a researcher is interested in the relationship between racial group membership and participation in voluntary groups, community-service organizations, and so forth. Do blacks and whites vary in their level of involvement in volunteer groups? We have two variables here (race and number of memberships) and, for the sake of simplicity, assume that both are simple dichotomies; that is, people have been classified as either black or white and as either high or low in their level of involvement in voluntary associations.

By convention, the independent variable (the variable that is taken to be the cause) is placed in the columns and the dependent variable in the rows. In the example at hand, race is the causal variable (the question was, "Is membership *affected by* race?"), and each column will represent a score on this variable. Each row, on the other hand, will represent a score on level of membership (high or low). Table 11.1 displays the outline of the bivariate table for a sample of 100 people.

Note some details of the table. First, subtotals have been added to each column and row. These are called the row or column **marginals,** and, in this case, they tell us that 50 members of the sample were black and 50 were white (the column marginals) and 50 were rated as high in participation and 50 were rated low (the row marginals). Second, the total number of cases in the sample ($N = 100$) is reported at the intersection of the row and column marginals. Finally, take careful note of the labeling of the table. Each row and column is identified, and the table has a descriptive title that includes the names of the variables with the dependent variable listed first. Clear, complete labels and concise titles should be included in *all* tables, graphs, and charts.

As you have noticed, Table 11.1 lacks one piece of crucial information: the numbers of each racial group that rated high or low on the dependent variable. To finish the table, we need to classify each member of the sample in terms of both their race and their level of participation, keep count of how often each combination of scores occurs, and record these numbers in the appropriate cell of the table. Since each of our variables (race and participation rates) has two scores,

TABLE 11.1 RATES OF PARTICIPATION IN VOLUNTARY ASSOCIATIONS BY RACIAL GROUP FOR 100 SENIOR CITIZENS

	Racial Group		
Participation Rates	Black	White	
High			50
Low			50
	50	50	100

there are four possible combinations of scores, each corresponding to a cell in the table. For example, blacks with high levels of participation would be counted in the upper left-hand cell, whites with low levels of participation would be counted in the lower right-hand cell, and so forth. When we are finished counting, each cell will display the number of times each combination of scores occurred.

Finally, note how the bivariate table could be expanded to accommodate variables with more scores. If we wished to include more groups in the test (e.g., Asian Americans or Hispanic Americans), we would simply add additional columns to the table. More elaborate dependent variables could also be easily accommodated. If we had measured participation rates with three categories (e.g., high, moderate, and low) rather than two, we would simply add an additional row to the table.

11.3 THE LOGIC OF CHI SQUARE

Chi square is a test for the independence of the relationship between the variables. We have encountered the term *independence* in connection with the requirements for the two-sample case (Chapter 9) and for the ANOVA test (Chapter 10). In those situations, we noted that independent random samples are gathered such that the selection of a particular case for one sample has no effect on the probability that any particular case will be selected for the other sample.

In the context of chi square, the concept of **independence** takes on a slightly different meaning because it refers to the relationship between the variables, not the samples. Two variables are independent if the classification of a case into a particular category of one variable has no effect on the probability that the case will fall into any particular category of the second variable. For example, race and participation in a voluntary association would be independent of each other if the classification of a person as black or white has no effect on their classification as high or low on participation. In other words, the variables would be independent if level of participation and race were completely unrelated to each other.

Consider Table 11.1 again. If these two variables are truly independent, the cell frequencies will be determined solely by random chance and we would find that, just as an honest coin will show heads about 50% of the time when flipped, about half of the black respondents will rank high on participation and half will rank low. The same pattern would hold for the 50 white respondents, and therefore each of the four cells would have about 25 cases in it, as illustrated in Table 11.2. This pattern of cell frequencies indicates that the racial classification of the subjects has no effect on the probability that they would be either high or low in participation. The probability of being classified as high or low would be 0.5 for both blacks and whites, and the variables would therefore be independent.

TABLE 11.2 THE CELL FREQUENCIES THAT WOULD BE EXPECTED IF RATES OF PARTICIPATION AND RACIAL GROUP WERE INDEPENDENT

	Racial Group		
Participation Rates	Black	White	
High	25	25	50
Low	25	25	50
	50	50	100

The null hypothesis for chi square is that the variables are independent. Under the assumption that the null hypothesis is true, the cell frequencies we would expect to find if only random chance were operating are computed. These frequencies, called **expected frequencies** (symbolized f_e), are then compared, cell by cell, with the frequencies actually observed in the table (**observed frequencies,** symbolized f_o). If the null hypothesis is true and the variables are independent, then there should be little difference between the expected and observed frequencies. If the null hypothesis is false, however, there should be large differences between the two. The greater the differences between expected (f_e) and observed (f_o) frequencies, the less likely that the variables are independent and the more likely that we will be able to reject the null hypothesis.

11.4 THE COMPUTATION OF CHI SQUARE

As with all tests of hypothesis, with chi square we compute a test statistic, χ^2**(obtained),** from the sample data and then place that value on the sampling distribution of all possible sample outcomes. Specifically, the χ^2(obtained) will be compared with the value of χ^2**(critical)** that will be determined by consulting a chi square table (Appendix C) for a particular alpha level and degrees of freedom. Prior to conducting the formal test of hypothesis, let us take a moment to consider the calculation of chi square, as defined by Formula 11.1.

FORMULA 11.1

$$\chi^2(\text{obtained}) = \Sigma\frac{(f_o - f_e)^2}{f_e}$$

Where: f_o = the cell frequencies observed in the bivariate table
f_e = the cell frequencies that would be expected if the variables were independent

We must work on a cell-by-cell basis to solve this formula. The formula tells us to subtract the expected frequency from the observed frequency for each cell, square the result, divide by the expected frequency for that cell, and then sum the resultant values for all cells.

This formula requires an expected frequency for each cell in the table. In Table 11.2, the marginals are the same value for all rows and columns, and the expected frequencies are obvious by intuition: $f_e = 25$ for all four cells. In the more usual case, the expected frequencies will not be obvious, marginals will be unequal, and we must use Formula 11.2 to find the expected frequency for each cell:

FORMULA 11.2

$$f_e = \frac{(\text{Row marginal} \times \text{Column marginal})}{N}$$

That is, the expected frequency for any cell is equal to the total number of cases in the row in which the cell is located (the row marginal) times the total number of cases in the column in which the cell is located (the column marginal) divided by the total number of cases in the table (N).

An example using Table 11.3 should clarify these procedures. A random sample of 100 social work majors have been classified in terms of whether the Council on Social Work Education has accredited their undergraduate programs (the column or independent variable) and whether they were hired in social work positions within three months of graduation (the row or dependent variable).

TABLE 11.3 EMPLOYMENT OF 100 SOCIAL WORK MAJORS BY ACCREDITATION STATUS OF UNDERGRADUATE PROGRAM

	Accreditation Status		
Employment Status	Accredited	Not Accredited	Totals
Working as a social worker	30	10	40
Not working as a social worker	25	35	60
Totals	55	45	100

TABLE 11.4 EXPECTED FREQUENCIES FOR TABLE 11.3

	Accreditation Status		
Employment Status	Accredited	Not Accredited	Totals
Working as a social worker	22	18	40
Not working as a social worker	33	27	60
Totals	55	45	100

TABLE 11.5 COMPUTATIONAL TABLE FOR TABLE 11.3

(1)	(2)	(3)	(4)	(5)
f_o	f_e	$f_o - f_e$	$(f_o - f_e)^2$	$(f_o - f_e)^2/f_e$
30	22	8	64	2.91
10	18	−8	64	3.56
25	33	−8	64	1.94
35	27	8	64	2.37
$N = 100$	$N = 100$	0		χ^2(obtained) = 10.78

Beginning with the upper left-hand cell (graduates of accredited programs who are working as social workers), the expected frequency for this cell, using Formula 11.2, is (40 × 55)/100, or 22. For the other cell in this row (graduates of nonaccredited programs who are working as social workers), the expected frequency is (40 × 45)/100, or 18. For the two cells in the bottom row, the expected frequencies are (60 × 55)/100, or 33, and (60 × 45)/100, or 27, respectively. The expected frequencies for all four cells are displayed in Table 11.4.

The value for chi square for these data can now be found by solving Formula 11.1. It will be helpful to use a computing table, such as Table 11.5, to organize the several steps required to compute chi square. The table lists the observed frequencies (f_o) in column 1 in order from the upper left-hand cell to the lower right-hand cell, moving left to right across the table and top to bottom. Column 2 lists the expected frequencies (f_e) in exactly the same order. Double-check to make sure that you have listed the cell frequencies in the same order for both of these columns. The complete procedure for computing χ^2 is presented in the One Step at a Time box at the end of the box.

ONE STEP AT A TIME **Computing Chi Square**

Step	*Operation*
1.	Prepare a computational table like Table 11.5. List the observed frequencies (f_o) in column 1. The total of column 1 is the number of cases (N).

Find the expected frequencies (f_e) using Formula 11.2.

2.	Start with the upper left-hand cell of the bivariate table and multiply the row marginal by the column marginal.
3.	Divide the quantity you found in Step 2 by N. The result is the expected frequency (f_e) for that cell. Record this value in the second column of your computational table. Double-check to make sure that you record the value of f_e in the same row as the observed frequency for that cell.
4.	Repeat Steps 2 and 3 for each cell in the table. Double-check to make sure that you are using the correct row and column marginals. Record each f_e in column 2 of the computational table.
5.	Find the total of the expected frequencies column. This total *must* equal the total of the observed frequencies column (which is the same as N). If the two totals do not match (within rounding error), recompute the expected frequencies.

Find chi square using Formula 11.1.

6.	For each cell, subtract the expected frequency (f_e) from the observed frequency (f_o) and list these values in the third column of the computational table. Find the total of this column. If this total does not equal zero, you have made a mistake and need to check your computations.
7.	Square each value in the third column of the table and record the result in the fourth column, labeled $(f_o - f_e)^2$.
8.	Divide each value in column 4 by the expected frequency for that cell and record the result in the fifth column, labeled $(f_o - f_e)^2/f_e$.
9.	Find the total of the fifth column. This value is χ^2(obtained).

Note that the totals for columns 1 and 2 (f_o and f_e) are exactly the same. This will always be the case. If the totals do not match, you have made a computational error (probably in the calculation of the expected frequencies). Also note that the sum of column 3 will always be zero, another convenient way to check your math to this point.

This sample value for chi square must still be tested for its significance. *(For practice in computing chi square, see Problem 11.1.)*

11.5 THE CHI SQUARE TEST FOR INDEPENDENCE

As always, the five-step model for significance testing will provide the framework for organizing our decision making. The data presented in Table 11.3 will serve as our example.

Step 1. Making Assumptions and Meeting Test Requirements. Note that we make no assumptions at all about the shape of the sampling distribution.

Model: Independent random samples
Level of measurement is nominal

Step 2. Stating the Null Hypothesis. As stated previously, the null hypothesis in the case of chi square states that the two variables are independent. If the null is true, the differences between the observed and expected frequencies will be

small. As usual, the research hypothesis directly contradicts the null. Thus, if we reject H_0, the research hypothesis will be supported.

H_0: The two variables are independent
(H_1: The two variables are dependent)

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region. The sampling distribution of sample chi squares, unlike the Z and t distributions, is positively skewed, with higher values of sample chi squares in the upper tail of the distribution (to the right). Thus, with the chi square test, the critical region is established in the upper tail of the sampling distribution.

Values for χ^2(critical) are given in Appendix C. This table is similar to the t table, with alpha levels arrayed across the top and degrees of freedom down the side. A major difference, however, is that degrees of freedom (df) for chi square are found by the following formula:

FORMULA 11.3

$$df = (r - 1)(c - 1)$$

Where: r is the number of rows
c is the number of columns

A table with two rows and two columns (a 2 × 2 table) has one degree of freedom regardless of the number of cases in the sample.[1] A table with two rows and three columns would have $(2 - 1)(3 - 1)$, or two degrees of freedom. Our sample problem involves a 2 × 2 table with $df = 1$, so if we set alpha at 0.05, the critical chi square score would be 3.841. Any value for the sample statistic—χ^2(obtained)—greater than 3.841 would cause us to reject the null hypothesis. Summarizing these decisions, we have

Sampling distribution = χ^2 distribution
Alpha = 0.05
Degrees of freedom = 1
χ^2(critical) = 3.841

Step 4. Computing the Test Statistic. The mechanics of these computations were introduced in Section 11.4. As you recall, we had

$$\chi^2(\text{obtained}) = \sum \frac{(f_o - f_e)^2}{f_e}$$

$$\chi^2(\text{obtained}) = 10.78$$

[1]Degrees of freedom are the number of values in a distribution that are free to vary for any particular statistic. A 2 × 2 table has one degree of freedom because, for a given set of marginals, once one cell frequency is determined, all other cell frequencies are fixed (that is, they are no longer free to vary). In Table 11.3, for example, if any cell frequency is known, all others are determined. If the upper left-hand cell is known to be 30, the remaining cell in that row must be 10, since there are 40 cases total in the row and 40 − 30 = 10. Once the frequencies of the cells in the top row are established, cell frequencies for the bottom row are determined by subtraction from the column marginals. Incidentally, this relationship can be used to good advantage when computing expected frequencies. For example, in a 2 × 2 table, only one expected frequency needs to be computed. The f_e's for all other cells can then be found by subtraction.

Step 5. Making a Decision and Interpreting the Results of the Test. Comparing the test statistic with the critical region,

$$\chi^2(\text{obtained}) = 10.78$$
$$\chi^2(\text{critical}) = 3.841$$

we see that the test statistic falls into the critical region, and therefore we reject the null hypothesis of independence. The pattern of cell frequencies observed in Table 11.3 is unlikely to have occurred by chance alone. The variables are dependent. Specifically, based on these sample data, the probability of securing employment in the field of social work is dependent on the accreditation status of the program. *(For practice in conducting and interpreting the chi square test for independence, see Problems 11.2–11.15.)*

Let us stress exactly what the chi square test does and does not tell us. A significant chi square means that the variables are (very likely) dependent on each other in the population: accreditation status makes a difference in whether or not a person is working as a social worker. What chi square does not tell us is the exact nature of the relationship. In our example, it does not tell us if it is the graduates of the accredited programs or the nonaccredited programs who are more likely to be working as social workers. To make this determination, we must perform some additional calculations. We can figure out how the independent variable (accreditation status) is affecting the dependent variable (employment as a social worker) by computing **column percentages** or by calculating percentages within each column of the bivariate table. This procedure is analogous to calculating percentages for frequency distributions (see Chapter 2).

To calculate column percentages, divide each cell frequency by the total number of cases in the column (the column marginal) and multiply the result by 100. For Table 11.3, starting in the upper left-hand cell, we see that there are 30 cases in this cell and 55 cases in the column. In other words, 30 of the 55 graduates of accredited programs are working as social workers. The column percentage for this cell is therefore $(30/55) \times 100 = 54.55\%$. For the lower left-hand cell, the column percentage is $(25/55) \times 100 = 45.45\%$. For the two cells in the right-hand column (graduates of nonaccredited programs), the column percentages are $(10/45) \times 100 = 22.22$ and $(35/45) \times 100 = 77.78$. All column percentages are displayed in Table 10.6.

Column percentages help to make the relationship between the two variables more obvious. Using Table 11.6, we can easily see that nearly 55% of the students from accredited programs are working as social workers versus about 22% of the students from nonaccredited programs. We already knew that this relationship is significant (unlikely to be caused by random chance), and now,

TABLE 11.6 COLUMN PERCENTAGES FOR TABLE 11.3

	Accreditation Status		
Employment Status	Accredited	Not Accredited	Totals
Working as a social worker	54.55%	22.22%	40.00%
Not working as a social worker	45.45%	77.78%	60.00%
Totals	100.00%	100.00%	100.00%
	(55)	(45)	

ONE STEP AT A TIME **Computing Column Percentages**

Step	*Operation*
1.	Start with the upper left-hand cell. Divide the cell frequency (the number of cases in the cell) by the total number of cases in the column (or the column marginal). Multiply the result by 100 to convert to a percentage.
2.	Move down one cell and repeat Step 1. Continue moving down the column until you have converted all cell frequencies to percentages.
3.	Move one column to the right. Start with the cell in the top row and repeat Step 1, making sure that you are using the correct column total in the denominator of the fraction.
4.	Continue moving down this column until you have converted all cell frequencies to percentages.
5.	Continue these operations, moving from one column to the next, until you have converted all cell frequencies to percentages.

with the aid of column percentages, we know how the two variables are related. According to these results, graduating from an accredited program would be a decided advantage for people seeking to enter the social work profession.

Let's summarize by highlighting two points.

1. Chi square is a test of statistical significance. It tests the null hypothesis that the variables are independent in the population. If we reject the null hypothesis, we are concluding, with a known probability of error (equal to the alpha level), that the variables are dependent on each other in the population. In the terms of our example, this means that accreditation status makes a difference in the likelihood of finding work as a social worker. By itself, however, chi square does not tell us the exact nature of the relationship.
2. Computing column percentages allows us to examine the bivariate relationship in more detail. By comparing the column percentages for the various scores of the independent variable, we can see exactly how the independent variable affects the dependent variable. In this case, the column percentages reveal that graduates of accredited programs are more likely to find work as social workers. We will explore column percentages more extensively when we discuss bivariate association in Chapter 12.

Application 11.1

Do members of different groups have different levels of narrow-mindedness? A random sample of 47 white and black Americans have been rated as high or low on a scale that measures intolerance of viewpoints or belief systems different from their own. The results are as follows.

	Group		
Intolerance	White	Black	Totals
High	15	5	20
Low	10	17	27
Totals	25	22	47

(continued next page)

Application 11.1 *(continued)*

The frequencies we would expect to find if the null hypothesis (H_0: the variables are independent) were true are as follows.

	Group		
Intolerance	White	Black	Totals
High	10.64	9.36	20.00
Low	14.36	12.64	27.00
Totals	25.00	22.00	47.00

Expected frequencies are found on a cell-by-cell basis by the formula

$$f_e = (\text{Row marginal} \times \text{Column marginal})/N$$

and the calculation of chi square will be organized into a computational table.

(1)	(2)	(3)	(4)	(5)
f_o	f_e	$f_o - f_e$	$(f_o - f_e)^2$	$(f_o - f_e)^2/f_e$
15	10.64	4.36	19.01	1.79
5	9.36	−4.36	19.01	2.03
10	14.36	−4.36	19.01	1.32
17	12.64	4.36	19.01	1.50
$N = 47$	$N = 47.00$	0.00	χ^2(obtained) = 6.64	

$$\chi^2(\text{obtained}) = 6.64$$

Step 1. Making Assumptions and Meeting Test Requirements.

Model: Independent random samples
Level of measurement is nominal

Step 2. Stating the Null Hypothesis.

H_0: The two variables are independent
(H_1: The two variables are dependent)

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region.

Sampling distribution = χ^2 distribution
Alpha = 0.05
Degrees of freedom = 1
χ^2(critical) = 3.841

Step 4. Computing the Test Statistic.

$$\chi^2(\text{obtained}) = \sum \frac{(f_o - f_e)^2}{f_e}$$

$$\chi^2(\text{obtained}) = 6.64$$

Step 5. Making a Decision and Interpreting the Results of the Test. With an obtained χ^2 of 6.64, we would reject the null hypothesis of independence. For this sample, there is a statistically significant relationship between group membership and intolerance.

To complete the analysis, it would be useful to know exactly how the two variables are related. We can determine this by computing and analyzing column percentages.

	Group		
Intolerance	White	Black	Totals
High	60.00%	22.73%	43.00%
Low	40.00%	77.27%	57.00%
Totals	100.00%	100.00%	100.00%

The column percentages show that 60% of whites in this sample are high on intolerance versus only 23% of blacks. We have already concluded that the relationship is significant, and now we know the pattern of the relationship: the white respondents were more likely to be high on intolerance.

11.6 THE CHI SQUARE TEST: AN ADDITIONAL EXAMPLE

Up to this point, we have confined our attention to 2 × 2 tables. For purposes of illustration, we will work through the computational routines and decision-making process for a larger table. As you will see, larger tables require more computations (because they have more cells), but in all other essentials they are handled in the same way as the 2 × 2 table.

A researcher is concerned with the possible effects of marital status on the academic progress of college students. Do married students, with their extra burden of family responsibilities, suffer academically as compared to unmarried students? Is academic performance dependent on marital status? A random sample of 453 students is gathered, and each student is classified as either married

TABLE 11.7 GRADE POINT AVERAGE (GPA) BY MARITAL STATUS FOR 453 COLLEGE STUDENTS

	Marital Status		
GPA	Married	Not Married	Totals
Good	70	90	160
Average	60	110	170
Poor	45	78	123
Totals	175	278	453

TABLE 11.8 EXPECTED FREQUENCIES FOR TABLE 11.6

	Marital Status		
GPA	Married	Not Married	Totals
Good	61.8	98.2	160
Average	65.7	104.3	170
Poor	47.5	75.5	123
Totals	175.0	278.0	453

TABLE 11.9 COMPUTATIONAL TABLE FOR TABLE 11.6

(1)	(2)	(3)	(4)	(5)
f_o	f_e	$f_o - f_e$	$(f_o - f_e)^2$	$(f_o - f_e)^2/f_e$
70	61.8	8.2	67.24	1.09
90	98.2	−8.2	67.24	0.69
60	65.7	−5.7	32.49	0.49
110	104.3	5.7	32.49	0.31
45	47.5	−2.5	6.25	0.13
78	75.5	2.5	6.25	0.08
$N = 453$	$N = 453.0$	0.0		χ^2(obtained) = 2.79

or unmarried, and—using grade point average (GPA) as a measure—as a good, average, or poor student. Results are presented in Table 11.7.

For the top left-hand cell (married students with good GPAs) the expected frequency would be (160 × 175)/453, or 61.8. For the other cell in this row, expected frequency is (160 × 278)/453, or 98.2. In similar fashion, all expected frequencies are computed (being very careful to use the correct row and column marginals) and displayed in Table 11.8.

The next step is to solve the formula for χ^2(obtained), being very careful to be certain that we are using the proper f_o's and f_e's for each cell. Once again, we will use a computational table (Table 11.9) to organize the calculations and then test the obtained chi square for its statistical significance. Remember that obtained chi square is equal to the total of column 5.

The value of the obtained chi square (2.79) can now be tested for its significance.

Step 1. Making Assumptions and Meeting Test Requirements.

Model: Independent random samples
Level of measurement is nominal

Step 2. Stating the Null Hypothesis.

H_0: The two variables are independent
(H_1: The two variables are dependent)

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region.

Sampling distribution = χ^2 distribution
Alpha = 0.05
Degrees of freedom = $(r - 1)(c - 1) = (3 - 1)(2 - 1) = 2$
χ^2(critical) = 5.991

Step 4. Computing the Test Statistic.

$$\chi^2(\text{obtained}) = \sum \frac{(f_o - f_e)^2}{f_e}$$

$$\chi^2(\text{obtained}) = 2.79$$

Step 5. Making a Decision and Interpreting the Results of the Test. The test statistic, χ^2(obtained) = 2.79, does not fall into the critical region, which, for alpha = 0.05, $df = 2$, begins at χ^2(critical) of 5.991. Therefore, we fail to reject the null. The observed frequencies are not significantly different from the frequencies we would expect to find if the variables were independent and only random chance were operating. Based on these sample results, we can conclude that the academic performance of college students is not dependent on their marital status.

Even though we failed to reject the null hypothesis, we still might compute column percentages to see if there is any pattern to the relationship. To do this, divide each cell frequency by its column total and then multiply by 100. Starting with the upper left-hand cell, we see that 70 of the 175 married students have "good" GPAs. The column percentage for this cell is $(70/175) \times 100 = 40.00\%$. For the middle cell in this column (married students with "average" GPAs), the column percentage is $(60/175) \times 100 = 34.28\%$, and for the bottom cell, the column percentage is $(45/175) \times 100 = 25.71\%$. Repeat these operations for the right-hand column, and make sure you are using the correct column marginal in your computations. Table 11.10 presents the column percentages for Table 11.7

TABLE 11.10 COLUMN PERCENTAGES FOR TABLE 11.7

	Marital Status		
GPA	Married	Not Married	Totals
Good	40.00%	32.37%	160
Average	34.29%	39.57%	170
Poor	25.71%	28.06%	123
Totals	100.00%	100.00%	453
	(175)	(278)	

The differences in percentage from column to column are relatively small (and not significant), but we can see that married students are slightly more likely to have good GPAs (40% vs. about 32%) and that nonmarried students have a very small tendency to have more poor GPAs.

11.7 THE LIMITATIONS OF THE CHI SQUARE TEST

Like any other test, chi square has limits, and you should be aware of several potential difficulties. First, even though chi square is very flexible and handles many different types of variables, it becomes difficult to interpret when the variables have many categories. For example, two variables with five categories each would generate a 5 × 5 table with 25 cells—far too many combinations of scores to be easily absorbed or understood. As a very rough rule of thumb, the chi square test is easiest to interpret and understand when both variables have four or fewer scores.

Two further limitations of the test are related to sample size. When sample size is small, it can no longer be assumed that the sampling distribution of all possible sample outcomes is accurately described by the chi square distribution. For chi square, a small sample is defined as one in which a high percentage of the cells have expected frequencies (f_e) of 5 or less. Various rules of thumb have been developed to help the researcher decide what constitutes a "high percentage of cells." Probably the safest course is to take corrective action whenever *any* of the cells have expected frequencies of 5 or less.

In the case of 2 × 2 tables, the value of χ^2(obtained) can be adjusted by applying Yates's correction for continuity, the formula for which is

FORMULA 11.4

$$\chi_c^2 = \Sigma \frac{(|f_o - f_e| - 0.5)^2}{f_e}$$

Where: χ_c^2 = corrected chi square
$|f_o - f_e|$ = the absolute values of the difference between the observed and expected frequency for each cell

The correction factor is applied by reducing the absolute value[2] of the term $(f_o - f_e)$ by 0.5 before squaring the difference and dividing by the expected frequency for the cell.

For tables larger than 2 × 2, there is no correction formula for computing χ^2(obtained) for small samples. It may be possible to combine some of the categories of the variables and thereby increase cell sizes. Obviously, however, this course of action should be taken only when it is sensible to do so. In other words, distinctions that have clear theoretical justifications should not be erased merely to conform to the requirements of a statistical test. When you feel that categories cannot be combined to build up cell frequencies, and the percentage of cells with expected frequencies of 5 or less is small, it is probably justifiable to continue with the uncorrected chi square test as long as the results are regarded with a suitable amount of caution.

A second potential problem related to sample size occurs with large samples. I pointed out in Chapter 9 that all tests of hypothesis are sensitive to sample size. That is, the probability of rejecting the null hypothesis increases as the number of cases increases, regardless of any other factor. It turns out that chi square is especially sensitive to sample size and that larger samples may lead to

[2]Absolute values ignore plus and minus signs.

the decision to reject the null when the actual relationship is trivial. In fact, chi square is more responsive to changes in sample size than other test statistics, since the value of χ^2(obtained) will increase at the same rate as sample size. That is, if sample size is doubled, the value of χ^2(obtained) will be doubled. (For an illustration of this principle, see Problem 11.14.)

You should be aware of this relationship between sample size and the value of chi square because it, once again, raises the distinction between statistical significance and theoretical importance. On one hand, tests of significance play a crucial role in research. As long as we are working with random samples, we must know if our research results could have been produced by mere random chance.

BECOMING A CRITICAL CONSUMER: Reading the Professional Literature

As was the case with ANOVA, it is extremely unlikely that you would encounter a chi square test in everyday life or in the popular media, so I will again confine this section to the professional research literature. The article I will use as an example addresses the impact of advances in technology and communication on gender inequality in developing nations: Does access to the Internet reduce the productivity gap between male and female scientists? The table below shows some of the differences between men and women in a sample of over 1,000 scientists drawn from Ghana, Kenya, and India in two different years.

As usual, results are presented in highly abbreviated form. The table below presents results using both chi square tests (for the percentage with access to computers and email) and *t* tests (for average numbers of external and local contacts and articles published in journals). The significance of the relationships are indicated using asterisks (**) after the statement of the sample statistics. Looking at the results for 2000, we can see that the gender differences in access to computers and email were *not* significant but that differences in number of external and local contacts and average number of publications *were* significant. In other words, male and female scientists in these nations had equal access to technology and communication channels, but men had significantly more external contacts and publications.

Access to Resources and Productivity by Gender and Time Period

	1994		2000	
Variable	Male	Female	Male	Female
% with access to personal computers	62.7%	55.0%	77.5%	72.1%
% with access to email	3.0%	5.0%	67.8%	65.1%
Number of external contacts	2.37	1.81	0.83	0.48**
Number of local contacts	2.16	2.32	1.37	1.90**
Articles in international journals	2.33	1.30	2.26	1.05**

**$p < 0.01$

These findings are interesting, in part, because they seem to show that globalization and the spread of the Internet impacts men and women differently. Women scientists are significantly more tied to their local area, less physically mobile, and less productive than male scientists. The great promise of modern technology seems to benefit males more than females, a pattern that is consistent with persistent gender inequality in the developing world, and, indeed, around the globe. Want to learn more? The citation for the article is given below.

Miller, Paige, Soorymoorthy, R., Anderson, Meredith, Palackal, Antony, and Shrum, Wesley. 2006. "Gender and Science in Developing Areas: Has the Internet Reduced Inequality?" *Social Science Quarterly* 87: 679–689.

On the other hand, like any other statistical technique, tests of hypothesis are limited in the range of questions they can answer. Specifically, these tests will tell us whether our results are statistically significant or not. They will not necessarily tell us if the results are important in any other sense. To deal more directly with questions of importance, we must use an additional set of statistical techniques called measures of association. We previewed these techniques in this chapter when we used column percentages and measures of association, and they will be the subject of Part III of this text.

SUMMARY

1. The chi square test for independence is appropriate for situations in which the variables of interest have been organized into table format. The null hypothesis is that the variables are independent or that the classification of a case into a particular category on one variable has no effect on the probability that the case will be classified into any particular category of the second variable.
2. Since chi square is nonparametric and requires only nominally measured variables, its model assumptions are easily satisfied. Furthermore, since it is computed from bivariate tables in which the number of rows and columns can be easily expanded, the chi square test can be used in many situations in which other tests are inapplicable.
3. In the chi square test, we first find the frequencies that would appear in the cells if the variables were independent (f_e) and then compare those frequencies, cell by cell, with the frequencies actually observed in the cells (f_o). If the null is true, expected and observed frequencies should be quite close in value. The greater the difference between the observed and expected frequencies, the greater the possibility of rejecting the null.
4. The chi square test has several important limitations. It is often difficult to interpret when tables have many (more than four or five) dimensions. Also, as sample size (N) decreases, the chi square test becomes less trustworthy, and corrective action may be required. Finally, with very large samples, we may declare relatively trivial relationships to be statistically significant. As is the case with all tests of hypothesis, statistical significance is not the same thing as "importance" in any other sense. As a general rule, statistical significance is a necessary but not sufficient condition for theoretical or practical importance.

SUMMARY OF FORMULAS

FORMULA 11.1 Chi square (obtained): $\chi^2(\text{obtained}) = \sum \frac{(f_o - f_e)^2}{f_e}$

FORMULA 11.2 Expected frequencies: $f_e = (\text{Row marginal} \times \text{Column marginal})/N$

FORMULA 11.3 Degrees of freedom, bivariate tables: $df = (r - 1)(c - 1)$

FORMULA 11.4 Yates's correction for continuity: $\chi_c^2 = \sum \frac{(|f_o - f_e| - 0.5)^2}{f_e}$

GLOSSARY

Bivariate table. A table that displays the joint frequency distributions of two variables.

Cells. The cross-classification categories of the variables in a bivariate table.

χ^2(critical). The score on the sampling distribution of all possible sample chi squares marking the beginning of the critical region.

χ^2(obtained). The test statistic as computed from sample results.

Chi square test. A nonparametric test of hypothesis for variables that have been organized into a bivariate table.

Column. The vertical dimension of a bivariate table. By convention, each column represents a score on the independent variable.

Column percentages. Percentages calculated with each column of a bivariate table.

Expected frequency (f_e). The cell frequencies that would be expected in a bivariate table if the variables were independent.

Independence. The null hypothesis in the chi square test. Two variables are independent if, for all cases, the classification of a case on one variable has no effect on the probability that the case will be classified in any particular category of the second variable.

Marginals. The row and column subtotals in a bivariate table.

Nonparametric. A "distribution-free" test. These tests do not assume a normal sampling distribution.

Observed frequency (f_o). The cell frequencies actually observed in a bivariate table.

Row. The horizontal dimension of a bivariate table, conventionally representing a score on the dependent variable.

PROBLEMS

(Problems are labeled with the social science discipline from which they are drawn: SOC for sociology, SW for social work, PS for political science, CJ for criminal justice, PA for public administration, and GER for gerontology.)

11.1 For each table below, calculate the obtained chi square. (*HINT: Calculate the expected frequencies for each cell with Formula 11.2. Double-check to make sure you are using the correct row and column marginals for each cell. It may be helpful to record the expected frequencies in table format as well: see Tables 11.2, 11.4, and 11.7. Next, use a computational table to organize the calculation for Formula 11.1: see Tables 11.5 and 11.9. For each cell subtract expected frequency from observed frequency and record the result in column 3. Square the value in column 3 and record the result in column 4, and then divide the value in column 4 by the expected frequency for that cell and record the result in column 5. Remember that the sum of column 5 in the computational table is obtained chi square. As you proceed, double-check to make sure that you are using the correct values for each cell.*)

a.

20	25	45
25	20	45
45	45	90

b.

10	15	25
20	30	50
30	45	75

c.

25	15	40
30	30	60
55	45	100

d.

20	45	65
15	20	35
35	65	100

11.2 SOC A sample of 25 cities have been classified as high or low on their homicide rates and on the number of handguns sold within the city limits. Is there a relationship between these two variables? Explain your results in a sentence or two.

	Homicide Rate		
Volume of Gun Sales	Low	High	Totals
High	8	5	13
Low	4	8	12
Totals	12	13	25

11.3 SW A local politician is concerned that a program for the homeless in her city is discriminating against blacks and other minorities. The data below were taken from a random sample of black and white homeless people.

	Race		
Received Services?	Black	White	Totals
Yes	6	7	13
No	4	9	13
Totals	10	16	26

a. Is there a statistically significant relationship between race and whether or not the person has received services from the program?
b. Compute column percentages for the table to determine the pattern of the relationship. Which group was more likely to get services?

11.4 PS Many analysts have noted a "gender gap" in elections for the U.S. presidency, with women more likely to vote for the Democratic candidate. A sample of university faculty were asked about their political party preference. Do their responses indicate a significant relationship between gender and party preference?

	Gender		
Party Preference	Male	Female	Totals
Democrats	10	15	25
Republicans	15	10	25
Totals	25	25	50

a. Is there a statistically significant relationship between gender and party preference?
b. Compute column percentages for the table to determine the pattern of the relationship. Which gender is more likely to prefer the Democrats?

11.5 PA Is there a relationship between salary levels and unionization for public employees? The data below represent this relationship for fire departments in a random sample of 100 cities of roughly the same size. Salary data have been dichotomized at the median. Summarize your findings.

	Status		
Salary	Union	Non-union	Totals
High	21	29	50
Low	14	36	50
Totals	35	65	100

a. Is there a statistically significant relationship between these variables?
b. Compute column percentages for the table to determine the pattern of the relationship. Which group was more likely to get high salaries?

11.6 SOC A program of pet therapy has been running at a local nursing home. Are the participants in the program more alert and responsive than nonparticipants? The results, drawn from a random sample of residents, are reported below.

	Status		
Alertness	Participants	Nonparticipants	Totals
High	23	15	38
Low	11	18	29
Totals	34	33	67

a. Is there a statistically significant relationship between participation and alertness?
b. Compute column percentages for the table to determine the pattern of the relationship. Which group was more likely to be alert?

11.7 SOC The state department of education has rated a sample of local school systems for compliance with state-mandated guidelines for quality. Is the quality of a school system significantly related to the affluence of the community as measured by per capita income?

	Per Capita Income		
Quality	Low	High	Totals
Low	16	8	24
High	9	17	26
Totals	25	25	50

a. Is there a statistically significant relationship between these variables?
b. Compute column percentages for the table to determine the pattern of the relationship. Are high or low income communities more likely to have high-quality schools?

11.8 CJ A local judge has been allowing some individuals convicted of "driving under the influence" to work in a hospital emergency room as an alternative to fines, suspensions, and other penalties. A random sample of offenders has been drawn. Do participants in this program have lower rates of recidivism for this offense?

	Status		
Recidivist?	Participants	Nonparticipants	Totals
Yes	60	123	183
No	55	108	163
Totals	115	231	346

a. Is there a statistically significant relationship between these variables?

b. Compute column percentages for the table to determine the pattern of the relationship. Which group is more likely to be re-arrested for driving under the influence?

11.9 SOC Is there a relationship between length of marriage and satisfaction with marriage? The necessary information has been collected from a random sample of 100 respondents drawn from a local community. Write a sentence or two explaining your decision.

	Length of Marriage (in years)			
Satisfaction	Less than 5	5–10	More than 10	Totals
Low	10	20	20	50
High	20	20	10	50
Totals	30	40	30	100

a. Is there a statistically significant relationship between these variables?

b. Compute column percentages for the table to determine the pattern of the relationship. Which group is more likely to be highly satisfied?

11.10 PS Is there a relationship between political ideology and class standing? Are upper-class students significantly different from underclass students on this variable? The table below reports the relationship between these two variables for a random sample of 267 college students.

	Class Standing		
Ideology	Underclass	Upper-Class	Totals
Liberal	43	40	83
Moderate	50	50	100
Conservative	40	44	84
Totals	133	134	267

a. Is there a statistically significant relationship between these variables?

b. Compute column percentages for the table to determine the pattern of the relationship. Which group is more likely to be conservative?

11.11 SOC At a large urban college, about half of the students live off campus in various arrangements, and the other half live in dormitories on campus. Is academic performance dependent on living arrangements? The results based on a random sample of 300 students are presented below.

	Residential Status			
GPA	Off Campus with Roommates	Off Campus with Parents	On Campus	Totals
Low	22	20	48	90
Moderate	36	40	54	130
High	32	10	38	80
Totals	90	70	140	300

a. Is there a statistically significant relationship between these variables?

b. Compute column percentages for the table to determine the pattern of the relationship. Which group is more likely to have a high GPA?

11.12 SOC An urban sociologist has built up a database describing a sample of the neighborhoods in her city and has developed a scale by which each area can be rated for the "quality of life" (this includes measures of pollution, noise, open space, services available, and so on). She has also asked samples of residents of these areas about their level of satisfaction with their neighborhoods. Is there significant agreement between the sociologist's objective ratings of quality and the respondents' self-reports of satisfaction?

	Quality of Life			
Satisfaction	Low	Moderate	High	Totals
Low	21	15	6	42
Moderate	12	25	21	58
High	8	17	32	57
Totals	41	57	59	157

a. Is there a statistically significant relationship between these variables?

b. Compute column percentages for the table to determine the pattern of the relationship. Which group is most likely to say that their satisfaction is high?

11.13 SOC Does support for the legalization of marijuana vary by region of the country? The table displays the relationship between the two variables for a random sample of 1,020 adult citizens. Is the relationship significant?

	Region				
Legalize?	North	Midwest	South	West	Totals
Yes	60	65	42	78	245
No	245	200	180	150	775
Totals	305	265	222	228	1,020

a. Is there a statistically significant relationship between these variables?

b. Compute column percentages for the table to determine the pattern of the relationship. Which region is most likely to favor the legalization of marijuana?

11.14 SOC A researcher is concerned with the relationship between attitudes toward violence and violent behavior. If attitudes "cause" behavior (a very debatable proposition), then people who have positive attitudes toward violence should have high rates of violent behavior. A pretest was conducted on 70 respondents and, among other things, the respondents were asked, "Have you been involved in a violent incident of any kind over the past six months?" The researcher established the following relationship.

	Attitude Toward Violence		
Involvement	Favorable	Unfavorable	Totals
Yes	16	19	35
No	14	21	35
Totals	30	40	70

The chi square calculated on these data is 0.23, which is not significant at the 0.05 level (confirm this conclusion with your own calculations). Undeterred by this result, the researcher proceeded with the project and gathered a random sample of 7,000. In terms of percentage distributions, the results for the full sample were exactly the same as for the pretest.

	Attitude Toward Violence		
Involvement	Favorable	Unfavorable	Totals
Yes	1,600	1,900	3,500
No	1,400	2,100	3,500
Totals	3,000	4,000	7,000

However, the chi square obtained is a very healthy 23.4 (confirm with your own calculations). Why is the full-sample chi square significant when the pretest was not? What happened? Do you think that the second result is important?

11.15 SOC Some results from a survey administered to a nationally representative sample are presented below. For each table, conduct the chi square test of significance and compute column percentages. Write a sentence or two of interpretation for each test.

a. Support for the legal right to an abortion for any reason by age:

	Age			
Support?	Younger than 30	30–49	50 and Older	Totals
Yes	154	360	213	727
No	179	441	429	1,049
Totals	333	801	642	1,776

b. Support for the death penalty for people convicted of homicide by age:

	Age			
Support?	Younger than 30	30–49	50 and Older	Totals
Favor	361	867	675	1,903
Oppose	144	297	252	693
Totals	505	1,164	927	2,596

c. Fear of walking alone at night by age:

	Age			
Fear?	Younger than 30	30–49	50 and Older	Totals
Yes	147	325	300	772
No	202	507	368	1,077
Totals	349	832	668	1,849

d. Support for legalizing marijuana by age:

	Age			
Legalize?	Younger than 30	30–49	50 and Older	Totals
Should	128	254	142	524
Should Not	224	534	504	1,262
Totals	352	788	646	1,786

e. Support for suicide when a person has an incurable disease by age:

	Age			
Support?	Younger than 30	30–49	50 and Older	Totals
Yes	225	537	367	1,129
No	107	270	266	643
Totals	332	807	633	1,772

YOU ARE THE RESEARCHER: Understanding Political Beliefs

Two projects are presented below, and you are urged to complete both to apply your understanding of the chi square test. In the first, you will examine the sources of people's beliefs about some of the most hotly debated topics in U.S. society: capital punishment, assisted suicide, gay marriage, and immigration. In the second, you will compare various independent variables to see which has the most significant relationship with your chosen dependent variable.

We will use a new procedure called **Crosstabs** to produce bivariate tables, chi square, and column percentages. This procedure is very commonly used in social science research at all levels, and you will see many references to **Crosstabs** in chapters to come.

Begin by clicking **Analyze,** then click **Descriptive Statistics** and **Crosstabs.** The **Crosstabs** dialog box will appear with the variables listed in a box on the left. Highlight the name(s) of your dependent variable(s) and click the arrow to move the variable name into the **Rows** box. Next, find the name of your independent variable(s) and move it into the **Columns** box. SPSS will process all combinations of variables in the row and column boxes at one time.

Click the **Statistics** button at the bottom of the window and click the box next to chi-square. Return to the **Crosstabs** window, click the **Cells** button, and select "column" in the **Percentages** box. This will generate column percentages for the table. Return to the **Crosstabs** window and click **Continue** and **OK** to produce your output.

I will demonstrate this command by examining the relationship between gender (*sex*) and support for legal abortion "for any reason" (*abany*). Gender is my independent variable, and I listed it in the column box; I placed *abany* in the row box. Here is the output:

ABANY ABORTION IF WOMAN WANTS FOR ANY REASON * SEX RESPONDENTS SEX CROSSTABULATION

			RESPONDENTS SEX		
			MALE	FEMALE	Total
ABORTION IF WOMAN WANTS FOR ANY REASON	YES	Count % within RESPONDENTS SEX	128 46.9%	148 41.5%	276 43.8%
	NO	Count % within RESPONDENTS SEX	145 53.1%	209 58.5%	354 56.2%
	Total	Count % within RESPONDENTS SEX	273 100.0%	357 100.0%	630 100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	1.853[a]	1	.173		
Continuity Correction[b]	1.639	1	.200		
Likelihood Ratio	1.852	1	.174		
Fisher's Exact Test				.195	.100
Linear-by-Linear Association	1.850	1	.174		
N of Valid Cases	630				

[a]0 cells (0%) have expected count less than 5. The minimum expected count is 119.60. [b]Computed only for a 2 × 2 table.

Read the crosstab table cell by cell. Each cell displays the number of cases in the cell and its column percentage. For example, starting with the upper left-hand cell, there were 128 respondents who were male and who said "yes" to *abany,* and these were 46.9% of all men in the sample. In contrast, 148 of the females (41.5%) also supported legal abortion "for any reason." We can see immediately that the column percentages are similar and that *sex* and *abany* will not have a significant relationship.

The results of the chi square test are reported in the output block that follows the table. The value of chi square (obtained) is 1.853, and there was 1 degree of freedom. The exact significance of the chi square, reported in the column labeled "Asymp. Sig (2-sided)," is 0.173. This is well above the standard indicator of a significant result (alpha = 0.05), so we may conclude, as we saw with the column percentages, that there is no statistically significant relationship between these variables. Support for legal abortion is not dependent on gender.

PROJECT 1: Explaining Beliefs

In this project, you will analyze beliefs about capital punishment (*cappun*), assisted suicide (*letdie1*), gay marriage (*marhomo*), and immigration (*letin1*). You will select an independent variable, use SPSS to generate chi squares and column percentages, and analyze and interpret your results.

STEP 1: Choose an Independent Variable

Select an independent variable that seems likely to be an important cause of people's attitudes about the death penalty, assisted suicide, gay marriage, and immigration. Be sure to select an independent variable that has *only* two to five categories. If you select an independent variable with more than five scores, use the **recode** command to reduce the number of categories. You might consider gender, level of education (use *degree*), religion, or age (the recoded version, as discussed in Chapter 10) as possible independent variables, but there are many others. Record the variable name and state exactly what the variable measures in the space below:

SPSS Name	What Exactly Does This Variable Measure?

STEP 2: Stating Hypotheses

State hypotheses about the relationships you expect to find between your independent variable and each of the four dependent variables. State these hypotheses in terms of which category of the independent variable you expect to be associated with which category of the dependent variable (for example, "I expect that men will be more supportive of the legal right to an abortion for any reason.").

1.
2.
3.
4.

STEP 3: Running Crosstabs

Click **Analyze → Descriptives → Crosstabs.** Place the four dependent variables (*cappun, letdie1, letin1,* and *marhomo*) in the **Rows:** box and the independent variable you selected in the **Columns:** box. Click the **Statistics** button to get chi square and the **Cells** button for column percentages.

STEP 4: Recording Results

Your output will consist of four tables, and it will be helpful to summarize your results in the following table. Remember that the significance of the relationship is found in the column labeled "Asymp. Sig (2-sided)" in the second box in the output.

Dependent Variable	Chi Square	Degrees of Freedom	Significance
cappun			
letdie1			
letin1			
marhomo			

STEP 5: Analyzing and Interpreting Results

Write a short summary of results for each test in which you do the following.

1. Identify the variables being tested, the value and significance of chi square, *N,* and the pattern (if any) of the column percentages. In the professional research literature, you might find the results reported as follows: For a sample of 630 respondents, there was no significant relationship between gender and support for abortion (Chi square = 1.853, $df = 1$, $p > 0.05$). About 47% of the men supported the legal right to an abortion "for any reason" versus about 42% of the women.
2. Explain if your hypotheses were supported, and if relevant, how they were supported.

PROJECT 2: Exploring the Impact of Various Independent Variables

In this project, you will examine the relative ability of a variety of independent variables to explain or account for a single dependent variable. You will again use the

Crosstabs procedure in SPSS to generate chi squares and column percentages and use the value of alpha to judge which independent variable has the most important relationship with your dependent variable.

STEP 1: Choosing Variables

Select a dependent variable. You may use any of the four from Project 1 or select a new dependent variable from the 2006 GSS. Be sure that your dependent variable has no more than five values or scores. Use the **recode** command as necessary to reduce the number of categories. Good choices for dependent variables include any measure of attitudes or opinions. *Do not* select characteristics such as race, sex, or religion as dependent variables.

Select three independent variables that seem likely to be important causes of the dependent variable you selected. Your independent variable should have no more than five or six categories. You might consider gender, level of education, religiosity, or age (the recoded version as seen in Chapter 10) as possibilities, but there are many others.

Record the variable names and state exactly what each variable measures in the table below.

SPSS Name	What Exactly Does This Variable Measure?
Dependent Variable	
Independent Variables	

STEP 2: Stating Hypotheses

State hypotheses about the relationships you expect to find between your independent variables and the dependent variable. State these hypotheses in terms of which category of the independent variable you expect to be associated with which category of the dependent variable (for example, "I expect that men will be more supportive of the legal right to an abortion for any reason.").

1.

2.

3.

STEP 3: Running Crosstabs

Click **Analyze → Descriptives → Crosstabs.** Place your dependent variable in the **Rows:** box and all three of your independent variables in the **Columns:** box. Click the **Statistics** button to get chi square and the **Cells** button for column percentages.

STEP 4: Recording Results

Your output will consist of three tables, and it will be helpful to summarize your results in the following table. Remember that the significance of the relationship is found in the column labeled "Asymp. Sig (2-sided)" in the second box in the output.

Independent Variables	Chi square	Degrees of Freedom	Significance

STEP 5: Analyzing and Interpreting Results

Write a short summary of results of each test in which you do the following.

1. Identify the variables being tested, the value and significance of chi square, *N,* and the pattern (if any) of the column percentages. In the professional research literature, you might find the results reported as follows: For a sample of 630 respondents, there was no significant relationship between gender and support for abortion (Chi square = 1.853, $df = 1$, $p > 0.05$). About 47% of the men supported the legal right to an abortion "for any reason" versus about 42% of the women.
2. Explain if your hypotheses were supported, and if relevant, how they were supported.
3. Explain which independent variable had the most significant relationship (lowest value in the "Asymp. Sig 2-tailed" column) with your dependent variable.

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Part III

Bivariate Measures of Association

The chapters in Part III cover the computation and analysis of a class of statistics known as measure of association. These statistics are extremely useful in scientific research and are commonly reported in the professional literature. They provide, in a single number, an indication of the strength and—if applicable—direction of a bivariate relationship.

It is important to remember the difference between statistical significance, covered in Part II, and association, the topic of Part III. Tests for statistical significance provide answers to certain questions: Were the differences or relationships observed in the sample caused by mere random chance? What is the probability that the sample results reflect patterns in the populations from which the samples were selected? Measures of association address a different set of questions: How strong is the relationship between the variables? What is the direction or pattern of the relationship?

Thus, the information provided by measures of association complements tests of significance. Association and statistical significance are two different things, and while the most satisfying results are those that are *both* statistically significant and strong, it is common to find mixed results: relationships that are statistically significant, but weak; not statistically significant, but strong; and so forth.

Chapter 12 introduces the basic ideas behind analysis of association in terms of bivariate tables and column percentages and presents measures of association appropriate for nominal level variables. Chapter 13 presents measures of association for variables measured at the ordinal level, and Chapter 14 presents Pearson's *r,* the most important measure of association and the only one designed for interval-ratio level variables.

12

Introduction to Bivariate Association and Measures of Association for Variables Measured at the Nominal Level

LEARNING OBJECTIVES

By the end of this chapter, you will be able to:

1. Explain how we can use measures of association to describe and analyze the importance of relationships (vs. their statistical significance).
2. Define *association* in the context of bivariate tables and in terms of changing conditional distributions.
3. List and explain the three characteristics of a bivariate relationship: existence, strength, and pattern or direction.
4. Investigate a bivariate association by properly calculating percentages for a bivariate table and interpreting the results.
5. Compute and interpret measures of association for variables measured at the nominal level.

12.1 STATISTICAL SIGNIFICANCE AND THEORETICAL IMPORTANCE

As we have seen over the past several chapters, tests of statistical significance are extremely important in social science research. As long as social scientists must work with random samples rather than populations, these tests are indispensable for dealing with the possibility that our research results are the products of mere random chance. However, tests of significance are, typically, only the first step in the analysis of research results. These tests do have limitations, and statistical significance is not necessarily the same thing as relevance or importance. Furthermore, all tests of significance are affected by sample size: tests performed on large samples may result in decisions to reject the null hypothesis when, in fact, the observed differences are quite minor.

Now we will turn our attention to **measures of association.** Whereas tests of significance detect nonrandom relationships, measures of association provide information about the strength and direction of relationships, and this information allows us to assess the importance of relationships and test the power and validity of our theories. The theories that guide scientific research are almost always stated in cause-and-effect terms (for example, variable *X* causes variable *Y*). For example, recall our discussion of the contact hypothesis in Chapter 1. In that theory, the causal (or independent) variable was equal status contacts between groups and the effect (or dependent) variable was the level of individual prejudice. The theory asserts that equal-status contact between members of different groups *causes* prejudice to decline. If the theory is true, we should expect to find a strong relationship between variables that measure equal status contacts and variables that measure prejudice. Furthermore, we should find that prejudice declines as involvement increases. Measures of association help us trace

causal relationships between variables, and they are our most important and powerful statistical tools for documenting, measuring, and analyzing cause and effect relationships.

As useful as they are, measures of association, like any class of statistics, do have their limitations. Most importantly, these statistics cannot *prove* that two variables are causally related. Even if there is a strong (and statistically significant) association between two variables, we cannot necessarily conclude that one variable is a cause of the other. A common adage in the social sciences is that correlation (or association) is not the same thing as causation, and you would do well to keep this caution in mind. We can use a statistical association between variables as evidence for a causal relationship, but association by itself is not proof that a causal relationship exists.

Another important use for measures of association is prediction. If two variables are associated, we can predict the score of a case on one variable from the score of that case on the other variable. For example, if equal status contacts and prejudice are associated, we can predict that people who have experienced many such contacts will be less prejudiced than those who have had few or no contacts. Note that *prediction* and *causation* can be two separate things. If variables are associated, we can predict from one to the other even if the variables are not causally related.

This chapter will introduce the concept of **association** between variables in the context of bivariate tables and then demonstrate how to use percentages to analyze associations between variables. We will then proceed to the logic, calculation, and interpretation of several widely used measures of association. By the end of this chapter, you will have an array of statistical tools you can use to analyze the strength and direction of associations between variables.

12.2 ASSOCIATION BETWEEN VARIABLES AND BIVARIATE TABLES

Most generally, two variables are said to be associated if the distribution of one of them changes under the various categories or scores of the other. For example, suppose that an industrial sociologist was concerned with the relationship between job satisfaction and productivity for assembly-line workers. If these two variables are associated, then scores on productivity will change under the different conditions of satisfaction. Highly satisfied workers will have different scores on productivity than do workers who are low on satisfaction, and levels of productivity will vary by levels of satisfaction.

This relationship will become clearer with the use of bivariate tables. As you recall (see Chapter 11), bivariate tables display the scores of cases on two different variables. By convention, the **independent** or ***X* variable** (that is, the variable taken as causal) is arrayed in the columns and the **dependent** or ***Y* variable** in the rows.[1] That is, each column of the table (the vertical dimension) represents a score or category of the independent variable (X), and each row (the horizontal dimension) represents a score or category of the dependent variable (Y).

Table 12.1 displays a relationship between productivity and job satisfaction for a fictitious sample of 173 factory workers. We focus on the columns to detect the presence of an association between variables displayed in table format. Each column shows the pattern of scores on the dependent variable for each score

[1]In the material that follows, we will often, for the sake of brevity, refer to the independent variable as X and the dependent variable as Y.

TABLE 12.1 PRODUCTIVITY BY JOB SATISFACTION (frequencies)

	Job Satisfaction (*X*)			
Productivity (*Y*)	Low	Moderate	High	Totals
Low	30	21	7	58
Moderate	20	25	18	63
High	10	15	27	52
Totals	60	61	52	173

on the independent variable. For example, the left-hand column indicates that 30 of the 60 workers who were low on job satisfaction were low on productivity, 20 were moderately productive, and 10 were highly productive. The middle column shows that 21 of the 61 moderately satisfied workers were low on productivity, 25 were moderately productive, and 15 were high on productivity. Of the 52 workers who are highly satisfied (the right-hand column), 7 were low on productivity, 18 were moderate, and 27 were high.

By reading the table from column to column we observe the effects of the independent variable on the dependent variable (provided, of course, that the table is constructed with the independent variable in the columns). These "within-column" frequency distributions are called the **conditional distributions of *Y*,** since they display the distribution of scores on the dependent variable (*Y*) for each condition (or score) of the independent variable (*X*).

Table 12.1 indicates that productivity and satisfaction are associated: the distribution of scores on *Y* (productivity) changes across the various conditions of *X* (satisfaction). For example, half of the workers who were low on satisfaction were also low on productivity (30 out of 60). On the other hand, over half of the workers who were high on satisfaction were high on productivity (27 out of 52).

Although it is intended to be a test of significance, the chi square statistic provides another way to detect the existence of an association between two variables that have been organized into table format. Any nonzero value for obtained chi square indicates that the variables are associated. For example, the obtained chi square for Table 12.1 is 24.2, a value that affirms our previous conclusion, based on the conditional distributions of *Y,* that an association of some sort exists between job satisfaction and productivity.

Often, the researcher will have already conducted a chi square test before considering matters of association. In such cases, it will not be necessary to inspect the conditional distributions of *Y* to ascertain whether or not the two variables are associated. If the obtained chi square is zero, the two variables are independent and not associated. Any value other than zero indicates some association between the variables. Remember, however, that statistical significance and association are two different things. It is perfectly possible for two variables to be associated (as indicated by a nonzero chi square) but still be independent (if we fail to reject the null hypothesis).

In this section, we have defined, in a general way, the concept of association between two variables. We have also shown two different ways to detect the presence of an association. In the next section, we will extend the analysis beyond questions of the mere presence or absence of an association and, in a systematic way, show how additional very useful information about the relationship between two variables can be developed.

12.3 THREE CHARACTERISTICS OF BIVARIATE ASSOCIATIONS

Bivariate associations possess three different characteristics, each of which must be analyzed for a full investigation of the relationship. Investigating these characteristics may be thought of as a process of finding answers to three questions:

1. Does an association exist?
2. If an association does exist, how strong is it?
3. What is the pattern and/or the direction of the association?

We will consider each of these questions separately.

Does an Association Exist? We have already discussed the general definition of association, and we have seen that we can detect an association by observing the conditional distributions of Y in a table or by using chi square. In Table 12.1, we know that the two variables are associated to some extent because the conditional distributions of productivity (Y) are different across the various categories of satisfaction (X) and because the chi square statistic is a nonzero value.

Comparisons from column to column in Table 12.1 are relatively easy to make because the column totals are roughly equal. This will not usually be the case and it is helpful to compute percentages to control for varying column totals. These **column percentages,** introduced in Chapter 11, are computed within each column separately and make the pattern of association more visible.

The general procedure for detecting association with bivariate tables is to compute percentages within each column (vertically or down each column) and then compare column to column across the table (horizontally or across the rows). See the One Step at a Time box in Chapter 11 on computing column percentages for a review. Table 12.2 presents column percentages calculated from the data in Table 12.1. Note that this table reports the row and column marginals in parentheses. Besides controlling for any differences in column totals, tables in percentage form are usually easier to read because changes in the conditional distributions of Y are easier to detect.

In Table 12.2, we can see that the largest cell changes position from column to column. For workers who are low on satisfaction, the single largest cell is in the top row (low on productivity). For the middle column (moderate on satisfaction), the largest cell is in the middle row (moderate on productivity); for the right-hand column (high on satisfaction), it is in the bottom row (high on productivity). Even a cursory glance at the conditional distributions of Y in Table 12.2 reinforces our conclusion that an association does exist between these two variables.

TABLE 12.2 PRODUCTIVITY BY JOB SATISFACTION (Percentages)

	Job Satisfaction (X)			
Productivity (Y)	Low	Moderate	High	Totals
Low	50.00%	34.43%	13.46%	33.52% (58)
Moderate	33.33%	40.98%	34.62%	36.42% (63)
High	16.67%	24.59%	51.92%	30.06% (52)
Totals	100.00%	100.00%	100.00%	100.00%
	(60)	(61)	(52)	(173)

TABLE 12.3 PRODUCTIVITY BY HEIGHT (an illustration of no association)

	Height (*X*)		
Productivity (*Y*)	Short	Medium	Tall
Low	33.33%	33.33%	33.33%
Moderate	33.33%	33.33%	33.33%
High	33.33%	33.33%	33.33%
Totals	100.00%	100.00%	100.00%

TABLE 12.4 PRODUCTIVITY BY HEIGHT (an illustration of perfect association)

	Height (*X*)		
Productivity (*Y*)	Short	Medium	Tall
Low	0%	0%	100%
Moderate	0%	100%	0%
High	100%	0%	0%
Totals	100%	100%	100%

If two variables are not associated, then the conditional distributions of *Y* will not change across the columns. The distribution of *Y* would be the same for each condition of *X*. Table 12.3 illustrates a perfect "non-association" between the height of the workers and their productivity. Table 12.3 is only one of many patterns that indicate no association. The important point is that the conditional distributions of *Y* are the same. Levels of productivity do not change at all for the various heights, and therefore, no association exists between these variables. Also, the obtained chi square computed from this table would have a value of zero, again indicating no association.

How Strong Is the Association? Once we establish the existence of the association, we need to develop some idea of how strong the association is. This is essentially a matter of determining the amount of change in the conditional distributions of *Y*. At one extreme, of course, there is the no association, where the conditional distributions of *Y* do not change at all (see Table 12.3). At the other extreme is a perfect association, the strongest possible relationship. In general, a perfect association exists between two variables if each value of the dependent variable is associated with one and only one value of the independent variable.[2] In a bivariate table, all cases in each column would be located in a single cell and there would be no variation in *Y* for a given value of *X* (see Table 12.4).

A perfect relationship would be taken as very strong evidence of a causal relationship between the variables, at least for the sample at hand. In fact, the results presented in Table 12.4 indicate that, for this sample, height is the sole cause of

[2]Each measure of association that will be introduced in this and the following chapters incorporates its own definition of a "perfect association," and these definitions vary somewhat, depending on the specific logic and mathematics of the statistic. That is, for different measures computed from the same table, some measures will possibly indicate perfect relationships when others will not. We will note these variations in the mathematical definitions of a perfect association at the appropriate times.

productivity. Also, in the case of a perfect relationship, predictions from one variable to the other can be made without error. If we know that a particular worker is short, for example, we could be sure that he or she is highly productive.

Of course, the huge majority of relationships fall somewhere between the two extremes of no association and perfect association. We need to develop some way of describing these intermediate relationships consistently and meaningfully. For example, Tables 12.1 and 12.2 show that there is an association between productivity and job satisfaction. How could this relationship be described in terms of strength? How close is the relationship to perfect? How far away from no association?

To answer these questions, researchers rely on statistics called *measures of association,* which provide precise, objective indicators of the strength of a relationship. Virtually all of these statistics are designed so that they have a lower limit of 0.00 and an upper limit of 1.00 ($\pm$1.00 for ordinal and interval-ratio measures of association). A measure that equals 0.00 indicates no association between the variables (the conditional distributions of Y do not vary), and a measure of 1.00 ($\pm$1.00 in the case of ordinal and interval-ratio measures) indicates a perfect relationship. The exact meaning of values between 0.00 and 1.00 varies from measure to measure, but for all measures, the closer the value is to 1.00, the stronger the relationship (the greater the change in the conditional distributions of Y).

We will begin to consider the many different measures of association used in social research later in this chapter. At this point, we will consider the **maximum difference,** a less formal way of assessing the strength of a relationship based on comparing column percentages across the rows. This technique is "quick and dirty": it is easy to apply (at least for small tables) but limited in its usefulness. To compute the maximum difference, compute the column percentages as usual and then skim the table across each of the rows to find the largest difference—in any row—between column percentages. For example, the largest difference in column percentages in Table 12.2 is in the top row between the "Low" column and the "High" column: 50.00% − 13.46% = 36.54%. The maximum difference in the middle row is between "Moderate" and "Low" (40.98% − 33.33% = 7.65%) and, in the bottom row, it is between "High" and "Low" (51.92% − 16.67% = 35.25%). Both of these values are less than the difference in the top row.

Once you have found the maximum difference, you can use the scale presented in Table 12.5 to describe the strength of the relationship. Using this scale, we can describe the relationship between productivity and job satisfaction in Table 12.2 as strong.

Be aware that the relationships between the size of the maximum difference and the descriptive terms (weak, moderate, and strong) in Table 12.5 are arbitrary and approximate. We will get more precise and useful information

TABLE 12.5 THE RELATIONSHIP BETWEEN THE MAXIMUM DIFFERENCE AND THE STRENGTH OF THE RELATIONSHIP

Maximum Difference	Strength
If the maximum difference is:	The strength of the relationship is:
Less then 10 percentage points	Weak
Between 10 and 30 percentage points	Moderate
More than 30 percentage points	Strong

when we compute and analyze the measures of association that we begin to discuss later in this chapter. Also, maximum differences are easiest to find and most useful for smaller tables. In large tables, with many (say, more than three) columns and rows, it will probably be too cumbersome to bother with this statistic, and we would use measures of association only as indicators of the strength for these tables. Finally, note that the maximum difference is based on only two values (the high and low column percentages within any row). Like the range (see Chapter 5), this statistic may give a misleading impression of the overall strength of the relationship. Within these limits, however, the maximum difference can provide a useful, quick, and easy way of characterizing the strength of relationships (at least for smaller tables).

As a final caution, do not mistake chi square as an indicator of the strength of a relationship. Even very large values for chi square do not necessarily mean that the relationship is strong. Remember that significance and association are two separate matters and that chi square, by itself, is not a measure of association. While a nonzero value indicates that there is some association between the variables, the magnitude of chi square bears no particular relationship to the strength of the association. *(For practice in computing percentages and judging the existence and strength of an association, see any of the problems at the end of this chapter.)*

What Is the Pattern and/or the Direction of the Association? Investigating the pattern of the association requires that we ascertain which values or categories of one variable are associated with which values or categories of the other. We have already remarked on the pattern of the relationship between productivity and satisfaction. Table 12.2 indicates that low scores on satisfaction are associated with low scores on productivity, moderate satisfaction with moderate productivity, and high satisfaction with high productivity.

When one or both variables in a bivariate table are nominal in level of measurement, our analysis will end with a consideration of the *pattern* of the relationship. However, when both variables are ordinal in level of measurement, we can go on to describe the association in terms of *direction*.[3] The direction of the association can be either positive or negative. An association is positive if the variables vary in the same direction. That is, in a **positive association,** high scores on one variable are associated with high scores on the other variable, and low scores on one variable are associated with low scores on the other. In a positive association, as one variable increases in value, the other also increases; and as one variable decreases, the other also decreases. Table 12.6 displays, with fictitious data, a positive relationship between education and use of public libraries. As education increases (as you move from left to right across the table), library use also increases (the percentage of "High" users increases). The association between job satisfaction and productivity, as displayed in Tables 12.1 and 12.2, is also a positive association.

In a **negative association,** the variables vary in opposite directions. High scores on one variable are associated with low scores on the other, and increases in one variable are accompanied by decreases in the other. Table 12.7

[3]Variables measured at the nominal level have no numerical order to them (by definition). Therefore, associations including nominal level variables, while they may have a pattern, cannot be said to have a direction.

TABLE 12.6 LIBRARY USE BY EDUCATION (an illustration of a positive relationship)

	Education		
Library Use	Low	Moderate	High
Low	60%	20%	10%
Moderate	30%	60%	30%
High	10%	20%	60%
Total	100%	100%	100%

TABLE 12.7 AMOUNT OF TELEVISION VIEWING BY EDUCATION (an illustration of a negative relationship)

	Education		
Television Viewing	Low	Moderate	High
Low	10%	20%	60%
Moderate	30%	60%	30%
High	60%	20%	10%
Totals	100%	100%	100%

displays a negative relationship, again with fictitious data, between education and television viewing. The amount of television viewing decreases as education increases. In other words, as you move from left to right across the top of the table (as education increases), the percentage of heavy viewers decreases.

Measures of association for ordinal and interval-ratio variables are designed so that they will take on positive values for positive associations and negative values for negative associations. Thus, a measure of association preceded by a plus sign indicates a positive relationship between the two variables, with the value +1.00 indicating a perfect positive relationship. A negative sign indicates a negative relationship, with −1.00 indicating a perfect negative relationship. We will consider the direction of relationships in more detail in Chapter 13.

Application 12.1

Why are many Americans attracted to movies that emphasize graphic displays of violence? One idea is that "slash" movie fans feel threatened by violence in their daily lives and use these movies as a means of coping with their fears. In the safety of the theater, violence can be vicariously experienced, and feelings and fears can be expressed privately. Also, highly violent movies almost always, as a necessary plot element, provide a role model of one character who does deal with violence successfully (usually, of course, with more violence).

Is frequency of attendance at high-violence movies associated with fear of violence? The following table reports the joint frequency distributions of "Fear" and "Attendance" in percentages for a fictitious sample of 600.

ATTENDANCE BY FEAR

	Fear		
Attendance	Low	Moderate	High
Rare	50%	20%	30%
Occasional	30%	60%	30%
Frequent	20%	20%	40%
Totals	100%	100%	100%
	(200)	(200)	(200)

(continued next page)

Application 12.1 *(continued)*

The conditional distributions of attendance (Y) do change across the values of fear (X), so these variables are associated. The clustering of cases in the diagonal from upper left to lower right suggests a substantial relationship in the predicted direction. People who are low on fear attend violent movies infrequently, and people who are high on fear are frequent attendees. Since the maximum difference in column percentages in the table is 30 (in both the top and middle rows), the relationship can be characterized as moderate to strong.

These results do suggest an important relationship between fear and attendance. Notice, however, that these results pose an interesting causal problem. The table supports the idea that fearful and threatened people attend violent movies as a coping mechanism (X causes Y), but it is also consistent with the reverse causal argument: attendance at violent movies increases fears for one's personal safety (Y causes X). The results support *both* causal arguments and remind us that association is not the same thing as causation.

12.4 INTRODUCTION TO MEASURES OF ASSOCIATION

The column percentages provide very useful information about the bivariate association and should always be computed and analyzed. However, they can be awkward and cumbersome to use, especially for larger tables. Measures of association, on the other hand, characterize the strength (and, for ordinal level variables, the direction) of bivariate relationships in a single number, a more compact and convenient format for interpretation and discussion.

There are many measures of association, but we will confine our attention to a few of the most widely used. We will cover these statistics by the level of measurement for which they are most appropriate. In this chapter, we will consider measures appropriate for nominal variables, and in the next chapter we will cover measures of association for ordinal level variables. Finally, in Chapter 14, we will consider Pearson's *r,* a measure of association or correlation for interval-ratio level variables. For relationships with variables at different levels of measurement (for example, one nominal level variable and one ordinal level variable), we generally use the measure of association appropriate for the lower level of measurement.

12.5 MEASURES OF ASSOCIATION FOR VARIABLES MEASURED AT THE NOMINAL LEVEL: CHI SQUARE-BASED MEASURES

When working with nominal level variables, social science researchers rely heavily on measures of association based on the value of chi square. When the value of chi square is already known, these measures are easy to calculate. To illustrate, let us reconsider Table 11.3, which displayed, with fictitious data, a relationship between accreditation and employment for social work majors. For the sake of convenience, this table is reproduced here as Table 12.8.

TABLE 12.8 EMPLOYMENT OF 100 SOCIAL WORK MAJORS BY ACCREDITATION STATUS OF UNDERGRADUATE PROGRAM (fictitious data)

	Accreditation Status		
Employment Status	Accredited	Not Accredited	Totals
Working as a social worker	30	10	40
Not working as a social worker	25	35	60
Totals	55	45	100

TABLE 12.9 EMPLOYMENT BY ACCREDITATION STATUS (percentages)

	Accreditation Status		
Employment Status	Accredited	Not Accredited	Totals
Working as a social worker	54.55%	22.22%	40.00%
Not working as a social worker	45.45%	77.78%	60.00%
Totals	100.00%	100.00%	100.00%

We saw in Chapter 11 that this relationship is statistically significant ($\chi^2 = 10.78$, which is significant at $\alpha = 0.05$), but the question now concerns the *strength* of the association. A brief glance at Table 12.8 shows that the conditional distributions of employment status do change, so the variables are associated. To emphasize this point, it is always helpful to calculate column percentages, as in Table 12.9.

So far, we know that the relationship between these two variables is statistically significant and that there is an association of some kind between accreditation and employment. To assess the strength of the association, we will compute a **phi (ϕ).** This statistic is a frequently used chi square–based measure of association appropriate for 2 × 2 tables (that is, tables with two rows and two columns).

Calculating Phi. One of the attractions of phi is that it is easy to calculate. Simply divide the value of the obtained chi square by N and take the square root of the result. Expressed in symbols, the formula for phi is

FORMULA 12.1

$$\phi = \sqrt{\frac{\chi^2}{N}}$$

For the data displayed in Table 12.8, the chi square was 10.78. Therefore, phi is

$$\phi = \sqrt{\frac{\chi^2}{N}}$$

$$\phi = \sqrt{\frac{10.78}{100}}$$

$$\phi = 0.33$$

For a 2 × 2 table, phi ranges in value from 0 (no association) to 1.00 (perfect association). The closer to 1.00, the stronger the relationship, and the closer to 0.00, the weaker the relationship. For Table 12.8, we already knew that the relationship was statistically significant at the 0.05 level. Phi, as a measure of association, adds information about the strength of the relationship. As for the pattern of the association, the column percentages in Table 12.9 show that graduates of accredited programs were more often employed as social workers.

Calculating Cramer's *V*. For tables larger than 2 × 2 (specifically, for tables with more than two columns and more than two rows), the upper limit of phi can exceed 1.00. This makes phi difficult to interpret, and a more general form

TABLE 12.10 ACADEMIC ACHIEVEMENT BY CLUB MEMBERSHIP

Academic Achievement	Membership: Fraternity or Sorority	Membership: Other Organization	Membership: No Memberships	Totals
Low	4	4	17	25
Moderate	15	6	4	25
High	4	16	5	25
Totals	23	26	26	75

of the statistic called **Cramer's *V*** must be used for larger tables. The formula for Cramer's V is

FORMULA 12.2

$$V = \sqrt{\frac{\chi^2}{(N)(\min\ r - 1,\ c - 1)}}$$

where: $(\min\ r - 1,\ c - 1)$ = the minimum value of $r - 1$ (number of rows minus 1) or $c - 1$ (number of columns minus 1)

In words, to calculate *V,* find the lesser of the number of rows minus 1 $(r - 1)$ or the number of columns minus 1 $(c - 1)$; multiply this value by N, divide the result into the value of chi square, and then find the square root. Cramer's V has an upper limit of 1.00 for any size table and will be the same value as phi if the table has either two rows or two columns. Like phi, Cramer's V can be interpreted as an index that measures the strength of the association between two variables.

To illustrate the computation of *V,* suppose you had gathered the data displayed in Table 12.10, which shows the relationship between membership in student organizations and academic achievement for a sample of college students.

The obtained chi square for this table is 31.5, a value that is significant at the 0.05 level. Cramer's V is

$$V = \sqrt{\frac{\chi^2}{(N)(\min\ r - 1,\ c - 1)}}$$

$$V = \sqrt{\frac{31.50}{(75)(2)}}$$

$$V = \sqrt{\frac{31.50}{150}}$$

$$V = \sqrt{0.21}$$

$$V = 0.46$$

Since Table 12.10 has the same number of rows and columns, we may use either $(r - 1)$ or $(c - 1)$ in the denominator. In either case, the value of the denominator is N multiplied by $(3 - 1)$, or 2. Column percentages are presented in Table 12.11 to help identify the pattern of this relationship. Fraternity and sorority members tend to be moderate, members of other organizations tend to be high, and nonmembers tend to be low in academic achievement.

TABLE 12.11 ACADEMIC ACHIEVEMENT BY CLUB MEMBERSHIP (percentages)

	Membership			
Academic Achievement	Fraternity or Sorority	Other Organization	No Memberships	Totals
Low	17.39	15.39	65.39	33.33%
Moderate	65.22	23.08	15.39	33.33%
High	17.39	61.54	19.23	33.33%
Totals	100.00	100.01	100.01	99.99%

TABLE 12.12 THE RELATIONSHIP BETWEEN THE VALUE OF NOMINAL LEVEL MEASURES OF ASSOCIATION AND THE STRENGTH OF THE RELATIONSHIP

Value	Strength
If the value is:	The strength of the relationship is:
less than 0.10	Weak
between 0.11 and 0.30	Moderate
greater than 0.30	Strong

Interpreting Phi and Cramer's *V*. It will be helpful to have some general guidelines for interpreting the value of measures of association for nominal level variables similar to the guidelines we used for interpreting the maximum difference in column percentages. For phi and Cramer's *V*, the general relationship between the value of the statistic and the strength of the relationship is presented in Table 12.12. As was the case for Table 12.5, the relationships in Table 12.12 are arbitrary and meant as general guidelines only. Using these guidelines, we can characterize the relationships in Table 12.8 ($\phi = 0.33$) and Table 12.10 ($V = 0.46$) as strong.

ONE STEP AT A TIME — Calculating and Interpreting Phi and Cramer's *V*

Step ***Operation***

To calculate phi, solve Formula 12.1:

1. Divide the value of chi square by *N*.
2. Take the square root of the quantity you found in Step 1.
3. Consult Table 12.12 to help interpret the value of phi.

To calculate Cramer's V, solve Formula 12.2:

1. Determine the number of rows (r) and columns (c) in the table. Subtract 1 from the lesser of these two numbers to find (min, $r - 1$, $c - 1$).
2. Multiply the value you found in Step 1 by *N*.
3. Divide the value of chi square by the quantity you found in Step 2.
4. Take the square root of the quantity you found in Step 3.
5. Consult Table 12.12 to help interpret the value of *V*.

Application 12.2

A random sample of students at a large urban university have been classified as either "Traditional" (18–23 years of age and unmarried) or "Nontraditional" (24 or older or married). Subjects have also been classified as "Vocational," if their primary motivation for college attendance is career or job oriented, or "Academic," if their motivation is to pursue knowledge for its own sake. Are these two variables associated?

MOTIVATION FOR COLLEGE ATTENDANCE BY TYPE OF STUDENT

	Type		
Motivation	Traditional	Nontraditional	Totals
Vocational	25	60	85
Academic	75	15	90
Totals	100	75	175

Always begin your analysis of bivariate tables by computing column percentages (assuming that the independent variable is in the columns). This will allow you to detect the pattern of the association and, by finding the maximum difference, to assess the strength of the association. Finally, we will calculate an appropriate measure of association.

MOTIVATION FOR COLLEGE ATTENDANCE BY TYPE OF STUDENT

	Type	
Motivation	Traditional	Nontraditional
Vocational	25.00%	80.00%
Academic	75.00%	20.00%
Totals	100.00%	100.00%

The maximum difference is 55, which indicates a strong relationship between these two variables. The pattern is quite clear: traditional students are more likely to be academically motivated, and nontraditional students are more vocationally motivated.

Since this is a 2×2 table, we can compute phi as a measure of association. The chi square for the table is 51.89. so phi is

$$\phi = \sqrt{\frac{\chi^2}{N}}$$

$$\phi = \sqrt{\frac{51.89}{175}}$$

$$\phi = \sqrt{0.30}$$

$$\phi = 0.55$$

The value of phi, like the maximum difference, indicates a strong relationship between the two variables.

The Limitations of Phi and *V*. One limitation of phi and Cramer's V is that they are only general indicators of the strength of the relationship. Of course, the closer these measures are to 0.00, the weaker the relationship, and the closer to 1.00, the stronger the relationship. Values between 0.00 and 1.00 can be described as weak, moderate, or strong, according to the general convention introduced earlier, but have no direct or meaningful interpretation. On the other hand, phi and V are easy to calculate (once the value of chi square has been obtained) and are commonly used indicators of the importance of an association.[4] *(For practice, phi and Cramer's* V *can be computed for any of the problems at the end of this chapter. These measures are most appropriate for relationships in which at least one variable is nominal in level of measurement: especially Problems 12.2–12.4, 12.7, 12.8a, and 12.9. Problems with 2 × 2 tables will minimize computations. Remember that for tables that have either two rows or two columns, phi and Cramer's* V *will have the same value.)*

[4]Two other chi square–based measures of association, T'^2 and C (the contingency coefficient), are sometimes reported in the literature. Both of these measures have serious limitations. T'^2 has an upper limit of 1.00 only for tables with an equal number of rows and columns, and the upper limit of C varies, depending on the dimensions of the table. These characteristics make these measures more difficult to interpret and thus less useful than phi or Cramer's V.

12.6 LAMBDA: A PROPORTIONAL REDUCTION IN ERROR MEASURE OF ASSOCIATION FOR NOMINAL LEVEL VARIABLES

The Logic of Proportional Reduction in Error. In recent years, a group of measures based on a logic known as **proportional reduction in error (PRE)** has been developed to complement the older chi square–based measures of association. Most generally stated, the logic of these measures requires us to make two different predictions about the scores of cases. In the first prediction, we ignore information about the independent variable and, therefore, make many errors in predicting the score on the dependent variable. In the second prediction, we take account of the score of the case on the independent variable to help predict the score on the dependent variable. If there is an association between the variables, we will make fewer errors when taking the independent variable into account. PRE measures of association express the proportional reduction in errors between the two predictions. Applying these general thoughts to the case of nominal level variables will make the logic clearer.

For nominal level variables, we first predict the category into which each case will fall on the dependent variable (Y) while ignoring the independent variable (X). Since we would be predicting blindly in this case, we would make many errors (that is, we would often predict the value of a case on the dependent variable incorrectly).

The second prediction allows us to take the independent variable into account. If the two variables are associated, the additional information supplied by the independent variable will reduce our errors of prediction (that is, we should misclassify fewer cases). The stronger the association between the variables, the greater the reduction in errors. In the case of a perfect association, we would make no errors at all when predicting a score on Y from a score on X. When there is no association between the variables, on the other hand, knowledge of the independent variable will not improve the accuracy of our predictions. We would make just as many errors of prediction with knowledge of the independent variable as we did without knowledge of the independent variable.

An illustration should make these principles clearer. Suppose you were placed in the rather unusual position of having to predict whether each of the next 100 people you meet will be shorter or taller than 5 feet 9 inches in height under the condition that you would have no knowledge about these people at all. With absolutely no information about these people, your predictions will be wrong quite often (you will frequently misclassify a tall person as short and vice versa).

Now assume that you must go through this ordeal twice; but, on the second round, you know the sex of the person whose height you must predict. Since height is associated with sex and females are, on the average, shorter than males, the optimal strategy would be to predict that all females are short and all males are tall. You will still make errors on this second round, but if the variables are associated, the number of errors will be fewer. That is, using information about the independent variable will reduce the number of errors (if, of course, the two variables are related). How can these unusual thoughts be translated into a useful statistic?

Lambda. One hundred individuals have been categorized by gender and height, and the data are displayed in Table 12.13. It is clear, even without percentages, that the two variables are associated. To measure the strength of this association, a PRE measure called **lambda** (symbolized by the Greek letter λ) will be calculated. Following the logic introduced in the previous section, we must find two quantities. First, the number of prediction errors made while ignoring the independent variable (gender) must be found. Next, we will find

TABLE 12.13 HEIGHT BY GENDER

	Gender		
Height	Male	Female	Totals
Tall	44	8	52
Short	6	42	48
Totals	50	50	100

the number of prediction errors made while taking gender into account. These two sums will then be compared to derive the statistic.

First, the information given by the independent variable (gender) can be ignored by working only with the row marginals. Two different predictions can be made about height (the dependent variable) by using these marginals. We can predict either that all subjects are tall or that all subjects are short.[5] For the first prediction (all subjects are tall), 48 errors will be made. That is, for this prediction, all 100 cases would be placed in the first row. Since only 52 of the cases actually belong in this row, this prediction would result in (100 − 52), or 48, errors. If we had predicted that all subjects were short, on the other hand, we would have made 52 errors (100 − 48 = 52). We will take the *lesser* of these two numbers and refer to this quantity as E_1 for the number of errors made while ignoring the independent variable. So, $E_1 = 48$.

In the second step in the computation of lambda, we predict a score for Y (height) again, but this time we take X (gender) into account. To do this, follow the same procedure as in the first step, but this time move from column to column. Since each column is a category of X, we thus take X into account in making our predictions. For the left-hand column (males), we predict that all 50 cases will be tall and make 6 errors (50 − 44 = 6). For the second column (females), our prediction is that all females are short, and 8 errors will be made. By moving from column to column, we have taken X into account and have made a total of 14 errors of prediction, a quantity we will label E_2 ($E_2 = 6 + 8 = 14$).

If the variables are associated, we will make fewer errors under the second procedure than under the first, or, in other words, E_2 will be smaller than E_1. In this case, we made fewer errors of prediction while taking gender into account ($E_2 = 14$) than while ignoring gender ($E_1 = 48$), so gender and height are clearly associated. Our errors were reduced from 48 to only 14. To find the *proportional* reduction in error, use Formula 13.3:

FORMULA 12.3

$$\lambda = \frac{E_1 - E_2}{E_1}$$

For the sample problem, the value of lambda would be

$$\lambda = \frac{E_1 - E_2}{E_1}$$

$$\lambda = \frac{48 - 14}{48}$$

[5]Other predictions are possible, of course, but these are the only two permitted by lambda.

$$\lambda = \frac{34}{48}$$
$$\lambda = 0.71$$

The value of lambda ranges from 0.00 to 1.00. Of course, a value of 0.00 means that the variables are not associated at all (E_1 is the same as E_2), and a value of 1.00 means that the association is perfect (E_2 is zero, and scores on the dependent variable can be predicted without error from the independent variable). Unlike phi or *V,* however, the numerical value of lambda between the extremes of 0.00 and 1.00 has a precise meaning: it is an index of the extent to which the independent variable (X) helps us to predict (or, more loosely, understand) the dependent variable (Y). When multiplied by 100, the value of lambda indicates the strength of the association in terms of the percentage reduction in error. Thus, the lambda above would be interpreted by concluding that knowledge of gender improves our ability to predict height by 71%. Or, we are 71% better off knowing gender when attempting to predict height.

An Additional Example of Calculating and Interpreting Lambda. In this section, we will work through another example in order to state the computational routine for lambda in general terms. Suppose a researcher was concerned with the relationship between religious denomination and attitude toward capital punishment and had collected the data presented in the table below.

Find E_1, the number of errors made while ignoring X (religion, in this case). Subtract the largest row total from N. For Table 12.14 E_1 will be

$$E_1 = N - \text{(Largest row total)}$$
$$E_1 = 130 - 50$$
$$E_1 = 80$$

To find E_2, begin with the left-hand column (Catholics) and subtract the largest cell frequency from the column total. Repeat this procedure for each column in the table and then add the subtotals together:

$$\text{For Catholics: } 35 - 14 = 21$$
$$\text{For protestants: } 25 - 12 = 13$$
$$\text{For others: } 40 - 25 = 15$$
$$\text{For none: } 30 - 14 = 16$$
$$E_2 = 65$$

TABLE 12.14 ATTITUDE TOWARD CAPITAL PUNISHMENT BY RELIGIOUS DENOMINATION (fictitious data)

	Religion				
Attitude	Catholic	Protestant	Other	None	Totals
Favors	10	9	5	14	38
Neutral	14	12	10	6	42
Opposed	11	4	25	10	50
Totals	35	25	40	30	130

ONE STEP AT A TIME **Calculating and Interpreting Lambda**

Step	*Operation*
1.	To find E_1, subtract the largest row subtotal (marginal) from *N*.
2.	To find E_2, start with the far left hand column and subtract the largest cell frequency in the column from the column total. Repeat this step for all columns in the table.
3.	Add up all the values you found in Step 2. The result is E_2.
4.	Subtract E_2 from E_1.
5.	Divide the quantity you found in Step 5 by E_1. The result is lambda.
6.	To interpret lambda, multiply the value of lambda by 100. This percentage tells us the extent to which our predictions of the dependent variable are improved by taking the independent variable into account. In addition, lambda may be interpreted using the descriptive terms in Table 12.12.

Substitute the values of E_1 and E_2 into Formula 12.3:

$$\lambda = \frac{80 - 65}{80}$$

$$\lambda = \frac{15}{80}$$

$$\lambda = 0.19$$

A lambda of 0.19 means that we are 19% better off using religion to predict attitude toward capital punishment (as opposed to predicting blindly). Or, we could say: Knowledge of a respondent's religious denomination improves the accuracy of our predictions by a factor of 19%. At best, this relationship is moderate in strength (see Table 12.12).

The Limitations of Lambda. Lambda has two characteristics that should be stressed. First, lambda is asymmetric. This means that the value of the statistic will vary, depending on which variable is taken as independent. For example, for Table 12.14, the value of lambda would be 0.14 if attitude toward capital punishment had been taken as the independent variable (verify this with your own computation). Thus, you should exercise some caution in the designation of an independent variable. If you consistently follow the convention of arraying the independent variable in the columns and compute lambda as outlined above, the asymmetry of the statistic should not be confusing.

Second, when one of the row totals is much larger than the others, lambda can be misleading. It can be 0.00 even when other measures of association are greater than 0.00 and the conditional distributions for the table indicate that there is an association between the variables. This anomaly is a function of the way lambda is calculated and suggests that great caution should be exercised in the interpretation of lambda when the row marginals are very unequal. In fact, in the case of very unequal row marginals, a chi square–based measure of association would be the preferred measure of association. *(For practice in computing lambda, see any of the problems at the end of this chapter or Chapter 11. As with phi and Cramer's* V*, it's probably a good idea to start with small samples and 2 × 2 tables.)*

BECOMING A CRITICAL CONSUMER: Reading Percentages

The first step in analyzing bivariate tables should always be to compute and analyze column percentages. These will give you more detail about the relationship than measures of association such as phi and lambda, which should be regarded as summary statements about the relationship. Remember that percentages, although among the more humble of statistics, are not necessarily simple and that they can be miscalculated and misunderstood. Errors can occur when there is confusion about which variable is the cause (or independent variable) and which is the effect (or dependent variable). A closely related error can happen when the researcher asks the wrong questions about the relationship.

To illustrate these errors, let's review the proper method for analyzing bivariate relationships with tables. Recall that, by convention, we array the independent variable in the columns, the dependent variable in the rows, and compute percentages within each column. When we follow this procedure, we are asking, Does *Y* (the dependent variable) vary by *X* (the independent variable)? or, Is *Y* caused by *X*? We conclude that there is evidence for a causal relationship if the values of *Y* change under the different values of *X*.

To illustrate further, consider Table 1, which shows the relationship between race and support for affirmative action for the 2006 General Social Survey, a representative national sample. Race must be the independent or causal variable in this relationship. A person's race may shape their attitudes and opinions, but the reverse cannot be true: a person's opinion cannot cause their race. Race is the column variable in Table 1, and the percentages are computed in the proper direction. A quick inspection shows that support for affirmative action varies by race: there may be a causal relationship between these variables. The maximum difference between the columns is about 31 percentage points, indicating that the relationship is moderate to strong.

What if we had misunderstood this causal relationship? If we had computed percentages within each row, for example, we would be treating race as the dependent variable. We would be asking, Does race vary by support for affirmative action? Table 2 shows the results of asking this question.

TABLE 1 SUPPORT FOR AFFIRMATIVE ACTION BY RACIAL GROUP Frequencies and (*Percentages*)

Support Affirmative Action?	Racial Group		
	White	Black	
Yes	158 (*11.5%*)	113 (*43.0%*)	271
No	1,221 (*88.5%*)	150 (*57.0%*)	1,371
	1,379 (*100.0%*)	263 (*100.0%*)	1,642

TABLE 2 ROW PERCENTAGES FOR TABLE 1

Support Affirmative Action?	Racial Group		
	White	Black	
Yes	58.3	41.7	100.0%
No	89.0	11.0	100.0%

A casual glance at the top row of the table might seem to indicate a causal relationship since 58% of the supporters of affirmative action are white and only about 42% are blacks. If we looked *only* at the top row of the table (as people sometimes do), we would conclude that whites are more supportive of affirmative action than blacks. But the second row shows that whites are also the huge majority (89%) of those who *oppose* the policy. How can this be? The row percentages in this table simply reflect the fact that whites vastly outnumber blacks in the sample: whites outnumber blacks in both rows because there are five times as many whites in the sample. Computing percentages within the rows would make sense only if race could vary by attitude or opinion, and Table 2 could easily lead to false conclusions about this relationship.

Professional researchers sometimes compute percentages in the wrong direction or ask a question about the relationship incorrectly; you should always check bivariate tables to make sure that the analysis agrees with the patterns in the table.

SUMMARY

1. Analyzing the association between variables provides information that is complementary to tests of significance. The latter are designed to detect nonrandom relationships, whereas measures of association are designed to quantify the importance or strength of a relationship.
2. Relationships between variables have three characteristics: the existence of an association, the strength of the association, and the direction or pattern of the association. These three characteristics can be investigated by calculating percentages for a bivariate table in the direction of the independent variable (vertically) and then comparing them in the opposite direction (horizontally). It is often useful (as well as quick and easy) to assess the strength of a relationship by finding the maximum difference in column percentages in any row of the table.
3. Tables 12.1 and 12.2 can be analyzed in terms of these three characteristics. Clearly, a relationship does exist between job satisfaction and productivity, since the conditional distributions of the dependent variable (productivity) are different for the three different conditions of the independent variable (job satisfaction). Even without a measure of association, we can see that the association is substantial in that the change in Y (productivity) across the three categories of X (satisfaction) is marked. The maximum difference of 36.54% confirms that the relationship is substantial (moderate to strong). Furthermore, the relationship is positive in direction. Productivity increases as job satisfaction rises, and workers who report high job satisfaction tend also to be high on productivity. Workers with little job satisfaction tend to be low on productivity.
4. Given the nature and strength of the relationship, it could be predicted with fair accuracy that highly satisfied workers tend to be highly productive ("happy workers are busy workers"). These results might be taken as evidence of a causal relationship between these two variables, but they cannot, by themselves, prove that a causal relationship exists: association is not the same thing as causation. In fact, although we have presumed that job satisfaction is the independent variable, we could have argued the reverse causal sequence ("busy workers are happy workers"). The results presented in Tables 12.1 and 12.2 are consistent with both causal arguments.
5. Phi and Cramer's V and lambda are measures of association, and each is appropriate for a specific situation. Phi is used for nominal level variables in a 2×2 table and Cramer's V is used for tables larger than 2×2. Lambda is a proportional reduction in error (PRE) measure appropriate for nominal level variables. These statistics express information about the strength of the relationship *only*. In all cases, be sure to analyze the column percentages as well as the measure of association in order to maximize the information you have about the relationship.

SUMMARY OF FORMULAS

FORMULA 12.1 Phi ϕ $= \sqrt{\frac{\chi^2}{N}}$

FORMULA 12.2 Cramer's V $V = \sqrt{\frac{\chi^2}{(N)(\min\ r - 1,\ c - 1)}}$

FORMULA 12.3 Lambda λ $= \frac{E_1 - E_2}{E_1}$

GLOSSARY

Association. The relationship between two (or more) variables. Two variables are said to be associated if the distribution of one variable changes for the various categories or scores of the other variable.

Column percentages. Percentages computed with each column of a bivariate table.

Conditional distribution of *Y*. The distribution of scores on the dependent variable for a specific score or category of the independent variable when the variables have been organized into table format.

Cramer's *V*. A chi square–based measure of association. Appropriate for nominally measured variables

that have been organized into a bivariate table of any number of rows and columns.

Dependent variable. In a bivariate relationship, the variable that is taken as the effect.

Independent variable. In a bivariate relationship, the variable that is taken as the cause.

Lambda. A proportional reduction in error (PRE) measure of association for variables measured at the nominal level that have been organized into a bivariate table.

Maximum difference. A way to assess the strength of an association between variables that have been organized into a bivariate table. The maximum difference is the largest difference between column percentages for any row of the table.

Measures of association. Statistics that quantify the strength of the association between variables.

Negative association. A bivariate relationship where the variables vary in opposite directions. As one variable increases, the other decreases, and high scores on one variable are associated with low scores on the other.

Phi (ϕ). A chi square–based measure of association. Appropriate for nominally measured variables that have been organized into a 2×2 bivariate table.

Positive association. A bivariate relationship in which the variables vary in the same direction. As one variable increases, the other also increases, and high scores on one variable are associated with high scores on the other.

Proportional reduction in error (PRE). The logic that underlies the definition and computation of lambda. The statistic compares the number of errors made when predicting the dependent variable while ignoring the independent variable with the number of errors made while taking the independent variable into account.

X. Symbol used for any independent variable.

Y. Symbol used for any dependent variable.

PROBLEMS

(Problems are labeled with the social science discipline from which they are drawn: SOC for sociology, SW for social work, PS for political science, CJ for criminal justice, PA for public administration, and GER for gerontology.)

12.1 PA Various supervisors in the city government of Shinbone, Kansas, have been rated on the extent to which they practice authoritarian styles of leadership and decision making. The efficiency of each department has also been rated, and the results are summarized below. Use column percentages, the maximum difference, and measures of association to describe the strength and pattern of this association.

	Authoritarianism		
Efficiency	Low	High	Totals
High	10	12	22
Low	17	5	22
Totals	27	17	44

12.2 SOC The administration of a local college campus has proposed an increase in the mandatory student fee in order to finance an upgrading of the intercollegiate football program. A member of the faculty has completed a survey on the issues. Is there any association between support for raising fees and the gender, discipline, or tenured status of the faculty? Use column percentages, the maximum difference and measures of association to describe the strength and pattern of these associations.

a. Support for raising fees by gender:

	Gender		
Support	Males	Females	Totals
For	12	8	20
Against	15	12	27
Totals	27	20	47

b. Support for raising fees by discipline:

	Discipline		
Support	Liberal Arts	Science & Business	Totals
For	6	13	19
Against	14	14	28
Totals	20	27	47

c. Support for raising fees by tenured status:

	Status		
Support	Tenured	Nontenured	Totals
For	15	4	19
Against	18	10	28
Totals	33	14	47

12.3 PS How consistent are people in their voting habits? Do people vote for the same party from election to election? Below are the results of a poll in which people were asked if they had voted Democrat or Republican in each of the last two presidential elections. Use column percentages, the maximum difference, and measures of association to describe the strength and pattern of the association.

	2000 Election		
2004 Election	Democrat	Republican	Totals
Democrat	117	23	140
Republican	17	178	195
Totals	134	201	335

12.4 SOC A needs assessment survey has been distributed in a large retirement community. Residents were asked to check off the services or programs they thought should be added. Use column percentages, the maximum difference, and measures of association to describe the strength and direction of the association. Write a few sentences describing the relationship.

	Gender		
More Parties?	Males	Females	Totals
Yes	321	426	747
No	175	251	426
Totals	496	677	1,173

12.5 SW As the state director of mental health programs, you note that some local mental health facilities have very high rates of staff turnover. You believe that part of this problem is a result of the fact that some of the local directors have very little training in administration and poorly developed leadership skills. Before implementing a program to address this problem, you collect some data to make sure that your beliefs are supported by the facts. Is there a relationship between staff turnover and the administrative experience of the directors? Use column percentages, the maximum difference, and measures of association to describe the strength and direction of the association. Write a few sentences describing the relationship.

	Director Experienced?		
Turnover	No	Yes	Totals
Low	4	9	13
Moderate	9	8	17
High	15	5	20
Totals	28	22	50

12.6 CJ About half the neighborhoods in a large city have instituted programs to increase citizen involvement in crime prevention. Do these areas experience less crime? Write a few sentences describing the relationship in terms of pattern and strength of the association. Use column percentages, the maximum difference, and measures of association to describe the strength and direction of the association. Write a few sentences describing the relationship.

	Program		
Crime Rate	No	Yes	Totals
Low	29	15	44
Moderate	33	27	60
High	52	45	97
Totals	114	87	201

12.7 GER A survey of senior citizens who live in either a housing development specifically designed for retirees or an age-integrated neighborhood has been conducted. Is type of living arrangement related to sense of social isolation?

	Living Arrangement		
Sense of Isolation	Housing Development	Integrated Neighborhood	Totals
Low	80	30	110
High	20	120	140
Totals	100	150	250

12.8 SOC A researcher has conducted a survey on sexual attitudes for a sample of 317 teenagers. The respondents were asked whether they considered premarital sex to be "always wrong" or "OK under certain circumstances." The tables below summarize the relationship between responses to this item and several other variables. For each table, assess the strength and pattern of the relationship, and write a paragraph interpreting these results.

a. Attitudes toward premarital sex by gender:

	Gender		
Premarital Sex	Female	Male	Totals
Always wrong	90	105	195
Not always wrong	65	57	122
Totals	155	162	317

b. Attitudes toward premarital sex by courtship status:

	Ever "Gone Steady"		
Premarital Sex	No	Yes	Totals
Always wrong	148	47	195
Not always wrong	42	80	122
Totals	190	127	317

c. Attitudes toward premarital sex by social class:

	Social Class		
Premarital Sex	Blue Collar	White Collar	Totals
Always wrong	72	123	195
Not always wrong	47	75	122
Totals	119	198	317

12.9 SOC Below are five dependent variables cross-tabulated against gender as an independent variable. Use column percentages, the maximum difference, and an appropriate measure of association to analyze these relationships. Summarize the results of your analysis in a paragraph that describes the strength and pattern of each relationship.

a. Support for the legal right to an abortion by gender:

	Gender		
Right to Abortion?	Male	Female	Totals
Yes	310	418	728
No	432	618	1,050
Totals	742	1,036	1,778

b. Support for capital punishment by gender:

	Gender		
Capital Punishment?	Male	Female	Totals
Favor	908	998	1,906
Oppose	246	447	693
Totals	1,154	1,445	2,559

c. Approval of suicide for people with incurable disease by gender:

	Gender		
Right to Suicide?	Male	Female	Totals
Yes	524	608	1,132
No	246	398	644
Totals	770	1,006	1,776

d. Support for sex education in public schools by gender:

	Gender		
Sex Education?	Male	Female	Totals
Favor	685	900	1,585
Oppose	102	134	236
Totals	787	1,034	1,821

e. Support for traditional gender roles by gender:

Women Should Take Care of Running Their Homes and Leave Running the Country to Men	Gender		
	Male	Female	Totals
Agree	116	164	280
Disagree	669	865	1,534
Totals	785	1,029	1,814

YOU ARE THE RESEARCHER: Understanding Political Beliefs, Part II

At the end of Chapter 11, you investigated possible causes of people's beliefs on four controversial issues. Now, you will extend your analysis by using **Crosstabs** to get the statistics presented in this chapter. There will be two projects, and, once again, we will begin with a brief demonstration using the relationship between *abany* and *sex*.

With the 2006 GSS loaded, click **Analyze, Descriptive Statistics,** and **Crosstabs** and name *abany* as the row (dependent) variable and *sex* as the column (independent) variable. Click the **Cells** button and request column percentages by clicking the box next to **Column** in the **Percentages** box. Also, click the **Statistics** button and request chi square, phi, Cramer's *V,* and lambda by clicking the appropriate boxes. Click **Continue** and **OK,** and your task will be executed.

As you will see, SPSS generates a good deal of output for this procedure and I have condensed the information into a single table, based on how this test might be presented in the professional research literature. Note that the cells of the bivariate table include both frequencies and percentages and that all the statistical information is presented in a single, short line beneath the table.

SUPPORT FOR LEGAL ABORTION (*abany*) BY GENDER

		Respondent's Sex		
		Male	Female	Total
Should a Woman Be Able To Get a Legal Abortion if She Wants it for Any Reason?	*Yes*	128 46.9%	148 41.5%	276 43.8%
	No	145 53.1%	209 58.5%	354 56.2%
	Total	273 100%	357 100%	630 100%

Chi square = 1.853 (*df* = 1, *p* > 0.05); Phi = 0.054; Lambda = 0.00

The three questions about bivariate association introduced in this chapter can provide a useful framework for reading and analyzing these results.

1. *Is there an association?* The column percentages in the bivariate table change so there is an association between support for abortion and gender. This conclusion is verified by the fact that chi square is a nonzero value.
2. *How strong is the association?* We can assess strength in several different ways. First, the maximum difference is 46.9 − 41.5 or 5.4 percentage points. (This calculation is based on the top row, but since this is a 2 × 2 table, using the bottom row would result in exactly the same value). According to Table 12.5, this means that the relationship between the variables is weak.

   Second, we can (and should) use a measure of association to assess the strength of the relationship. We have a choice of two different measures: lambda and (since this is a 2 × 2 table) phi. Looking in the **Directional Measures** output box, we see several different values for lambda. Recall that lambda is an asymmetric measure and that it changes value depending on which variable is seen as dependent. In this case, the dependent variable is "abortion if woman wants for any reason" and the associated lambda is reported as 0.000. This indicates that there is no association between the variables, but we have already seen that the variables *are* associated, if only weakly. Remember that lambda can be zero when the variables are associated but the row totals in the table are very unequal. That's *not* the problem here: lambda is a little misleading, but it's still telling us that the relationship is weak.

   Turning to phi (under **Symmetric Measures**), we see a value of 0.054. This indicates that the relationship is weak (see Table 12.12), a conclusion that is consistent with the value of the maximum difference and the fact that chi square is low in value and that the relationship is not significant at the 0.05 level.
3. *What is the pattern of the relationship?* Since sex is a nominal level variable, the relationship cannot have a direction (that is, it cannot be positive or negative). We can, however, discuss the pattern of the relationship: how values of the variables seem to go together. Although the difference is small (5.4%), males are more supportive of abortion than females.

In summary, using chi square, the column percentages, and phi, we can say that the relationship between support for legal abortion and gender is weak and not statistically significant. If we were searching for an important cause of attitudes about abortion, we would discard this independent variable and seek another.

Your turn.

PROJECT 1 Explaining Beliefs

In this project, you will once again analyze beliefs about capital punishment (*cappun*), assisted suicide (*letdie1*), gay marriage (*marhomo*), and immigration (*letin1*). You will select an independent variable other than the one you used in Chapter 11 and use SPSS to generate chi square, column percentages, phi or Cramer's *V,* and lambda. You will use all of these statistics to help analyze and interpret your results.

STEP 1: Choose an Independent Variable

Select an independent variable that seems likely to be an important cause of people's attitudes about the death penalty, assisted suicide, gay marriage, and immigration. Be sure to select an independent variable that has *only* two to five categories, and use the **recode** command if necessary. You might consider gender, level of education, religion, or age (the recoded version; see Chapter 10) as possibilities, but there are many others. Record the variable name and state exactly what the variable measures in the table below.

SPSS Name	What Exactly Does This Variable Measure?

STEP 2: Stating Hypotheses

State hypotheses about the relationships you expect to find between your independent variable and each of the four dependent variables. State these hypotheses in terms of which category of the independent variable you expect to be associated with which category of the dependent variable (for example, "I expect that men will be more supportive of the legal right to an abortion for any reason").

1.

2.

3.

4.

STEP 3: Running Crosstabs

Click **Analyze → Descriptives → Crosstabs** and place the four dependent variables (*cappun, letdie1, letin1,* and *marhomo*) in the **Rows:** box and the independent variable you selected in the **Columns:** box. Click the **Statistics** button to get chi square, phi, Cramer's *V,* and lambda and the **Cells** button for column percentages.

STEP 4: Recording Results

These commands will generate a lot of output, and it will be helpful to summarize your results in the following table.

Dependent Variable	Chi Square Significant at < 0.05?	Maximum Difference	Phi or Cramer's *V*	Lambda
cappun				
letdie1				
letin1				
marhomo				

STEP 5: Analyzing and Interpreting Results

Write a short summary of results for each dependent variable. The summary needs to identify the variables being tested, the results of the chi square test, and the strength and pattern of the relationship. It is probably best to characterize the relationship in general terms and then cite the statistical values in parentheses. For example, we might summarize our test of the relationship between gender and support for abortion as follows: "The relationship between gender and support for abortion was not significant and weak (chi square = 1.853, $df = 1$, $p < 0.05$; phi = 0.05). Men were slightly more supportive of the right to a legal abortion for any reason than women." You should also note whether or not your hypotheses were supported.

PROJECT 2: Exploring the Impact of Various Independent Variables

In this project, you will examine the relative ability of a variety of independent variables to explain or account for a single dependent variable. You will again use the **Crosstabs** procedure in SPSS to generate statistics and use the alpha levels and measures of association to judge which independent variable has the most important relationship with your dependent variable.

STEP 1: Choosing Variables

Select a dependent variable. You may use any of the four from Project 1 in this chapter or select a new dependent variable from the 2006 GSS. Be sure that your dependent variable has no more than five values or scores. Good choices for dependent variables include any measure of attitudes or opinions. Do not select characteristics such as race, sex, or religion as dependent variables.

Select three independent variables that seem likely to be important causes of the dependent variable you selected. Your independent variable should have no more than four or five categories. You might consider gender, level of education, religion, or age (the recoded version; see Chapter 10) as possibilities, but there are many others.

Record the variable names, and state exactly what each variable measures in the table below.

SPSS Name	What Exactly Does This Variable Measure?
Dependent Variable	
Independent Variables	

STEP 2: Stating Hypotheses

State hypotheses about the relationships you expect to find between your independent variables and the dependent variable. State these hypotheses in terms of which category of the independent variable you expect to be associated with which category of the dependent variable (for example, "I expect that men will be more supportive of the legal right to an abortion for any reason").

1.

2.

3.

STEP 3: Running Crosstabs

Click **Analyze → Descriptives → Crosstabs** and place your dependent variable in the **Rows:** box and all three of your independent variables in the **Columns:** box. Click the **Statistics** button to get chi square, phi, Cramer's *V,* and lambda. Click the **Cells** button for column percentages.

STEP 4: Recording Results

Your output will consist of three tables, and it will be helpful to summarize your results in the following table. Remember that the significance of the relationship is found in the column labeled "Asymp. Sig (2-sided)" of the second box in the output.

Independent Variables	Chi Square Significant at < 0.05?	Maximum Difference	Phi or Cramer's *V*	Lambda

STEP 5: Analyzing and Interpreting Results

Write a short summary of results of each test using the same format as in Project 1. Remember to explain whether or not your hypotheses were supported. Finally, assess which of the independent variables had the most important relationship with your dependent variable. Use the alpha (or the "Asymp. Sig 2-tailed") level and the value of the measures of association to make this judgment.

13 Association Between Variables Measured at the Ordinal Level

LEARNING OBJECTIVES

By the end of this chapter, you will be able to:

1. Calculate and interpret gamma and Spearman's rho.
2. Explain the logic of proportional reduction in error in terms of gamma.
3. Use gamma and Spearman's rho to analyze and describe a bivariate relationship in terms of the three questions introduced in Chapter 12.

13.1 INTRODUCTION

There are two common types of ordinal level variables. Some have many possible scores and look, at least at first glance, like interval-ratio level variables. We will call these *continuous ordinal variables*. An attitude scale that incorporates many different items and, therefore, has many possible values would produce this type of variable.

The second type, which we will call a *collapsed ordinal variable*, has only a few (no more than five or six) values or scores and can be created either by collecting data in collapsed form or by collapsing a continuous ordinal scale. For example, we can produce collapsed ordinal variables by measuring social class as upper, middle, or lower or by reducing the scores on an attitude scale into just a few categories (such as high, moderate, and low).

A number of measures of association have been invented for use with collapsed ordinal level variables. Rather than attempt to cover all of these statistics, we will concentrate on **gamma (*G*)** for "collapsed" ordinal variables, and for "continuous" ordinal variables, we will use a statistic called **Spearman's rho (r_s).** We will cover gamma first and treat Spearman's rho toward the end of this chapter.

This chapter will expand your understanding of how bivariate associations can be described and analyzed, but it is important to remember that we are still trying to answer the three questions raised in Chapter 12: Are the variables associated? How strong is the association? What is the direction of the association?

13.2 PROPORTIONAL REDUCTION IN ERROR

For nominal level variables, the logic of proportional reduction in error (PRE) was based on two different "predictions" of the scores of cases on the dependent variable (Y): one that ignored the independent variable (X) and a second that took the independent variable into account. The value of lambda showed the extent to which taking the independent variable into account improved our accuracy when predicting the score of the dependent variable. The PRE logic for variables measured at the ordinal level is similar, and gamma, like lambda, measures the proportional reduction in error gained by predicting one variable while taking the other into account. The major difference lies in the way predictions are made.

In the case of gamma, we predict the order of pairs of cases rather than a score on the dependent variable. That is, we predict whether one case will have a higher or lower score than the other. First, we predict the order of a pair of cases on the dependent variable while ignoring their order on the independent variable. Second, we predict the order on the dependent variable while taking into account the order on the independent variable.

As an illustration, assume that a researcher is concerned about the causes of burnout (that is, demoralization and loss of commitment) among elementary school teachers and wonders about the relationship between levels of burnout and years of service. One way to state the research question would be to ask if teachers with more years of service have higher levels of burnout. Another way to ask the same question is, Do teachers who *rank higher* on years of service also *rank higher* on burnout? If we knew that teacher A had more years of service than teacher B, would we be able to predict that teacher A is also more "burned out" than teacher B? That is, would knowledge of the order of this pair of cases on one variable help us predict their order on the other?

If the two variables are associated, we will reduce our errors when our predictions about one of the variables are based on knowledge of the other. Furthermore, the stronger the association, the fewer the errors we will make. When there is no association between the variables, gamma will be 0.00, and knowledge of the order of a pair of cases on one variable will not improve our ability to predict their order on the other. A gamma of ± 1.00 denotes a perfect relationship: the order of all pairs of cases on one variable would be predictable without error from their order on the other variable.

In Chapter 12, we learned how to analyze the pattern of the relationship between nominal level variables. That is, we looked to see which value on one variable (e.g., "male" on the variable gender) was associated with which value on the other variable (e.g., "tall" on the variable height).

Recall that a defining characteristic of variables measured at the ordinal level is that the scores or values can be rank ordered from high to low or from more to less (see Chapter 1). This means that relationships between ordinal level variables can have a direction as well as a pattern. In terms of the logic of gamma, the overall relationship between the variables is *positive* if cases tend to be ranked in the same order on both variables. For example, if case A is ranked above case B on one variable, it would also be ranked above case B on the second variable. The relationship suggested above between years of service and burnout would be a *positive* relationship.

In a *negative* relationship, the order of the cases would be reversed between the two variables. If case A ranked above case B on one variable, it would tend to rank below case B on the second variable. If there is a negative relationship between prejudice and education, and case A was more educated than case B (or ranked *above* case B on education), then case A would be less prejudiced (or would rank *below* case B on prejudice).

13.3 GAMMA

Computation. Table 13.1 summarizes the relationship between length of service and burnout for a fictitious sample of 100 teachers. To compute gamma, two sums are needed. First, we must find the number of pairs of cases that are ranked the same on both variables (we will label this $\boldsymbol{N_s}$) and then the number

TABLE 13.1 BURNOUT BY LENGTH OF SERVICE (fictitious data)

	Length of Service			
Burnout	Low	Moderate	High	Totals
Low	20	6	4	30
Moderate	10	15	5	30
High	8	11	21	40
Totals	38	32	30	100

of pairs of cases ranked differently on the variables ($\boldsymbol{N_d}$). We find these sums by working with the cell frequencies.

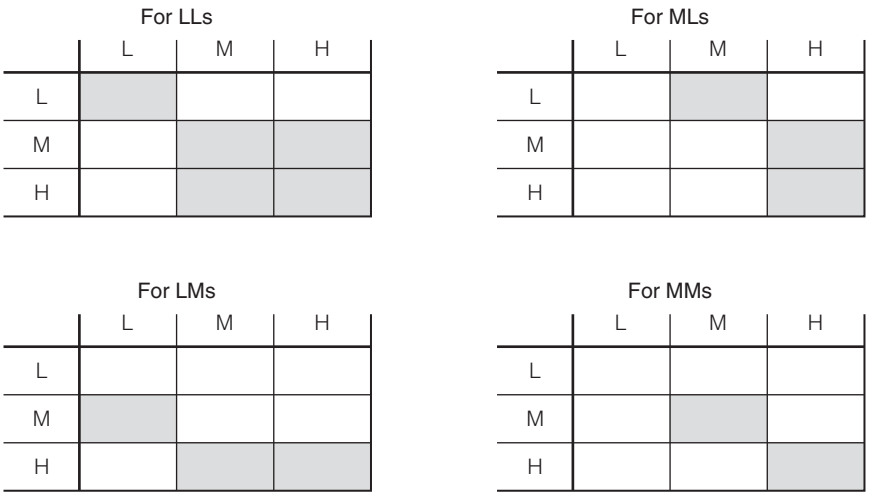
To find the number of pairs of cases ranked the same (N_s), begin with the cell containing the cases that were ranked the lowest on both variables. In Table 13.1, this would be the upper left-hand cell. (*Note:* Not all tables are constructed with values increasing from left to right across the columns and from top to bottom across the rows. When using other tables, always be certain that you have located the proper cell.) The 20 cases in the upper left-hand cell all rank low on both burnout and length of service, and we will refer to these cases as *low-lows,* or LLs.

Now form a pair of cases by selecting one case from this cell and one from any other cell—for example, the middle cell in the table. All 15 cases in this cell are moderate on both variables and, following our practice above, can be labeled *moderate-moderates,* or MMs. Any pair of cases formed between these two cells will be ranked the same on both variables. That is, all LLs are lower than all MMs on both variables (on *X,* low is less than moderate, and on *Y,* low is less than moderate). The total number of pairs of cases is given by multiplying the cell frequencies. So, the contribution of these two cells to the total N_s is (20)(15), or 300.

Gamma ignores all pairs of cases that are tied on either variable. For example, any pair of cases formed between the LLs and any other cell in the top row (low on burnout) or the left-hand column (low on length of service) will be tied on one variable. Also, any pair of cases formed within any cell will be tied on both *X* and *Y.* Gamma ignores all pairs of cases formed within the same row, column, or cell. This means that in computing N_s, we will work with only the pairs of cases that can be formed between each cell and the cells below and to the right of the cell.

In summary, to find the total number of pairs of cases ranked the same on both variables (N_s), multiply the frequency in each cell by the total of all frequencies below and to the right of that cell. Repeat this procedure for each cell and add the resultant products. The total of these products is N_s. This procedure is displayed in Figure 13.1 for each cell in Table 13.1. Note that none of the cells in the bottom row or the right-hand column can contribute to N_s because they have no cells below and to the right of them. Figure 13.1 shows the direction of multiplication for each of the four cells that in a 3×3 table can contribute to N_s. Computing N_s for Table 13.1, we find that a total of 1,831 pairs of cases are ranked the same on both variables.

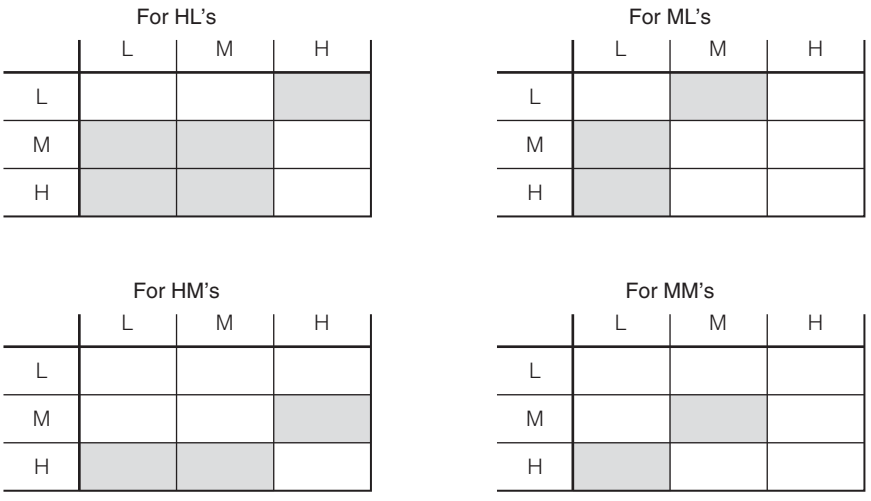
FIGURE 13.1 COMPUTING N_s IN A 3 × 3 TABLE



	Contribution to N_s
For LLs, 20(15 + 5 + 11 + 21)	= 1,040
For MLs, 6(21 + 5)	= 156
For HLs, 4(0)	= 0
For LMs, 10(11 + 21)	= 320
For MMs, 15(21)	= 315
For HMs, 5(0)	= 0
For LHs, 8(0)	= 0
For MHs, 11(0)	= 0
For HHs, 21(0)	= 0
	N_s = 1,831

Our next step is to find the number of pairs of cases ranked differently (N_d) on both variables. To find the total number of pairs of cases ranked in different order on the variables, multiply the frequency in each cell by the total of all frequencies below and to the left of that cell. Note that the pattern for computing N_d is the reverse of the pattern for N_s. This time we begin with the upper right-hand cell (high-lows, or HLs) and multiply the number of cases in the cell by the total frequency of cases below and to the left. The four cases in the upper right-hand cell are low on Y and high on X, and if a pair is formed with any case from this cell and any cell below and to the left, the cases will be ranked differently on the two variables. For example, if a pair is formed between any HL case and any case from the middle cell (moderate-moderates, or MMs), the HL case would be less than the MM case on Y (*low* is less than *moderate*) but more than the MM case on X (*high* is greater than *moderate*). The computation of N_d is detailed below and shown graphically in Figure 13.2. In the computations, we have omitted cells that cannot contribute to N_d because they have no cells below and to the left of them.

FIGURE 13.2 COMPUTING N_d IN A 3 × 3 TABLE



	Contribution to N_d
For HLs, 4(10 + 15 + 8 + 11)	= 176
For MLs, 6(10 + 8)	= 108
For HMs, 5(8 + 11)	= 95
For MMs, 15(8)	= 120
	N_d = 499

Table 13.1 has 499 pairs of cases ranked in different order and 1,831 pairs of cases ranked in the same order. The formula for computing gamma is

FORMULA 13.1

$$G = \frac{N_s - N_d}{N_s + N_d}$$

Where: N_s = the number of pairs of cases ranked the same on both variables
N_d = the number of pairs of cases ranked differently on the two variables

For Table 13.1, the value of gamma would be

$$G = \frac{N_s - N_d}{N_s + N_d}$$

$$G = \frac{1{,}831 - 499}{1{,}831 + 499}$$

$$G = \frac{1{,}332}{2{,}330}$$

$$G = 0.57$$

Interpretation. A gamma of 0.57 indicates that we would make 57% fewer errors if we predicted the order of pairs of cases on one variable from the order of pairs of cases on the other (as opposed to predicting order while ignoring the other variable.) Length of service is associated with degree of burnout, and the relationship is positive. Knowing the respective rankings of two teachers on

TABLE 13.2 THE RELATIONSHIP BETWEEN THE VALUE OF GAMMA AND THE STRENGTH OF THE RELATIONSHIP

Value	Strength
If the value is:	The strength of the relationship is:
Between 0.00 and 0.30	Weak
Between 0.31 and 0.60	Moderate
Greater than 0.61	Strong

length of service (case A is higher on length of service than case B) will help us predict their ranking on burnout (we would predict that case A will also be higher than case B on burnout).

Table 13.2 provides some additional assistance for interpreting gamma in a format similar to Tables 12.5 and 12.12. As before, the table presents general guidelines only and the relationship between the values and the descriptive terms are arbitrary. Note, in particular, that the strength of the relationship is independent of its direction. That is, a gamma of -0.35 is exactly as strong as a gamma of $+0.35$, but opposite in direction.

To use the computational routine for gamma presented above, you must arrange the table in the manner of Table 13.1, with the column variable increasing in value as you move from left to right and the row variable increasing from top to bottom. Be careful to construct your tables according to this format, and if you are working with data already in table format, you may have to rearrange the table or rethink the direction of patterns. Gamma is a symmetrical measure of association; that is, the value of gamma will be the same regardless of which variable is taken as independent. *(To practice computing and interpreting gamma, see Problems 13.1–13.10. Begin with some of the smaller, 2×2 tables until you are comfortable with these procedures.)*

13.4 DETERMINING THE DIRECTION OF RELATIONSHIPS

Nominal measures of association, such as phi and lambda, measure only the strength of a bivariate association. Ordinal measures of association, such as gamma, are more sophisticated and add information about the overall direction of the relationship (positive or negative). In one way, it is easy to determine direction: If the sign of the statistic is a plus, the direction is positive, and a minus sign indicates a negative relationship. Often, however, direction is confusing when working with ordinal-level variables, and it will be helpful if we focus on the matter specifically. We'll discuss positive relationships first and then relationships in the negative direction.

With gamma, a positive relationship means that the scores of cases tend to be ranked in the same order on both variables and that the variables change in the same direction. Cases tend to have scores in the same range on both variables (i.e., low scores go with low scores, moderate with moderate, and so forth), and as scores on one variable increase (or decrease), scores on the other variable also increase (or decrease). Table 13.3 illustrates the general shape of a positive relationship. In a positive relationship, cases tend to fall along a diagonal from upper left of the bivariate table to lower right (assuming, of course, that tables have been constructed with the column variable increasing from left to right and the row variable from top to bottom).

TABLE 13.3 A GENERALIZED POSITIVE RELATIONSHIP

	Variable *X*		
Variable *Y*	Low	Moderate	High
Low	**X**		
Moderate		**X**	
High			**X**

TABLE 13.4 STATE STRUCTURE BY DEGREE OF STRATIFICATION (Frequencies)*

	Degree of Stratification		
Type of State	Low	Medium	High
Stateless	77	5	0
Semi-state	28	15	4
State	12	19	26
Totals	117	39	30

*Data are from the Human Relation Area File, standard cross-cultural sample.

TABLE 13.5 STATE STRUCTURE BY DEGREE OF STRATIFICATION (Percentages)

	Degree of Stratification		
Type of State	Low	Medium	High
Stateless	65.8%	12.8%	0.0%
Semistate	23.9%	38.5%	13.3%
State	10.3%	48.7%	86.7%
Totals	100.0%	100.0%	100.0%

Tables 13.4 and 13.5 present an example of a positive relationship using actual data from 186 preindustrial societies from around the globe. Each society has been rated on its degree of stratification or inequality and the type of political institution it has. In a society that is low on inequality, people are essentially equal in terms of wealth and power. The degree to which people are unequal increases from left to right across the columns of the table. In a "stateless" society, there is no formal political institution or government, but the political institution becomes more elaborate and stronger as you read down the rows from top to bottom.

The gamma for this Table is 0.86, so the relationship is strong and positive. Most cases fall in the diagonal from upper left to lower right. The percentages in Table 13.5 make it clear that societies with little inequality tend to be stateless and that the political institution becomes more elaborate as inequality increases. The great majority of the least-stratified societies had no political institution, and none of the highly stratified societies were stateless.

Negative relationships are the opposite of positive relationships. low scores on one variable are associated with high scores on the other and high scores with low scores. This pattern means that the cases tend to fall along a diagonal from lower left to upper right (at least for all tables in this text). Table 13.6

TABLE 13.6 A GENERALIZED NEGATIVE RELATIONSHIP

	Variable *X*		
Variable *Y*	Low	Moderate	High
Low			**X**
Moderate		**X**	
High	**X**		

TABLE 13.7 APPROVAL OF COHABITATION BY CHURCH ATTENDANCE (Frequencies)

	Attendance		
Approval	Never	Monthly or Yearly	Weekly
Low	37	186	195
Moderate	25	126	46
High	156	324	52
Totals	218	636	293

TABLE 13.8 APPROVAL OF COHABITATION BY CHURCH ATTENDANCE (Percentages)

	Attendance		
Approval	Never	Monthly or Yearly	Weekly
Low	17.0%	29.3%	66.6%
Moderate	11.5%	19.8%	15.7%
High	71.6%	50.9%	17.8%
Totals	100.1%	100.1%	100.1%

illustrates a generalized negative relationship. The cases with higher scores on variable *X* tend to have lower scores on variable *Y*, and scores on *Y* decrease as scores on *X* increases.

Tables 13.7 and 13.8 present an example of a negative relationship using data taken from a recent public opinion poll administered to a representative sample of U.S. citizens. The independent variable is church attendance, and the dependent variable is approval of cohabitation (Is it all right for a couple to live together without intending to get married?). Note that rates of attendance increase from left to right, and approval of cohabitation increases from top to bottom of the table.

Once again, the percentages in Table 13.8 make the pattern obvious. The great majority of people who do not attend church ("Never") were high on approval of cohabitation, and most people who were high on attendance were low on approval. As attendance increases, approval of cohabitation tends to decrease. The gamma for this table is -0.57, indicating a moderate to strong negative relationship between attendance and approval of this living arrangement.

You should be aware of an additional complication. The coding for ordinal level variables, such as approval of cohabitation, is arbitrary. A higher score may mean "more" or "less" of the variable being measured. For example, if we

ONE STEP AT A TIME **Computing and Interpreting Gamma**

Step ***Operation***

Computation

1. Double-check to make sure that the table is arranged with the column variable increasing from left to right and the row variable increasing from top to bottom.
2. To compute N_s, start with the upper left-hand cell. Multiply the number of cases in this cell by the total number of cases in all cells below and to the right. Repeat this process for each cell in the table. Add up these subtotals to find N_s.
3. To compute N_d, start with the upper right-hand cell. Multiply the number of cases in this cell by the total number of cases in all cells below and to the left. Repeat this process for each cell in the table. Add up these subtotals to find N_d.
4. Subtract N_d from N_s.
5. Add N_d and N_s.
6. Divide the quantity you found in Step 4 by the quantity you found in Step 5. The result is gamma.

Interpretation

7. To interpret the *strength* of the relationship, always begin with the column percentages: the bigger the change in column percentages, the stronger the relationship.
8. Next, you can use gamma to interpret the strength of the relationship in two ways:
   a. Use Table 13.2 to describe strength in general terms.
   b. To use the logic of proportional reduction in error, multiply gamma by 100. This value represents the percentage by which we improve our prediction of the dependent variable by taking into account the independent variable.
9. To interpret the *direction* of the relationship, always begin with the pattern of the column percentages. If the cases tend to fall in a diagonal from upper-left to lower-right, the relationship is positive. If the cases tend to fall in a diagonal from lower-left to upper-right, the relationship is negative.
10. The sign of the gamma also tells the direction of the relationship; however, be very careful when interpreting direction with ordinal level variables. Remember that coding schemes for these variables are arbitrary, and a positive gamma may mean that the actual relationship is negative and vice versa.

measured social class as upper, middle, and lower, we could assign scores to the categories in either of two ways:

A	B
(1) Upper	(3) Upper
(2) Middle	(2) Middle
(3) Lower	(1) Lower

While coding scheme B might seem preferable (because higher scores go with higher class position), *both* schemes are perfectly legitimate, and the direction of gamma will change, depending on which scheme is selected. Using scheme B, we would find positive relationships between social class and education: as education increased, so would class. Using scheme A, however, the same relationship would appear to be negative because the numerical scores (1, 2, 3) are coded in reverse order: the highest social class is assigned the lowest score, and

Application 13.1

A group of 40 nations have been rated as high or low on religiosity (based on the percentage of a random sample of citizens that described themselves as "a religious person") and as high or low in their support for single mothers (based on the percentage of a random sample of citizens who said they would approve of a woman choosing to be a single parent). Are more religious nations less approving of single mothers?

APPROVAL OF SINGLE MOTHERS BY RELIGIOSITY OF NATION

	Religiosity		
Approval	Low	High	Totals
Low	4 *(26.67%)*	9 *(36.00%)*	13
High	11 *(73.33%)*	16 *(64.00%)*	27
Totals	15 *(100.00%)*	25 *(100.00%)*	40

The column percentages show that nations that rank higher on religiosity are also less approving of single mothers. The maximum difference of about 10 suggests a weak to moderate relationship.

Since both variables are ordinal in level of measurement, we can use gamma to measure the strength and direction of the relationship. The number of pairs of cases ranked in the same order on both variables (N_s) would be

$$N_s = 4(16) = 64$$

The number of pairs of cases ranked in different order on both variables (N_d) would be

$$N_d = 9(11) = 99$$

Gamma is

$$G = \frac{N_s - N_d}{N_s + N_d} = \frac{64 - 99}{64 + 99} = \frac{-35}{163} = -0.21$$

A gamma of −0.21 means that, when predicting the order of pairs of cases on the dependent variable (approval of single mothers), we would make 21% fewer errors by taking into account the independent variable (religiosity). There is a moderate to weak negative association between these two variables. As religiosity increases, approval decreases (or, more religious nations are less approving of single mothers).

so forth. If you didn't check the coding scheme, you might conclude that the negative gamma means that class decreases as education increases when, actually, the opposite is true.

Unfortunately, this source of confusion cannot be avoided when working with ordinal level variables. Coding schemes will always be arbitrary for these variables, and you need to exercise additional caution when interpreting the direction of ordinal level variables.

13.5 SPEARMAN'S RHO (r_s)

To this point, we have considered ordinal variables that have a limited number of categories (possible values) and are presented in tables. However, many ordinal level variables have a broad range of scores and many distinct values. Such data may be collapsed into a few broad categories (such as high, moderate, and low), organized into a bivariate table, and analyzed with gamma. Collapsing scores in this manner may be beneficial and desirable in many instances, but some important distinctions between cases may be obscured or lost as a consequence.

For example, suppose a researcher wished to test the claim that jogging is beneficial not only physically, but also psychologically. Do joggers have an enhanced sense of self-esteem? To deal with this issue, 10 female joggers are measured on two scales, the first measuring involvement in jogging and the other measuring self-esteem. Scores are reported in Table 13.9.

These data could be collapsed and a bivariate table produced. We could, for example, dichotomize both variables to create only two values (high and low) for both variables. Although collapsing scores in this way is certainly legitimate

TABLE 13.9 THE SCORES OF 10 SUBJECTS ON INVOLVEMENT IN JOGGING AND A MEASURE OF SELF-ESTEEM

Joggers	Involvement in Jogging (X)	Self-esteem (Y)
Wendy	18	15
Debbie	17	18
Phyllis	15	12
Stacy	12	16
Evelyn	10	6
Tricia	9	10
Christy	8	8
Patsy	8	7
Marsha	5	5
Lynn	1	2

and often necessary,[1] two difficulties with this practice must be noted. First, the scores seem continuous, and there are no obvious or natural division points in the distribution that would allow us to distinguish, in a nonarbitrary fashion, between high and low scores. Second, and more important, grouping these cases into broader categories will lose information. That is, if both Wendy and Debbie are classified as "high" on involvement, the fact that they had different scores on the variable would be obscured. If these differences are important and meaningful, then we should opt for a measure of association that permits the retention of as much detail and precision in the scores as possible.

Computation. Spearman's rho (r_s) is a measure of association for ordinal level variables that have a broad range of many different scores and few ties between cases on either variable. Scores on ordinal level variables cannot, of course, be manipulated mathematically except for judgments of "greater than" or "less than." To compute Spearman's rho, cases are first ranked from high to low on each variable, and then the ranks (not the scores) are manipulated to produce the final measure. Table 13.10 displays the original scores and the rankings of the cases on both variables.

To rank the cases, first find the highest score on each variable and assign it rank 1. Wendy has the high score on X (18) and is thus ranked number 1. Debbie, on the other hand, is highest on Y and is ranked first on that variable. All other cases are then ranked in descending order of scores. If any cases have the same score on a variable, assign them the average of the ranks they would have used had they not been tied. Christy and Patsy have identical scores of 8 on involvement. Had they not been tied, they would have used ranks 7 and 8. The average of these two ranks is 7.5, and this average of used ranks is assigned to all tied cases. (For example, if Marsha had also had a score of 8, three ranks—7, 8, and 9—would have been used, and all three tied cases would have been ranked eighth.)

The formula for Spearman's rho is

FORMULA 13.2

$$r_s = 1 - \frac{6\Sigma D^2}{N(N^2 - 1)}$$

Where $= \Sigma D^2$ the sum of the differences in ranks, the quantity squared

[1]For example, collapsing scores may be advisable when the researcher is not sure that fine distinctions between scores are meaningful.

TABLE 13.10 COMPUTING SPEARMAN'S RHO

	Involvement (X)	Rank	Self-Image (Y)	Rank	D	D^2
Wendy	18	1	15	3	−2.0	4
Debbie	17	2	18	1	1.0	1
Phyllis	15	3	12	4	−1.0	1
Stacey	12	4	16	2	2.0	4
Evelyn	10	5	6	8	−3.0	9
Tricia	9	6	10	5	1.0	1
Christy	8	7.5	8	6	1.5	2.25
Patsy	8	7.5	7	7	0.5	0.25
Marsha	5	9	5	9	0	0
Lynn	1	10	2	10	0	0
					$\Sigma D = 0$	$\Sigma D^2 = 22.50$

To compute ΣD^2, the rank of each case on Y is subtracted from its rank on X (D is the difference between rank on Y and rank on X). A column has been provided in Table 13.10 so that these differences may be recorded on a case-by-case basis. Note that the sum of this column (ΣD) is 0. That is, the negative differences in rank are equal to the positive differences, as will always be the case. You should find the total of this column as a check on your computations to this point. If the ΣD is not equal to 0, you have made a mistake either in ranking the cases or in subtracting the differences.

In the column headed D^2, each difference is squared to eliminate negative signs. The sum of this column is ΣD^2, and this quantity is entered directly into the formula. For our sample problem:

$$r_s = 1 - \frac{6\Sigma D^2}{N(N^2 - 1)}$$

$$r_s = 1 - \frac{6(22.5)}{10(100 - 1)}$$

$$r_s = 1 - \frac{135}{990}$$

$$r_s = 1 - 0.14$$

$$r_s = 0.86$$

Interpretation. Spearman's rho is an index of the strength of association between the variables; it ranges from 0 (no association) to ±1.00 (perfect association). A perfect positive association ($r_s = +1.00$) would exist if there were no disagreements in ranks between the two variables (if cases were ranked in exactly the same order on both variables). A perfect negative relationship ($r_s = -1.00$) would exist if the ranks were in perfect disagreement (if the case ranked highest on one variable were lowest on the other, and so forth). A Spearman's rho of 0.86 indicates a strong, positive relationship between these two variables. The respondents who were highly involved in jogging also ranked high on self-image. These results are supportive of claims regarding the psychological benefits of jogging.

Spearman's rho is an index of the relative strength of a relationship, and values between 0 and ±1.00 have no direct interpretation. However, if the value of rho is squared, a PRE interpretation is possible. Rho squared (r_s^2) is the proportional reduction in errors of prediction when predicting rank on one variable based on rank on the other variable, as compared to predicting rank while ignoring the other variable. In the example above, r_s was 0.86 and r_s^2 would be 0.74. Thus, our errors of prediction would be reduced by 74% if, when predicting the rank of a subject on self-image, the rank of the subject on involvement in jogging were taken into account. (*For practice in computing and interpreting Spearman's rho, see Problems 13.11–13.14. Problem 13.11 has the fewest number of cases and is probably a good choice for a first attempt at these procedures.*)

ONE STEP AT A TIME — Computing and Interpreting Spearman's Rho

Step ***Operation***

Computation

1. Set up a computing table like Table 13.10 to help organize the computations. In the far left-hand column, list the cases in order, with the case with the highest score on the independent variable (*X*) stated first.
2. In the next column, list the scores on *X.*
3. In the third column, list the rank of each case on *X,* beginning with rank 1 for the highest score. If any cases have the same score, assign them the average of the ranks they would have used had they not been tied.
4. In the fourth and fifth columns, repeat Steps 2 and 3 for the scores of the cases on the dependent variable (*Y*). List the scores on *Y* in the fourth column, and then, in the fifth column, rank the cases on *Y* from high to low. Start by assigning the rank of 1 to the case with the highest score on *Y* and assign any tied cases the average of the ranks they would have used had they not been tied.
5. For each case, subtract the rank on *Y* from the rank on *X* and write the difference (*D*) in the sixth column. Add this column. If the sum is not zero, you have made a mistake and need to recompute.
6. Square the value of each *D* and record the result in the seventh column.
7. Add column 7 to find ΣD^2, and substitute the result into the numerator of Formula 13.2.
8. Multiply the ΣD^2 (the total of column 7 in the computing table) by 6.
9. Square *N* and subtract 1 from the result.
10. Multiply the quantity you found in Step 9 by *N*.
11. Divide the quantity you found in Step 8 by the quantity you found in Step 10.
12. Subtract the quantity you found in Step 11 from 1. The result is r_s.

Interpretation

13. To interpret the strength of Spearman's rho, you can do either of the following:
    - **a.** Use Table 13.2 to characterize the strength of the relationship in general terms.
    - **b.** Square the value of r_s and multiply the result by 100. This value represents the percentage by which we improve our prediction of the dependent variable by taking into account the independent variable.
14. To interpret the direction of the relationship, look at the sign of r_s; however, be careful when interpreting direction with ordinal level variables. Remember that coding schemes for these variables are arbitrary and a positive r_s may mean that the actual relationship is negative and vice versa.

Application 13.2

Five cities have been rated on an index that measures the quality of life. Also, the percentage of the population that has moved into each city over the past year has been determined. Have cities with higher quality-of-life scores attracted more new residents? The table below summarizes the scores, ranks, and differences in ranks for each of the five cities.

Spearman's rho for these variables is

$$r_s = 1 - \frac{6\Sigma D^2}{N(N^2 - 1)}$$

$$r_s = 1 - \frac{(6)(4)}{5(25 - 1)}$$

$$r_s = 1 - \left(\frac{24}{120}\right)$$

$$r_s = 1 - 0.20$$

$$r_s = 0.80$$

These variables have a strong, positive association. The higher the quality-of-life score, the greater the percentage of new residents. The value of r_s^2 is 0.64 ($0.80^2 = 0.64$), which indicates that we will make 64% fewer errors when predicting rank on one variable from rank on the other, as opposed to ignoring rank on the other variable.

City	Quality of Life	Rank	% New Residents	Rank	D	D^2
A	30	1	17	1	0	0
B	25	2	14	3	−1	1
C	20	3	15	2	1	1
D	10	4	3	5	−1	1
E	2	5	5	4	1	1
					$\Sigma D = 0$	$\Sigma D^2 = 4$

SUMMARY

1. Measures of association for variables with collapsed (gamma) and continuous (Spearman's rho) ordinal variables were covered. Both measures summarize the overall strength and direction of the association between the variables.
2. Gamma is a PRE-based measure that shows the improvement in our ability to predict the order of pairs of cases on one variable from the order of pairs of cases on the other variable, as opposed to ignoring the order of the pairs of cases on the other variable.
3. Spearman's rho is computed from the ranks of the scores of the cases on two continuous ordinal variables and, when squared, can be interpreted by the logic of PRE.

SUMMARY OF FORMULAS

FORMULA 13.1 Gamma $G = \dfrac{N_s - N_d}{N_s + N_d}$

FORMULA 13.2 Spearman's rho $r_s = 1 - \dfrac{6\Sigma D^2}{N(N^2 - 1)}$

GLOSSARY

Gamma (G). A measure of association appropriate for variables measured with collapsed ordinal scales that have been organized into table format; G is the symbol for gamma.

N_d. The number of pairs of cases ranked in different order on two variables.

N_s. The number of pairs of cases ranked in the same order on two variables.

Spearman's rho (r_s). A measure of association appropriate for ordinally measured variables that are continuous in form; r_s is the symbol for Spearman's rho.

PROBLEMS

(Problems are labeled with the social science discipline from which they are drawn: SOC for sociology, SW for social work, PS for political science, CJ for criminal justice, PA for public administration, and GER for gerontology.)

For Problems 13.1–13.10, calculate column percentages and use the percentages to help analyze the strength and direction of the association.

13.1 SOC A small sample of non-English-speaking immigrants to the United States has been interviewed about their level of assimilation. Is the pattern of adjustment affected by length of residence in the United States? For each table, compute gamma and summarize the relationship in terms of strength and direction. (*HINT: In 2 × 2 tables, only two cells can contribute to N_s or N_d. To compute N_s, multiply the number of cases in the upper left-hand cell by the number of cases in the lower right-hand cell. For N_d, multiply the number of cases in the upper right-hand cell by the number of cases in the lower left-hand cell.*)

a. Facility in English:

	Length of Residence		
English Facility	Less than Five Years (Low)	More than Five Years (High)	Totals
Low	20	10	30
High	5	15	20
Totals	25	25	50

b. Total family income:

	Length of Residence		
Income	Less than Five Years (Low)	More than Five Years (High)	Totals
Below national average (1)	18	8	26
Above national average (2)	7	17	24
Totals	25	25	50

c. Extent of contact with country of origin:

	Length of Residence		
Contact	Less than Five Years (Low)	More than Five Years (High)	Totals
Rare (1)	5	20	25
Frequent (2)	20	5	25
Totals	25	25	50

13.2 CJ A random sample of 150 cities has been classified as small, medium, or large by population and as high or low on crime rate. Is there a relationship between city size and crime rate?

	City Size			
Crime Rate	Small	Medium	Large	Totals
Low	21	17	8	46
High	29	33	42	104
Totals	50	50	50	150

Describe the strength and direction of the relationship.

13.3 SOC Some research has shown that families vary by how they socialize their children to sports, games, and other leisure-time activities. In middle-class families, such activities are carefully monitored by parents and are, in general, dominated by adults (for example, Little League baseball). In working-class families, children more often organize and initiate such activities themselves, and parents are much less involved (for example, sandlot or playground baseball games). Are the data below consistent with these findings? Summarize your conclusions in a few sentences.

As a Child, Did You Play Mostly Organized or Sandlot Sports?	Social Class		
	White Collar	Blue Collar	Totals
Organized	155	123	278
Sandlot	101	138	239
Totals	256	261	517

13.4 Is there a relationship between education and support for women in the paid labor force? Is the relationship between the variables different for different nations? The World Values Survey has been administered to random samples drawn from Canada, the United States, and Mexico. Respondents were asked if they agree or disagree that both husbands and wives should contribute to the family income. Compute column percentages and gamma for each table. Is there a relationship? Describe the strength and direction of the relationship. Which educational level is most supportive of women being in the paid labor

force? How does the relationship change from nation to nation?

a. Canada

Husbands and Wives Should Contribute to Income	Education			
	Low	Moderate	High	Totals
Agree	352	682	365	1,399
Disagree	98	204	154	456
Totals	450	886	519	1,855

b. United States

Husbands and Wives Should Contribute to Income	Education			
	Low	Moderate	High	Totals
Agree	177	239	380	796
Disagree	54	107	203	364
Totals	231	346	583	1,160

c. Mexico

Husbands and Wives Should Contribute to Income	Education			
	Low	Moderate	High	Totals
Agree	718	471	140	1,329
Disagree	105	48	14	167
Totals	823	519	154	1,496

13.5 PA All applicants for municipal jobs in Shinbone, Kansas, are given an aptitude test, but the test has never been evaluated to see if test scores are in any way related to job performance. The following table reports aptitude test scores and job performance ratings for a random sample of 75 city employees.

Efficiency Ratings	Test Scores			
	Low	Moderate	High	Totals
Low	11	6	7	24
Moderate	9	10	9	28
High	5	9	9	23
Totals	25	25	25	75

a. Are these two variables associated? Describe the strength and direction of the relationship in a sentence or two.

b. Should the aptitude test continue to be administered? Why or why not?

13.6 SW A sample of children has been observed and rated for symptoms of depression. Their parents have been rated for authoritarianism. Is there any relationship between these variables? Write a few sentences stating your conclusions.

Symptoms of Depression	Authoritarianism			
	Low	Moderate	High	Totals
Few	7	8	9	24
Some	15	10	18	43
Many	8	12	3	23
Totals	30	30	30	90

13.7 SOC Are prejudice and level of education related? State your conclusion in a few sentences.

Prejudice	Level of Education				
	Elementary School	High School	Some College	College Graduate	Totals
Low	48	50	61	42	201
High	45	43	33	27	148
Totals	93	93	94	69	349

13.8 SOC In a recent survey, a random sample of respondents was asked to indicate how happy they were with their situations in life. Are their responses related to income level? Describe the strength and direction of the relationship.

Happiness	Income			
	Low	Moderate	High	Totals
Not happy	101	82	36	219
Pretty happy	40	227	100	367
Very happy	216	198	203	617
Totals	357	507	339	1,203

13.9 The tables below test the relationship between income and a set of dependent variables. For each table, calculate percentages and gamma. Describe the strength and direction of each relationship in a few sentences. *Be careful in interpreting direction.*

a. Support for the legal right to an abortion by income:

Right to an Abortion?	Income			
	Low	Moderate	High	Totals
Yes	220	218	226	664
No	366	299	250	915
Totals	586	517	476	1,579

b. Support for capital punishment by income:

Capital Punishment?	Income			
	Low	Moderate	High	Totals
Favor	567	574	552	1,693
Oppose	270	183	160	613
Totals	837	757	712	2,306

c. Approval of suicide for people with an incurable disease by income:

Right to Suicide?	Income			
	Low	Moderate	High	Totals
Approve	343	341	338	1,022
Oppose	227	194	147	568
Totals	570	535	485	1,590

d. Support for sex education in public schools by income:

Sex Education?	Income			
	Low	Moderate	High	Totals
For	492	478	451	1,421
Against	85	68	53	206
Totals	577	546	504	1,627

e. Support for traditional gender roles by income:

Women Should Take Care of Running Their Homes and Leave Running the Country to Men	Income			
	Low	Moderate	High	Totals
Agree	130	71	39	240
Disagree	448	479	461	1,388
Totals	578	550	500	1,628

13.10 SOC A random sample of 11 neighborhoods in Shinbone, Kansas, has been rated by an urban sociologist on a quality-of-life scale (which includes measures of affluence, availability of medical care, and recreational facilities) and a social cohesion scale. The results are presented below in scores. Higher scores indicate higher quality of life and greater social cohesion. Are the two variables associated? What is the strength and direction of the association? Summarize the relationship in a sentence or two. (*HINT: Don't forget to square the value of Spearman's rho for a PRE interpretation.*)

Neighborhood	Quality of Life	Social Cohesion
Queens Lake	17	8.8
North End	40	3.9
Brentwood	47	4.0
Denbigh Plantation	90	3.1
Phoebus	35	7.5
Kingswood	52	3.5
Chesapeake Shores	23	6.3
Windsor Forest	67	1.7
College Park	65	9.2
Beaconsdale	63	3.0
Riverview	100	5.3

13.11 SW Several years ago, a job-training program began, and a team of social workers screened the candidates for suitability for employment. Now the screening process is being evaluated, and the actual work performance of a sample of hired candidates has been rated. Did the screening process work? Is there a relationship between the original scores and performance evaluation on the job?

Case	Original Score	Performance Evaluation
A	17	78
B	17	85
C	15	82
D	13	92
E	13	75
F	13	72
G	11	70
H	10	75
I	10	92
J	10	70
K	9	32
L	8	55
M	7	21
N	5	45
O	2	25

13.12 SOC Below are the scores of a sample of 15 nations on a measure of ethnic diversity (the higher the number, the greater the diversity) and a measure of economic inequality (the higher the score, the greater the inequality). Are these variables related? Are ethnically diverse nations more economically unequal?

Nation	Diversity	Inequality
India	91	29.7
South Africa	87	58.4
Kenya	83	57.5
Canada	75	31.5
Malaysia	72	48.4
Kazakstan	69	32.7
Egypt	65	32.0
United States	63	41.0
Sri Lanka	57	30.1
Mexico	50	50.3
Spain	44	32.5
Australia	31	33.7
Finland	16	25.6
Ireland	4	35.9
Poland	3	27.2

13.13 Twenty ethnic, racial, or national groups were rated by a random sample of white and black students on a social distance scale. Lower scores represent less social distance and less prejudice. How similar are these rankings?

		Average Social Distance Scale Score	
	Group	White Students	Black Students
1	White Americans	1.2	2.6
2	English	1.4	2.9
3	Canadians	1.5	3.6
4	Irish	1.6	3.6
5	Germans	1.8	3.9
6	Italians	1.9	3.3
7	Norwegians	2.0	3.8
8	American Indians	2.1	2.7
9	Spanish	2.2	3.0
10	Jews	2.3	3.3
11	Poles	2.4	4.2
12	Black Americans	2.4	1.3
13	Japanese	2.8	3.5
14	Mexicans	2.9	3.4
15	Koreans	3.4	3.7
16	Russians	3.7	5.1
17	Arabs	3.9	3.9
18	Vietnamese	3.9	4.1
19	Turks	4.2	4.4
20	Iranians	5.3	5.4

YOU ARE THE RESEARCHER: Exploring Sexual Attitudes and Behavior

Two projects are presented to help you apply the skills developed in this chapter. Both focus on sex, a subject of great fascination to many people. The first project uses bivariate tables and gamma to explore the possible causes of attitudes and opinions about premarital sex. The second uses Spearman's rho to investigate sexual behavior, specifically the number of different sexual partners people have had over the past five years.

PROJECT 1: Who Approves of Premarital Sex?

What type of person is most likely to oppose sex before marriage? In this exercise, you will take attitudes toward premarital sex (*premarsx*) as the dependent variable. The wording and coding scheme for the variable are presented in the table below. Remember that the coding scheme for ordinal level variables is arbitrary. In this case, higher scores indicate greater support for premarital sex and lower scores mean greater disapproval. Keep the coding scheme in mind as you analyze the direction of the relationships you find.

There's been a lot of discussion about the way morals and attitudes about sex are changing in this country. If a man and a woman have sex relations before marriage, do you think this is

1	Always wrong
2	Almost always wrong
3	Sometimes wrong
4	Not wrong at all

STEP 1: Choosing Independent Variables

Select four variables from the 2006 GSS that you think might be important causes of attitudes toward premarital sex and list the variables in the table below. Your independent variables *cannot* be nominal in level of measurement and should have no more than four to five categories or scores. Some of the ordinal level variables in which you might be interested (such as *attend*, a measure of church attendance) have more than four to five categories and should be recoded, as should interval-ratio variables such as *age* or *income06*. See Chapter 10 for instructions on recoding. Select independent variables that seem likely to be an important cause of people's attitudes about sex. Be sure to note the coding scheme for each variable.

SPSS Variable Name	What Exactly Does This Variable Measure?

STEP 2: Stating Hypotheses

State hypotheses about the relationships you expect to find between your independent variables and *premarsx*. State these hypotheses in terms of the direction of the relationship you expect to find. For example, you might hypothesize that your dependent variables—approval of premarital sex—will decline as age increases (a negative relationship).

1.

2.

3.

4.

STEP 3: Running Crosstabs

Click **Analyze → Descriptives → Crosstabs** and place the dependent variable (*premarsx*) in the **Rows:** box and the independent variables you selected in the **Columns:** box. Click the **Statistics** button to get chi square and gamma and the **Cells** button to get column percentages.

STEP 4: Recording Results

These commands will generate a lot of output, and it will be helpful to summarize your results in the following table.

Independent Variable	Chi Square Significant at < 0.05? (Write *Yes* or *No*)	Gamma (Be sure to note the sign or direction as well as numerical value)

STEP 5: Analyzing and Interpreting Results

For each independent variable, analyze and interpret the significance, strength, and direction of the relationship. For example, you might say the following: There was a moderate negative relationship between age and approval of premarital sex (chi square = 4.26; $df = 2$; $p < 0.05$. Gamma = 0.35). Older respondents tended to be more opposed. Be sure to *explain* the direction of the relationship and don't just characterize it as negative or positive. *Be careful when interpreting direction with ordinal variables*.

Independent Variable	Interpretation
1	
2	
3	
4	

STEP 6: Testing Hypotheses

Write a few sentences of overall summary for this test. Were your hypotheses supported? Which independent variable had the strongest relationship with *premarsx*? Why do you think this is so?

PROJECT 2: Sexual Behavior

In this exercise, your dependent variable will be *partnrs5,* which measures how many different sex partners a person has had over the past five years. The wording and coding scheme for the variable are presented in the table below. For this variable, higher scores mean a greater number of partners, so interpreting the direction of relationships should be relatively straightforward.

How Many Sex Partners Have You Had Over the Past Five Years?	
0	No partners
1	1 partner
2	2 partners
3	3 partners
4	4 partners
5	5–10 partners
6	11–20 partners
7	21–100 partners
8	More than 100 partners

Note that, like the *sexfreq* variable we used in Chapter 10, *partnrs5* is interval-ratio for the first five categories and then becomes ordinal for higher scores. In other words, the variable has a true zero point (a score of 0 means no sex partners at all) and increases by equal, defined units from one partner through four partners. Higher scores, however, represent broad categories, not exact numbers.

This mixture of levels of measurement makes Spearman's rho an appropriate statistic to use to measure the strength and direction of relationships

To access Spearman's rho, click **Analyze → Correlate → Bivariate,** and the **Bivariate Correlations** window will open. The variables are listed in the window on the left. Below that window is a box labeled **Correlation Coefficients.** Click **Spearman** from the options to get Spearman's rho. Next, select *partnrs5* and your independent variables from the list on the left; click the arrow to move them to the **Variables:** window on the right. SPSS will compute Spearman's rho for all pairs of variables included in the **Variables:** window.

Let's take a brief look at the output produced by this procedure. To provide an example, I looked at the relationships between *chldidel* (the respondent's perception of the ideal number of children for a family), *attend* (frequency of church attendance), and *income06*. I take *chldidel* as the dependent variable and hypothesize that it will have a positive relationship with *attend* (the greater the religiosity, the greater the value placed on a large family) and a negative relationship with *income06* (the higher the income, the less the respondent will value a large family).

The output from the **Correlate** procedure is a table showing the bivariate correlations of all possible combinations of variables, including the relationship of the variable with itself. The table is called a correlation matrix and will look like the table below. We will deal with the correlation matrix in more detail in Chapter 14. (Note that this table has been edited to fit this space and will not look exactly like the SPSS output).

Correlations

		Ideal Number of Children	How Often r Attends Religious Services	Total Family Income
Ideal Number of Children	Correlation Coefficient	1.000	.089*	-.144**
	Sig. (2-tailed)	.	.039	.002
	N	537	534	460
How Often r Attends Religious Services	Correlation Coefficient	.089*	1.000	.103**
	Sig. (2-tailed)	.039	.	.000
	N	534	1419	1204
Total Family Income	Correlation Coefficient	-.144**	.103**	1.000
	Sig. (2-tailed)	.002	.000	.
	N	460	1204	1205

*Correlation is significant at the 0.05 level (2-tailed).
**Correlation is significant at the 0.01 level (2-tailed).

Since we have three variables, the table has nine cells, and there are three pieces of information in each cell: the value of Spearman's rho, the statistical significance of the r_s, and the number of cases. The cell on the upper left shows the relationship between *chldidel* and itself, the next cell to the right shows the relationship between *chldidel* and *attend,* and so forth.

The output shows a weak positive relationship between *chldidel* and *attend* and a weak negative relationship between *chldidel* and *income06.* Both of these relationships are in the direction I predicted, but they are weak and provide very minimal (if any) support for my hypotheses.

Now it's your turn.

STEP 1: Choosing Independent Variables for *partnrs5*

Select four variables from the 2006 GSS that you think might be important causes of this dimension of sexual behavior. Your independent variables *cannot* be nominal in level of measurement and should have more than three categories or scores. List the variable and describe exactly what they measure in the table below.

SPSS Variable Name	What Exactly Does This Variable Measure?

STEP 2: Stating Hypotheses

State hypotheses about the relationships you expect to find between your independent variables and *partnrs5*. State these hypotheses in terms of the direction of the relationship you expect to find. For example, you might hypothesize that number of sexual partners will decline as age increases.

1.

2.

3.

4.

STEP 3: Running Bivariate Correlations

Click **Analyze → Bivariate → Correlate** and place all variables in the **Variables:** box. Click **OK** to get your results.

STEP 4: Recording Results

Use the table below to summarize your results. Enter the r_s for each independent variable in each cell. Ignore correlations of variables with themselves and redundant information.

	Independent Variables
	1. _____ 2. _____ 3. _____ 4. _____
partnrs5	

STEP 5: Analyzing and Interpreting Results

Write a short summary of results for each independent variable. Your summary needs to identify the variables being tested and the strength and direction of the relationship. It is probably best to characterize the relationship in general terms and then cite the statistical values in parentheses. Be sure to note whether or not your hypotheses were supported. Be careful when interpreting direction and refer back to the coding scheme to make sure you understand the relationship.

14 Association Between Variables Measured at the Interval-Ratio Level

LEARNING OBJECTIVES

By the end of this chapter, you will be able to:

1. Interpret a scattergram.
2. Calculate and interpret slope (*b*), *Y* intercept (*a*), and Pearson's *r* and r^2.
3. Find and explain the least-squares regression line and use it to predict values of *Y*.
4. Explain the concepts of total, explained, and unexplained variance.
5. Use regression and correlation techniques to analyze and describe a bivariate relationship in terms of the three questions introduced in Chapter 12.

14.1 INTRODUCTION

This chapter presents a set of statistical techniques for analyzing the association or correlation between variables measured at the interval-ratio level.[1] As we shall see, these techniques are rather different in their logic and computation from those covered in Chapters 12 and 13. Let me stress at the outset, therefore, that we are still asking the same three questions: Is there a relationship between the variables? How strong is the relationship? What is the direction of the relationship? You might become preoccupied with some of the technical details and computational routines in this chapter, so remind yourself occasionally that our ultimate goals are unchanged: we are trying to understand bivariate relationships, explore possible causal ties between variables, and improve our ability to predict scores.

14.2 SCATTERGRAMS

As we have seen over the past several chapters, properly percentaged tables provide important information about bivariate associations between nominal and ordinal level variables. In addition to measures of association such as phi or gamma, the conditional distributions and patterns of cell frequency almost always provide useful information and a better understanding of the relationship between variables.

In the same way, the usual first step in analyzing a relationship between interval-ratio variables is to construct and examine a type of graph called a **scattergram.** Like bivariate tables, scattergrams allow us to quickly identify several important features of the relationship. An example will illustrate their construction and use. Suppose a researcher is interested in analyzing how dual-wage-earner families (that is, families where both husband and wife have jobs outside the home) cope with housework. Specifically, the researcher wonders if the number of children in the family is related to the amount of time the husband contributes to housekeeping chores. The relevant data for a sample of 12 dual-wage-earner families are displayed in Table 14.1.

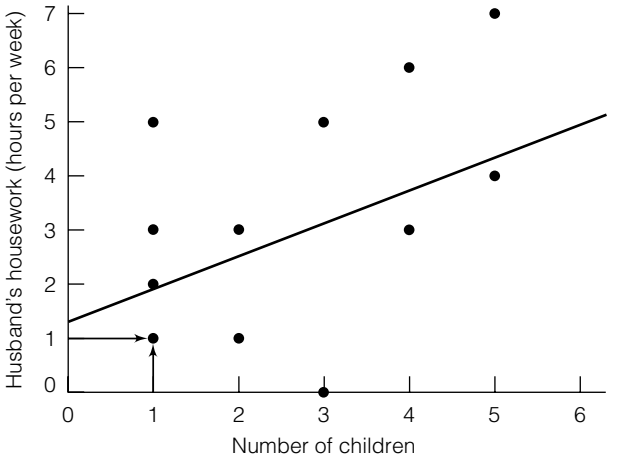
[1]The term *correlation* is commonly used instead of *association* when discussing the relationship between interval-ratio variables. We will use the two terms interchangeably.

TABLE 14.1 NUMBER OF CHILDREN AND HUSBAND'S CONTRIBUTION TO HOUSEWORK (fictitious data)

Family	Number of Children	Hours per Week Husband Spends on Housework
A	1	1
B	1	2
C	1	3
D	1	5
E	2	3
F	2	1
G	3	5
H	3	0
I	4	6
J	4	3
K	5	7
L	5	4

Constructing Scattergrams. A scattergram, like a bivariate table, has two dimensions. The scores of the independent (X) variable are arrayed along the horizontal axis and the scores of the dependent (Y) variable along the vertical axis. Each dot on the scattergram represents a case in the sample and is located at a point determined by the scores of the case. The scattergram in Figure 14.1 shows the relationship between "number of children" and "husband's housework" for the sample of 12 families presented in Table 14.1. Family A has a score of 1 on the X variable (number of children) and 1 on the Y variable (husband's housework) and is represented by the dot above the score of 1 on the X axis and directly to the right of the score of 1 on the Y axis. All 12 cases are similarly represented by dots on Figure 14.1. Also note that, as with all tables, graphs, and charts, the scattergram is clearly titled and both axes are labeled.

FIGURE 14.1 HUSBAND'S HOUSEWORK BY NUMBER OF CHILDREN



Interpreting Scattergrams. The overall pattern of the dots or cases summarizes the nature of the relationship between the two variables. The clarity of the pattern can be enhanced by drawing a straight line through the cluster of dots such that the line touches every dot or comes as close to doing so as possible. In Section 14.3, a precise technique for fitting this line to the pattern of the dots will be explained. For now, an "eyeball" approximation will suffice. This summarizing line is called the **regression line** and has already been added to the scattergram in Figure 14.1.

Scattergrams, even when they are crudely drawn, can be used for a variety of purposes. They provide at least impressionistic information about the existence, strength, and direction of the relationship and can also be used to check the relationship for linearity (that is, how well the pattern of dots can be approximated with a straight line). Finally, the scattergram can be used to predict the score of a case on one variable from the score of that case on the other variable. Let's return to the three questions first asked in Chapter 12 and see how we can use the scattergram to answer them.

- **Does a relationship exist?** To ascertain the existence of a relationship, we can return to the basic definition of an association stated in Chapter 12: two variables are associated if the distributions of Y (the dependent variable) change for the various conditions of X (the independent variable). In Figure 14.1, scores on X (number of children) are arrayed along the horizontal axis. The dots above each score on X are the scores (or conditional distributions) of Y. That is, the dots represent scores on Y for each value of X. Figure 14.1 shows that there is a relationship because these conditional distributions of Y (the dots above each score on X) are different for the different values of X. That is, the dots (scores on Y) above the X score of 1 look different from the dots above the X score of 2. The existence of an association is further reinforced by the fact that the regression line lies at an angle to the X axis. If these two variables had not been associated, the conditional distributions of Y would not have changed and the regression line would have been parallel to the horizontal axis.
- **How strong is the relationship?** The strength of the bivariate association can be judged by observing the spread of the dots around the regression line. In a perfect association, every single dot would be on the regression line. The more the dots are clustered around the regression line, the stronger the association.
- **What is the direction of the relationship?** The direction of the relationship can be detected by observing the angle of the regression line. Figure 14.1 shows a positive relationship: as X (number of children) increases, husband's housework (Y) also increases. Husbands in families with more children tend to do more housework. If the relationship had been negative, the regression line would have sloped in the opposite direction to indicate that high scores on one variable were associated with low scores on the other.

To summarize these points about the existence, strength, and direction of the relationship, Figure 14.2 shows a perfect positive and a perfect negative relationship and a "zero relationship" between two variables.

Linearity. One key assumption underlying the statistical techniques introduced later in this chapter is that the two variables have an essentially **linear**

FIGURE 14.2 PERFECT POSITIVE, PERFECT NEGATIVE, AND ZERO RELATIONSHIPS

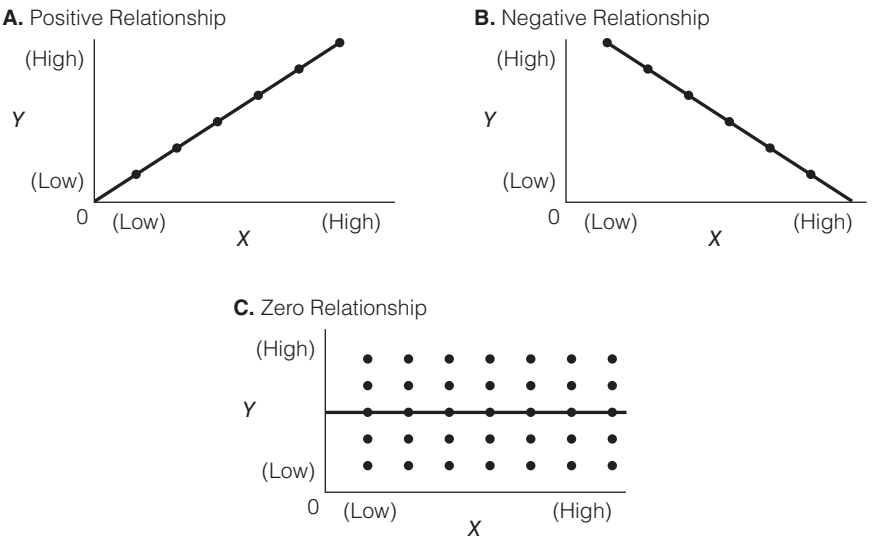
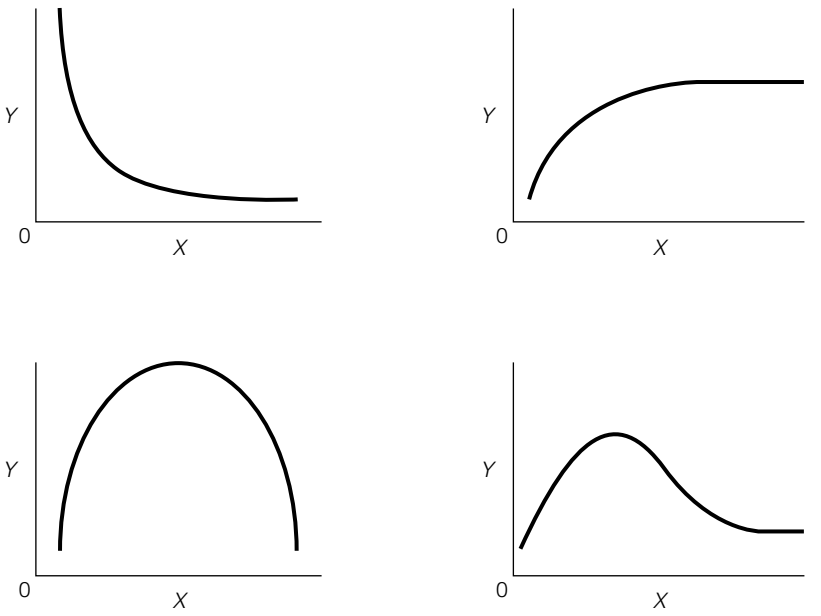


FIGURE 14.3 SOME NONLINEAR RELATIONSHIPS



relationship. In other words, the observation points or dots in the scattergram must form a pattern that can be approximated with a straight line. Significant departures from linearity would require the use of statistical techniques beyond the scope of this text. Examples of some common curvilinear relationships are presented in Figure 14.3. If the scattergram shows that the variables have a nonlinear relationship, the techniques described in this chapter should be used with

great caution or not at all. Checking for the linearity of the relationship is perhaps the most important reason for constructing at least a crude, hand-drawn scattergram before proceeding with the statistical analysis. If the relationship is nonlinear, you might need to treat the variables as if they were ordinal rather than interval-ratio in level of measurement. *(For practice in constructing and interpreting scattergrams, see Problems 14.1–14.4.)*

14.3 REGRESSION AND PREDICTION

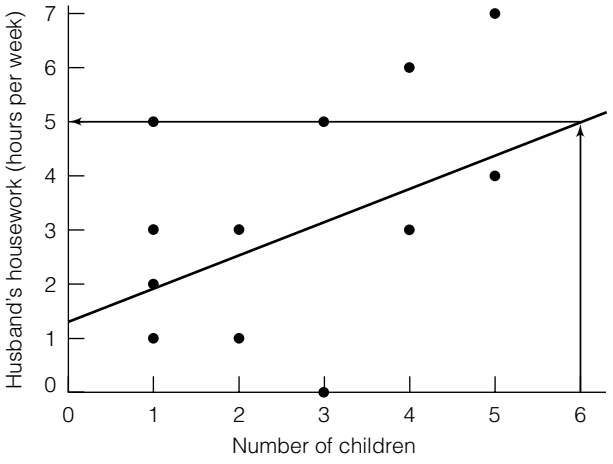
Prediction. A final use of the scattergram is to predict scores of cases on one variable from their score on the other. To illustrate, suppose that, based on the relationship between number of children and husband's housework displayed in Figure 14.1, we wish to predict the number of hours of housework a husband with a family of six children would do each week. The sample has no families with six children, but if we extend the axes and regression line in Figure 14.1 to incorporate this score, a prediction is possible. Figure 14.4 reproduces the scattergram and illustrates how the prediction would be made.

The predicted score on Y—which is symbolized as $\boldsymbol{Y'}$ to distinguish predictions of Y from actual Y scores—is found by first locating the relevant score on X ($X = 6$ in this case) and then drawing a straight line from that point to the regression line. From the regression line, another straight line parallel to the X axis is drawn across to the Y axis. The predicted Y score (Y') is found at the point where this line crosses the Y axis. In our example, we would predict that, in a dual-wage-earner family with six children, the husband would devote about five hours per week to housework.

The Regression Line. Of course, this prediction technique is crude, and the value of Y' can change, depending on how accurately the freehand regression line is drawn. One way to eliminate this source of error would be to find the straight line that most accurately summarizes the pattern of the observation points and so best describes the relationship between the two variables. How can the "best-fitting" straight line be found?

Recall that our criterion for the freehand regression line was that it touch all the dots or come as close to doing so as possible. Also, recall that the dots

FIGURE 14.4 PREDICTING HUSBAND'S HOUSEWORK



above each value of X can be thought of as conditional distributions of Y, the dependent variable. Within each conditional distribution of Y, the mean is the point around which the variation of the scores is at a minimum. In Chapter 4, we noted that the mean of any distribution of scores is the point around which the variation of the scores, as measured by squared deviations, is minimized:

$$\Sigma(X_i - \bar{X})^2 = \text{minimum}$$

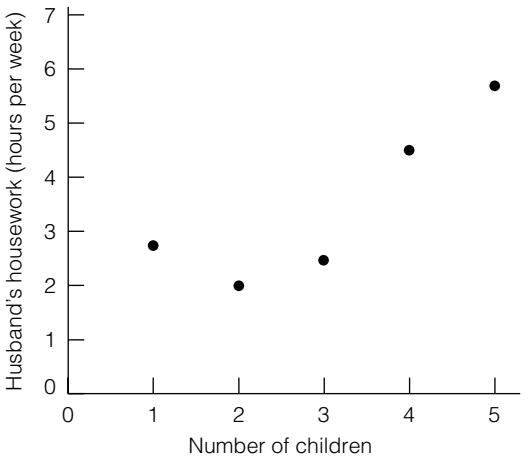
Thus, a regression line that is drawn so that it passes through each **conditional mean of *Y*** would be the straight line that comes as close as possible to all the scores.

Conditional means are found by summing all Y values for each value of X and then dividing by the number of cases. For example, four families had one child ($X = 1$), and the husbands of these four families devoted 1, 2, 3, and 5 hours per week to housework. Thus, for $X = 1$, $Y = 1$, 2, 3, and 5, and the conditional mean of Y for $X = 1$ is 2.75 ($11/4 = 2.75$). Husbands in families with one child worked an average of 2.75 hours per week doing housekeeping chores. Conditional means of Y can be computed in the same way for each value of X and are displayed in Table 14.2 and plotted in Figure 14.5.

TABLE 14.2 CONDITIONAL MEANS OF *Y* (husband's housework) FOR VARIOUS VALUES OF *X* (number of children)

Number of Children (X)	Husband's Housework (Y)	Conditional Means of Y
1	1,2,3,5	2.75
2	3,1	2.00
3	5,0	2.50
4	6,3	4.50
5	7,4	5.50

FIGURE 14.5 CONDITIONAL MEANS OF *Y*



Let us quickly remind ourselves of the reason for these calculations. We are seeking the single best-fitting regression line for summarizing the relationship between X and Y, and we have seen that a line drawn through the conditional means of Y will minimize the spread of the observation points. It will come as close to all the scores as possible and will therefore be the single best-fitting regression line.

Now, a line drawn through the points on Figure 14.5 (the conditional means of Y) will be the best-fitting line we are seeking, but you can see from the scattergram that the line will not be straight. In fact, only rarely (when there is a perfect relationship between X and Y) will conditional means fall in a perfectly straight line. Since we still must meet the condition of linearity, we will revise our criterion and define the regression line as the unique straight line that touches all conditional means of Y or comes as close to doing so as possible. Formula 14.1 defines the least-squares regression line, or the single straight regression line that best fits the pattern of the data points.

FORMULA 14.1

$$Y = a + bX$$

Where: Y = score on the dependent variable
a = the Y intercept or the point where the regression line crosses the Y axis
b = the slope of the regression line or the amount of change produced in Y by a unit change in X
X = score on the independent variable

The formula introduces two new concepts. First, the ***Y* intercept (*a*)** is the point at which the regression line crosses the vertical, or Y, axis. Second, the **slope (*b*)** of the least-squares regression line is the amount of change produced in the dependent variable (Y) by a unit change in the independent variable (X). Think of the slope of the regression line as a measure of the effect of the X variable on the Y variable. If the variables have a strong association, then changes in the value of X will be accompanied by substantial changes in the value of Y, and the slope (b) will have a high value. The weaker the effect of X on Y (the weaker the association between the variables), the lower the value of the slope (b). If the two variables are unrelated, the least-squares regression line would be parallel to the X axis, and b would be 0.00 (the line would have no slope).

With the least-squares formula (Formula 14.1), we can predict values of Y in a much less arbitrary and impressionistic way than through mere eyeballing. This will be so, remember, because the least-squares regression line as defined by Formula 14.1 is the single straight line that best fits the data because it comes as close as possible to all of the conditional means of Y. Before seeing how predictions of Y can be made, however, we must first calculate a and b. *(For practice in using the regression line to predict scores on Y from scores on X, see Problems 14.1–14.3 and 14.5.)*

14.4 COMPUTING *a* AND *b*

In this section, we cover how to compute and interpret the coefficients in the equation for the regression line: the slope (b) and the Y intercept (a). Since the value of b is needed to compute a, we begin with the computation of the slope.

Computing the Slope (*b*). The formula for the slope is

FORMULA 14.2

$$b = \frac{\Sigma(X - \bar{X})(Y - \bar{Y})}{\Sigma(X - \bar{X})^2}$$

The numerator of this formula is called the *covariation* of X and Y. It is a measure of how X and Y vary together, and its value will reflect both the direction and strength of the relationship. The denominator is simply the sum of the squared deviations around the mean of X.

The calculations necessary for computing the slope should be organized into a computational table, as in Table 14.3, which has a column for each of the four quantities needed to solve the formula. The data are from the dual-wage-earner family sample (see Table 14.1).

In Table 14.3, the first column lists the original X scores for each case, and the second column shows the deviations of these scores around their mean. The third and fourth columns repeat this information for the Y scores and the deviations of the Y scores. Column 5 shows the covariation of the X and Y scores. The entries in this column are found by multiplying the deviation of the X score (column 2) by the deviation of the Y score (column 4) for each case. Finally, the entries in column 6 are found by squaring the value in column 2 for each case.

Table 14.3 gives us all the quantities we need to solve Formula 14.2. Substitute the total of column 5 in Table 14.3 in the numerator and the total of column 6 in the denominator.

$$b = \frac{\Sigma(X - \bar{X})(Y - \bar{Y})}{\Sigma(X - \bar{X})^2}$$

$$b = \frac{18.33}{26.67}$$

$$b = 0.69$$

A slope of 0.69 indicates that, for each unit change in X, there is an increase of 0.69 units in Y. For our example, the addition of each child (an increase of

TABLE 14.3 COMPUTATION OF THE SLOPE (*b*)

1 (X)	2 ($X - \bar{X}$)	3 Y	4 ($Y - \bar{Y}$)	5 ($X - \bar{X}$) ($Y - \bar{Y}$)	6 $(X - \bar{X})^2$
1	−1.67	1	−2.33	3.89	2.79
1	−1.67	2	−1.33	2.22	2.79
1	−1.67	3	−0.33	0.55	2.79
1	−1.67	5	1.67	−2.79	2.79
2	−0.67	3	−0.33	0.22	0.45
2	−0.67	1	−2.33	1.56	0.45
3	0.33	5	1.67	0.55	0.11
3	0.33	0	−3.33	−1.10	0.11
4	1.33	6	2.67	3.55	1.77
4	1.33	3	−0.33	−0.44	1.77
5	2.33	7	3.67	8.55	5.43
5	2.33	4	0.67	1.56	5.43
32	−0.04	40	0.04	18.33	26.67
			$\bar{X} = 32/12 = 2.67$		
			$\bar{Y} = 40/12 = 3.33$		

ONE STEP AT A TIME **Computing the Slope (*b*)**

Step	*Operation*
1.	Set up a computing table like Table 14.3 to help organize the computations. List the scores of the cases on the independent variable (X) in column 1.
2.	Compute the mean of X ($\overline{X}$) by dividing the total of column 1 (ΣX) by the number of cases (N).
3.	Subtract the mean of X ($\overline{X}$) from each X score and list the results in column 2.
4.	Find the sum of column 2. This value must be zero (except for rounding error). If this sum is not zero, you have made a mistake in computations.
5.	List the score of each case on Y in column 3. Compute the mean of Y ($\overline{Y}$) by dividing the total of column 3 (ΣY) by the number of cases (N).
6.	Subtract the mean of Y ($\overline{Y}$) from each Y score and list the results in column 4.
7.	Find the sum of column 4. This value must be zero (except for rounding error). If this sum is not zero, you have made a mistake in computations.
8.	For each case, multiply the value in column 2 by the value in column 4. Place the result in column 5. Find the sum of this column.
9.	Square each value in column 2 and place the result in column 6. Find the sum of this column.
10.	Divide the sum of column 5 by the sum of column 6. The result is the slope.

one unit in X) results in an increase of 0.69 hour of housework being done by the husband (an increase of 0.69 units—or hours—in Y).

Computing the *Y* Intercept (*a*). Once the slope has been calculated, finding the intercept (a) is relatively easy. To compute the mean of X and the mean of Y, divide the sums of columns 1 and 2 of Table 14.3 by N and enter these figures into Formula 14.3:

FORMULA 14.3

$$a = \overline{Y} - b\overline{X}$$

For our sample problem, the value of a would be

$$a = \overline{Y} - b\overline{X}$$
$$a = 3.33 - (0.69)(2.67)$$
$$a = 3.33 - 1.84$$
$$a = 1.49$$

Thus, the least-squares regression line will cross the Y axis at the point where Y equals 1.49.

ONE STEP AT A TIME **Computing the *Y* Intercept (*a*)**

Step	*Operation*
1.	The values for the mean of X and Y were computed while finding b.
2.	Multiply the slope (b) by the mean of X ($\overline{X}$).
3.	Subtract the value you found in Step 2 from the mean of Y ($\overline{Y}$). This value is a, or the Y intercept.

ONE STEP AT A TIME **Using the Regression Line to Predict Scores on *Y***

Step	*Operation*
1.	Choose a value for *X*. Multiply this value by the value of the slope (*b*).
2.	Add the value you found in Step 1 to the value of *a,* the *Y* intercept. The resulting value is the predicted score on *Y*.

Stating the Least-Squares Regression Line. Now that we have values for the slope and the *Y* intercept, we can state the full least-squares regression line for our sample of 12 families:

$$Y = a + bX$$
$$Y = (1.49) + (0.69)X$$

Predicting Scores on *Y* with the Least-Squares Regression Line. The regression formula can be used to estimate, or predict, scores on *Y* for any value of *X*. In Section 14.3, we used the freehand regression line to predict a score on *Y* (husband's housework) for a family with six children ($X = 6$). Our prediction was that, in families of six children, husbands would contribute about five hours per week to housekeeping chores. By using the least-squares regression line, we can see how close our impressionistic, eyeball prediction was.

$$Y' = a + bX$$
$$Y' = (1.49) + (0.69)(6)$$
$$Y' = (1.49) + (4.14)$$
$$Y' = 5.63$$

Based on the least-squares regression line, we would predict that in a dual-wage-earner family with six children, husbands would devote 5.63 hours a week to housework. What would our prediction of husband's housework be for a family of seven children ($X = 7$)?

Note that our predictions of *Y* scores are basically "educated guesses." We will be unlikely to predict values of *Y* exactly except in the (relatively rare) case where the bivariate relationship is perfect and perfectly linear. Note also, however, that the accuracy of our predictions will increase as relationships become stronger. This is because the dots are more clustered around the least-squares regression line in stronger relationships. *(The slope and Y intercept may be computed for any problem at the end of this chapter, but see Problems 14.1–14.5 in particular. These problems have smaller data sets and will provide good practice until you are comfortable with these calculations.)*

14.5 THE CORRELATION COEFFICIENT (PEARSON'S *r*)

I pointed out in Section 14.4 that the slope of the least-squares regression line (*b*) is a measure of the effect of *X* on *Y*. Since the slope is the amount of change produced in *Y* by a unit change in *X, b* will increase in value as the relationship increases in strength. However, *b* does not vary between zero and one and is therefore awkward to use as a measure of association. Instead, researchers rely heavily (almost exclusively) on a statistic called **Pearson's *r*,** or the correlation coefficient, to measure association between interval-ratio variables. Like gamma,

the ordinal measures of association discussed in Chapter 13, Pearson's *r* varies from 0.00 to ±1.00, with 0.00 indicating no association and +1.00 and −1.00 indicating perfect positive and perfect negative relationships, respectively. The formula for Pearson's *r* is

FORMULA 14.4

$$r = \frac{\Sigma(X - \bar{X})(Y - \bar{Y})}{\sqrt{[\Sigma(X - \bar{X})^2][\Sigma(Y - \bar{Y})^2]}}$$

Both the numerator (the covariation of *X* and *Y*) and the first term in the denominator (the sum of the deviations around the mean of *X*, squared) were used in Formula 14.2 to solve for the slope. To solve Formula 14.4, we can re-use the computing table we used to compute the slope (Table 14.3) and add a column for the new term in the denominator (the sum of the deviations around the mean of *Y*, squared). The revised computing table is presented as Table 14.4.

For our sample problem involving dual-wage-earner families, the quantities displayed in Table 14.4 can be substituted directly into Formula 14.4:

$$r = \frac{\Sigma(X - \bar{X})(Y - \bar{Y})}{\sqrt{[\Sigma(X - \bar{X})^2][\Sigma(Y - \bar{Y})^2]}}$$

$$r = \frac{18.33}{\sqrt{(26.67)(50.67)}}$$

$$r = \frac{18.33}{\sqrt{1{,}351.37}}$$

$$r = \frac{18.33}{36.76}$$

$$r = 0.50$$

An *r* value of 0.50 indicates a moderately strong, positive linear relationship between the variables. As the number of children in the family increases, the hourly contribution of husbands to housekeeping duties also increases. *(Every problem at the end of this chapter requires the computation of Pearson's r. It is probably a good idea to practice with smaller data sets and easier computations first—see Problem 14.1 in particular.)*

TABLE 14.4 COMPUTATION OF PEARSON'S *r*

1 (X)	2 $(X - \bar{X})$	3 Y	4 $(Y - \bar{Y})$	5 $(X - \bar{X})(Y - \bar{Y})$	6 $(X - \bar{X})^2$	7 $(Y - \bar{Y})^2$
1	−1.67	1	−2.33	3.89	2.79	5.43
1	−1.67	2	−1.33	2.22	2.79	1.77
1	−1.67	3	−0.33	0.55	2.79	0.11
1	−1.67	5	1.67	−2.79	2.79	2.79
2	−0.67	3	−0.33	0.22	0.45	0.11
2	−0.67	1	−2.33	1.56	0.45	5.43
3	0.33	5	1.67	0.55	0.11	2.79
3	0.33	0	−3.33	−1.10	0.11	11.09
4	1.33	6	2.67	3.55	1.77	7.13
4	1.33	3	−0.33	−0.44	1.77	0.11
5	2.33	7	3.67	8.55	5.43	13.47
5	2.33	4	0.67	1.56	5.43	0.45
32	−0.04	40	0.04	18.33	26.67	50.67

ONE STEP AT A TIME **Computing Pearson's *r***

These instructions assume you have already constructed a computing table for the slope (see Table 14.3).

Step	*Operation*
1.	Add a column to the computing table you used to compute the slope (b). Square the value of $(Y - \bar{Y})$ and record the result in this new column (column 7).
2.	Find the sum of column 7.
3.	Multiply the sum of column 6 or $\Sigma(X - \bar{X})^2$ by the sum of column 7 or $\Sigma(Y - \bar{Y})^2$.
4.	Take the square root of the value you found in Step 3.
5.	Divide the quantity you found in step 4 into the sum of column 5 (the covariation $(X - \bar{X})(Y - \bar{Y})$. The result is Pearson's r.

14.6 INTERPRETING THE CORRELATION COEFFICIENT: r^2

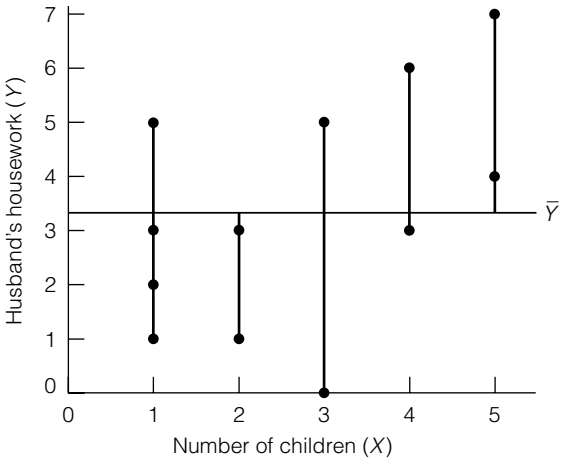
Pearson's r is an index of the strength of the linear relationship between two variables. A value of 0.00 indicates no linear relationship and a value of ± 1.00 indicates a perfect linear relationship, but values between these extremes have no direct interpretation. We can, of course, describe relationships in terms of how closely they approach the extremes (for example, coefficients approaching 0.00 can be described as "weak" and those approaching ± 1.00 as "strong"), but this description is somewhat subjective. Also, we can use the guidelines stated in Table 13.2 for gamma to attach descriptive words to the specific values of Pearson's r. In other words, values between 0.00 and 0.30 would be described as weak, values between 0.30 and 0.60 would be moderate, and values greater than 0.60 would be strong. Remember, of course, that these labels are arbitrary guidelines and will not be appropriate or useful in all possible research situations.

The Coefficient of Determination. Fortunately, we can develop a less arbitrary, more direct interpretation of r by calculating an additional statistic called the **coefficient of determination.** This statistic, which is simply the square of Pearson's r **(r^2),** can be interpreted with logic akin to proportional reduction in error (PRE). As you recall, the logic of PRE measures of association is to predict the value of the dependent variable under two different conditions. First, Y is predicted while ignoring the information supplied by X, and second, the independent variable is taken into account. With r^2, both the method of prediction and the construction of the final statistic are somewhat different and require the introduction of some new concepts.

Predicting *Y* without *X*. When working with variables measured at the interval-ratio level, the predictions of the Y scores under the first condition (while ignoring X) will be the mean of the Y. Given no information on X, this prediction strategy will be optimal because we know that the mean of any distribution is closer to all the scores than any other point in the distribution. I remind you of the principle of minimized variation introduced in Chapter 4 and expressed as

$$\Sigma(Y - \bar{Y})^2 = \text{minimum}$$

FIGURE 14.6 PREDICTING *Y* WITHOUT *X* (dual-career families)



The scores of any variable vary less around the mean than around any other point. If we predict the mean of *Y* for every case, we will make fewer errors of prediction than if we predict any other value for *Y.*

Of course, we will still make many errors in predicting *Y* even if we faithfully follow this strategy. The amount of error is represented in Figure 14.6, which displays the relationship between number of children and husband's housework with the mean of *Y* noted. The vertical lines from the actual scores to the predicted score represent the amount of error we would make when predicting *Y* while ignoring *X.*

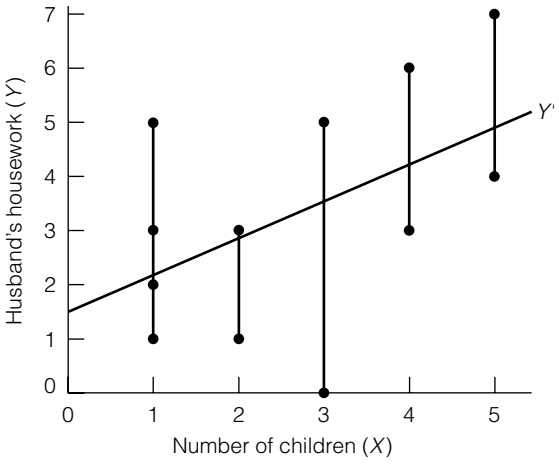
We can define the extent of our prediction error under the first condition (while ignoring *X*) by subtracting the mean of *Y* from each actual *Y* score and squaring and summing these deviations. The resultant figure, which can be noted as $\Sigma(Y - \overline{Y})^2$, is called the **total variation** in *Y.* We now have a visual representation (Figure 14.6) and a method for calculating the error we incur by predicting *Y* without knowledge of *X.* As we shall see below, we do not need to actually calculate the total variation to find the value of the coefficient of determination, r^2.

Predicting *Y* with *X.* Our next step will be to determine the extent to which knowledge of *X* improves our ability to predict *Y.* If the two variables have a linear relationship, then predicting scores on *Y* from the least-squares regression equation will incorporate knowledge of *X* and reduce our errors of prediction. So, under the second condition, our predicted *Y* score for each value of *X* will be

$$Y' = a + bX$$

Figure 14.7 displays the data from the dual-career families with the regression line, as determined by the above formula, drawn in. The vertical lines from each data point to the regression line represent the amount of error in predicting *Y* that remains even after *X* has been taken into account.

FIGURE 14.7 PREDICTING *Y* WITH *X* (dual-career families)



Explained, Unexplained, and Total Variation. We can precisely define the reduction in error that results from taking *X* into account. Two different sums can be found and then compared with the total variation of *Y* to construct a statistic that will indicate the improvement in prediction.

The first sum, called the **explained variation,** represents the improvement in our ability to predict *Y* when taking *X* into account. This sum is found by subtracting $\overline{Y}$ (our predicted *Y* score without *X*) from the score predicted by the regression equation (Y', or the *Y* score predicted with knowledge of *X*) for each case and then squaring and summing these differences. These operations can be summarized as $\Sigma(Y' - \overline{Y})^2$, and the resultant figure could then be compared with the total variation in *Y* to ascertain the extent to which our knowledge of *X* improves our ability to predict *Y*. Specifically, it can be shown mathematically that

FORMULA 14.5

$$r^2 = \frac{\Sigma(Y' - \overline{Y})^2}{\Sigma(Y - \overline{Y})^2} = \text{Explained variation/Total variation}$$

Thus, the coefficient of determination, or r^2, is the proportion of the total variation in *Y* attributable to or explained by *X*. Like other PRE measures, r^2 indicates precisely the extent to which *X* helps us predict, understand, or explain *Y*.

Earlier, we referred to the improvement in predicting *Y* with *X* as the explained variation. The use of this term suggests that some of the variation in *Y* will be "unexplained" or not attributable to the influence of *X*. In fact, the vertical lines in Figure 14.7 represent the **unexplained variation,** or the difference between our best prediction of *Y* with *X* and the actual scores. The unexplained variation is thus the scattering of the actual scores around the regression line and can be found by subtracting the predicted *Y* scores from the actual *Y* scores for each case and then squaring and summing the differences. These operations can be summarized as $\Sigma(Y - Y')^2$, and the resultant sum would measure the amount of error in predicting *Y* that remains even after *X* has been taken into account. The proportion of the total variation in *Y* unexplained by *X* can be

Application 14.1

Are nations that have more educated populations more tolerant? Are more educated nations therefore less likely to see homosexuality as wrong? Random samples from 10 nations have been asked if they agree that homosexuality is "never justifiable." Information has also been gathered on the extent of literacy in each nation. How are these variables related? The data are presented in the table below. The independent variable (X) is the percentage of the adult population that is literate, and the dependent variable (Y) is the percentage of the population that feels homosexuality is "never justified." Columns are included for all sums necessary to compute the slope (b), the Y intercept, and Pearson's r.

COMPUTATION OF PEARSON'S r

	1 X	2 $(X - \overline{X})$	3 Y	4 $(Y - \overline{Y})$	5 $(X - \overline{X})(Y - \overline{Y})$	6 $(X - \overline{X})^2$	7 $(Y - \overline{Y})^2$
China	91	−1.2	82	41.8	−50.16	1.44	1,747.24
Argentina	97	4.8	36	−4.2	−20.16	23.04	17.64
United States	99	6.8	31	−9.2	−62.56	46.24	84.64
Japan	99	6.8	42	1.8	12.24	46.24	3.24
Mexico	91	−1.2	48	7.8	−9.36	1.44	60.84
India	61	−31.2	50	9.8	−305.76	973.44	96.04
South Africa	86	−6.2	46	5.8	−35.96	38.44	33.64
Finland	100	7.8	28	−12.2	−95.16	60.84	148.84
France	99	6.8	22	−18.2	−123.76	46.24	331.24
Germany	99	6.8	17	−23.2	−157.76	46.24	538.24
Totals	922	0.0	402	0.0	−848.4	1,283.6	3,061.60
				$\overline{X} = 92.2$			
				$\overline{Y} = 40.2$			

Source: Data are from the World Values Survey and the Human Development Report published by the United Nations.

The slope (b) is

$$b = \frac{\Sigma(X - \overline{X})(Y - \overline{Y})}{\Sigma(X - \overline{X})^2}$$

$$b = \frac{-848.4}{1283.6}$$

$$b = -0.66$$

A slope of –0.66 means that for every increase in literacy (a unit change in X), there is a decrease of 0.66 point in the percentage of people who feel that homosexuality is never justified.

The Y intercept (a) is

$$a = \overline{Y} - b\overline{X}$$
$$a = 40.2 - (-0.66)(92.2)$$
$$a = 40.2 - (-60.85)$$
$$a = 101.05$$

The least-squares regression equation is

$$Y = a + bX$$
$$Y = 101.05 + (-0.66)X$$

The correlation coefficient is

$$r = \frac{\Sigma(X - \overline{X})(Y - \overline{Y})}{\sqrt{[\Sigma(X - \overline{X})^2][\Sigma(Y - \overline{Y})^2]}}$$

$$r = \frac{-848.4}{\sqrt{(1{,}283.6)(3{,}061.6)}}$$

$$r = \frac{-848.4}{\sqrt{3{,}929{,}869.76}}$$

$$r = \frac{-848.4}{1{,}982.39}$$

$$r = -0.43$$

For these 10 nations, literacy and disapproval of homosexuality have a moderate negative relationship. Disapproval of homosexuality decreases as literacy increases. The coefficient of determination, r^2, is $(0.43)^2$, or 0.19. This indicates that 19% of the variance in attitude toward homosexuality is explained by literacy for this sample of 10 nations.

found by subtracting the value of r^2 from 1.00. Unexplained variation is usually attributed to the influence of some combination of other variables, measurement error, and random chance.

As you may have recognized by this time, the explained and unexplained variations bear a reciprocal relationship with each other. As one of these sums increases in value, the other decreases. Furthermore, the stronger the linear relationship between X and Y, the greater the value of the explained variation and the lower the unexplained variation. In the case of a perfect relationship ($r = \pm 1.00$), the unexplained variation would be 0 and r^2 would be 1.00. This would indicate that X explains or accounts for all the variation in Y and that we could predict Y from X without error. On the other hand, when X and Y are not linearly related ($r = 0.00$), the explained variation would be 0 and r^2 would be 0.00. In such a case, we would conclude that X explains none of the variation in Y and does not improve our ability to predict Y.

Relationships intermediate between these two extremes can be interpreted in terms of how much X increases our ability to predict or explain Y. For the dual-career families, we calculated an r of 0.50. Squaring this value yields a coefficient of determination of 0.25 ($r^2 = 0.25$), which indicates that number of children (X) explains 25% of the total variation in husband's housework (Y). When predicting the number of hours per week that husbands in such families would devote to housework, we will make 25% fewer errors by basing the predictions on number of children and predicting from the regression line, as opposed to ignoring this variable and predicting the mean of Y for every case. Also, 75% of the variation in Y is unexplained by X and presumably due to some combination of the influence of other variables, measurement error, and random chance. *(For practice in the interpretation of r^2, see any of the problems at the end of this chapter.)*

14.7 THE CORRELATION MATRIX

Social science research projects usually include many variables, and the data analysis phase of a project often begins with the examination of a **correlation matrix,** a table that shows the relationships between all possible pairs of variables. The correlation matrix gives a quick, easy-to-read overview of the interrelationships in the data set and may suggest strategies or "leads" for further analysis. These tables are commonly included in the professional research literature, and it will be useful to have some experience reading them.

An example of a correlation matrix, using cross-national data, is presented in Table 14.5. The matrix uses variable names as rows and columns, and the cells in the table show the bivariate correlation (usually a Pearson's *r*) for each combination of variables. Note that the row headings duplicate the column headings. To read the table, begin with GDP per capita, the variable in the far left-hand column (column 1) and top row (row 1). Read down column 1 or across row 1 to see the correlations of this variable with all other variables, including the correlation of GDP per capita with itself (1.00) in the top cell. To see the relationships between other variables, move from column to column or row to row.

Note that the diagonal from upper left to lower right of the matrix presents the correlation of each variable with itself. Values along this diagonal will always be exactly 1.00, and since this information is not useful, it could easily be deleted from the table.

TABLE 14.5 A CORRELATION MATRIX SHOWING INTERRELATIONSHIPS FOR FIVE VARIABLES ACROSS 161 NATIONS

	(1) GDP per Capita	(2) Inequality	(3) Unemployment Rate	(4) Literacy Rate	(5) Voter Turnout
(1) GDP per Capita	1.00	−0.43	−0.34	0.46	0.28
(2) Inequality	−0.43	1.00	0.33	−0.15	−0.36
(3) Unemployment Rate	−0.34	0.33	1.00	−0.48	−0.28
(4) Literacy Rate	0.46	−0.15	−0.48	1.00	0.40
(5) Voter Turnout	0.28	−0.36	−0.28	0.40	1.00

VARIABLES:

(1) *GDP per Capita:* Gross domestic product (the total value of all goods and services) divided by population size. This variable is an indicator of the level of affluence and prosperity in the society. Higher scores mean greater prosperity.

(2) *Inequality:* An index of income inequality. Higher scores mean greater inequality

(3) *Unemployment Rate:* The annual rate of joblessness.

(4) *Literacy Rate:* Number of people over 15 able to read and write per 1,000 population.

(5) *Voter Turnout:* Percentage of eligible voters who participated in the most recent election.

Also note that the cells below and to the left of the diagonal are redundant with the cells above and to the right of the diagonal. For example, look at the second cell down (row 2) in column 1. This cell displays the correlation between GDP per capita and inequality, as does the cell in the top row (row 1) of column 2. In other words, the cells below and to the left of the diagonal are mirror images of the cells above and to the right of the diagonal. Commonly, research articles in the professional literature will delete the redundant cells in order to make the table more readable.

What does this matrix tell us? Starting at the upper left of the table (column 1), we can see that GDP per capita has a moderate negative relationship with inequality and unemployment rate, which means that more affluent nations tend to have less inequality and lower rates of joblessness. GDP per capita also has a moderate positive relationship with literacy (more affluent nations have higher levels of literacy) and a weak to moderate positive relationship with voter turnout (more affluent nations tend to have higher levels of participation in the electoral process).

To assess the other relationships in the data set, move from column to column and row to row, one variable at a time. For each subsequent variable, there will be one less cell of new information. For example, consider inequality, the variable in column 2 and row 2. We have already noted its moderate negative relationship with GDP per capita and, of course, we can ignore the correlation of the variable with itself. This leaves only three new relationships, which can be read by moving down column 2 or across row 2. Inequality has a positive moderate relationship with unemployment (the greater the inequality, the greater the unemployment), a weak negative relationship with literacy (nations with more inequality tend to have lower literacy rates), and a moderate negative relationship with voter turnout (the greater the inequality, the lower the turnout).

For unemployment, the variable in column 3, there are only two new relationships: a moderate negative correlation with literacy (the higher the unemployment, the lower the literacy) and a weak to moderate negative relationship

with voter turnout (the higher the unemployment rate, the lower the turnout). For voter turnout, the variable in column 5, there is only one new relationship. Voter turnout has a moderate positive relationship with literacy (turnout increases as literacy goes up).

In closing, we should note that the cells in a correlation matrix will often include other information in addition to the bivariate correlations. It is common, for example, to include the number of cases on which the correlation is based and, if relevant, an indication of the statistical significance of the relationship.

BECOMING A CRITICAL CONSUMER: Correlation, Causation, and Cancer

Causation—how variables affect each other—is a central concern of the scientific enterprise. Virtually every social science theory argues that some variable(s) cause some other variable(s), and the central goal of social research is to ascertain the strength and direction of these causal relationships.

Causation is not just a concern of science: we encounter claims about causal relationships between variables in the popular media and in everyday conversation. For example, you might hear a news commentator say that a downturn in the economy will lead to higher crime rates, or that higher gasoline prices will cause people to change their driving habits and result in fewer highway deaths, or that the attractions of cable TV have led to lower rates of community involvement. How can we know when such causal claims are true? How can we judge the credibility of arguments that one variable causes another?

Probably the most obvious evidence for a causal relationship between two variables comes from measures of association, the statistics covered in this part of the text. Any of the measures introduced in this or previous chapters—phi, gamma, Pearson's *r,* etc.—can be used as evidence for the existence of a causal association. The larger the value of the measure, the stronger the evidence for causation, and measures of zero—or close to zero—make it extremely difficult to argue for a causal relationship.

However, even very strong correlations are not proof of causation. A common adage in social science research is that correlation is not the same thing as causation, and, in fact, I made this point when we first took up the topic of bivariate association at the beginning of Chapter 12.

If correlation by itself doesn't prove causation, what other evidence is required? To build a case for causation beyond the strength of the association, we generally need to satisfy two more tests. First, we should be able to show that the independent variable occurred before the dependent variable in time and, second, that no other variable can explain the bivariate relationship. Let's explore these criteria by seeing how they have been applied to one of the most serious public health problems that has affected (and continues to affect) U.S. society: smoking and cancer.

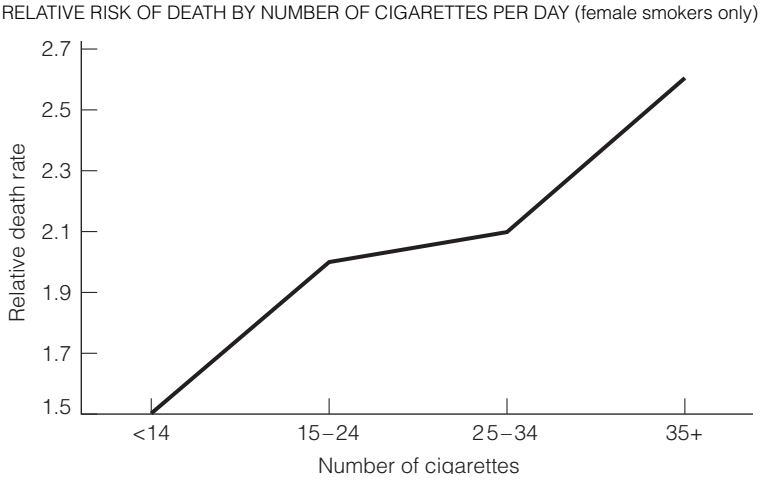
Today, virtually everyone knows that smoking tobacco causes cancer. However, this information was not part of the common wisdom just a few generations ago. As recently as the 1950s, about half of all men smoked, and smoking was equated with sophistication and mature adulthood, not illness and disease. Since that time, medical research has established links between smoking, cancer, and a number of other health risks, and, just as importantly, these connections have been widely broadcast by both public and private agencies. The effect has been dramatic: now, fewer than 25% of adults smoke.

Statistics, especially measures of association like Pearson's *r,* played an important role in establishing the links that led to the public campaign against smoking. The most convincing studies followed large samples of individuals over long periods of time. Researchers collected a variety of medical information for each respondent, including smoking habits and the incidence of cancer and other health problems. For example, one study conducted by the office of the U.S. Surgeon General studied women from 1976 to

(continued next page)

BECOMING A CRITICAL CONSUMER *(continued)*

1988 and, in the graph below, the connection between smoking and cancer is clear. The graph plots number of cigarettes per day for the smokers against the relative risk of contracting cancer. (The relative risk is the actual cancer death rate for the smokers compared to the cancer death rate for nonsmokers, controlling for age and a number of medical conditions).



Source: http://www.cdc.gov/tobacco/sgr/sgr_forwomen/pdfs/chp3.pdf.

Even though this relationship is not perfectly linear, it is clear that there is a strong correlation between number of cigarettes smoked and cancer risk. Women who smoked more than 35 cigarettes a day had almost twice the relative risk of women who smoked less than 14 cigarettes a day. It was graphs like this—along with strong measures of association and tons of other medical evidence, of course—that persuaded the medical profession and the government that smoking was a very significant public health risk.

Some argued against the growing evidence of a causal link between smoking and cancer by arguing, quite legitimately, that correlation is not the same thing as causation. Researchers developed a convincing response to this objection. First, since the best studies followed people across time, researchers were able to demonstrate a *time order* between the variables. These studies started with a large number of smokers in good health. If the smokers developed cancer later in the study, cancer must be the dependent variable. That is, in this design, smoking can be the cause of cancer, but the reverse cannot be true: given the time order, getting cancer could not have caused people to start smoking.

By itself, time order doesn't prove that the relationship is causal. Other variables might have been involved and might explain both why people smoke and why they get cancer. For example, maybe very nervous or anxious people smoke to "calm their nerves" and are more likely to contract cancer because of their nervousness, not because of their smoking. If this were the case, the relationship between smoking and cancer would be spurious or false: what looks like a bivariate, causal relationship between the two variables actually would be caused by a third variable (nervousness or anxiety).

How did the researchers respond to this possibility? The best, most convincing studies used various means to control for and examine the effect of third variables, such as anxiety, family history, gender, race, and so forth, and were able to statistically eliminate the possibility that these factors affected the bivariate relationship. We will

BECOMING A CRITICAL CONSUMER *(continued)*

examine some of the multivariate statistics used by these studies in the next chapter.

In addition, the case against smoking was further reinforced by several other statistical relationships, including the fact that people who smoke are more at risk for cancer (e.g., see the graph above) and the fact that people who quit smoking get a health dividend: they are less likely to contract cancer. Finally, tests show that the chemicals in tobacco smoke cause cancer in rats and other animals. Taken together, these various studies provided overwhelming evidence for a causal relationship.

Whenever you encounter a claim that one variable causes another, think through these questions: How strong is the measure of association? Can it be shown that the independent variable occurred first? What other variables might affect the relationship? Have they been controlled for? The greater the extent to which a relationship satisfies these criteria, the stronger the case for a causal relationship.

14.8 CORRELATION, REGRESSION, LEVEL OF MEASUREMENT, AND DUMMY VARIABLES

Correlation and regression are very powerful and useful techniques, so much so that they are often used to analyze relationships between variables that are not interval-ratio in level of measurement. This practice is generally not a problem for continuous ordinal level variables that have a broad range of possible scores, even though the variables may lack true zero points and equal distances from score to score. We considered an example of these types of variables when we discussed Spearman's rho in Chapter 13.

Researchers also use correlation and regression when working with "collapsed" ordinal variables. These are variables that have a limited number of scores (usually between two and five), such as survey items that ask respondents about their support for capital punishment or gay marriage. As was the case with continuous ordinal variables, this violation of level of measurement is not particularly a problem as long as results are treated with a suitable amount of caution.

While researchers have a good deal of leeway in including ordinal level variables for correlation and regression, this flexibility does not extend to nominal level variables. Computing correlation or regression coefficients for variables such as marital status or religious denomination simply does not make sense. Why? Remember that the "scores" of nominal level variables are not numbers and have no mathematical quality. We might represent a Protestant with a score of 2 and a Catholic with a score of 1, but the former score is not twice as much as the latter. The scores of nominal level variables are labels, not numbers. Because these variables are nonmathematical, it makes no sense to compute a slope or discuss positive or negative relationships.

This is an unfortunate situation. Many of the variables that are most important in everyday social life—gender, marital status, race or ethnicity—are nominal in level of measurement and cannot be included in a regression equation or a correlational analysis, two of the most powerful and sophisticated tools available for social science research.

Fortunately, researchers have developed a way to solve this problem and include nominal level variables by creating **dummy variables.** Dummy variables can be any level of measurement, including nominal, and have exactly two categories, one coded as 0 and the other as 1. Treated this way, any nominal

level variable that can be meaningfully represented as a dichotomy can be included in a regression equation. For example, we could score gender, with males coded as 0 and females coded as 1; race, with whites coded as 0 and blacks as 1 (we will need additional dummy variables to include other racial or ethnic groups); or religious denomination, with Catholics coded as 1 and Protestants coded as 0 (again, we would need additional dummy variables to account for other religious groupings).

To illustrate, imagine that we were concerned with the relationship between race and education as measured by number of years of schooling completed. If we coded whites as 0 and blacks as 1, we could compute a slope and Y intercept, write a regression equation using race as an independent variable, and examine the correlation between the two variables. Suppose we measured the education of a sample of black (coded as 1) and white (coded as 0) Americans and found the following regression equation:

$$Y = a + bX$$
$$Y = (12.0) + (-0.5)(X)$$

Education is the dependent (or Y) variable, and race (X) is the independent variable. The regression line crosses the vertical axis of the scattergram at the point where $Y = 12.0$. The value for the slope ($b = -0.5$) indicates a negative relationship: as race "increases" (or moves toward the higher score associated with being black), education tends to decrease. In other words, the black respondents in this sample averaged fewer years of schooling than the white respondents. Note that the *sign* of the slope (b) would have been positive had we reversed the coding scheme and labeled whites as 1 and blacks as 0, but the *value* of b would have stayed exactly the same. The coding scheme for dummy variables is arbitrary, and, as with ordinal level variables, the researcher needs to be clear about what the values of a dummy variable indicate.

We can also use Pearson's r to assess the strength and direction of relationships with dummy variables. If we found an r of -0.23 between race and education, we would conclude that there was a weak to moderate negative relationship between these variables for this sample. Consistent with the sign of the slope, we could also say that education decreased as race increased or moved from white to black. Also, using the coefficient of determination, we can say that race explains or accounts for 5% ($r^2 = 0.23^2 = 0.05$) of the variance in education. *(For experience in working with dummy variables, see Problem 14.9.)*

SUMMARY

This summary is based on the example used throughout the chapter.

1. We began with a question: Is the number of children in dual-wage-earner families related to the number of hours per week husbands devote to housework? We presented the observations in a scattergram (Figure 14.1), and our visual impression was that the variables were associated in a positive direction. The pattern formed by the observation points in the scattergram could be approximated with a straight line; thus, the relationship was roughly linear.
2. Values of Y can be predicted with the freehand regression line, but predictions are more accurate if the least-squares regression line is used. The least-squares regression line is the line that best fits the data by minimizing the variation in Y. Using the formula that defines the least-squares regression line ($Y = a + bX$), we found a slope (b) of 0.69, which indicates that each additional child (a unit

change in X) is accompanied by an increase of 0.69 hour of housework per week for the husbands. We also predicted, based on this formula, that in a dual-wage-earner family with six children ($X = 6$), husbands would contribute 5.63 hours of housework a week ($Y' = 5.63$ for $X = 6$).

3. Pearson's r is a statistic that measures the overall linear association between X and Y. Our impression from the scattergram of a substantial positive relationship was confirmed by the computed r of 0.50. We also saw that this relationship yields an r^2 of 0.25, which indicates that 25% of the total variation in Y (husband's housework) is accounted for or explained by X (number of children).
4. We acquired a great deal of information about this bivariate relationship. We know the strength and direction of the relationship and have also identified the regression line that best summarizes the effect of X on Y. We know the amount of change we can expect in Y for a unit change in X. In short, we have a greater volume of more precise information about this association between interval-ratio variables than we ever did about associations between ordinal or nominal variables. This is possible, of course, because the data generated by interval-ratio measurement are more precise and flexible than those produced by ordinal or nominal measurement techniques.

SUMMARY OF FORMULAS

FORMULA 14.1 Least-squares regression line: $Y = a + bX$

FORMULA 14.2 Definitional formula for the slope: $b = \dfrac{\Sigma(X - \overline{X})(Y - \overline{Y})}{\Sigma(X - \overline{X})^2}$

FORMULA 14.3 Y intercept: $a = \overline{Y} - b\overline{X}$

FORMULA 14.4 Definitional formula for Pearson's r: $r = \dfrac{\Sigma(X - \overline{X})(Y - \overline{Y})}{\sqrt{[\Sigma(X - \overline{X})^2][\Sigma(Y - \overline{Y})^2]}}$

FORMULA 14.5 Coefficient of determination: $r^2 = \dfrac{\Sigma(Y' - \overline{Y})^2}{\Sigma(Y - \overline{Y})^2}$

GLOSSARY

Coefficient of determination (r^2). The proportion of all variation in Y that is explained by X. Found by squaring the value of Pearson's r.

Conditional means of *Y*. The mean of all scores on Y for each value of X.

Correlation matrix. A table showing the correlations between all possible combinations of variables.

Dummy variable. A nominal level variable that has been recoded into exactly two categories (zero and one) for inclusion in regression equations.

Explained variation. The proportion of all variation in Y that is attributed to the effect of X. Equal to $\Sigma(Y' - \overline{Y})^2$.

Linear relationship. A relationship between two variables in which the observation points (dots) in the scattergram can be approximated with a straight line.

Pearson's *r* (*r*). A measure of association for variables that have been measured at the interval-ratio level.

Regression line. The single, best-fitting straight line that summarizes the relationship between two variables. Regression lines are fitted to the data points by the least-squares criterion, whereby the line touches all conditional means of Y or comes as close to doing so as possible.

Scattergram. Graphic display device that depicts the relationship between two variables.

Slope (*b*). The amount of change in one variable per unit change in the other; b is the symbol for the slope of a regression line.

Total variation. The spread of the Y scores around the mean of Y. Equal to $\Sigma(Y - \overline{Y})^2$.

Unexplained variation. The proportion of the total variation in Y that is not accounted for by X. Equal to $\Sigma(Y - Y')^2$.

***Y* intercept (*a*).** The point where the regression line crosses the Y axis.

***Y'*.** Symbol for predicted score on Y.

PROBLEMS

(Problems are labeled with the social science discipline from which they are drawn: SOC for sociology, SW for social work, PS for political science, CJ for criminal justice, PA for public administration, and GER for gerontology.)

14.1 PS Why does voter turnout vary from election to election? For municipal elections in five different cities, information has been gathered on the percentage of registered voters who actually voted, unemployment rate, average years of education for the city, and the percentage of all political ads that used "negative campaigning" (personal attacks, negative portrayals of the opponent's record, etc.). For each relationship:

a. Draw a scattergram and a freehand regression line.
b. Compute the slope (b) and find the Y intercept (a). (*HINT: Remember to compute b before computing a. A computing table such as Table 14.3 is highly recommended.*)
c. State the least-squares regression line and predict the voter turnout for a city in which the unemployment rate was 12, a city in which the average years of schooling was 11, and an election in which 90% of the ads were negative.
d. Compute r and r^2. (*HINT: A computing table such as Table 14.3 is highly recommended. If you constructed one for computing b, you already have most of the quantities you will need to solve for r.*)
e. Describe the strength and direction of the relationships in a sentence or two. Which factor had the strongest effect on turnout?

Turnout and Unemployment:

City	Turnout	Unemployment Rate
A	55	5
B	60	8
C	65	9
D	68	9
E	70	10

Turnout and Level of Education

City	Turnout	Average Years of School
A	55	11.9
B	60	12.1
C	65	12.7
D	68	12.8
E	70	13.0

Turnout and Negative Campaigning

City	Turnout	% of Negative Ads
A	55	60
B	60	63
C	65	55
D	68	53
E	70	48

14.2 SOC Occupational prestige scores for a sample of fathers and their oldest son and oldest daughter are shown in the table.

Family	Father's Prestige	Son's Prestige	Daughter's Prestige
A	80	85	82
B	78	80	77
C	75	70	68
D	70	75	77
E	69	72	60
F	66	60	52
G	64	48	48
H	52	55	57

Analyze the relationship between father's and son's prestige and the relationship between father's and daughter's prestige. For each relationship:

a. Draw a scattergram and a freehand regression line.
b. Compute the slope (b) and find the Y intercept (a).
c. State the least-squares regression line. What prestige score would you predict for a son whose father had a prestige score of 72? What prestige score would you predict for a daughter whose father had a prestige score of 72?
d. Compute r and r^2.

e. Describe the strength and direction of the relationships in a sentence or two. Does the occupational prestige of the father have an impact on his children? Does it have the same impact for daughters as it does for sons?

14.3 GER The residents of a housing development for senior citizens have completed a survey in which they indicated how physically active they are and how many visitors they receive each week. Are these two variables related for the 10 cases reported here? Draw a scattergram and compute r and r^2. Find the least-squares regression line. What would be the predicted number of visitors for a person whose level of activity was a 5? How about a person who scored 18 on level of activity?

Case	Level of Activity	Number of Visitors
A	10	14
B	11	12
C	12	10
D	10	9
E	15	8
F	9	7
G	7	10
H	3	15
I	10	12
J	9	2

14.4 PS The variables below were collected for a random sample of 10 precincts during the last national election. Draw scattergrams and compute r and r^2 for each combination of variables. Write a paragraph interpreting the relationship between these variables.

Precinct	Percent Democrat	Percent Minority	Voter Turnout
A	50	10	56
B	45	12	55
C	56	8	52
D	78	15	60
E	13	5	89
F	85	20	25
G	62	18	64
H	33	9	88
I	25	0	42
J	49	9	36

14.5 SOC/CJ The table below presents the scores of 10 states on each of six variables: three measures of criminal activity and three measures of population structure. Crime rates are number of incidents per 100,000 population as of 2005.

	Crime Rates			Population*		
State	Homicide	Robbery	Car Theft	Growth	Density	Mobility
Maine	1	24	102	4	43	86
New York	5	183	186	2	409	89
Ohio	5	163	361	1	280	85
Iowa	1	39	185	2	53	84
Virginia	6	99	211	8	193	84
Kentucky	5	88	211	4	106	84
Texas	6	157	409	13	90	81
Arizona	8	144	924	20	54	81
Washington	3	92	784	9	96	80
California	7	176	713	8	234	84

**Growth:* percentage change in population from 2000 to 2006; *Density:* population per square mile of land area, 2006; *Mobility:* percentage of population in same house as in 2004.

Source: U.S. Bureau of the Census, 2008. *Statistical Abstract of the United States, 2008. Crime Rates:* p. 193. *Population:* pp. 18 and 36. Accessed from http://www.census.gov/compendia/statab/2008edition.html.

For each combination of crime rate and population characteristic:

a. Draw a scattergram and a freehand regression line.
b. Compute the slope (b) and find the Y intercept (a).
c. State the least-squares regression line. What homicide rate would you predict for a state with a growth rate of -1? What robbery rate would you predict for a state with a population density of 250? What auto theft rate would you predict for a state in which 90% of the population had not moved since 2004?
d. Compute r and r^2.
e. Describe the strength and direction of each of these relationships in a sentence or two.

14.6 Data on three variables have been collected for 15 nations as of 2007. The variables are fertility rate (average number of children born to each woman), average life expectancy for females, and percent urban.

Nation	Fertility	Life Expectancy for Females	Percent Urban
Niger	7.1	55	17
Cambodia	3.4	65	15
Guatemala	4.4	73	47
Ghana	4.4	59	44
Bolivia	3.7	67	63
Egypt	3.1	73	43
Dominican Republic	2.9	74	65
Mexico	2.4	78	75
Vietnam	2.1	73	27
Turkey	2.2	74	66
United States	2.1	80	79
China	1.6	74	44
United Kingdom	1.8	81	90
Japan	1.3	86	79
Italy	1.4	84	68

Source: 2007 World Population Data Sheet. http://www.prb.org/.

a. Compute r and r^2 for each combination of variables.
b. Summarize these relationships in terms of strength and direction.

14.7 SOC The basketball coach at a small local college believes that his team plays better and scores more points in front of larger crowds. The number of points scored and attendance for all home games last season are reported below. Do these data support the coach's argument?

Game	Points Scored	Attendance
1	54	378
2	57	350
3	59	320
4	80	478
5	82	451
6	75	250
7	73	489
8	53	451
9	67	410
10	78	215
11	67	113
12	56	250
13	85	450
14	101	489
15	99	472

14.8 SOC The table below presents the scores of 15 states on three variables. Compute r and r^2 for each combination of variables. Write a paragraph interpreting the relationship among these three variables.

State	Per Capita Expenditures on Education, 2005	Percent High School Graduates, 2006*	Rank in Per Capita Income, 2006
Arkansas	1,194	83	48
Colorado	1,659	90	8
Connecticut	2,197	88	1
Florida	1,374	87	20
Illinois	1,740	88	13
Kansas	1,571	90	21
Louisiana	1,431	80	34
Maryland	1,623	87	4
Michigan	1,911	90	27
Mississippi	1,226	81	50
Nebraska	1,338	91	23
New Hampshire	1,673	92	7
North Carolina	1,280	84	36
Pennsylvania	1,710	88	18
Wyoming	2,045	91	6

*Based on percentage of population age 25 and older.

Source: United States Bureau of the Census, *Statistical Abstracts of the United States: 2008*. pp. 147, 163, 439.

14.9 SOC Fifteen individuals were selected from a nationally representative sample, and their scores on five variables are reproduced below. Is there a relationship between occupational prestige and age? Between occupational prestige and gender? Between church attendance and number of children? Between number of children and hours

of TV watching? Between age and hours of TV watching? Between hours of TV watching and gender? Between age and number of children? Between hours of TV watching and occupational prestige? Variables and codes are explained in detail in Appendix G.

Occupational Prestige	Number of Children	Age	Church Attendance	Hours of TV per Day	Gender
32	3	34	3	1	1
50	0	41	0	3	1
17	0	52	7	2	1
69	3	67	0	5	0
17	0	40	0	5	0
52	0	22	2	3	1
32	3	31	0	4	0
50	0	23	8	4	0
19	9	64	1	6	1
37	4	55	0	2	0
14	3	66	5	5	1
51	0	22	6	0	0
45	0	19	3	7	0
44	0	21	4	1	1
46	4	58	2	0	1

YOU ARE THE RESEARCHER: Who Surfs the Internet? Who Succeeds in Life?

Two projects are presented to help you apply the statistical skills developed in this chapter. In the first, you will analyze the correlates of time spent on the World Wide Web. You will select four independent variables and assess their impact on *wwwhr* (number of hours per week spent on the Internet). In the second project, you will choose either *income06* or *prestg80* as your dependent variable. Both variables measure social class standing, and you will select independent variables that you believe might be associated with or might predict an individual's level of success.

In both projects, you will have the option to conduct an additional analysis in which you analyze males and females separately to see if the pattern of correlation varies by sex. You will use an SPSS command called **Split File,** which allows us to examine subgroups within the sample independently.

There is no need for a detailed demonstration of how to generate Pearson's *r* with SPSS. We will use the same command used in Chapter 13 to generate Spearman's rho. Briefly, click **Analyze → Correlate → Bivariate.** The **Bivariate Correlations** dialog box will open and "Pearson" will already be checked. Find your variables in the list on the left and transfer them to the **Variables:** window; click **OK,** and SPSS will produce a correlation matrix showing the relationship of all variables with each other. Recall that the cells in the output present three pieces of information: the value of the measure of association (Pearson's *r* in this case), its statistical significance, and the number of cases.

Optional Project. The **Split File** command is used when we want to see if variables are related in the same way for various subgroups in the sample. To demonstrate this command, let's look at the relationship between religiosity (measured by *attend*) and support for gay marriage (*marhomo*). The correlation between these

variables for the entire sample is 0.35. This is a moderate, positive relationship: as frequency of church attendance rises (as score on *attend* goes up), opposition to gay marriage also rises (score on *marhomo* increases). Would the same relationship hold for both males and females?

To observe the effect of sex on the bivariate relationship, we split the GSS sample into two subfiles. Men and women will then be processed separately, and SPSS will produce one correlation matrix for men and another for women.

Click **Data** from the main menu, and then click **Split File.** On the **Split File** window, click the button next to "organize output by groups." This will generate separate outputs for men and women. Select *sex* from the variable list, and click the arrow to move the variable name into the **Groups Based On** window. Click **OK,** and all procedures requested will be done separately for men and women. To restore the full sample, call up the **Split File** window again and click the **Reset** button. For now, click **Statistics → Correlate → Bivariate** and rerun the correlation *attend* and *marhomo*.

The output shows a correlation of 0.40 for males and 0.33 for females. This result suggests that religiosity has a slightly more powerful impact for males than for females, but the difference for the sexes is actually rather minor. For both groups, there is a moderate, positive relationship between religiosity and opposition to gay marriage. Be sure to return to the **Split File** window and click **Reset** to restore the full sample before starting the projects.

PROJECT 1: Who Uses the Internet?

In this exercise, your dependent variable will be *wwwhr,* which measures how many hours per week the respondent spends on the Internet. Scores on *wwwhr* range from 0 (about 40% of the sample) to a high of 75 hours (almost double the number of hours in a typical work week).

STEP 1: Choosing Independent Variables

Select four variables from the 2006 GSS that you think might be important causes of *wwwh*. Your independent variables *cannot* be nominal in level of measurement unless you recode the variable into a dummy variable (see below). You may use any interval-ratio or ordinal level variables with more than three categories or scores.

Dummy Variables. To include a dummy variable in your analysis, recode the variable so that it has only two values: 0 and 1. Some possibilities include recoding *sex* so that males = 0 and females = 1; *racecen1* so that whites = 0 and nonwhites = 1; or *relig* so that Protestants = 0 and non-Protestants = 1. See Chapter 10 for instructions on recoding.

Once you have selected your variables, list them in the table below and describe exactly what they measure.

SPSS Variable Name	What Exactly Does This Variable Measure?

STEP 2: Stating Hypotheses

State hypotheses about the relationships you expect to find between your independent variables and *wwwhr*. State these hypotheses in terms of the direction of the relationship you expect to find. For example, you might hypothesize that hours spent on the Internet will decline as age increases.

1.

2.

3.

4.

STEP 3: Running Bivariate Correlations

Click **Analyze → Bivariate → Correlate** and place all variables in the **Variables:** box. Click **OK** to get your results.

STEP 4: Recording Results

Use the table below to summarize your results. Enter the *r* for each independent variable in each cell. As you read the correlation matrix, ignore correlations of variables with themselves and any redundant information.

	Independent Variables
	1. _____ 2. _____ 3. _____ 4. _____
wwwhr	

STEP 5. Analyzing and Interpreting Results

Write a short summary of results for each independent variable. Your summary needs to identify the variables being tested and the strength and direction of the relationship. It is probably best to characterize the relationship in general terms and then cite the statistical values in parentheses. Be sure to note whether or not your hypotheses were supported. *Be careful when interpreting direction* and refer back to the coding scheme to make sure you understand the relationship.

STEP 6: Optional Analysis Using Split Files Command

You can analyze the correlations for different subgroups by splitting the sample. *Do not* use any of your independent variables to split the sample, and follow the instructions above to use the **Split Files** command. Disregard the results for very small groups. Are the variables related in essentially the same way for the subgroups as for the entire sample? If not, what experiences might account for the differences?

PROJECT 2: Who Succeeds?

For this exercise, choose either income (*income06*) or occupational prestige (*prestg80*) as your dependent variable. Most Americans would regard these variables as measures of success in life. What are the correlates and antecedents of affluence and prestige?

STEP 1: Choosing Independent Variables

Select four variables from the 2006 GSS that you think might be important causes of the dependent variable you selected. An obvious choice is education (use *educ,* the interval-ratio version, not *degree,* the ordinal version). Remember that independent variables *cannot* be nominal in level of measurement unless you recode the variable into a dummy variable (see below). You may use any interval-ratio or ordinal level variables with more than three categories or scores.

Dummy Variables. To include a dummy variable in your analysis, recode the variable so that it has only two values: 0 and 1. Some possibilities include recoding *sex* so that males = 0 and females = 1; *racecen1* so that whites = 0 and non-whites = 1; or *relig* so that Protestants = 0 and non-Protestants =1. See Chapter 10 for instructions on recoding.

Once you have selected your variables, list them in the table below and describe exactly what they measure.

SPSS Variable Name	What Exactly Does This Variable Measure?

STEP 2: Stating Hypotheses

State hypotheses about the relationships you expect to find between your independent variables and the dependent variable you selected. State these hypotheses in terms of the direction of the relationship you expect to find. For example, you might hypothesize that income will increase as age increases.

1.

2.

3.

4.

STEP 3: Running Bivariate Correlations

Click **Analyze → Bivariate → Correlate** and place all variables in the **Variables:** box. Click **OK** to get your results.

STEP 4: Recording Results

Use the table below to summarize your results. Enter the *r* for each independent variable in each cell. As you read the correlation matrix, ignore correlations of variables with themselves and any redundant information.

	Independent Variables
Dependent Variable: _____	1. _____ 2. _____ 3. _____ 4. _____

STEP 5: Analyzing and Interpreting Results

Write a short summary of results for each independent variable. Your summary needs to identify the variables being tested and the strength and direction of the relationship. It is probably best to characterize the relationship in general terms and then cite the statistical values in parentheses. Be sure to note whether or not your hypotheses were supported. *Be careful when interpreting direction* and refer back to the coding scheme to make sure you understand the relationship.

STEP 6: Optional Analysis Using Split Files Command

You can analyze the correlations for different subgroups by splitting the sample. *Do not* use any of your independent variables to split the sample, and follow the instructions above to use the **Split Files** command. Disregard the results for very small groups. Are the variables related in essentially the same way for the subgroups as for the entire sample? If not, what experiences might account for the differences?

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Part IV Multivariate Techniques

Chapter 15 introduces multivariate analytical techniques, which are statistics that allow us to analyze the relationships between more than two variables at a time. These statistics are extremely useful for probing possible causal relationships between variables and are commonly reported in the professional literature. In particular, the chapter introduces regression analysis, which is the basis for many of the most popular and powerful statistical techniques in use today. These techniques are designed to be used with variables measured at the interval-ratio level of measurement, and the mathematics underlying these statistics can become very complicated. For this reason, the chapter focuses on the simplest possible applications and stresses interpretation.

15 Partial Correlation and Multiple Regression and Correlation

LEARNING OBJECTIVES

By the end of this chapter, you will be able to:

1. Compute and interpret partial correlation coefficients.
2. Find and interpret the least-squares multiple regression equation with partial slopes.
3. Calculate and interpret the multiple correlation coefficient (R^2).
4. Explain the limitations of partial and multiple regression analysis.

15.1 INTRODUCTION

Very few (if any) worthwhile research questions can be answered through a statistical analysis of only two variables. Social science research is, by nature, multivariate and often involves the simultaneous analysis of scores of variables. Some of the most powerful and widely used statistical tools for multivariate analysis are introduced in this chapter. We will cover techniques that are used to analyze causal relationships and to make predictions, both crucial endeavors in any science.

These techniques are based on Pearson's r (see Chapter 14) and are most appropriately used with high-quality, precisely measured interval-ratio level variables. As we have noted, such data are relatively rare in the social sciences, and the techniques presented in this chapter are commonly used with variables measured at the ordinal level and with nominal-level variables in the form of dummy variables (see Section 14.8).

We will first consider partial correlation analysis, a technique that allows us to examine bivariate relationship while controlling for a third variable. The second technique involves multiple regression and correlation and allows the researcher to assess the effects, separately and in combination, of more than one independent variable on the dependent variable.

Throughout this chapter, we will focus our attention on research situations involving three variables. This is the least complex application of these techniques, but extensions to situations involving four or more variables are relatively straightforward. To deal efficiently with the computations required by the more complex applications, I refer you to any of the computerized statistical packages (such as SPSS) probably available on your campus.

15.2 PARTIAL CORRELATION

In Chapter 14, we used Pearson's r to measure the strength and direction of bivariate relationships. To provide an example, we looked at the relationship between husbands' contribution to housework (the dependent or Y variable) and the number of children (the independent or X variable) for a sample of 12 families. We found a positive relationship of moderate strength ($r = +0.50$) and concluded that husbands tend to make a larger contribution to housework as the number of children increases.

You might wonder, as researchers commonly do, if this relationship always holds true for *all* types of families? For example, might husbands in strongly religious families respond differently from those in less religious families? Would

husbands from families that were politically conservative behave differently from husbands from more liberal families? How about more educated husbands? Would they respond differently from less educated husbands? Or perhaps husbands in some ethnic or racial groups would have different responses from husbands in other groups. We can address these kinds of issues by a technique called **partial correlation** in which we observe how the bivariate relationship changes when a third variable, such as religiosity, education, or ethnicity, is introduced. Third variables are often referred to as *Z* variables or **control variables.**

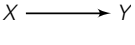
Partial correlation proceeds by first computing Pearson's *r* for the bivariate (or zero-order) relationship and then computing the partial (or first-order) correlation coefficient. If the partial correlation coefficient differs from the zero-order correlation coefficient, we conclude that the third variable does have an effect on the bivariate relationship. If, for example, well-educated husbands and less well-educated husbands respond differently to an additional child, the partial correlation coefficient will differ in strength (and, perhaps, in direction) from the bivariate correlation coefficient.

Before considering matters of computation, we will consider the relationships between the partial and bivariate correlation coefficients and what they might mean. There are three possible patterns, and we will consider each in turn.

Types of Relationships

Direct Relationship. One possible outcome is that the partial correlation coefficient is essentially the same value as the bivariate coefficient. Imagine, for example, that after we controlled for husbands' education, we found a partial correlation coefficient of +0.49 compared to the zero-order Pearson's *r* of +0.50. This would mean that the third variable (husbands' education) has no effect on the relationship between number of children and husbands' hours of housework. In other words, regardless of their education, husbands respond in a similar way to additional children. This outcome is consistent with the conclusion that there is a **direct** or casual relationship (see Figure 15.1) between *X* and *Y* and that the third variable (*Z*) is irrelevant to the investigation. In this case, the researcher would discard that particular *Z* variable from further consideration, but might well run additional tests with other likely control variables (e.g., the researcher might control for the religion or ethnicity of the family).

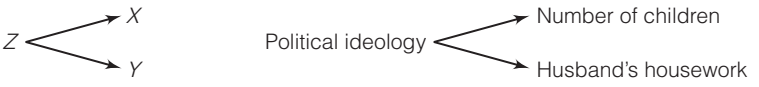
FIGURE 15.1 A DIRECT RELATIONSHIP BETWEEN *X* AND *Y*



Spurious and Intervening Relationships. A second possible outcome occurs when the partial correlation coefficient is much weaker than the bivariate correlation, perhaps even dropping to zero. This outcome is consistent with two different relationships between the variables. The first is called a **spurious relationship:** the control variable (*Z*) is a cause of both the independent (*X*) and dependent (*Y*) variable (see Figure 15.2). This outcome would mean that there is no actual relationship between *X* and *Y* and that they appear to be related only because both are dependent on a common cause (*Z*). Once *Z* is taken into account, the apparent relationship between *X* and *Y* disappears.

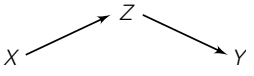
What would a spurious relationship look like? Imagine that we controlled for the political ideology of the parents in our 12-family sample and found that the partial correlation coefficient was much weaker than the bivariate Pearson's *r*.

FIGURE 15.2 A SPURIOUS RELATIONSHIP BETWEEN *X* AND *Y*



This would indicate that the number of children does not actually change the husband's contribution to housework (or, the relationship between *X* and *Y* is not direct). Rather, political ideology is the mutual cause of both of the other variables: more conservative families would be more likely to follow traditional gender role patterns (in which husbands contribute less to housework) and have more children.

FIGURE 15.3 AN INTERVENING RELATIONSHIP BETWEEN *X* AND *Y*

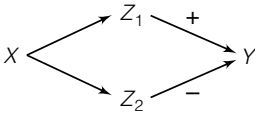


This pattern (partial correlation much weaker than the bivariate correlation) is also consistent with an **intervening** relationship between the variables (see Figure 15.3). In this situation, *X* and *Y* are not linked directly but are casually connected through the *Z* variable. Again, once *Z* is controlled, the apparent relationship between *X* and *Y* disappears.

How can we tell the difference between spurious and intervening relationships? This distinction cannot be made on statistical grounds: spurious and intervening relationships look exactly the same in terms of statistics. The researcher may be able to distinguish between these two relationships in terms of the time order of the variables (i.e., which came first) or theoretical grounds, but not on statistical grounds.

Interaction. A final possible relationship between variables should be mentioned even though it *cannot* be detected by partial correlation analysis. This relationship, called **interaction,** occurs when the relationship between *X* and *Y* changes markedly under the various values of *Z*. For example, if we controlled for social class and found that husbands in middle-class families increased their contribution to housework as the number of children increased whereas husbands in working-class families did just the reverse, we would conclude that there was interaction between these three variables. In other words, there would be a positive relationship between *X* and *Y* for one category of *Z* and a negative relationship for the other category, as illustrated in Figure 15.4.

FIGURE 15.4 AN INTERACTIVE RELATIONSHIP BETWEEN *X*, *Y*, AND *Z*



Computing and Interpreting the Partial Correlation Coefficient

Terminology and Formula. The formula for partial correlation requires some new terminology. We will be dealing with more than one bivariate relationship and need to differentiate between them with subscripts. Thus, the symbol r_{yx} will refer to the correlation coefficient between variable *Y* and variable *X*, r_{yz} will refer to the correlation coefficient between *Y* and *Z*, and r_{xz} to the correlation coefficient between *X* and *Z*. Recall that correlation coefficients calculated for bivariate relationships are often referred to as **zero-order correlations.**

Partial correlation coefficients, or first-order partials, are symbolized as $r_{yx.z}$. The variable to the right of the dot is the control variable. Thus, $r_{yx.z}$ refers to the partial correlation coefficient that measures the relationship between

variables X and Y while controlling for variable Z. The formula for the first-order partial is

FORMULA 15.1

$$r_{yx.z} = \frac{r_{yx} - (r_{yz})(r_{xz})}{\sqrt{1 - r_{yz}^2}\sqrt{1 - r_{xz}^2}}$$

Note that you must first calculate the zero-order coefficients between all possible pairs of variables (variables X and Y, X and Z, and Y and X) before solving this formula.

Computation. To illustrate the computation of a first-order partial, we will return to the relationship between number of children (X) and husbands' contributions to housework (Y) for 12 dual-career families. The zero-order r between these two variables ($r_{yx} = 0.50$) indicated a moderate, positive relationship (as number of children increased, husbands tended to contribute more to housework). Suppose the researcher wished to investigate the possible effects of husbands' education on the bivariate relationship. The original data (from Table 14.1) and the scores of the 12 families on the new variable are presented in Table 15.1.

The zero-order correlations, as presented in Table 15.2, indicate that husbands' contributions to housework is positively related to number of children ($r_{yx} = 0.50$), that better-educated husbands tend to do less housework ($r_{yz} = -0.30$), and that families with better-educated husbands have fewer children ($r_{xz} = -0.47$).

TABLE 15.1 SCORES ON THREE VARIABLES FOR 12 DUAL-WAGE-EARNER FAMILIES AND ZERO-ORDER CORRELATIONS

Family	Husbands' Housework (Y)	Number of Children (X)	Husbands' Years of Education (Z)
A	1	1	12
B	2	1	14
C	3	1	16
D	5	1	16
E	3	2	18
F	1	2	16
G	5	3	12
H	0	3	12
I	6	4	10
J	3	4	12
K	7	5	10
L	4	5	16

TABLE 15.2 ZERO-ORDER CORRELATIONS

	Husbands' Housework (Y)	Number of Children (X)	Husbands' Years of Education (Z)
Husbands' Housework (Y)	1.00	0.50	−0.30
Number of Children (X)		1.00	−0.47
Husbands' Years of Education (Z)			1.00

ONE STEP AT A TIME **Computing and Interpreting Partial Correlations**

Computation

Step	*Operation*
1.	Compute Pearson's *r* for all pairs of variables, Be clear about which variable is independent (*X*), which is dependent (*Y*), and which is the control (*Z*).

To compute the partial correlation coefficient, solve Formula 15.1:

2. Multiply r_{yz} by r_{xz}.
3. Subtract the value you found in Step 2 from r_{yx}.
4. Square the value of r_{yz}.
5. Subtract the quantity you found in Step 4 from 1.
6. Take the square root of the quantity you found in Step 5.
7. Square the value of r_{xz}.
8. Subtract the quantity you found in Step 7 from 1.
9. Take the square root of the quantity you found in Step 8.
10. Multiply the quantity you found in Step 6 by the quantity you found in Step 9.
11. Divide the quantity you found in Step 3 by the quantity you found in Step 10.

Interpretation

12. Compare the value of r_{yx} with $r_{yx.z}$. Choose the scenario below that comes closest to describing the relationship between the two values:
    - **a.** The partial correlation coefficient is roughly the same value (no less than, say, 0.10 lower) as the bivariate correlation. This is evidence that the control variable (*Z*) has no effect and that the relationship between *X* and *Y* is direct.
    - **b.** The partial correlation coefficient is much less (say, more than 0.10 less) than the bivariate correlation. This is evidence that the control variable (*Z*) changes the relationship between *X* and *Y*. The relationship between *X* and *Y* is either spurious (*Z* causes both *X* and *Y*) or intervening (*X* and *Y* are linked by *Z*).
13. Be aware that *X*, *Y*, and *Z* may have an interactive relationship in which the relationship between *X* and *Y* changes for each category of *Z*. Partial correlation analysis cannot detect interactive relationships.

Is the relationship between husbands' housework and number of children affected by husbands' years of education? Substituting the zero-order correlations into Formula 15.1, we would have

$$r_{yx.z} = \frac{r_{yx} - (r_{yz})(r_{xz})}{\sqrt{1 - r_{yz}^2}\sqrt{1 - r_{xz}^2}}$$

$$r_{yx.z} = \frac{(0.50) - (-0.30)(-0.47)}{\sqrt{1 - (0.30)^2}\sqrt{1 - (0.47)^2}}$$

$$r_{yx.z} = \frac{(0.50) - (0.14)}{\sqrt{1 - 0.09}\sqrt{1 - 0.22}}$$

$$r_{yx.z} = \frac{0.36}{\sqrt{0.91}\sqrt{0.78}}$$

$$r_{yx.z} = \frac{0.36}{(0.95)(0.88)}$$

$$r_{yx.z} = \frac{0.36}{0.84}$$

$$r_{yx.z} = 0.43$$

Interpretation. The first-order partial ($r_{yx.z} = 0.43$), which measures the strength of the relationship between husbands' housework (Y) and number of children (X) while controlling for husbands' education (Z), is lower in value than the zero-order coefficient ($r_{yx} = 0.50$), but the difference in the two values is not great. This result suggests a direct relationship between variables X and Y. That is, when controlling for husbands' education, the statistical relationship between husbands' housework and number of children is essentially unchanged. Regardless of education, husbands' hours of housework increase with the number of children.

Our next step in statistical analysis would probably be to select another control variable. The more the bivariate relationship retains its strength across a series of controls for third variables (Zs), the stronger the evidence for a direct relationship between X and Y. (*For practice in computing and interpreting partial correlation coefficients, see Problems 15.1–15.3.*)

15.3 MULTIPLE REGRESSION: PREDICTING THE DEPENDENT VARIABLE

In Chapter 14, the least-squares regression line was introduced as a way of describing the overall linear relationship between two interval-ratio variables and of predicting scores on Y from scores on X. This line was the best-fitting line to summarize the bivariate relationship and was defined by the formula:

FORMULA 15.2

$$Y = a + bX$$

Where: a = the Y intercept
b = the slope

The least-squares regression line can be modified to include (theoretically) any number of independent variables. This technique is called **multiple regression.** For ease of explication, we will confine our attention to the case involving two independent variables. The least-squares multiple regression equation for two independent variables is

FORMULA 15.3

$$Y = a + b_1X_1 + b_2X_2$$

Where: b_1 = the partial slope of the linear relationship between the first independent variable and Y
b_2 = the partial slope of the linear relationship between the second independent variable and Y

Some new notation and some new concepts are introduced in this formula. First, while the dependent variable is still symbolized as Y, the independent variables are differentiated by subscripts. Thus, X_1 identifies the first independent variable and X_2 the second. The symbol for the slope (b) is also subscripted to identify the independent variable with which it is associated.

Partial Slopes. A major difference between the multiple and bivariate regression equations concerns the slopes (b's). In the case of multiple regression, the b's are called **partial slopes,** and they show the amount of change in Y for a unit change in one independent variable while controlling for the effects of the other independent variable(s) in the equation. The partial slopes are thus analogous to partial correlation coefficients and represent the direct effect of the associated independent variable on Y.

Computing Partial Slopes. The partial slopes for the independent variables are determined by Formula 15.4 and Formula 15.5.[1] The subscripts attached to the symbols in the formula (b, s, r) identify the variables: Y is the dependent variable, 1 refers to the first independent variable, and 2 to the second independent variable.

FORMULA 15.4

$$b_1 = \left(\frac{s_y}{s_1}\right)\left(\frac{r_{y1} - r_{y2}r_{12}}{1 - r_{12}^2}\right)$$

FORMULA 15.5

$$b_2 = \left(\frac{s_y}{s_2}\right)\left(\frac{r_{y2} - r_{y1}r_{12}}{1 - r_{12}^2}\right)$$

Where: b_1 = the partial slope of X_1 on Y
b_2 = the partial slope of X_2 on Y
s_y = the standard deviation of Y
s_1 = the standard deviation of the first independent variable (X_1)
s_2 = the standard deviation of the second independent variable (X_2)
r_{y1} = the bivariate correlation between Y and X_1
r_{y2} = the bivariate correlation between Y and X_2
r_{12} = the bivariate correlation between X_1 and X_2

To illustrate the computation of the partial slopes, we will assess the combined effects of number of children (X_1) and husbands' education (X_2) on husbands' contribution to housework. All the relevant information can be calculated from Table 15.1 and is reproduced below:

Husbands' Housework	Number of Children	Husbands' Education
$\bar{Y} = 3.3$ $s_y = 2.1$	$\bar{X}_1 = 2.7$ $s_1 = 1.5$	$\bar{X}_2 = 13.7$ $s_2 = 2.6$
Zero-Order Correlations		
$r_{y1} = 0.50$ $r_{y2} = -0.30$ $r_{12} = -0.47$		

The partial slope for the first independent variable, number of children or X_1, is

$$b_1 = \left(\frac{s_y}{s_1}\right)\left(\frac{r_{y1} - r_{y2}r_{12}}{1 - r_{12}^2}\right)$$

$$b_1 = \left(\frac{2.1}{1.5}\right)\left(\frac{0.50 - (-0.30)(-0.47)}{1 - (-0.47)^2}\right)$$

$$b_1 = (1.4)\left(\frac{0.50 - 0.14}{1 - 0.22}\right)$$

$$b_1 = (1.4)\left(\frac{0.36}{0.78}\right)$$

$$b_1 = (1.4)(0.46)$$

$$b_1 = 0.65$$

[1]Partial slopes can be computed from zero-order slopes, but Formulas 15.4 and 15.5 are somewhat easier to use.

For the second independent variable, husbands' education or X_2, the partial slope is

$$b_2 = \left(\frac{s_y}{s_2}\right)\left(\frac{r_{y2} - r_{y1}r_{12}}{1 - r_{12}^2}\right)$$

$$b_2 = \left(\frac{2.1}{2.6}\right)\left(\frac{-0.30 - (0.50)(-0.47)}{1 - (-0.47)^2}\right)$$

$$b_2 = (0.81)\left(\frac{-0.30 - (-0.24)}{1 - 0.22}\right)$$

$$b_2 = (0.81)\left(\frac{-0.30 + 0.24}{0.78}\right)$$

$$b_2 = (0.81)\left(\frac{-0.06}{0.78}\right)$$

$$b_2 = (0.81)(-0.08)$$

$$b_2 = -0.07$$

ONE STEP AT A TIME — Computing and Interpreting Partial Slopes

(*Note:* These procedures apply when there are two independent variables and one dependent variable. For more complex situations, use a computerized statistical package such as SPSS to do the calculations.)

Computation

Step ***Operation***

To compute the partial slope of the first independent variable, solve Formula 15.4:

1. Divide s_y by s_1.
2. Multiply r_{y2} by r_{12}.
3. Subtract the value you found in Step 2 from r_{y1}.
4. Square r_{12}.
5. Subtract the quantity you found in Step 4 from 1.
6. Divide the quantity you found in Step 3 by the value you found in Step 5.
7. Multiply the quantity you found in Step 6 by the quantity you found in Step 1. This value is the partial slope associated with the first independent variable.

To compute the partial slope of the second independent variable, solve Formula 15.5:

8. Divide s_y by s_2.
9. Multiply r_{y1} by r_{12}.
10. Subtract the value you found in Step 9 from r_{y2}.
11. Square r_{12}.
12. Subtract the quantity you found in Step 11 from 1.
13. Divide the quantity you found in Step 10 by the value you found in Step 12.
14. Multiply the quantity you found in Step 13 by the quantity you found in Step 8. This value is the partial slope associated with the second independent variable.

Interpretation

15. The value of a partial slope is the increase in the value of *Y* for a unit increase in the value of the associated independent variable while controlling for the effect of the other independent variable.

ONE STEP AT A TIME **Computing the *Y* intercept**

Step	*Operation*
Find the Y intercept by solving Formula 15.6:	
1.	Multiply $\overline{X}_2$, the mean of the second independent variable, by b_2.
2.	Multiply $\overline{X}_1$, the mean of the first independent variable, by b_1.
3.	Subtract the quantity you found in Step 1 from the quantity you found in Step 2.
4.	Subtract the quantity you found in Step 3 from $\overline{Y}$, the mean of *Y*. The result is *a,* the *Y* intercept.

Finding the Y Intercept. Now that partial slopes have been determined for both independent variables, the *Y* intercept (*a*) can be found. Note that *a* is calculated from the mean of the dependent variable (symbolized as $\overline{Y}$) and the means of the two independent variables ($\overline{X}_1$ and $\overline{X}_2$).

FORMULA 15.6

$$a = \overline{Y} - b_1\overline{X}_1 - b_2\overline{X}_2$$

Substituting the proper values for the example problem at hand, we would have

$$\begin{aligned} a &= \overline{Y} - b_1\overline{X}_1 - b_2\overline{X}_2 \\ a &= 3.3 - (0.65)(2.7) - (-0.07)(13.7) \\ a &= 3.3 - (1.8) - (-1.0) \\ a &= 3.3 - 1.8 + 1.0 \\ a &= 2.5 \end{aligned}$$

The Least Squares Multiple Regression Line and Predicting Y′. For our example problem, the full least-squares multiple regression equation is

$$\begin{aligned} Y &= a + b_1X_1 + b_2X_2 \\ Y &= 2.5 + (0.65)X_1 + (-0.07)X_2 \end{aligned}$$

As was the case with the bivariate regression line, this formula can be used to predict scores on the dependent variable from scores on the independent variables. For example, what would be our best prediction of the husband's housework (*Y′*) for a family of four children ($X_1 = 4$) where the husband had completed 11 years of schooling ($X_2 = 11$)? Substituting these values into the least-squares formula, we would have

$$\begin{aligned} Y' &= 2.5 + (0.65)(4) + (-0.07)(11) \\ Y' &= 2.5 + 2.6 - 0.77 \\ Y' &= 4.3 \end{aligned}$$

Our prediction would be that this husband would contribute 4.3 hours per week to housework. This prediction is, of course, a kind of "educated guess" that is unlikely to be perfectly accurate. However, we will make fewer errors of prediction using the least-squares line (and, thus, incorporating information from the independent variables) than we would using any other method of prediction (assuming, of course, that there is a linear association between the independent and the dependent variables). (*For practice in predicting Y scores and in computing slopes and the Y intercept, see Problems 15.1–15.6.*)

ONE STEP AT A TIME **Using the Multiple Regression Line to Predict Scores on *Y***

Step	*Operation*
1.	Choose a value for X_1. Multiply this value by the value of b_1.
2.	Choose a value for X_2. Multiply this value by the value of b_2.
3.	Add the values you found in Steps 1 and 2 to the value of *a,* the *Y* intercept. The result is the predicted score on *Y*.

15.4 MULTIPLE REGRESSION: ASSESSING THE EFFECTS OF THE INDEPENDENT VARIABLES

The least-squares multiple regression equation (Formula 15.3) is used to isolate the separate effects of the independent variables and to predict scores on the dependent variable. However, in many situations, using this formula to determine the relative importance of the various independent variables will be awkward—especially when the independent variables differ in terms of units of measurement (e.g., number of children vs. years of education). When the independent variables are measured in different units, a comparison of the partial slopes will not necessarily tell us which independent variable has the strongest effect and is thus the most important. Comparing the partial slopes of variables that differ in units of measurement is like comparing apples and oranges.

The comparability of the independent variables can be increased by converting all variables in the equation to a common scale or unit of measurement. This will eliminate variations in the values of the partial slopes that are solely a function of differences in units of measurement. We can, for example, standardize the independent variables by changing their scores to *Z* scores, as we did in Chapter 6. Each distribution of scores would then have a mean of 0 and a standard deviation of 1, and comparisons between the independent variables would be much more meaningful.

Computing the Standardized Regression Coefficients

Beta-Weights. To standardize the variables to the normal curve, we could actually convert all scores into the equivalent *Z* scores and then re-compute the slopes and the *Y* intercept. This would require a good deal of work; fortunately, a shortcut is available for computing the slopes of the standardized scores directly. These **standardized partial slopes** are called **beta-weights** and are symbolized ***b**.** The beta-weights show the amount of change in the standardized scores of *Y* for a one-unit change in the standardized scores of each independent variable while controlling for the effects of all other independent variables.

Formulas and Computation for Beta-Weights. When we have two independent variables, the beta-weight for each is found by using Formula 15.7 and Formula 15.8:

FORMULA 15.7

$$b_1^* = b_1\left(\frac{s_1}{s_y}\right)$$

FORMULA 15.8

$$b_2^* = b_2\left(\frac{s_2}{s_y}\right)$$

We can now compute the beta-weights for our sample problem to see which of the two independent variables (number of children and husbands' education) has the stronger effect on the dependent variable (husbands' hours of housework). For the first independent variable, number of children (X_1):

$$b_1^* = b_1\left(\frac{s_1}{s_y}\right)$$
$$b_1^* = (0.65)\left(\frac{1.5}{2.1}\right)$$
$$b_1^* = (0.65)(0.71)$$
$$b_1^* = 0.46$$

For the second independent variable, husbands' years of education (X_2):

$$b_2^* = b_2\left(\frac{s_2}{s_y}\right)$$
$$b_2^* = (-0.07)\left(\frac{2.6}{2.1}\right)$$
$$b_2^* = (-0.07)(1.24)$$
$$b_2^* = -0.09$$

Interpreting Beta-Weights Comparing the value of the beta-weights for our example problem, we see that number of children ($b_1^* = 0.46$) has a stronger effect on the husband's housework than does the husband's education ($b_2^* = -0.09$). Furthermore, the net effect (after controlling for the effect of education) of the first independent variable is positive while the net effect of the second

ONE STEP AT A TIME **Computing and Interpreting Beta-Weights (*b**)**

(*Note:* These procedures apply when there are two independent variables and one dependent variable. For more complex situations, use a computerized statistical package such as SPSS to do the calculations.)

Computation

Step ***Operation***

To compute the beta-weight associated with the first independent variable, solve Formula 15.7:

1. Divide s_1 by s_y.
2. Multiply the quantity you found in Step 1 by b_1. This value is the beta-weight associated with the first independent variable.

To compute the beta-weight associated with the second independent variable, solve Formula 15.8:

3. Divide s_2 by s_y.
4. Multiply the quantity you found in Step 3 by b_2. This value is the beta-weight associated with the second independent variable.

Interpretation

5. A beta-weight (or standardized partial slope) shows the change in the value of *Y* for a unit increase in the value of the associated independent variable while controlling for the effect of the other independent variable after all variables have been standardized (or transformed to *Z* scores).

independent variable (after controlling for the effect of number of children) is negative. We can conclude that number of children is the more important of the two variables and that husbands' contribution to housework increases as the number of children increases regardless of husbands' years of education.

The Standardized Least-Squares Regression Line Using standardized scores, the least-squares regression equation can be written as

FORMULA 15.9

$$Zy = a_z + b_1^* Z_1 + b_2^* Z_2$$

Where: Z indicates that all scores have been standardized to the normal curve

The standardized regression equation can be further simplified by dropping the term for the Y intercept, since this term will always be zero when scores have been standardized. Remember that a is the point where the regression line crosses the Y axis and is equal to the mean of Y when all independent variables equal 0. This relationship can be seen by substituting 0 for all independent variables in Formula 15.6:

$$a = \overline{Y} - b_1\overline{X}_1 - b_2\overline{X}_2$$
$$a = \overline{Y} - b_1(0) - b_2(0)$$
$$a = \overline{Y}$$

Since the mean of any standardized distribution of scores is zero, the mean of the standardized Y scores will be zero and the Y intercept will also be zero ($a = \overline{Y} = 0$). Thus, Formula 15.9 simplifies to

FORMULA 15.10

$$Zy = b_1^* Z_1 + b_2^* Z_2$$

The standardized regression equation for our example problem, with beta-weights noted, would be

$$Zy = (0.46)Z_1 + (-0.09)Z_2$$

and it is immediately obvious that the first independent variable has a much stronger direct effect on Y than the second independent variable.

Summary Multiple regression analysis permits the researcher to summarize the linear relationship among two or more independents and a dependent variable. The unstandardized regression equation (Formula 15.2) permits values of Y to be predicted from the independent variables in the original units of the variables. The standardized regression equation (Formula 15.10) allows the researcher to easily assess the relative importance of the various independent variables by comparing the beta-weights. (*For practice in computing and interpreting beta-weights, see any of the problems at the end of this chapter. It is probably a good idea to start with Problem 15.1 since it has the smallest data set and the least complex computations.*)

15.5 MULTIPLE CORRELATION

We use multiple regression equations to disentangle the separate direct effects of each independent variable on the dependent. Using **multiple correlation** techniques, we can also ascertain the *combined* effects of all independent variables on the dependent variable. We do so by computing the **multiple correlation coefficient (R)** and the **coefficient of multiple determination (R)^2.**

The value of the latter statistic represents the proportion of the variance in Y that is explained by all the independent variables combined.

In terms of zero-order correlation, we have seen that number of children (X_1) explains a proportion of 0.25 of the variance in Y ($r^2_{y1} = (0.50)^2 = 0.25$) by itself and that the husband's education explains a proportion of 0.09 of the variance in Y ($r^2_{y2} = (-0.30)^2 = 0.09$). The zero-order correlations cannot be simply added together to ascertain their combined effect on Y because the two independents are also correlated with each other; therefore, they will "overlap" in their effects on Y and explain some of the same variance. This overlap is eliminated in Formula 15.11:

FORMULA 15.11

$$R^2 = r^2_{y1} + r^2_{y2.1}(1 - r^2_{y1})$$

Where: R^2 = the coefficient of multiple determination
r^2_{y1} = the zero-order correlation between Y and X_1, the quantity squared
$r^2_{y2.1}$ = the partial correlation of Y and X_2, while controlling for X_1, the quantity squared

The first term in this formula (r^2_{y1}) is the coefficient of determination for the bivariate relationship between Y and X_1. It represents the amount of variation in Y explained by X_1 by itself. To this quantity we add the amount of the variation remaining in Y (given by $1 - r^2_{y1}$) that can be explained by X_2 after the effect of X_1 is controlled ($r^2_{y2.1}$). Basically, Formula 15.11 allows X_1 to explain as much of Y as it can and then adds in the effect of X_2 after X_1 is controlled (thus eliminating the overlap in the variance of Y that X_1 and X_2 have in common).

Computing and Interpreting R and R^2 To observe the combined effects of number of children (X_1) and husbands' years of education (X_2) on husbands' housework (Y), we need two quantities. The correlation between X_1 and Y ($r_{y1} = 0.50$) has already been found. Before we can solve Formula 15.11, we must first calculate the partial correlation of Y and X_2 while controlling for X_1 ($r_{y2.1}$):

$$r_{y2.1} = \frac{r_{y2} - (r_{y1})(r_{12})}{\sqrt{1 - r^2_{y1}}\sqrt{1 - r^2_{12}}}$$

$$r_{y2.1} = \frac{(-0.30) - (0.50)(-0.47)}{\sqrt{1 - (0.50)^2}\sqrt{1 - (-0.47)^2}}$$

$$r_{y2.1} = \frac{(-0.30) - (-0.24)}{\sqrt{0.75}\sqrt{0.78}}$$

$$r_{y2.1} = \frac{-0.06}{0.77}$$

$$r_{y2.1} = -0.08$$

Formula 15.11 can now be solved for our sample problem:

$$R^2 = r^2_{y1} + r^2_{y2.1}(1 - r^2_{y1})$$
$$R^2 = (0.50)^2 + (-0.08)^2(1 - 0.50^2)$$
$$R^2 = 0.25 + (0.006)(1 - 0.25)$$
$$R^2 = 0.25 + 0.005$$
$$R^2 = 0.255$$

ONE STEP AT A TIME **Computing and Interpreting the Coefficient of Multiple Determination (R^2)**

(*Note:* These procedures apply when there are two independent variables and one dependent variable. For more complex situations, use a computerized statistical package such as SPSS to do the calculations.)

Computation

Step ***Operation***

To compute the multiple correlation coefficient (R^2), solve Formula 15.11:

1. Find the value of the partial correlation coefficient for $r_{y2.1}$.
2. Square the value you found in Step 1.
3. Square the value of r_{y1}.
4. Subtract the quantity you found in Step 3 from 1.
5. Multiply the quantity you found in Step 4 by the quantity you found in Step 2.
6. Add the value you found in Step 5 to the value you found in Step 3. The result is the coefficient of multiple determination (R^2).

Interpretation

7. The coefficient of multiple determination (R^2) is the total variation in Y explained by all independent variables combined.

The first independent variable (X_1), number of children, explains 25% of the variance in Y by itself. To this total, the second independent (X_2), husbands' education, adds only a half a percent, for a total explained variance of 25.5%. In combination, the two independents explain a total of 25.5% of the variation in the dependent variable. (*For practice in computing and interpreting R and R^2, see any of the problems at the end of this chapter. It is probably a good idea to start with Problem 15.1 since it has the smallest data set and the least complex computations.*)

15.6 THE LIMITATIONS OF MULTIPLE REGRESSION AND CORRELATION

Partial correlation and multiple regression and correlation are very powerful tools for analyzing the interrelationships among three or more variables. The techniques presented in this chapter permit the researcher to predict scores on one variable from two or more other variables, to distinguish between independent variables in terms of the importance of their direct effects on a dependent, and to ascertain the total effect of a set of independent variables on a dependent variable: they are some of the most powerful and flexible statistical tools available to social science researchers.

Powerful tools are not cheap. They demand high-quality data, and measurement at the interval-ratio level is often difficult to accomplish. Furthermore, these techniques assume that the interrelationships among the variables follow a particular form. First, they assume that each independent variable has a linear relationship with the dependent variable. How well a given set of variables meets this assumption can be quickly checked with scattergrams.

Second, the techniques presented in this chapter assume that the effects of the independent variables are *additive*. This means that we must assume that

BECOMING A CRITICAL CONSUMER: Is Support for the Death Penalty Related to White Racism?

As is the case with many of the statistics covered in this text, it is unlikely that you will encounter reports using multiple regression in the popular press or in your everyday conversations. On the other hand, multiple regression analysis has become an extremely important and widely used tool in social science research, and it is very likely that you will have to deal with articles using this technique—or one of its many variants—in the professional research literature. These articles might appear to be hopelessly complex at first glance, well beyond the understanding of an undergraduate student. Indeed, they are written for other professionals in the field and assume a high level of statistical sophistication on the part of the reader.

Nevertheless, without denying the challenge, it is quite possible for non-professionals to distill the essence of these articles by following a few guidelines and looking for a few central elements. The key is to focus on the words, not the numbers. That is, read the text to see what the authors have to say about their results, not the (perhaps impenetrable) array of numbers and symbols. The details of the statistical analysis may be beyond your understanding, but you can almost always decipher the words.

Let's take a look at some actual research and see what we can make of it. We'll focus on a project that takes a serious look at a serious topic: the sharp racial difference in support for the death penalty in American society. Public opinion surveys show that capital punishment is supported by large majorities of white respondents, but only a minority of blacks. What accounts for this difference? One possibility is that support among whites is associated with anti-black racism and that opposition among blacks is associated with the perception that this ultimate sanction—indeed, the entire criminal justice system—is biased against blacks. The former possibility was investigated by sociologists James Unnever and Francis Cullen. Using a nationally representative sample of over 1,000 respondents, they attempted to ascertain the extent to which white support for the death penalty reflects a perception of blacks as threatening, criminally dangerous, and unintelligent.

The researchers constructed several different regression models with support for capital punishment as the dependent variable. They begin their analysis by reporting the bivariate relationship between race and support, summarized as Model 1 in the table below. Note that race is a dummy variable (see Section 14.8), with blacks coded as 1 and whites as 0. Thus, the negative values of the regression coefficients indicate that, as expected, whites are more supportive of the death penalty. In other words, support decreases as race "increases," or moves toward the higher scores associated with black respondents. Note also that the relationship is statistically significant at the 0.001 level.

Unnever and Cullen examined several different multiple regression models, each showing the relationship of support for capital punishment with different sets of independent variables. Across these models, they introduced almost a score of independent variables, including many measures of the social and political traits that previous research has shown to be correlated with support for capital punishment. For example, support has been shown to be greater among males, Southerners, political conservatives, and the more religious. The great power of regression analysis is that the researchers can examine the effects of these variables separately and while controlling for the effects of all other factors. Consider, for example, the fact that blacks historically have had fewer opportunities for education. This difference might mean that the apparent effect of race in Model 1 was really due to an educational difference between the two groups. If this were the case, the effect of race would disappear once education had been entered into the equation and thus controlled.

Model 4 in the table below summarizes some of the final results. I have included only 7 of the nearly 20 independent variables in the actual equation in the table. Note that, indeed, the effect of race on support for the death penalty is weaker once the other variables have been entered into the equation. The beta-weight

(continued next page)

BECOMING A CRITICAL CONSUMER *(continued)*

associated with race loses about a third of its strength (that is, it declines from −0.18 to −0.11) in Model 4 as compared to Model 1. This means that some of the difference between the races is spurious and due to the differences on these other variables (education, religiosity, etc.), not group membership itself. Still, the beta-weight associated with race is statistically significant and among the stronger values in Model 4. This means that, even after other variables have been accounted for, a racial difference in support for the death penalty persists.

Multiple Regression Results With Support for Capital Punishment as the Dependent Variable (N − 1,117)†

	Model 1		Model 4	
	Unstandardized Coefficient	Beta-Weight	Unstandardized Coefficient	Beta-weight
Race (black = 1)	−0.60	−0.18***	−0.36	−0.11***
Gender (male = 1)			−0.04	−0.01
Education			−0.01	−0.03
Religiosity			−0.05	−0.17***
Political Conservatism			0.04	0.07**
Income			0.02	0.04
Racism			0.05	0.11***
R^2	0.03***		0.15***	

*$p < 0.05$; **$p < 0.01$; ***$p < 0.001$

†Adapted from Table 1 in the original article

Model 4 also shows that support for the death penalty is significantly associated with white racism. Indeed the beta-weight shows that white racism is also one of the stronger predictors of support for the death penalty and that the relationship is positive: the greater the racism—with all other factors controlled—the greater the support for the death penalty.

Finally, note the value of R^2 of 0.15. This means that all of the independent variables combined explain 15% of the variance in support for capital punishment. This may strike you as low, especially given the large number of independent variables in the equation. Certainly, the researchers would have been gratified if their model had explained a larger percentage of the variance. However, values for R^2 in this range are common in social research, in large part because the strength of associations between variables is depressed by the fact that, unlike natural scientists conducting experiments in laboratories, we cannot control all the factors that might impact our results and by the high levels of measurement errors in our data.

Source: Unnever, James and Francis Cullen. 2007. "The Racial Divide in Support for the Death Penalty: Does Racism Matter?" *Social Forces:* 85 1281–1301.

the best prediction of the dependent variable (Y) can be obtained by simply adding up the scores of the independent variables, as reflected by the plus signs in Formula 15.3. Not all combinations of variables will conform to this assumption. For example, some variables have *interactive* relationships with each other. These are relationships in which some of the scores of the variables combine in unusual, nonadditive ways. For example, consider a survey of neighborhoods that found positive and linear bivariate relationships between

Application 15.1

The table below presents information on three variables for a small sample of 10 nations. The dependent variable is the percentage of respondents who said they are "very happy" on a survey administered to random samples from each nation. The independent variables measure health and physical well-being (life expectancy, or the number of years the average citizen can expect to live) and income inequality (the amount of total income that goes to the richest 20% of the population). Our expectation is that happiness will have a positive correlation with life expectancy (the greater the health, the happier the population) and a negative relationship with inequality (the greater the inequality, the greater the discontent and the lower the level of happiness). In this analysis, we will focus on R^2 and the beta-weights only.

The scores of the nations, along with descriptive statistics, are listed below.

Nation	Percent "Very Happy" (Y)	Life Expectancy (X_1)	Income Inequality (X_2)
Brazil	22	69	64
Belgium	37	79	37
Canada	32	80	39
China	25	73	39
Dom. Rep.	32	71	53
Ghana	26	58	47
India	23	65	46
Japan	23	81	36
Mexico	31	75	57
Ukraine	5	70	38
Mean =	*24.7*	*72.8*	*45.6*
Standard Deviation =	*9.4*	*6.8*	*9.6*

The zero-order correlations for these variables are given in the correlation matrix below:

	Happy	Life Expectancy	Inequality
Happiness	1.00	0.32	0.12
Life Expectancy		1.00	−0.39
GDP per capita			1.00

Consistent with our expectations, there is a positive, weak to moderate relationship between life expectancy and happiness. Unexpectedly, however, the relationship between inequality and happiness is positive, although weak in strength. The relationship between the two independent variables is moderate and negative, indicating that nations with more income inequality have lower life expectancy.

The combined effect of life expectancy and inequality on happiness is found by computing R^2:

$$R^2 = r^2_{y1} + r^2_{y2.1}(1 - r^2_{y1})$$
$$R^2 = (0.32)^2 + (0.27)^2(1 - 0.26^2)$$
$$R^2 = 0.10 + (0.07)(0.93)$$
$$R^2 = 0.10 + 0.07$$
$$R^2 = 0.17$$

By itself, life expectancy alone explains 10% of the variance in happiness. To this, income inequality adds another 7% for a total of 17%. This leaves about 83% of the variance unexplained, a sizeable proportion but not unusually large in social science research.

To assess the separate effects of the two independent variables, the beta-weights must be calculated. We need values for the unstandardized partial slopes to compute beta-weights, and we will simply report the values as 0.53 or X_1 (life expectancy) and (0.26) for X_2 (income inequality).

For the first independent variable (life expectancy):

$$b^*_1 = b_1\left(\frac{s_1}{s_y}\right)$$
$$b^*_1 = (0.53)\left(\frac{7.17}{8.77}\right)$$
$$b^*_1 = (0.53)(0.82)$$
$$b^*_1 = 0.43$$

For the second independent variable (income inequality):

$$b^*_2 = b_2\left(\frac{s_2}{s_y}\right)$$
$$b^*_2 = (0.26)\left(\frac{9.64}{8.77}\right)$$
$$b^*_2 = (0.26)(1.10)$$
$$b^*_2 = 0.28$$

Recall that the beta-weights show the effect of each independent variable on the dependent variable while controlling for the other independent variables in the equation. In this case, life expectancy has the stronger effect, and the relationship is positive. The effect of income inequality is also positive.

In summary, for these nations, level of happiness has a moderate positive relationship with life expectancy and a weaker positive relationship with income inequality. Taken together, the independent variables explain 17% of the variation in happiness.

poverty (X_1), racial residential segregation (X_2), and crime rates (Y). When combined in regression analysis, however, certain combinations of scores on the independent variables (for example, high levels of poverty combined with high levels of segregation) produced especially high crimes rates. In other words, poverty and segregation interact with each other, and neighborhoods that were high in poverty *and* segregation had much higher crime rates than areas that were impoverished but not racially segregated or areas that were segregated but not impoverished.

If there is interaction among the variables, it is not possible to accurately estimate or predict the dependent variable by simply adding the effects of the independents. There are techniques for identifying and handling interaction among variables, but these techniques are beyond the scope of this text.

Third, the techniques of multiple regression and correlation assume that the independent variables are uncorrelated. Strictly speaking, this condition means that the zero-order correlation among all pairs of independents should be zero. In practice, however, we act as if this assumption has been met if the intercorrelations among the independents are low.

To the extent that these assumptions are violated, the regression coefficients (especially partial and standardized slopes) and the coefficient of multiple determination (R^2) become less and less trustworthy and the techniques less and less useful. A careful inspection of the bivariate scattergrams will help to assess the reasonableness of the assumptions of partial and multiple correlation and regression.

Finally, we should note that we have covered only the simplest applications of partial correlation and multiple regression and correlation. In terms of logic and interpretation, the extensions to situations involving more independent variables are relatively straightforward. However, the computations for these situations are extremely complex. If you are faced with a situation involving more than three variables, turn to one of the computerized statistical packages that are commonly available on college campuses (e.g., SPSS and SAS). These programs require minimal computer literacy and can handle complex calculations in, literally, the blink of an eye. Efficient use of these packages will enable you to avoid drudgery and will free you to do what social scientists everywhere enjoy doing most: pondering the meaning of your results and, by extension, the nature of social life.

SUMMARY

1. Partial correlation involves controlling for third variables. Partial correlations permit the detection of direct and spurious or intervening relationships between X and Y.
2. Multiple regression includes statistical techniques by which predictions of the dependent variable from more than one independent variable can be made (by partial slopes and the multiple regression equation) and by which we can disentangle the relative importance of the independent variables (by standardized partial slopes).
3. The coefficient of multiple determination (R^2) summarizes the combined effects of all independents on the dependent variable in terms of the proportion of the total variation in Y that is explained by all of the independents.
4. Partial correlation and multiple regression and correlation are some of the most powerful tools available to the researcher and demand high-quality measurement and relationships among the variables that are linear and noninteractive. Furthermore, correlations among the independents must be low (preferably zero). Although the price is high, these techniques pay considerable dividends in the volume of precise and detailed information they generate about the interrelationships among the variables.

SUMMARY OF FORMULAS

FORMULA 15.1 Partial correlation coefficient: $$r_{yx.z} = \frac{r_{yx} - (r_{yz})(r_{xz})}{\sqrt{1 - r_{yz}^2}\sqrt{1 - r_{xz}^2}}$$

FORMULA 15.2 Least-squares regression line (bivariate): $$Y = a + bX$$

FORMULA 15.3 Least-squares multiple regression line: $$Y = a + b_1X_1 + b_2X_2$$

FORMULA 15.4 Partial slope for X_1: $$b_1 = \left(\frac{S_y}{S_1}\right)\left(\frac{r_{y1} - r_{y2}r_{12}}{1 - r_{12}^2}\right)$$

FORMULA 15.5 Partial slope for X_2: $$b_2 = \left(\frac{S_y}{S_2}\right)\left(\frac{r_{y2} - r_{y1}r_{12}}{1 - r_{12}^2}\right)$$

FORMULA 15.6 Y intercept: $$a = \overline{Y} - b_1\overline{X}_1 - b_2\overline{X}_2$$

FORMULA 15.7 Standardized partial slope (beta-weight) for X_1: $$b_1^* = b_1\left(\frac{S_1}{S_y}\right)$$

FORMULA 15.8 Standardized partial slope (beta-weight) for X_2: $$b_2^* = b_2\left(\frac{S_2}{S_y}\right)$$

FORMULA 15.9 Standardized least-squares regression line: $$Zy = a_z + b_1^*Z_1 + b_2^*Z_2$$

FORMULA 15.10 Standardized least-squares regression line (simplified): $$Zy = b_1^*Z_1 + b_2^*Z_2$$

FORMULA 15.11 Coefficient of multiple determination: $$R^2 = r_{y1}^2 + r_{y2.1}^2\,(1 - r_{y1}^2)$$

GLOSSARY

Beta-weights (*b).** Standardized partial slopes.

Coefficient of multiple determination (R^2). A statistic that equals the total variation explained in the dependent variable by all independent variables combined.

Control variable. A third or Z variable. A control variable is introduced in a statistical analysis to see if it affects the original bivariate relationship.

Direct relationship. A multivariate relationship in which the control variable has no effect on the bivariate relationship.

Interaction. A multivariate relationship in which a bivariate relationship changes substantially across the categories of the control variable.

Intervening relationship. A multivariate relationship in which the dependent and independent variables are linked through the control variable. Once the third variable is controlled, the relationship becomes substantially weaker.

Multiple correlation. A multivariate technique for examining the combined effects of more than one independent variable on a dependent variable.

Multiple correlation coefficient (R). A statistic that indicates the strength of the correlation between a dependent variable and two or more independent variables.

Multiple regression. A multivariate technique that breaks down the separate effects of the independent variables on the dependent variable; used to make predictions of the dependent variable.

Partial correlation. A multivariate technique for examining a bivariate relationship while controlling for other variables.

Partial correlation coefficient. A statistic that shows the relationship between two variables while controlling for other variables; $r_{yx.z}$ is the symbol for the partial correlation coefficient when controlling for one variable.

Partial slopes. In a multiple regression equation, the slope of the relationship between a particular independent variable and the dependent variable while controlling for all other independents in the equation.

Standardized partial slopes (beta-weights). The slope of the relationship between a particular independent variable and the dependent when all scores have been normalized.

Spurious relationship. A multivariate relationship in which there is no actual causal relationship between the dependent and independent variables. Both are caused by the control variable. Once the third variable is controlled, the relationship becomes substantially weaker.

Zero-order correlations. Correlation coefficients for bivariate relationships.

PROBLEMS

(Problems are labeled with the social science discipline from which they are drawn: SOC for sociology, SW for social work, PS for political science, CJ for criminal justice, PA for public administration, and GER for gerontology.)

15.1 PS In Problem 14.1 data regarding voter turnout in five cities were presented. For the sake of convenience, the data for three of the variables are presented again here along with descriptive statistics and zero-order correlations.

City	Turnout	Unemployment Rate	% Negative Ads
A	55	5	60
B	60	8	63
C	65	9	55
D	68	9	53
E	70	10	48
Mean =	63.6	8.2	55.8
s =	5.5	1.7	5.3

	Unemployment Rate	% Negative Ads
Turnout	0.95	−0.87
Unemployment Rate		−0.70

a. Compute the partial correlation coefficient for the relationship between turnout (Y) and unemployment (X) while controlling for the effect of negative advertising (Z). What effect does this control variable have on the bivariate relationship? Is the relationship between turnout and unemployment direct? (*HINT: Use Formula 15.1 and see Section 15.2.*)

b. Compute the partial correlation coefficient for the relationship between turnout (Y) and negative advertising (X) while controlling for the effect of unemployment (Z). What effect does this have on the bivariate relationship? Is the relationship between turnout and negative advertising direct? (*HINT: Use Formula 15.1 and see Section 15.2. You will need this partial correlation to compute the multiple correlation coefficient.*)

c. Find the unstandardized multiple regression equation with unemployment (X_1) and negative ads (X_2) as the independent variables. What turnout would be expected in a city in which the unemployment rate was 10% and 75% of the campaign ads were negative? (*HINT: Use Formulas 15.4 and 15.5 to compute the partial slopes and then use Formula 15.6 to find* a*, the Y intercept. The regression line is stated in Formula 15.3. Substitute 10 for X_1 and 75 for X_2 to compute predicted Y.*)

d. Compute beta-weights for each independent variable. Which has the stronger impact on turnout? (*HINT: Use Formulas 15.7 and 15.8 to calculate the beta-weights.*)

e. Compute the coefficient of multiple determination (R^2). How much of the variance in voter turnout is explained by the two independent variables combined? (*HINT: Use Formula 15.11. You calculated the partial correlation coefficient in part b of this problem.*)

f. Write a paragraph summarizing your conclusions about the relationships among these three variables.

15.2 SOC A scale measuring support for increases in the national defense budget has been administered to a sample. The respondents have also been asked to indicate how many years of school they have completed and how many

years, if any, they served in the military. Take "support" as the dependent variable.

Case	Support	Years of School	Years of Service
A	20	12	2
B	15	12	4
C	20	16	20
D	10	10	10
E	10	16	20
F	5	8	0
G	8	14	2
H	20	12	20
I	10	10	4
J	20	16	2

a. Compute the partial correlation coefficient for the relationship between support (Y) and years of school (X) while controlling for the effect of years of service (Z). What effect does the third variable (years of service or Z) have on the bivariate relationship? Is the relationship between support and years of school direct?

b. Compute the partial correlation coefficient for the relationship between support (Y) and years of service (X) while controlling for the effect of years of school (Z). What effect does this have on the bivariate relationship? Is the relationship between support and years of service direct? (*HINT: You will need this partial correlation to compute the multiple correlation coefficient.*)

c. Find the unstandardized multiple regression equation with school (X_1) and service (X_2) as the independent variables. What level of support would be expected in a person with 13 years of school and 15 years of service?

d. Compute beta-weights for each independent variable. Which has the stronger impact on turnout?

e. Compute the coefficient of multiple determination (R^2). How much of the variance in support is explained by the two independent variables? (*HINT: You calculated the partial correlation coefficient in part b of this problem.*)

f. Write a paragraph summarizing your conclusions about the relationships among these three variables.

15.3 SOC Data on civil strife (number of incidents), unemployment, and urbanization have been gathered for 10 nations. Take civil strife as the dependent variable. Compute the zero-order correlations among all three variables.

Number of Incidents of Civil Strife	Unemployment Rate	Percentage of Population Living in Urban Areas
0	5.3	60
1	1.0	65
5	2.7	55
7	2.8	68
10	3.0	69
23	2.5	70
25	6.0	45
26	5.2	40
30	7.8	75
53	9.2	80

a. Compute the partial correlation coefficient for the relationship between strife (Y) and unemployment (X) while controlling for the effect of urbanization (Z). What effect does this have on the bivariate relationship? Is the relationship between strife and unemployment direct?

b. Compute the partial correlation coefficient for the relationship between strife (Y) and urbanization (X) while controlling for the effect of unemployment (Z). What effect does this have on the bivariate relationship? Is the relationship between strife and urbanization direct? (*HINT: You will need this partial correlation to compute the multiple correlation coefficient.*)

c. Find the unstandardized multiple regression equation with unemployment (X_1) and urbanization (X_2) as the independent variables. What level of strife would be expected in a nation in which the unemployment rate was 10% and 90% of the population lived in urban areas?

d. Compute beta-weights for each independent variable. Which has the stronger impact on turnout?

e. Compute the coefficient of multiple determination (R^2). How much of the variance in strife is explained by the two independent variables?

f. Write a paragraph summarizing your conclusions about the relationships among these three variables.

15.4 SOC/CJ In Problem 14.5, crime and population data were presented for each of 10 states. The data are reproduced here.

	Crime Rates			Population		
State	Homicide	Robbery	Car Theft	Growth	Density	Mobility
Maine	1	24	102	4	43	86
New York	5	186	186	2	409	89
Ohio	5	163	361	1	280	85
Iowa	1	39	185	2	53	84
Virginia	6	99	211	8	193	84
Kentucky	5	88	211	4	106	84
Texas	6	157	409	13	90	81
Arizona	8	144	924	20	54	81
Washington	3	92	784	9	96	80
California	7	176	713	8	234	84

Take the three crime variables as the dependent variables (one at a time) and do the following.

a. Find the multiple regression equations (unstandardized) with growth and mobility as independent variables.
b. Make a prediction for each crime variable for a state with a 5% growth rate and a 85% rate of mobility.
c. Compute beta-weights for each independent variable in each equation and compare their relative effect on each dependent.
d. Compute R^2 for each crime variable, using the population variables as independent variables.
e. Write a paragraph summarizing your findings.

15.5 PS Problem 14.4 presented data on 10 precincts. The information is reproduced here.

Precinct	Percent Democrat	Percent Minority	Voter Turnout
A	50	10	56
B	45	12	55
C	56	8	52
D	78	15	60
E	13	5	89
F	85	20	25
G	62	18	64
H	33	9	88
I	25	0	42
J	49	9	36

Take voter turnout as the dependent variable and do the following.

a. Find the multiple regression equations (unstandardized).
b. What turnout would you expect for a precinct in which 0% of the voters were Democrats and 5% were minorities?
c. Compute beta-weights for each independent variable and compare their relative effect on turnout. Which was the more important factor?
d. Compute R^2.
e. Write a paragraph summarizing your findings.

15.6 SW Twelve families have been referred to a counselor, and she has rated each of them on a cohesiveness scale. Also, she has information on family income and number of children currently living at home. Take family cohesion as the dependent variable.

Family	Cohesion Score	Income	Number of Children
A	10	30,000	5
B	10	70,000	4
C	9	35,000	4
D	5	25,000	0
E	1	55,000	3
F	7	40,000	0
G	2	60,000	2
H	5	30,000	3
I	8	50,000	5
J	3	25,000	4
K	2	45,000	3
L	4	50,000	0

a. Find the multiple regression equations (unstandardized).
b. What level of cohesion would be expected in a family with an income of $20,000 and six children?
c. Compute beta-weights for each independent variable and compare their relative effect on cohesion. Which was the more important factor?
d. Compute R^2.
e. Write a paragraph summarizing your findings.

15.7 Problem 14.8 presented per capita expenditures on education for 15 states, along with rank on income per capita and the percentage of the population that has graduated from high school. The data are reproduced below.

State	Per Capita Expenditures on Education, 2005	Percent High School Graduates, 2006*	Rank in Per Capita Income, 2006
Arkansas	1,194	83	48
Colorado	1,659	90	8
Connecticut	2,197	88	1
Florida	1,374	87	20
Illinois	1,740	88	13
Kansas	1,571	90	21
Louisiana	1,431	80	34
Maryland	1,623	87	4
Michigan	1,911	90	27
Mississippi	1,226	81	50
Nebraska	1,338	91	23
New Hampshire	1,673	92	7
North Carolina	1,280	84	36
Pennsylvania	1,710	88	18
Wyoming	2,045	91	6

Take educational expenditures as the dependent variable.

a. Compute beta-weights for each independent variable and compare their relative effect on expenditures. Which was the more important factor?

b. Compute R^2.

c. Write a paragraph summarizing your findings.

15.8 SOC The scores on four variables for 20 individuals are reported below: hours of TV (average number of hours of TV viewing each day), occupational prestige (higher scores indicate greater prestige), number of children, and age. Take TV viewing as the dependent variable and select two of the remaining variables as independents.

a. Compute beta-weights for each of the independent variables you selected and compare their relative effect on the hours of television watching. Which was the more important factor?

b. Compute R^2.

c. Write a paragraph summarizing your findings.

Hours of TV	Occupational Prestige	Number of Children	Age
4	50	2	43
3	36	3	58
3	36	1	34
4	50	2	42
2	45	2	27
3	50	5	60
4	50	0	28
7	40	3	55
1	57	2	46
3	33	2	65
1	46	3	56
3	31	1	29
1	19	2	41
0	52	0	50
2	48	1	62
4	36	1	24
3	48	0	25
1	62	1	87
5	50	0	45
1	27	3	62

YOU ARE THE RESEARCHER: A Multivariate Analysis of Internet Use and Success

The two projects below continue the investigations begun in Chapter 14. You will use an SPSS procedure called **Regression** to analyze the combined effects of your independent variables (including dummy variables if you used any) on the dependent variables from Chapter 14. The **Regression** procedure produces

a much greater volume of output than the **Correlate** procedure used in Chapters 13 and 14. Among other things, **Regression** displays the slope (*b*) and the *Y* intercept (*a*), so we can use this procedure to find least-squares regression lines. In these projects, we will use very little of the power of the **Regression** command and we will be extremely economical in our choice of all the options available. I urge you to explore some of the variations and capabilities of this powerful data-analysis procedure on your own.

Let's begin with a demonstration using *marhomo* (approval of gay marriage) as the dependent variable and *attend* (church attendance) and *educ* as independent variables.

Click **Analyze, Regression,** and **Linear,** and the **Linear Regression** window will appear. Move *marhomo* into the **Dependent:** box and *attend* and *educ* into the **Independent(s):** box. If you wish, you may click the **Statistics** button and then click **Descriptives** to get zero-order correlations, means, and standard deviations for the variables. Click **Continue** and **OK,** and the following output will appear (descriptive information about the variables and the zero-order correlations are omitted to conserve space).

Model Summary

Model	*R*	*R* Square	Adjusted *R* Square	Std. Error of the Estimate
1	.400[a]	.160	.157	1.382

[a]Predictors: (Constant), HIGHEST YEAR OF SCHOOL COMPLETED, HOW OFTEN R ATTENDS RELIGIOUS SERVICES

ANOVA[b]

Model		Sum of Squares	Df	Mean Square	F	Sig.
1	Regression	232.244	2	116.122	60.825	.000[a]
	Residual	1219.918	639	1.909		
	Total	1452.162	641			

[a]Predictors: (Constant), HIGHEST YEAR OF SCHOOL COMPLETED, HOW OFTEN R ATTENDS RELIGIOUS SERVICES

[b]Dependent Variable: HOMOSEXUALS SHOULD HAVE RIGHT TO MARRY

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	*T*	Sig.
1	Constant)	3.815	.241		15.843	.000
	HOW OFTEN R ATTENDS RELIGIOUS SERVICES	.189	.019	.354	9.754	.000
	HIGHEST YEAR OF SCHOOL COMPLETED	-.089	.017	-.190	-5.247	.000

[a]Dependent Variable: HOMOSEXUALS SHOULD HAVE RIGHT TO MARRY

The "Model Summary" block reports the multiple *R* (.400) and *R* square (.160). The ANOVA output block shows the significance of the relationship (Sig. = .000). In the last output block, under "Unstandardized Coefficients," we see the *Y* intercept, reported as a constant of 3.815, and the slopes (B) of the independent variables on *marhomo*. From this information, we can build a regression equation (see Formula 15.3) to predict scores on *marhomo*:

$$Y = 3.185 + (0.189)(attend) + (-0.089)(educ)$$

Under "Standardized Coefficients," we find the standardized partial slopes (Beta). The beta for *attend* (.354) is greater in value than the beta for *educ* (−.190). This tells us that religiosity (*attend*) is the more important of the two independent variables. It has a positive relationship with *marhomo* after the effects of education have been controlled. As religiosity increases, disapproval of gay marriage increases. Education has a negative relationship with the dependent variable after religiosity has been controlled: as education increases, disapproval of gay marriage decreases.

Your turn.

PROJECT 1: Who Uses the Internet?

This project follows up on Project 1 from Chapter 14. The dependent variable is *wwwhr,* which, as you recall, measures the number of hours per week the respondent uses the internet.

STEP 1: Choosing Independent Variables

Choose two of the four independent variables you selected in Project 1 from Chapter 14. All other things being equal, you might choose the variables that had the strongest relationship with *wwwhr*. Remember that independent variables *cannot* be nominal in level of measurement unless you recode the variable into a dummy variable. You may use any interval-ratio or ordinal level variables with more than three categories or scores. Once you have selected your variables, list them in the table below and describe exactly what they measure.

SPSS Variable Name	What Exactly Does This Variable Measure?

STEP 2: Stating Hypotheses

Restate your hypotheses from Chapter 14 and add some ideas about the relative importance of your two independent variables. Which do you expect to have the stronger effect? Why?

1.

2.

STEP 3: Running the Regression Procedure

Click **Analyze → Regression → Linear** and place *wwwhr* in the **Dependent:** box and your selected independent variables in the **Independent(s):** box. Click **Statistics** and **Descriptives,** and then click **OK** to get your results.

STEP 4: Recording Results

Summarize your results by filling in the blanks below with information from the **Coefficients** box. The *Y* intercept (*a*) is reported in the top row of the box as a "constant." The slopes (*b*) are reported in the "Unstandardized Coefficients" column under "B," and the beta-weights (*b**) are reported in the "Standardized Coefficients" column under "Beta."

1. a = _____
2. **a.** Slope for your first independent variable (b_1) = _____.
   b. Slope for your second independent variable (b_2) = _____.
3. State the least squares multiple regression equation (see Formula 15.3).

$$Y = \_\_\_ + (\_\_)\, X_1 + (\_\_)\, X_2$$

4. **a.** Beta-weight of your first independent variable (b_1^*) = _____.
   b. Beta-weight of your second independent variable (b_2^*) = _____.
5. State the standardized least-square regression line (see Formula 15.10).

$$Z_y = (\_\_\_)\, Z_1 + (\_\_\_)\, Z_2$$

6. R^2 = _____

STEP 5: Analyzing and Interpreting Results

Write a short summary of your results. Your summary needs to identify your variables and distinguish between independent and dependent variables. You also need to report the strength and direction of relationships as indicated by the slopes and beta-weights and the total explained variance (R^2). How much of the variance does your first independent variable explain? How much more is explained by your second independent variable?

PROJECT 2: What Are the Correlates of Success?

STEP 1: Choosing Independent Variables

Use two of the four independent variables you selected in Project 2 from Chapter 14. All other things being equal, you might choose the variables that had the strongest relationship with the variable you selected to be dependent (either *income06* or *prestg80*). Remember that independent variables *cannot* be nominal in level of measurement unless you recode the variable into a dummy variable. You may use any interval-ratio or ordinal level variables with more than three categories or scores. Once you have selected your variables, list them in the table below and describe exactly what they measure.

SPSS Variable Name	What Exactly Does This Variable Measure?

STEP 2: Stating Hypotheses

Restate your hypotheses from Chapter 14 and add some ideas about the relative importance of your two independent variables. Which do you expect to have the stronger effect? Why?

1.

2.

STEP 3: Running the Regression Procedure

Click **Analyze → Regression → Linear** and place your dependent variable in the **Dependent:** box and your selected independent variables in the **Independent(s):** box. Click **Statistics** and **Descriptives,** and then click **OK** to get your results.

STEP 4: Recording Results

Summarize your results by filling in the blanks below with information from the **Coefficients** box. The *Y* intercept (*a*) is reported in the top row of the box as a "constant." The slopes are reported in the "Unstandardized Coefficients" column under "B," and the beta-weights are reported in the "Standardized Coefficients" column under "Beta."

1. $a =$ _____
2. **a.** Slope for your first independent variable (b_1) = _____.
   b. Slope for your second independent variable (b_2) = _____.
3. State the least-squares multiple regression equation (see Formula 15.3).

$$Y = \_\_\_ + (\_\_)\, X_1 + (\_\_)\, X_2$$

4. **a.** Beta-weight of your first independent variable (b_1^*) = _____.
   b. Beta-weight of your second independent variable (b_2^*) = _____.
5. State the standardized least-squares regression line (see Formula 15.10).

$$Z_y = (\_\_\_)\, Z_1 + (\_\_\_)\, Z_2$$

6. $R^2 =$ _____

STEP 5: Analyzing and Interpreting Results

Write a short summary of your results. Your summary needs to identify your variables and distinguish between independent and dependent variables. You also need to report the strength and direction of relationships as indicated by the slopes and beta-weights and the total explained variance (R^2). How much of the variance does your first independent variable explain? How much more is explained by your second independent variable?

APPENDIX A

Area Under the Normal Curve

Column (a) lists *Z* scores from 0.00 to 4.00. Only positive scores are displayed, but since the normal curve is symmetrical, the areas for negative scores will be exactly the same as areas for positive scores. Column (b) lists the proportion of the total area between the *Z* score and the mean. Figure A.1 displays areas of this type. Column (c) lists the proportion of the area beyond the *Z* score, and Figure A.2 displays this type of area.

FIGURE A.1 AREA BETWEEN MEAN AND *Z*

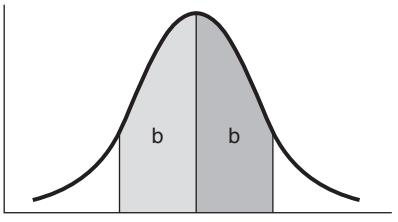
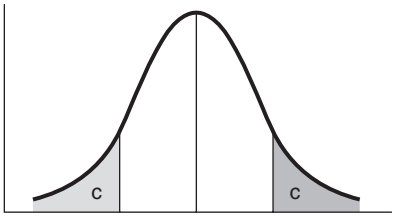


FIGURE A.2 AREA BEYOND *Z*



(a) *Z*	(b) Area Between Mean and *Z*	(c) Area Beyond *Z*
0.00	0.0000	0.5000
0.01	0.0040	0.4960
0.02	0.0080	0.4920
0.03	0.0120	0.4880
0.04	0.0160	0.4840
0.05	0.0199	0.4801
0.06	0.0239	0.4761
0.07	0.0279	0.4721
0.08	0.0319	0.4681
0.09	0.0359	0.4641
0.10	0.0398	0.4602
0.11	0.0438	0.4562
0.12	0.0478	0.4522
0.13	0.0517	0.4483
0.14	0.0557	0.4443
0.15	0.0596	0.4404
0.16	0.0636	0.4364
0.17	0.0675	0.4325
0.18	0.0714	0.4286
0.19	0.0753	0.4247
0.20	0.0793	0.4207
0.21	0.0832	0.4168
0.22	0.0871	0.4129
0.23	0.0910	0.4090
0.24	0.0948	0.4052
0.25	0.0987	0.4013

(a) *Z*	(b) Area Between Mean and *Z*	(c) Area Beyond *Z*
0.26	0.1026	0.3974
0.27	0.1064	0.3936
0.28	0.1103	0.3897
0.29	0.1141	0.3859
0.30	0.1179	0.3821
0.31	0.1217	0.3783
0.32	0.1255	0.3745
0.33	0.1293	0.3707
0.34	0.1331	0.3669
0.35	0.1368	0.3632
0.36	0.1406	0.3594
0.37	0.1443	0.3557
0.38	0.1480	0.3520
0.39	0.1517	0.3483
0.40	0.1554	0.3446
0.41	0.1591	0.3409
0.42	0.1628	0.3372
0.43	0.1664	0.3336
0.44	0.1700	0.3300
0.45	0.1736	0.3264
0.46	0.1772	0.3228
0.47	0.1808	0.3192
0.48	0.1844	0.3156
0.49	0.1879	0.3121
0.50	0.1915	0.3085

(a) *Z*	(b) Area Between Mean and *Z*	(c) Area Beyond *Z*
0.51	0.1950	0.3050
0.52	0.1985	0.3015
0.53	0.2019	0.2981
0.54	0.2054	0.2946
0.55	0.2088	0.2912
0.56	0.2123	0.2877
0.57	0.2157	0.2843
0.58	0.2190	0.2810
0.59	0.2224	0.2776
0.60	0.2257	0.2743
0.61	0.2291	0.2709
0.62	0.2324	0.2676
0.63	0.2357	0.2643
0.64	0.2389	0.2611
0.65	0.2422	0.2578
0.66	0.2454	0.2546
0.67	0.2486	0.2514
0.68	0.2517	0.2483
0.69	0.2549	0.2451
0.70	0.2580	0.2420
0.71	0.2611	0.2389
0.72	0.2642	0.2358
0.73	0.2673	0.2327
0.74	0.2703	0.2297
0.75	0.2734	0.2266
0.76	0.2764	0.2236
0.77	0.2794	0.2206
0.78	0.2823	0.2177
0.79	0.2852	0.2148
0.80	0.2881	0.2119
0.81	0.2910	0.2090
0.82	0.2939	0.2061
0.83	0.2967	0.2033
0.84	0.2995	0.2005
0.85	0.3023	0.1977
0.86	0.3051	0.1949
0.87	0.3078	0.1922
0.88	0.3106	0.1894
0.89	0.3133	0.1867
0.90	0.3159	0.1841
0.91	0.3186	0.1814
0.92	0.3212	0.1788
0.93	0.3238	0.1762
0.94	0.3264	0.1736
0.95	0.3289	0.1711
0.96	0.3315	0.1685
0.97	0.3340	0.1660
0.98	0.3365	0.1635
0.99	0.3389	0.1611
1.00	0.3413	0.1587
1.01	0.3438	0.1562
1.02	0.3461	0.1539

(a) *Z*	(b) Area Between Mean and *Z*	(c) Area Beyond *Z*
1.03	0.3485	0.1515
1.04	0.3508	0.1492
1.05	0.3531	0.1469
1.06	0.3554	0.1446
1.07	0.3577	0.1423
1.08	0.3599	0.1401
1.09	0.3621	0.1379
1.10	0.3643	0.1357
1.11	0.3665	0.1335
1.12	0.3686	0.1314
1.13	0.3708	0.1292
1.14	0.3729	0.1271
1.15	0.3749	0.1251
1.16	0.3770	0.1230
1.17	0.3790	0.1210
1.18	0.3810	0.1190
1.19	0.3830	0.1170
1.20	0.3849	0.1151
1.21	0.3869	0.1131
1.22	0.3888	0.1112
1.23	0.3907	0.1093
1.24	0.3925	0.1075
1.25	0.3944	0.1056
1.26	0.3962	0.1038
1.27	0.3980	0.1020
1.28	0.3997	0.1003
1.29	0.4015	0.0985
1.30	0.4032	0.0968
1.31	0.4049	0.0951
1.32	0.4066	0.0934
1.33	0.4082	0.0918
1.34	0.4099	0.0901
1.35	0.4115	0.0885
1.36	0.4131	0.0869
1.37	0.4147	0.0853
1.38	0.4162	0.0838
1.39	0.4177	0.0823
1.40	0.4192	0.0808
1.41	0.4207	0.0793
1.42	0.4222	0.0778
1.43	0.4236	0.0764
1.44	0.4251	0.0749
1.45	0.4265	0.0735
1.46	0.4279	0.0721
1.47	0.4292	0.0708
1.48	0.4306	0.0694
1.49	0.4319	0.0681
1.50	0.4332	0.0668
1.51	0.4345	0.0655
1.52	0.4357	0.0643
1.53	0.4370	0.0630
1.54	0.4382	0.0618

(a) Z	(b) Area Between Mean and Z	(c) Area Beyond Z
1.55	0.4394	0.0606
1.56	0.4406	0.0594
1.57	0.4418	0.0582
1.58	0.4429	0.0571
1.59	0.4441	0.0559
1.60	0.4452	0.0548
1.61	0.4463	0.0537
1.62	0.4474	0.0526
1.63	0.4484	0.0516
1.64	0.4495	0.0505
1.65	0.4505	0.0495
1.66	0.4515	0.0485
1.67	0.4525	0.0475
1.68	0.4535	0.0465
1.69	0.4545	0.0455
1.70	0.4554	0.0446
1.71	0.4564	0.0436
1.72	0.4573	0.0427
1.73	0.4582	0.0418
1.74	0.4591	0.0409
1.75	0.4599	0.0401
1.76	0.4608	0.0392
1.77	0.4616	0.0384
1.78	0.4625	0.0375
1.79	0.4633	0.0367
1.80	0.4641	0.0359
1.81	0.4649	0.0351
1.82	0.4656	0.0344
1.83	0.4664	0.0336
1.84	0.4671	0.0329
1.85	0.4678	0.0322
1.86	0.4686	0.0314
1.87	0.4693	0.0307
1.88	0.4699	0.0301
1.89	0.4706	0.0294
1.90	0.4713	0.0287
1.91	0.4719	0.0281
1.92	0.4726	0.0274
1.93	0.4732	0.0268
1.94	0.4738	0.0262
1.95	0.4744	0.0256
1.96	0.4750	0.0250
1.97	0.4756	0.0244
1.98	0.4761	0.0239
1.99	0.4767	0.0233
2.00	0.4772	0.0228
2.01	0.4778	0.0222
2.02	0.4783	0.0217
2.03	0.4788	0.0212
2.04	0.4793	0.0207
2.05	0.4798	0.0202
2.06	0.4803	0.0197
2.07	0.4808	0.0192
2.08	0.4812	0.0188
2.09	0.4817	0.0183
2.10	0.4821	0.0179
2.11	0.4826	0.0174
2.12	0.4830	0.0170
2.13	0.4834	0.0166
2.14	0.4838	0.0162
2.15	0.4842	0.0158
2.16	0.4846	0.0154
2.17	0.4850	0.0150
2.18	0.4854	0.0146
2.19	0.4857	0.0143
2.20	0.4861	0.0139
2.21	0.4864	0.0136
2.22	0.4868	0.0132
2.23	0.4871	0.0129
2.24	0.4875	0.0125
2.25	0.4878	0.0122
2.26	0.4881	0.0119
2.27	0.4884	0.0116
2.28	0.4887	0.0113
2.29	0.4890	0.0110
2.30	0.4893	0.0107
2.31	0.4896	0.0104
2.32	0.4898	0.0102
2.33	0.4901	0.0099
2.34	0.4904	0.0096
2.35	0.4906	0.0094
2.36	0.4909	0.0091
2.37	0.4911	0.0089
2.38	0.4913	0.0087
2.39	0.4916	0.0084
2.40	0.4918	0.0082
2.41	0.4920	0.0080
2.42	0.4922	0.0078
2.43	0.4925	0.0075
2.44	0.4927	0.0073
2.45	0.4929	0.0071
2.46	0.4931	0.0069
2.47	0.4932	0.0068
2.48	0.4934	0.0066
2.49	0.4936	0.0064
2.50	0.4938	0.0062
2.51	0.4940	0.0060
2.52	0.4941	0.0059
2.53	0.4943	0.0057
2.54	0.4945	0.0055
2.55	0.4946	0.0054
2.56	0.4948	0.0052
2.57	0.4949	0.0051
2.58	0.4951	0.0049

(a) Z	(b) Area Between Mean and Z	(c) Area Beyond Z
2.59	0.4952	0.0048
2.60	0.4953	0.0047
2.61	0.4955	0.0045
2.62	0.4956	0.0044
2.63	0.4957	0.0043
2.64	0.4959	0.0041
2.65	0.4960	0.0040
2.66	0.4961	0.0039
2.67	0.4962	0.0038
2.68	0.4963	0.0037
2.69	0.4964	0.0036
2.70	0.4965	0.0035
2.71	0.4966	0.0034
2.72	0.4967	0.0033
2.73	0.4968	0.0032
2.74	0.4969	0.0031
2.75	0.4970	0.0030
2.76	0.4971	0.0029
2.77	0.4972	0.0028
2.78	0.4973	0.0027
2.79	0.4974	0.0026
2.80	0.4974	0.0026
2.81	0.4975	0.0025
2.82	0.4976	0.0024
2.83	0.4977	0.0023
2.84	0.4977	0.0023
2.85	0.4978	0.0022
2.86	0.4979	0.0021
2.87	0.4979	0.0021
2.88	0.4980	0.0020
2.89	0.4981	0.0019
2.90	0.4981	0.0019
2.91	0.4982	0.0018
2.92	0.4982	0.0018
2.93	0.4983	0.0017
2.94	0.4984	0.0016
2.95	0.4984	0.0016
2.96	0.4985	0.0015
2.97	0.4985	0.0015
2.98	0.4986	0.0014
2.99	0.4986	0.0014
3.00	0.4986	0.0014
3.01	0.4987	0.0013
3.02	0.4987	0.0013
3.03	0.4988	0.0012
3.04	0.4988	0.0012
3.05	0.4989	0.0011
3.06	0.4989	0.0011
3.07	0.4989	0.0011
3.08	0.4990	0.0010
3.09	0.4990	0.0010
3.10	0.4990	0.0010
3.11	0.4991	0.0009
3.12	0.4991	0.0009
3.13	0.4991	0.0009
3.14	0.4992	0.0008
3.15	0.4992	0.0008
3.16	0.4992	0.0008
3.17	0.4992	0.0008
3.18	0.4993	0.0007
3.19	0.4993	0.0007
3.20	0.4993	0.0007
3.21	0.4993	0.0007
3.22	0.4994	0.0006
3.23	0.4994	0.0006
3.24	0.4994	0.0006
3.25	0.4994	0.0006
3.26	0.4994	0.0006
3.27	0.4995	0.0005
3.28	0.4995	0.0005
3.29	0.4995	0.0005
3.30	0.4995	0.0005
3.31	0.4995	0.0005
3.32	0.4995	0.0005
3.33	0.4996	0.0004
3.34	0.4996	0.0004
3.35	0.4996	0.0004
3.36	0.4996	0.0004
3.37	0.4996	0.0004
3.38	0.4996	0.0004
3.39	0.4997	0.0003
3.40	0.4997	0.0003
3.41	0.4997	0.0003
3.42	0.4997	0.0003
3.43	0.4997	0.0003
3.44	0.4997	0.0003
3.45	0.4997	0.0003
3.46	0.4997	0.0003
3.47	0.4997	0.0003
3.48	0.4997	0.0003
3.49	0.4998	0.0002
3.50	0.4998	0.0002
3.60	0.4998	0.0002
3.70	0.4999	0.0001
3.80	0.4999	0.0001
3.90	0.4999	<0.0001
4.00	0.4999	<0.0001

APPENDIX B

Distribution of *t*

Degrees of Freedom (*df*)	Level of Significance for One-tailed Test					
	0.10	0.05	0.025	0.01	0.005	0.0005
	Level of Significance for Two-tailed Test					
	0.20	0.10	0.05	0.02	0.01	0.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.598
3	1.638	2.353	3.182	4.541	5.841	12.941
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.859
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.405
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.767
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.358	2.617	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.291

Source: Table III of Fisher & Yates: *Statistical Tables for Biological, Agricultural and Medical Research,* published by Longman Group Ltd., London (1974), 6th edition (previously published by Oliver & Boyd Ltd., Edinburgh). Reprinted by permission of Pearson Education Limited.

APPENDIX C Distribution of Chi Square

df	099	0.98	0.95	0.90	0.80	0.70	0.50	0.30	0.20	0.10	0.05	0.02	0.01	0.001
1	$.0^{3}157$	$.0^{3}628$	.00393	.0158	.0642	.148	.455	1.074	1.642	2.706	3.841	5.412	6.635	10.827
2	.0201	.0404	.103	.211	.446	.713	1.386	2.408	3.219	4.605	5.991	7.824	9.210	13.815
3	.115	.185	.352	.584	1.005	1.424	2.366	3.665	4.642	6.251	7.815	9.837	11.341	16.268
4	.297	.429	.711	1.064	1.649	2.195	3.357	4.878	5.989	7.779	9.488	11.668	13.277	18.465
5	.554	.752	1.145	1.610	2.343	3.000	4.351	6.064	7.289	9.236	11.070	13.388	15.086	20.517
6	.872	1.134	1.635	2.204	3.070	3.828	5.348	7.231	8.558	10.645	12.592	15.033	16.812	22.457
7	1.239	1.564	2.167	2.833	3.822	4.671	6.346	8.383	9.803	12.017	14.067	16.622	18.475	24.322
8	1.646	2.032	2.733	3.490	4.594	5.527	7.344	9.524	11.030	13.362	15.507	18.168	20.090	26.125
9	2.088	2.532	3.325	4.168	5.380	6.393	8.343	10.656	12.242	14.684	16.919	19.679	21.666	27.877
10	2.558	3.059	3.940	4.865	6.179	7.267	9.342	11.781	13.442	15.987	18.307	21.161	23.209	29.588
11	3.053	3.609	4.575	5.578	6.989	8.148	10.341	12.899	14.631	17.275	19.675	22.618	24.725	31.264
12	3.571	4.178	5.226	6.304	7.807	9.034	11.340	14.011	15.812	18.549	21.026	24.054	26.217	32.909
13	4.107	4.765	5.892	7.042	8.634	9.926	12.340	15.119	16.985	19.812	22.362	25.472	27.688	34.528
14	4.660	5.368	6.571	7.790	9.467	10.821	13.339	16.222	18.151	21.064	23.685	26.873	29.141	36.123
15	5.229	5.985	7.261	8.547	10.307	11.721	14.339	17.322	19.311	22.307	24.996	28.259	30.578	37.697
16	5.812	6.614	7.962	9.312	11.152	12.624	15.338	18.418	20.465	23.542	26.296	29.633	32.000	39.252
17	6.408	7.255	8.672	10.085	12.002	13.531	16.338	19.511	21.615	24.769	27.587	30.995	33.409	40.790
18	7.015	7.906	9.390	10.865	12.857	14.440	17.338	20.601	22.760	25.989	28.869	32.346	34.805	42.312
19	7.633	8.567	10.117	11.651	13.716	15.352	18.338	21.689	23.900	27.204	30.144	33.687	36.191	43.820
20	8.260	9.237	10.851	12.443	14.578	16.266	19.337	22.775	25.038	28.412	31.410	35.020	37.566	45.315
21	8.897	9.915	11.591	13.240	15.445	17.182	20.337	23.858	26.171	29.615	32.671	36.343	38.932	46.797
22	9.542	10.600	12.338	14.041	16.314	18.101	21.337	24.939	27.301	30.813	33.924	37.659	40.289	48.268
23	10.196	11.293	13.091	14.848	17.187	19.021	22.337	26.018	28.429	32.007	35.172	38.968	41.638	49.728
24	10.856	11.992	13.848	15.659	18.062	19.943	23.337	27.096	29.553	33.196	36.415	40.270	42.980	51.179
25	11.524	12.697	14.611	16.473	18.940	20.867	24.337	28.172	30.675	34.382	37.652	41.566	44.314	52.620
26	12.198	13.409	15.379	17.292	19.820	21.792	25.336	29.246	31.795	35.563	38.885	42.856	45.642	54.052
27	12.879	14.125	16.151	18.114	20.703	22.719	26.336	30.319	32.912	36.741	40.113	44.140	46.963	55.476
28	13.565	14.847	16.928	18.939	21.588	23.647	27.336	31.391	34.027	37.916	41.337	45.419	48.278	56.893
29	14.256	15.574	17.708	19.768	22.475	24.577	28.336	32.461	35.139	39.087	42.557	46.693	49.588	58.302
30	14.953	16.306	18.493	20.599	23.364	25.508	29.336	33.530	36.250	40.256	43.773	47.962	50.892	59.703

Source: Table IV of Fisher & Yates: *Statistical Tables for Biological, Agricultural and Medical Research,* published by Longman Group Ltd., London (1974), 6th edition (previously published by Oliver & Boyd Ltd., Edinburgh). Reprinted by permission of Pearson Education Limited.

APPENDIX D

Distribution of *F*

$p = 0.05$

n_1 / n_2	1	2	3	4	5	6	8	12	24	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	238.9	243.9	249.0	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.37	19.41	19.45	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.84	8.74	8.64	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.04	5.91	5.77	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.82	4.68	4.53	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.15	4.00	3.84	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.73	3.57	3.41	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.44	3.28	3.12	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.23	3.07	2.90	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.07	2.91	2.74	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	2.95	2.79	2.61	2.40
12	4.75	3.88	3.49	3.26	3.11	3.00	2.85	2.69	2.50	2.30
13	4.67	3.80	3.41	3.18	3.02	2.92	2.77	2.60	2.42	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.70	2.53	2.35	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.64	2.48	2.29	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.59	2.42	2.24	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.55	2.38	2.19	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.51	2.34	2.15	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.48	2.31	2.11	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.45	2.28	2.08	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.42	2.25	2.05	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.40	2.23	2.03	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.38	2.20	2.00	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.36	2.18	1.98	1.73
25	4.24	3.38	2.99	2.76	2.60	2.49	2.34	2.16	1.96	1.71
26	4.22	3.37	2.98	2.74	2.59	2.47	2.32	2.15	1.95	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.30	2.13	1.93	1.67
28	4.20	3.34	2.95	2.71	2.56	2.44	2.29	2.12	1.91	1.65
29	4.18	3.33	2.93	2.70	2.54	2.43	2.28	2.10	1.90	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.27	2.09	1.89	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.18	2.00	1.79	1.51
60	4.00	3.15	2.76	2.52	2.37	2.25	2.10	1.92	1.70	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.02	1.83	1.61	1.25
∞	3.84	2.99	2.60	2.37	2.21	2.09	1.94	1.75	1.52	1.00

Values of n_1 and n_2 represent the degrees of freedom associated with the between and within estimates of variance, respectively.

Source: Table V of Fisher and Yates: *Statistical Tables for Biological, Agricultural and Medical Research,* published by Longman Group Ltd., London (1974), 6th edition (previously published by Oliver and Boyd Ltd., Edinburgh). Reprinted by permission of Pearson Education Limited.

$p = 0.01$

n_1 / n_2	1	2	3	4	5	6	8	12	24	∞
1	4052	4999	5403	5625	5764	5859	5981	6106	6234	6366
2	98.49	99.01	99.17	99.25	99.30	99.33	99.36	99.42	99.46	99.50
3	34.12	30.81	29.46	28.71	28.24	27.91	27.49	27.05	26.60	26.12
4	21.20	18.00	16.69	15.98	15.52	15.21	14.80	14.37	13.93	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.27	9.89	9.47	9.02
6	13.74	10.92	9.78	9.15	8.75	8.47	8.10	7.72	7.31	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.84	6.47	6.07	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.03	5.67	5.28	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.47	5.11	4.73	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.06	4.71	4.33	3.91
11	9.65	7.20	6.22	5.67	5.32	5.07	4.74	4.40	4.02	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.50	4.16	3.78	3.36
13	9.07	6.70	5.74	5.20	4.86	4.62	4.30	3.96	3.59	3.16
14	8.86	6.51	5.56	5.03	4.69	4.46	4.14	3.80	3.43	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.00	3.67	3.29	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	3.89	3.55	3.18	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.79	3.45	3.08	2.65
18	8.28	6.01	5.09	4.58	4.25	4.01	3.71	3.37	3.00	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.63	3.30	2.92	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.56	3.23	2.86	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.51	3.17	2.80	2.36
22	7.94	5.72	4.82	4.31	3.99	3.76	3.45	3.12	2.75	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.41	3.07	2.70	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.36	3.03	2.66	2.21
25	7.77	5.57	4.68	4.18	3.86	3.63	3.32	2.99	2.62	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.29	2.96	2.58	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.26	2.93	2.55	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.23	2.90	2.52	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.20	2.87	2.49	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.17	2.84	2.47	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	2.99	2.66	2.29	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.82	2.50	2.12	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.66	2.34	1.95	1.38
∞	6.64	4.60	3.78	3.32	3.02	2.80	2.51	2.18	1.79	1.00

Values of n_1 and n_2 represent the degrees of freedom associated with the between and within estimates of variance, respectively.

APPENDIX E Using Statistics: Ideas for Research Projects

This appendix presents outlines for four research projects, each of which requires you to use SPSS to analyze the 2006 General Social Survey that has been used throughout this text. The research projects should be completed at various intervals during the course, and each project permits a great deal of choice on the part of the student. The first project stresses description and should be done after completing Chapters 2–5. The second involves estimation and should be completed in conjunction with Chapter 7. The third project uses inferential statistics and should be done after completing Part II, and the fourth combines inferential statistics with measures of association (with an option for multivariate analysis) and should be done after Part III (or IV).

PROJECT 1—DESCRIPTIVE STATISTICS

1. Select five variables from the 2006 General Social Survey *other than* the ones you have used in the end-of-chapter exercises (*NOTE: Your instructor may specify a different number of variables*) and use the **Frequencies** command to get frequency distributions and summary statistics for each variable. Click the **Statistics** button on the **Frequencies** command window and request the mean, median, mode, standard deviation, and range. See Chapters 2–5 for guidelines and examples. Make a note of all relevant information when it appears on screen or make a hard copy. See Appendix G for a list of variables available in the 2006 GSS.
2. For each variable, get bar or line charts to summarize the overall shape of the distribution of the variable. See Chapter 3 for guidelines and examples.
3. Inspect the frequency distributions and graphs and choose appropriate measures of central tendency and, for ordinal and interval-ratio level variables, dispersion. Also, for interval-ratio and ordinal variables with many scores, check for skew both by using the line chart and by comparing the mean and median (see Chapter 4). Write a sentence or two of description for each variable, being careful to include a description of the overall shape of the distribution (see Chapters 2 and 3), the central tendency (Chapter 4), and the dispersion (Chapter 5). For nominal and ordinal level variables, be sure to explain any arbitrary numerical codes. For example, on the variable *class* in the 2006 GSS (see Appendix G), a 1 is coded as "lower class," a 2 indicates "working class," and so forth. This is an ordinal level variable, so you might choose to report the median as a measure of central tendency. If the median score on *class* were 2.45, for example, you might place that value in context by reporting that "the median is 2.45, about halfway between 'working class' and 'middle class'."
4. Below are examples of *minimal* summary sentences, using fictitious data:

   For a nominal-level variable (e.g., marital status), report the mode and some detail about the overall distribution: for example, "Most respondents

were married (57.5%), but divorced (17.4%) and single (21.3%) individuals were also common."

For an ordinal level variable (e.g., occupational prestige), use the median (and, perhaps, the mean and mode) and the range: for example, "The median prestige score was 44.3, and the range extended from 34 to 87. The most common score was 42, and the average score was 40.8."

For an interval-ratio level variable (e.g., age), use the mean (and, perhaps, the median or mode) and the standard deviation (and, perhaps, the range): for example, "Average age for this sample was 42.3. Respondents ranged from 18 to 94 years of age with a standard deviation of 15.37."

PROJECT 2—ESTIMATION

In this exercise, you will use the 2006 GSS sample to estimate the characteristics of the U.S. population. You will use SPSS to generate the sample statistics and then use either Formula 7.2 or 7.3 to find the confidence interval and state each interval in words.

A. Estimating Means

1. There are relatively few interval-ratio variables in the 2006 GSS. For this part of the project, you may also use ordinal variables that have *at least* three categories or scores. Choose a total of three variables that fit this description *other than* the variables you used in the exercises at the end of Chapter 7. (*NOTE: Your instructor may specify a different number of variables.*)
2. Use the **Descriptives** command to get means, standard deviations, and sample size (*N*), and use this information to construct 95% confidence intervals for each of your variables. Make a note of the mean, standard deviation, and sample size, or keep a hard copy. Use Formula 7.2 to compute the confidence intervals. Repeat this procedure for the remaining variables.
3. For each variable, write a summary sentence reporting the variable, the interval itself, the confidence level, and sample size. Write in plain English, as if you were reporting results in a newspaper. Most importantly, you should make it clear that you are estimating characteristics of the population of the entire United States. For example, a summary sentence might look like this: "Based on a random sample of 1,231, I estimate at the 95% level that U.S. drivers average between 64.46 and 68.22 miles per hour when driving on interstate highways."

B. Estimating Proportions

4. Choose three variables that are nominal or ordinal *other than* the variables you used in the exercises at the end of Chapter 7. (*NOTE: Your instructor may specify a different number of variables.*)
5. Use the **Frequencies** command to get the percentage of the sample in the various categories of each variable. Change the percentages (remember to use the "valid percents" column) to proportions and construct confidence intervals for one category of each variable (e.g., the % female for *sex*) using Formula 7.3.
6. For each variable, write a summary sentence reporting the variable, the interval, the confidence level, and sample size. Write in plain English, as if

you were reporting results in a newspaper. Remember to make it clear that you are estimating a characteristic of the United States population.

7. For any one of the intervals you constructed in Part A or B of this project, identify each of the following concepts and terms and briefly explain their role in estimation: sample, population, statistic, parameter, EPSEM, representative, confidence level.

PROJECT 3—SIGNIFICANCE TESTING

A. Two Sample *t* Test (Chapter 9)

1. Choose two different dependent variables from the interval-ratio or ordinal variables that have three or more scores. Choose independent variables that might logically be a cause of your dependent variables. Remember that, for a *t* test, independent variables can have *only* two categories, but you can still use independent variables with more than two categories by (a) using the **Grouping Variable** box to specify the exact categories (e.g., select scores of 1 and 5 on *marital* to compare married with never married respondents) or (b) collapsing the scores of variables with more than two categories by using the **Recode** command. Independent variables can be any level of measurement, and you may use the same independent variable for both tests.
2. Click **Analyze, Compare Means,** and then **Independent Samples T Test.** Name your dependent variable(s) in the **Test Variable** window and your independent variable in the **Grouping Variable** window. You will also need to specify the scores used to define the groups on the independent variable. See the exercises at the end of Chapter 9 for examples. Make a note of the test results (group means, obtained *t* score, significance, sample size) or keep a hard copy. Repeat the procedure for the second dependent variable.
3. Write up the results of the test. At a minimum, your report should clearly identify the independent and dependent variables, the sample statistics, the sample size (*N*), the value of the test statistic (Step 4), the results of the test (Step 5), and the alpha level you used.

B. Analysis of Variance (Chapter 10)

1. Choose two different dependent variables from the interval-ratio or ordinal variables that have three or more scores. Choose independent variables that might logically be a cause of your dependent variables and that have between three and five categories. You may use the same independent variables for both tests.
2. Click **Analyze, Compare Means,** and then **One-way Anova.** The **One-way Anova** window will appear. Find your dependent variable in the variable list on the left and click the arrow to move the variable name into the **Dependent List** box. Note that you can request more than one dependent variable at a time. Next, find the name of your independent variable and move it to the **Factor** box. Click **Options** and then click the box next to **Descriptive** in the **Statistics** box to request means and standard deviations. Click **Continue** and **OK.** Make a note of the test results or keep a hard copy. Repeat, if necessary, for your second dependent variable.

3. Write up the results of the test. At a minimum, your report should clearly identify the independent and dependent variables, the sample statistics (category means), the sample size (*N*), the value of the test statistic (Step 4), the results of the test (Step 5), the degrees of freedom, and the alpha level you used.

C. Chi Square (Chapter 11)

1. Choose two different dependent variables of any level of measurement that have five or fewer (preferably two or three) scores. For each dependent variable, choose an independent variable that might logically be a cause. Independent variables can be any level of measurement as long as they have five or fewer (preferably two or three) categories. Output will be easier to analyze if you use variables with few categories. You may use the same independent variable for both tests.
2. Click **Analyze, Descriptive Statistics,** and then **Crosstabs.** The **Crosstabs** dialog box will appear. Highlight your first dependent variable and move it into the **Rows** box. Next, highlight your independent variable and move it into the **Columns** box. Click the **Statistics** button at the bottom of the window and click the box next to chi square. Click **Continue** and **OK.** Make a note of the results or a get a hard copy. Repeat for your second dependent variable.
3. Write up the results of the test. At a minimum, your report should clearly identify the independent and dependent variables, the value of the test statistic (Step 4), the results of the test (Step 5), the sample size (*N*) and degrees of freedom, and the alpha level you used. It is almost always desirable to also report the column percentages.

PROJECT 4—ANALYZING THE STRENGTH AND SIGNIFICANCE OF RELATIONSHIPS

A. Using Bivariate Tables

1. From the 2006 GSS data set, select either
   a. One dependent variable and three independent variables (possible causes), or
   b. One independent variable and three possible dependent variables (possible effects).

   Variables can be from any level of measurement but must have only a few (two to five) categories or scores. Develop research questions or hypotheses about the relationships between all combinations of independent and dependent variables. Make sure that the causal links you suggest are sensible and logical.
2. Use the **Crosstabs** procedure to generate bivariate tables. See the exercises at the end of Chapters 11 and 12 for examples of how to use **Crosstabs**. Click **Analyze, Descriptive Statistics,** and **Crosstabs** and place your dependent variable(s) in the rows and independent variable(s) in the columns. On the **Crosstabs** dialog box, click the **Statistics** button and choose chi square, phi or *V*, and gamma for every table you request. On the **Crosstabs** dialog box, click the **Cells** button and get column percentages for every table you request. Make a note of the results as they appear on the screen or get hard copies.
3. Write a report that presents and analyzes these relationships. Be clear about which variables are dependent and which are independent. For each

combination of variables, report the test of significance and measure of association. In addition, for each relationship, report and discuss column percentages, pattern or direction of the relationship, and strength of the relationship.

B. Using Interval-Ratio Variables

1. From the 2006 GSS, select either
   a. One dependent variable and three independent variables (possible causes), or
   b. One independent variable and three dependent variables (possible effects).

   Variables should be interval-ratio in level of measurement, but you may use ordinal level variables as long as they have more than three scores. Develop research questions or hypotheses about the relationships between variables. Make sure that the causal links you suggest are sensible and logical.
2. Use the **Regression** procedures to analyze the bivariate relationships. Make a note of results (including r, r^2, slope, beta-weights, and a) as they appear on the screen or get hard copies.
3. Write a report that presents and analyzes these relationships. Be clear about which variables are dependent and which are independent. For each combination of variables, report the significance of the relationship (if relevant) and the strength and direction of the relationship. Include r, r^2, and the beta-weights in your report.
4. *OPTIONAL MULTIVARIATE ANALYSIS:* Pick one of the bivariate relationships you produced in Step 2 and find another logical independent variable. Run **Regression** again with both independent variables and analyze the results. How much improvement is there in the explained variance after the second independent variable is included? Write up the results of this analysis and include them in your summary paper for this project.

APPENDIX F An Introduction to SPSS for Windows

Computers have affected virtually every aspect of human society, and as you would expect, their impact on the conduct of social research has been profound. Researchers routinely use computers to organize data and compute statistics—activities that humans often find dull, tedious, and difficult but that computers accomplish with accuracy and ease. This division of labor allows social scientists to spend more time on analysis and interpretation—activities that humans typically enjoy but that are beyond the power of computers (so far, at least).

These days, the skills needed to use computers successfully are quite accessible, even for people with little or no experience. This appendix will prepare you to use a statistics program called SPSS for Windows (SPSS stands for Statistical Package for the Social Sciences). If you have used a mouse to "point and click" and run a computer program, you are ready to learn how to use this program. Even if you are completely unfamiliar with computers, you will find this program accessible. After you finish this appendix, you will be ready to do the exercises found at the end of most chapters of this text as well as the projects listed in Appendix E.

A word of caution before we begin: This appendix is intended only as an *introduction* to SPSS. It will give you an overview of the program and enough information so that you can complete the assignments in the text. It is unlikely, however, that this appendix will answer all your questions or provide solutions to all the problems you might encounter. So, this is a good place to tell you that SPSS has an extensive and easy-to-use "help" facility that will provide assistance as you request it. You should familiarize yourself with this feature and use it as needed. To get help, simply click on the **Help** command on the toolbar across the top of the screen.

SPSS is a **statistical package** (or **statpak**) or a set of computer programs that work with data and compute statistics as requested by the user (you). Once you have entered the data for a particular group of observations, you can easily and quickly produce an abundance of statistical information without doing any computations or writing any computer programs yourself.

Why bother to learn this technology? The truth is that the laborsaving capacity of computers is sometimes exaggerated, and there are research situations in which they are unnecessary. If you are working with a small number of observations or need only a few, uncomplicated statistics, then statistical packages are probably not going to be helpful. However, as the number of cases increases and as your requirements for statistics become more sophisticated, computers and statpaks will become more and more useful.

An example should make this point clearer. Suppose you have gathered a sample of 150 respondents and the *only* thing you want to know about these people is their average age. To compute an average, as you know, you add the scores and divide by the number of cases. How long do you think it would take you to add 150 two-digit numbers (ages) with a hand calculator? If you entered

the scores at the rate of one per second—60 scores a minute—it would take about 3 or 4 minutes to enter the ages and get the average. Even if you worked slowly and carefully and did the addition a second and third time to check your math, you could probably complete all calculations in less than 15 or 20 minutes. If this were all the information you needed, computers and statpaks would not save you any time.

Such a simple research project is not very realistic, however. Typically, researchers deal with not one but scores or even hundreds of variables, and samples have hundreds or thousands of cases. While you could add 150 numbers in perhaps 3 or 4 minutes, how long would it take to add the scores for 1,500 cases? What are the chances of adding 1,500 numbers without making significant errors of arithmetic? The more complex the research situation, the more valuable and useful statpaks become. SPSS can produce statistical information in a few keystrokes or clicks of the mouse that might take you minutes, hours, or even days to produce with a hand calculator.

Clearly, this is technology worth mastering by any social researcher. With SPSS, you can avoid the drudgery of mere computation, spend more time on analysis and interpretation, and conduct research projects with very large data sets. Mastery of this technology might be very handy indeed in your senior-level courses, in a wide variety of jobs, or in graduate school.

F.1 GETTING STARTED—DATABASES AND COMPUTER FILES

Before statistics can be calculated, SPSS must first have some data to process. A **database** is an organized collection of related information such as the responses to a survey. For purposes of computer analysis, a database is organized into a **file:** a collection of information that is stored under the same name in the memory of the computer, on a disk or flash drive, or in some other medium. Words as well as numbers can be saved in files. If you've ever used a word-processing program to type a letter or term paper, you probably saved your work in a file so that you could update or make corrections at a later time. Data can be stored in files indefinitely. Since it can take months to conduct a thorough data analysis, the ability to save a database is another advantage of using computers.

For the SPSS exercises in this text, we will use a database that contains some of the results of the General Social Survey (GSS) for 2006. This database contains the responses of a sample of adult Americans to questions about a wide variety of social issues. The GSS has been conducted regularly since 1972 and has been the basis for hundreds of research projects by professional social researchers. It is a rich source of information about public opinion in the United States and includes data on everything from attitudes about abortion to opinions on assisted suicide.

The GSS is especially valuable because the respondents are chosen so that the sample as a whole is representative of the entire U.S. population. A representative sample reproduces, in miniature form, the characteristics of the population from which it was taken (see Chapter 7). So, when you analyze the 2006 General Social Survey database, you are in effect analyzing U.S. society as of 2006. The data are real, and the relationships you will analyze reflect some of the most important and sensitive issues in American life.

The complete General Social Survey for 2006 includes hundreds of items of information (age, sex, opinion about such social issues as capital punishment, and so forth) for about 4,000 respondents. Some of you will be

using a student version of SPSS for Windows, which is limited in the number of cases and variables it can process. To accommodate these limits, I have reduced the database to about 50 items of information and fewer than 1,500 respondents.

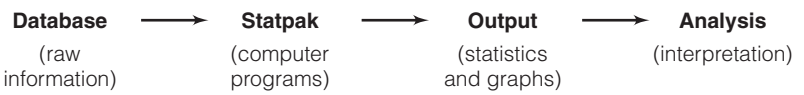
The GSS data file is summarized in Appendix G. Please turn to this appendix and familiarize yourself with it. Note that the variables are listed alphabetically by their variable names. In SPSS, the names of variables must be no more than eight characters long. In many cases, the resultant need for brevity is not a problem, and variable names (e.g., *age*) are easy to figure out. In other cases, the eight-character limit necessitates extreme abbreviation, and some variable names (such as *abany* or *fefam*) are not so obvious. Appendix G also shows the wording of the item that generated the variable. For example, the *abany* variable consists of responses to a question about legal abortion: should it be possible for a woman to have an abortion for "any reason"? Note that the variable name is formed from the question: should an *ab*ortion be possible for *any* reason? The *fefam* variable consists of responses to the statement "It is much better for everyone involved if the man is the achiever outside the home and the woman takes care of the home and family." Appendix G is an example of a code book for the database since it lists all the codes (or scores) for the survey items along with their meanings.

Notice that some of the possible responses to *abany* and *fefam* and other variables in Appendix G are labeled "Not applicable," "NA," or "DK." The first of these responses means that the item in question was not given to a respondent. The full GSS is very long, and to keep the time frame for completing the survey reasonable, not all respondents are asked every question. NA stands for "No Answer" and means that the respondent was asked the question but refused to answer. DK stands for "Don't Know," which means that the respondent did not have the requested information. All three of these scores are *Missing Values,* and as "noninformation," they should be eliminated from statistical analysis. Missing values are common on surveys, and as long as they are not too numerous, they are not a particular problem.

It's important that you understand the difference between a statpak (SPSS) and a database (the GSS) and what we are ultimately after here. A database consists of information. A statpak organizes the information in the database and produces statistics. Our goal is to apply the statpak to the database to produce output (for example, statistics and graphs) that we can analyze and use to answer questions. The process might be diagrammed as in Figure F.1.

Statpaks such as SPSS are general research tools that can be used to analyze databases of all sorts: they are not limited to the 2006 GSS. In the same way, the 2006 GSS could be analyzed with a statpak other than the one used in this text. Other widely used statpaks include Microcase, SAS, and Stata—each of which may be available on your campus.

FIGURE F.1 THE DATA ANALYSIS PROCESS



F.2 STARTING SPSS FOR WINDOWS AND LOADING THE 2006 GSS

If you are using the complete, professional version of SPSS for Windows, you will probably be working in a computer lab, and you can begin running the program immediately. If you are using the student version of the program on your personal computer, the first thing you need to do is install the software. Follow the instructions that came with the program and return to this appendix when installation is complete.

To start SPSS for Windows, find the icon (or picture) on the screen of your monitor that has an "SPSS" label attached to it. Use the computer mouse to move the arrow on the monitor screen over this icon and then double-click the left button on the mouse. This will start up the SPSS program.

After a few seconds, the SPSS for Windows screen will appear and ask, at the top of the screen, "What would you like to do?" Of the choices listed, the button next to "Open an existing file" will be checked (or preselected), and there will probably be a number of data sets listed in the window at the bottom of the screen. Find the 2006 General Social Survey data set, probably labeled *GSS06.sav* or something similar. If the data set is on a disk or flash drive, you will need to specify the correct drive. Check with your instructor to make sure that you know where to find the 2006 GSS.

Once you've located the data set, click on the name of the file with the left-hand button on the mouse, and SPSS will load the data. The next screen you will see is the SPSS Data Editor screen.

Note that there is a list of commands across the very top of the screen. These commands begin with **File** at the far left and end with **Help** at the far right. This is the main menu bar for SPSS. When you click any of these words, a **menu** of commands and choices will drop down. You tell SPSS what to do by clicking on your desired choices from these menus. Sometimes, submenus will appear, and you will need to specify your choices further.

SPSS provides the user with a variety of options for displaying information about the data file and output on the screen. I recommend that you tell the program to display lists of variables by name (e.g., *age, abany*) rather than labels (e.g, AGE OF RESPONDENT, ABORTION IF WOMAN WANTS FOR ANY REASON). Lists displayed this way will be easier to read and compare to Appendix G. To do this, click **Edit** on the main menu bar and then click **Options** from the drop-down submenu. A dialog box labeled "Options" will appear with a series of tabs along the top. The "General" options should be displayed but, if not, click on this tab. On the "General" screen, find the box labeled "Variable Lists" and, if they are not already selected, click "Display names" and "alphabetical" and then click **OK.** If you make changes, a message may appear on the screen that tells you that changes will take effect the next time a data file is opened.

In this section, you learned how to start up SPSS for Windows, load a data file, and set some of the display options for this program. These procedures are summarized in Table F.1.

TABLE F.1 SUMMARY OF COMMANDS

To start SPSS for Windows	Click the SPSS icon on the screen of the computer monitor.
To open a data file	Double-click on the data file name.
To set display options for lists of variables	Click **Edit** from the main menu bar, then click **Options.** On the "General" tab. make sure that "Display names" and "alphabetical" are selected and then click **OK.**

F.3 WORKING WITH DATABASES

Note that in the SPSS **Data Editor** window the data are organized into a two-dimensional grid with columns running up and down (vertically) and rows running across (horizontally). Each column is a variable or item of information from the survey. The names of the variables are listed at the tops of the columns. Remember that you can find the meaning of these variable names in the GSS 2006 code book in Appendix G.

Another way to decipher the meaning of variable names is to click **Utilities** on the menu bar and then click **Variables.** The Variables window opens. This window has two parts. On the left is a list of all variables in the database arranged in alphabetical order, with the first variable highlighted. On the right is the **Variable Information** window with information about the highlighted variable. The first variable is listed as *abany.* The **Variable Information** window displays a fragment of the question that was actually asked during the survey ("ABORTION IF WOMAN WANTS FOR ANY REASON") and shows the possible scores on this variable (a score of 1 = yes and a score of 2 = no), along with some other information.

The same information can be displayed for any variable in the data set. For example, find the variable *marital* in the list. You can do this by using the arrow keys on your keyboard or the slider bar on the right of the variable list window. You can also move through the list by typing the first letter of the variable name you are interested in. For example, type "m" and you will be moved to the first variable name in the list that begins with that letter. Now you can see that the variable measures marital status and that a score of "1" indicates that the respondent was married, and so forth. What do *prestg80* and *marhomo* measure? Close this window by clicking the **Close** button at the bottom of the window.

Examine the window displaying the 2006 GSS a little more. Each row of the window (reading across or from left to right) contains the scores of a particular respondent on all the variables in the database. Note that the upper left-hand cell is highlighted (outlined in a darker border than the other cells). This cell contains the score of respondent 1 on the first variable. The second row contains the scores of respondent 2, and so forth. You can move around in this window with the arrow keys on your keyboard. The highlight moves in the direction of the arrow, one cell at a time.

In this section, you learned to read information in the data display window and to decipher the meaning of variable names and scores. These commands are summarized in Table F.2, and we are now prepared to actually perform some statistical operations with the 2006 GSS database.

TABLE F.2 SUMMARY OF COMMANDS

To move around in the **Data Editor** window:	1. Click the cell you want to highlight, or 2. Use the arrow keys on your keyboard, or 3. Move the slider buttons, or 4. Click the arrows on the right-hand and bottom margins.
To get information about a variable:	1. From the menu bar, click **Utilities** and then click **Variables.** Scroll through the list of variable names until you highlight the name of the variable in which you are interested. Variable information will appear in the window on the right. 2. See Appendix G.

F.4 PUTTING SPSS TO WORK: PRODUCING STATISTICS

At this point, the database on the screen is just a mass of numbers with little meaning for you. That's okay because you will not have to actually read any information from this screen. Virtually all of the statistical operations you will conduct will begin by clicking the **Analyze** command from the menu bar, selecting a procedure and statistics, and then naming the variable or variables you would like to process.

To illustrate, let's have SPSS for Windows produce a frequency distribution for the variable *sex*. Frequency distributions are tables that display the number of times each score of a variable occurred in the sample (see Chapter 2). So, when we complete this procedure, we will know the number of males and females in the 2006 GSS sample.

With the 2006 GSS loaded, begin by clicking the **Analyze** command on the menu bar. From the menu that drops down, click **Descriptive Statistics** and then **Frequencies.** The **Frequencies** window appears with the variables listed in alphabetical order in the box on the left. The first variable (*abany*) will be highlighted. Use the slider button or the arrow keys on the right-hand margin of this box to scroll through the variable list until you highlight the variable *sex,* or type "s" to move to the approximate location.

Once the variable you want to process has been highlighted, click the arrow button in the middle of the screen to move the variable name to the box on the right-hand side of the screen. SPSS will produce frequency distributions for all variables listed in this box, but for now we will confine our attention to sex. Click the **OK** button in the upper right-hand corner of the **Frequencies** window, and in seconds, a frequency distribution will be produced.

SPSS sends all tables and statistics to the **Output** window or SPSS viewer. This window is now "closest" to you, and the **Data Editor** window is "behind" the **Output** window. If you wanted to return to the **Data Editor**, click on any part of it if it is visible and it will move to the "front" and the **Output** window will be "behind" it. To display the **Data Editor** window if it is not visible, minimize the **Output** window by clicking the "–" box in the upper right-hand corner.

Frequencies. The output from SPSS, slightly modified, is reproduced as Table F.3. What can we tell from this table? The score labels (male and female) are printed at the left with the number of cases (frequency) in each category of the variable one column to the right. As you can see, there are 644 males and 782 females in the sample. The next two columns give information about percentages, and the last column to the right displays cumulative percentages. We will defer a discussion of this last column until a later exercise.

TABLE F.3 AN EXAMPLE OF SPSS OUTPUT

		RESPONDENT'S SEX			
		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	MALE	644	45.2	45.2	45.2
	FEMALE	782	54.8	54.8	100.0
	Total	1,426	100.0	100.0	

One of the percentage columns is labeled "Percent" and the other is labeled "Valid Percent." The difference between these two columns lies in the handling of missing values. The "Percent" column is based on all cases, including people who did not respond to the item (NA) and people who said they did not have the requested information (DK). The "Valid Percent" column excludes all missing scores. Since we will almost always want to ignore missing scores, we will pay attention only to the "Valid Percent" column. Note that for sex, there are no missing scores (gender was determined by the interviewer), and the two columns are identical.

F.5 PRINTING AND SAVING OUTPUT

Once you've gone to the trouble of producing statistics, a table, or a graph, you will probably want to keep a permanent record. There are two ways to do this. First, you can print a copy of the contents of the **Output** window to take with you. To do this, click on **File** and then click **Print** from the **File** menu. Alternatively, find the icon of a printer (third from the left) in the row of icons just below the menu bar and click on it.

The other way to create a permanent record of SPSS output is to save the **Output** window to the computer's memory or to a disk. To do this, click **Save** from the **File** menu. The **Save** dialog box opens. Give the output a name (some abbreviation such as "freqsex" might do) and, if necessary, specify the name of the drive in which your disk is located. Click **OK,** and the table will be permanently saved.

F.6 ENDING YOUR SPSS FOR WINDOWS SESSION

Once you have saved or printed your work, you may end your SPSS session. Click on **File** from the menu bar and then click **Exit.** If you haven't already done so, you will be asked if you want to save the contents of the **Output** window. You may save the frequency distribution at this point if you wish. Otherwise, click **NO.** The program will close, and you will be returned to the screen from which you began.

APPENDIX G Code Book for the General Social Survey, 2006

The General Social Survey (GSS) is a public opinion poll that has been conducted regularly by the National Opinion Research Council. A version of the 2006 GSS is available at the web site for this text and is used for all end-of-chapter exercises. Our version of the 2006 GSS includes about 50 variables for a randomly selected subsample of about 1,500 of the original respondents. This code book lists each item in the data set. The variable names are those used in the data files. The questions have been reproduced exactly as they were asked (with a few exceptions to conserve space), and the numbers beside each response are the scores recorded in the data file.

The data set includes variables that measure demographic or background characteristics of the respondents, including sex, age, race, religion, and several indicators of socioeconomic status. Also included are items that measure opinion on such current and controversial topics as abortion, capital punishment, and homosexuality.

Most variables in the data set have codes for "missing data." These codes are italicized in the listings below for easy identification. The codes refer to various situations in which the respondent does not or cannot answer the question and are excluded from all statistical operations. The codes are: NAP, or "not applicable" (the respondent was not asked the question), DK, or "Don't Know" (the respondent didn't have the requested information), and NA, or "No Answer" (the respondent refused to answer).

Please tell me if you think it should be possible for a woman to get a legal abortion if . . .

abany She wants it for any reason.

1. Yes
2. No

0. NAP, 8. DK, 9. NA

abrape The pregnancy is the result of rape
(Same scoring as abany)

affrmact Some people say that because of past discrimination, blacks should be given preference in hiring and promotion. Others say that such preference is wrong because it discriminates against whites. Are you for or against preferential hiring and promotion of blacks?

1. Strongly supports preferences
2. Supports preferences
3. Opposes preferences
4. Strongly opposes preferences

0. NAP, 8. DK, 9. NA

age Age of respondent
18–89. Actual age in years
99. NA

attend How often do you attend religious services?
0. Never
1. Less than once per year
2. Once or twice a year
3. Several times per year
4. About once a month
5. 2–3 times a month
6. Nearly every week
7. Every week
8. Several times a week
9. DK or NA

cappun Do you favor or oppose the death penalty for persons convicted of murder?
1. Favor
2. Oppose
0. NAP, 8. DK, 9. NA

childs How many children have you had? Please count all that were born alive at any time (including any from a previous marriage).
0–7. Actual number
8. Eight or more
9. NA

chldidel What do you think is the ideal number of children for a family to have?
0–6 Actual values
–1. NAP, 8. As Many as Want, 9. DK, NA

class Subjective class identification
1. Lower class
2. Working class
3. Middle class
4. Upper class
0. NAP, 8. DK, 9. NA

degree Respondent's highest degree
0. Less than HS
1. High school
2. Assoc./Junior college
3. Bachelor's
4. Graduate
7. NAP, 8. DK 9. NA

educ Highest year of school completed
0–20. Actual number of years
97. NAP, 98. DK, 99. NA

fear Is there any area right around here—that is, within a mile—where you would be afraid to walk alone at night?
1. Yes
2. No
0. NAP, 8. DK, 9. NA

fefam It is much better for everyone involved if the man is the achiever outside the home and the woman takes care of the home and family.
1. Strongly agree
2. Agree
3. Disagree
4. Strongly disagree
0. NAP, 8. DK, 9. NA

fepresch — A preschool child is likely to suffer if his or her mother works.

1. Strongly agree
2. Agree
3. Disagree
4. Strongly Disagree

0. NAP, 8. DK, 9. NA

grass — Do you think the use of marijuana should be made legal or not?

1. Should
2. Should not

0. NAP, 8. DK, 9. NA

gunlaw — Would you favor or oppose a law which would require a person to obtain a police permit before he or she could buy a gun?

1. Favor
2. Oppose

0. NAP, 8. DK, 9. NA

happy — Taken all together, how would you say things are these days—would you say that you are very happy, pretty happy, or not too happy?

1. Very happy
2. Pretty happy
3. Not too happy

0. NAP, 8. DK, 9. NA

helppoor — Some people think that the [federal] government should do everything possible to improve the standard of living of all poor Americans. Other people think it is not the government's responsibility, and that each person should take care of him- or herself. Where would you put yourself on this scale?

1. Government action
2.
3. Agree with both
4.
5. People should help selves

0. NAP, 8. DK, 9. NA

hrs1 — How many hours did you work last week?

1–89. Actual hours

–1. NAP, 98. DK, 99. NA

income06 — Respondent's total family income from all sources

1. Less than 1,000
2. 1,000 to 2,999
3. 3,000 to 3,999
4. 4,000 to 4,999
5. 5,000 to 5,999
6. 6,000 to 6,999
7. 7,000 to 7,999
8. 8,000 to 9,999
9. 10,000 to 12,499
10. 12,500 to 14,999
11. 15,000 to 17,499
12. 17,500 to 19,999
13. 20,000 to 22,499
14. 22,500 to 24,999
15. 25,000 to 29,999
16. 30,000 to 34,999
17. 35,000 to 39,999
18. 40,000 to 49,999
19. 50,000 to 59,999
20. 60,000 to 74,999
21. 75,000 to 89,999
22. 90,000 to 109,999
23. 110,000 to 129,999
24. 130,000 to 149,999
25. 150,000 or more

98. DK, 99. NA

letdie1 — When a person has a disease that cannot be cured, do you think doctors should be allowed by law to end the patient's life by some painless means if the patient and his family request it?

1. Yes
2. No

0. NAP, 8. DK, 9. NA

letin1 — Do you think the number of immigrants to America nowadays should be

1. Increased a lot
2. Increased a little
3. Remain the same as it is
4. Reduced a little
5. Reduced a lot

0. NAP, 8. DK, 9. NA

marblk — What about a close relative marrying a black person? Would you be

1. Strongly in favor
2. In favor
3. Neither favor nor oppose
4. Oppose
5. Strongly oppose

0. NAP, 8. DK, 9. NA

marhomo — Homosexual couples should have the right to marry one another

1. Strongly agree
2. Agree
3. Neither agree or disagree
4. Disagree
5. Strongly disagree

0. NAP, 8. DK, 9. NA

marital — Are you currently married, widowed, divorced, separated, or have you never been married?

1. Married
2. Widowed
3. Divorced
4. Separated
5. Never married

9. NA

news — How often do you read the newspaper?

1. Every day
2. A few times a week
3. Once a week
4. Less than once a week
5. Never

0. NAP, 8. DK, 9. NA

obey — How important is it for a child to learn obedience to prepare him or her for life?

1. Most important
2. 2nd most important
3. 3rd most important
4. 4th most important
5. Least important

0. NAP, 8. DK, 9. NA

paeduc — Father's highest year of school completed

0–20. Actual number of years

97. NAP, 98. DK, 99. NA

papres80 — Prestige of father's occupation

17–86. Actual score

0. NAP, DK, NA

partnrs5 How many sex partners have you had over the past five years?

0. No partners
1. 1 partner
2. 2 partners
3. 3 partners
4. 4 partners
5. 5–10 partners
6. 11–20 partners
7. 21–100 partners
8. More than 100 partners
9. *1 or more, don't know the number, 95. Several, 98. DK, 99. NA, –1 NAP*

poleff3 The average citizen has considerable influence on politics

1. Strongly agree
2. Agree
3. Neither agree nor disagree
4. Disagree
5. Strongly disagree
0. *NAP, 8. DK, 9. NA*

polviews I'm going to show you a seven-point scale on which the political views that people might hold are arranged from extremely liberal to extremely conservative. Where would you place yourself on this scale?

1. Extremely liberal
2. Liberal
3. Slightly liberal
4. Moderate
5. Slightly conservative
6. Conservative
7. Extremely conservative
0. *NAP, 8. DK, 9. NA*

premarsx There's been a lot of discussion about the way morals and attitudes about sex are changing in this country. If a man and a woman have sex relations before marriage, do you think it is always wrong, almost always wrong, wrong only sometimes, or not wrong at all?

1. Always wrong
2. Almost always wrong
3. Wrong only sometimes
4. Not wrong at all
0. *NAP, 8. DK, 9. NA*

pres04 In 2004, did you vote for Kerry (the Democratic candidate) or Bush for the Republicans? (Includes only those who said they voted in this election)

1. Kerry
2. Bush
3. *Nader, 4. Other, 6. No presidential vote, 0. NAP, 8. DK, 9. NA*

prestg80 Prestige of respondent's occupation

17–86. Actual score
0. *NAP, DK, NA*

racecen1 Race of respondent

1. White
2. Black
3. American Indian or Alaska Native
4. Asian American and Pacific Islanders
5. Hispanic
0. *NAP, 98. DK*

region — Region of interview
1. New England (ME, VT, NH, MA, CT, RI)
2. Mid-Atlantic (NY, NJ, PA)
3. East North Central (WI, IL, IN, MI, OH)
4. West North Central (MN, IA, MO, ND, SD, NE, KS)
5. South Atlantic (DE, MY, WV, VA, NC, SC, GA, FL, DC)
6. East South Central (KY, TN, AL, MS)
7. West South Central (AK, OK, LA, TX)
8. Mountain (MT, ID, WY, NV, UT, CO, AR, NM)
9. Pacific (WA, OR, CA, AL, HI)

relexper — Has there been a turning point in your life when you made a new and personal commitment to religion?
1. Yes
2. No
0. NAP, 8. DK, 9. NA

relig — What is your religious preference? Is it Protestant, Catholic, Jewish, some other religion, or no religion?
1. Protestant
2. Catholic
3. Jewish
4. None
5. Other
8. DK, 9. NA

satjob — All in all, how satisfied would you say you are with your job?
1. Very satisfied
2. Moderately satisfied
3. A little dissatisfied
4. Very dissatisfied
0. NAP, 8. DK, 9. NA

scresrch — Recently, there has been controversy over whether the government should provide any funds for scientific research that uses "stem cells" taken from human embryos. Would you say the government
1. Definitely should fund such research
2. Probably should fund such research
3. Probably should not fund such research
4. Definitely should not fund such research
0. NAP, 8. DK, 9. NA

sex — Respondent's gender
1. Male
2. Female

sexfreq — About how many times did you have sex during the last 12 months?
0. Not at all
1. Once or twice
2. About once a month
3. 2 or 3 times a month
4. About once a week
5. 2 or 3 times a week
6. More than 3 times a week
–1. NAP, 8. DK, 9. NA

size — Size of place in thousands. Population figures from U.S. Census. Add three zeros to code for actual values.

spanking — Do you strongly agree, agree, disagree, or strongly disagree that it is sometimes necessary to discipline a child with a good, hard spanking?

1. Strongly agree
2. Agree
3. Disagree
4. Strongly disagree

0. NAP, 8. DK, 9. NA

tapphone — Suppose the government suspected that a terrorist act was about to happen. Do you think the authorities should have the right to tap people's telephone conversations?

1. Definitely should have the right
2. Probably should have the right
3. Probably should not have the right
4. Definitely should not have the right

0. NAP, 8. DK, 9. NA

tvhours — On the average day, about how many hours do you personally watch television?

00–22. Actual hours

–1. NAP, DK, NA

wwwhr — Not counting email, about how many hours per week do you use the web (or Internet)?

0–75 actual hours

–1. NAP, 998. DK, 999. NA

APPENDIX H Glossary of Symbols

The number in parentheses indicates the chapter in which the symbol is introduced.

a	Point at which the regression line crosses the *Y* axis (14)
ANOVA	The analysis of variance (10)
b	Slope of the regression line (14)
b_i	Partial slope of the linear relationship between the *i*th independent variable and the dependent variable (15)
b_i^*	Standardized partial slope of the linear relationship between the *i*th independent variable and the dependent variable (15)
df	Degrees of freedom (9)
f	Frequency (2)
F	The *F* ratio (10)
f_e	Expected frequency (11)
f_o	Observed frequency (11)
G	Gamma for a sample (13)
H_0	Null hypothesis (8)
H_1	Research or alternate hypothesis (8)
Md	Median (4)
Mo	Mode (4)
N	Number of cases (2)
N_d	Number of pairs of cases ranked in different order on two variables (13)
N_s	Number of pairs of cases ranked in the same order on two variables (13)
%	Percentage (2)
P	Proportion (2)
P_s	A sample proportion (7)
P_u	A population proportion (7)
PRE	Proportional reduction in error (12)
Q	Interquartile range (5)
r	Pearson's correlation coefficient for a sample (14)
r^2	Coefficient of determination (14)
R	Multiple correlation coefficient (15)
R	Range (5)
r_s	Spearman's rho for a sample (13)
$r_{xy.z}$	Partial correlation coefficient (15)
R^2	Coefficient of multiple determination (15)
s	Sample standard deviation (5)
SSB	The sum of squares between (10)
SST	The total sum of squares (10)
SSW	The sum of squares within (10)
s^2	Sample variance (5)
t	Student's *t* score (8)
V	Cramer's *V* (12)
X	Any independent variable (12)
X_i	Any score in a distribution (4)
$\overline{X}$	The mean of a sample (4)
Y	Any dependent variable (12)
Y'	A predicted score on *Y* (14)
Z	A control variable (15)
Z scores	Standard scores (6)

GREEK LETTERS

α	Probability of Type I error (8)
β	Probability of Type II error (8)
λ	Lambda (12)
μ	Mean of a population (4)
μ_p	Mean of a sampling distribution of sample proportions (7)
$\mu_{\bar{X}}$	Mean of a sampling distribution of sample means (7)
σ	Population standard deviation (5)
σ_p	Standard deviation of a sampling distribution of sample proportions (7)
σ_{p-p}	Standard deviation of the sampling distribution of difference in sample proportions (9)
$\sigma_{\bar{X}}$	Standard deviation of a sampling distribution of sample means (7)
$\sigma_{\bar{X}-\bar{X}}$	Standard deviation of the sampling distribution of the difference in sample means (9)
σ^2	Population variance (5)
Σ	"Summation of" (4)
ϕ	Phi (12)
χ^2	Chi square statistic (11)
χ_c^2	Chi square corrected by Yates's correction (11)

Answers to Odd-Numbered Computational Problems

In addition to answers, this section suggests some problem-solving strategies and provides some examples of how to interpret some statistics. You should try to solve and interpret the problems on your own before consulting this section.

In solving these problems, I let my calculator or computer do most of the work. I worked with whatever level of precision these devices permitted and didn't round off until the end or until I had to record an intermediate sum. I always rounded off to two places of accuracy (or, two places beyond the decimal point, or to 100ths). If you follow these same conventions, your answers will almost always match mine. However, there is no guarantee that our answers will always be exact matches. You should realize that small discrepancies might occur and that these differences almost always will be trivial. If the difference between your answer and mine doesn't seem trivial, you should double-check to make sure you haven't made an error or solve the problem again using a greater degree of precision.

Finally, please allow me a brief disclaimer about mathematical errors in this section. Let me assure you, first of all, that I know how important this section is for most students and that I worked hard to be certain that these answers are correct. Human fallibility being what it is, however, I know that I cannot make absolute guarantees. Should you find any errors, please let me know so I can make corrections in the future.

Chapter 1

1.5

a. Nominal

b. Ordinal (the categories can be ranked in terms of degree of honesty with "Returned the wallet with money" the "most honest")

c. Ordinal

d. Interval-ratio ("years" has equal intervals and a true zero point)

e. Interval-ratio

f. Interval-ratio

g. Nominal (the various patterns are different from each other but cannot be ranked from high to low)

h. Interval-ratio

i. Ordinal

j. Number of accidents: Interval ratio
Severity of accident: Ordinal

1.7

Variable	Level of Measurement	Application
a. Opinion	Ordinal	Inferential
b. Grade	Interval-ratio	Descriptive (two variables)
c. Party	Nominal	Inferential
Sex	Nominal	
Opinion	Ordinal	
d. Homicide Rate	Interval-ratio	Descriptive (two variables)
e. Satisfaction	Ordinal	Descriptive (one variable)

Chapter 2

2.1

a. Complex A: $\left(\frac{5}{20}\right) \times 100 = 25.00\%$

Complex B: $\left(\frac{10}{20}\right) \times 100 = 50.00\%$

b. Complex A: $4{:}5 = 0.80$
Complex B: $6{:}10 = 0.60$

c. Complex A: $\left(\frac{0}{20}\right) = 0.00$

Complex B: $\left(\frac{1}{20}\right) = 0.05$

d. $\left(\frac{6}{4+6}\right) = \left(\frac{6}{10}\right) = 60.00\%$

e. Complex A: 8:5 = 1.60
Complex B: 2:10 = 0.20

2.3 Bank robbery rate $= \left(\frac{47}{211{,}732}\right) \times 100{,}000 = 22.20$

Homicide rate $= \left(\frac{13}{211{,}732}\right) \times 100{,}000 = 6.14$

Auto theft rate $= \left(\frac{23}{211{,}732}\right) \times 100{,}000 = 10.86$

2.5 For sex:

Sex	Frequency
Male	9
Female	6
Total	15

For age, we will follow the procedure established in Section 2.5 (see the "One Step at a Time" box). Set $k = 10$. $R = 77 - 23$ or 54 so we can round off interval size to 5 ($i = 5$). The first interval will be 20–24 to include the low score of 23 and the highest interval will be 75–79.

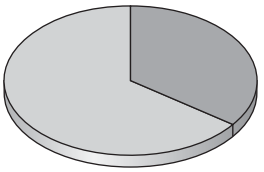
Age	Frequency
20–24	1
25–29	2
30–34	3
35–39	2
40–44	1
45–49	3
50–54	1
55–59	1
60–64	0
65–69	0
70–74	0
75–79	1
	15

2.9 Set $k = 10$. $R = 92 - 5$ or 87, so set i at 10.

Score	Frequency
0–9	3
10–19	7
20–29	6
30–39	0
40–49	2
50–59	2
60–69	3
70–79	0
80–89	0
90–99	2
	25

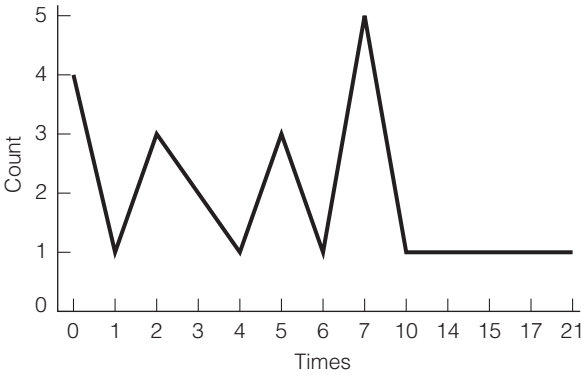
Chapter 3

3.1 **a. For Sex:**

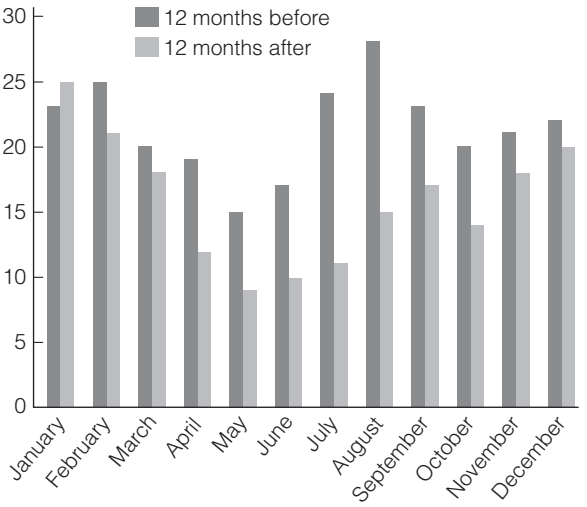


b. The graphs will have two peaks over 48 and 33 years of age.

3.3 Number of times left home:



3.7



3.9 **a.** Niger has the highest birth rate.
b. Ukraine and Sweden have the lowest birth rate.
c. Niger has the highest death rate.
d. Sweden has the lowest death rate.

Chapter 4

4.1 Region of Birth and Religion are nominal level variables, Legalization and Cafeteria Food are ordinal, and Out-of-Pocket Expenses and Movies are interval-ratio. The mode, the most common score, is the only measure of central tendency available for nominal level variables. For the two ordinal level variables, *don't forget to array the scores from high to low* before locating the median. There are 10 freshmen (N is even), so the median for freshmen will be the score halfway between the scores of the two middle cases. There are 11 seniors (N is odd), so the median for seniors will be the score of the middle case. To find the mean for the interval-ratio variables, add the scores and divide by the number of cases.

Variable	Freshmen	Seniors
Region of Birth:	Mode = North	Mode = North
Legalization:	Median = 3	Median = 5
Expenses:	Mean = 48.50	Mean = 63.00
Movies:	Mean = 5.80	Mean = 5.18
Food:	Median = 6	Median = 4
Religion:	Mode = Protestant	Mode = Protestant and None (4 cases each)

4.3

Variable	Level of Measurement	Measure of Central Tendency
Sex	Nominal	Mode = Male
Social Class	Ordinal	Median = Medium (the middle case is in this category)

continued on next column

Variable	Level of Measurement	Measure of Central Tendency
Number of Years in the Party	I-R	Mean = 26.15
Education	Ordinal	Median = High school
Marital Status	Nominal	Mode = Married
Number of Children	I-R	Mean = 2.39

4.5

Variable	Level of Measurement	Measure of Central Tendency
Marital Status	Nominal	Mode = Married
Race	Nominal	Mode = White
Age	I-R	Mean = 27.53
Attitude on Abortion	Ordinal	Median = 7

4.7

	2000	2006
Mean	$48,161.54	$63,223.08
Median	$47,300.00	$59,000.00

4.9 1997: Mean = 54.25
2007: Mean = 60.30

4.11 Pretest: Mean = 9.33, Md = 10
Posttest: Mean = 12.93, Md = 12

4.13 a. Mean = 24.41
Median = 25.5

Chapter 5

5.1 The high score is 50 and the low score is 10, so the range is 50 − 10 or 40. The standard deviation is 12.28.

5.3

	Out-of-Pocket Expenses		Number of Movies		Rating of Cafeteria Food	
	Freshmen	Seniors	Freshmen	Seniors	Freshmen	Seniors
Mean	48.5	63.0	5.8	5.19	5.5	4.55
R	32	50	14	14	10	8
s	9.21	15.19	5.06	4.04	3.35	2.50

5.5

	2000	2006
Mean	46,161.53	63,223.08
Median	47,300	59,000
Range	23,400	38,300
Stnd. Dev	6,753.64	10,845.00

In this time period, the mean and median increase. The distribution shows relatively little skew in 2000, but there is a positive skew in 2006. The latter is caused by the Northwest Territories, which has a much higher average income than the other provinces. The standard deviation and range also increase, a reflection of greater skew in 2006.

5.7

Variable	Statistic	Males	Females
Labor Force Participation	Mean	77.60	58.40
	Standard deviation	2.73	6.73
% High School Graduate	Mean	69.20	70.20
	Standard deviation	5.38	4.98
Mean Income	Mean	33,896.60	29,462.40
	Standard deviation	4,443.16	4,597.93

Males and females are very similar in terms of educational level, but females are less involved in the labor force and, on the average, earn almost $4,500 less than males per year. The females in these ten states are much more variable in their labor force participation but are similar to males in dispersion on the other two variables.

5.9

	1973	1975
Mean	12.21	19.06
Stnd Dev.	12.20	9.63

The average rate increases, but the amount of dispersion decreases. If you inspect the scores state by state, you will see that the rate goes up for every state but New York. This accounts for the rising average rate. Also note that in 1973, New York was very different from the other states. The strong positive skew in 1973 reflects the fact that New York had more liberal abortion laws than the other 19 states. Because of the *Roe v. Wade* Supreme Court decision, abortion laws in the other states became more liberal—the other states became more like NY—and this declining diversity is what accounts for the lower standard deviation in 1975.

Chapter 6

6.1

X_i	*Z* Score	% Area Above	% Area Below
5	−1.67	95.25	4.75
6	−1.33	90.82	9.18
7	−1.00	84.13	15.87
8	−0.67	74.86	25.14
9	−0.33	62.93	37.07
11	0.33	37.07	62.93
12	0.67	25.14	74.86
14	1.33	9.18	90.82
15	1.67	4.75	95.25
16	2.00	2.28	97.72
18	2.67	0.38	99.62

6.3

	Z Scores	Area
a.	0.10 & 1.10	32.45%
b.	0.60 & 1.10	13.86%
c.	0.60	27.43%
d.	0.90	18.41%
e.	0.60 & −0.40	38.11%
f.	0.10 & −0.40	19.52%
g.	0.10	53.98%
h.	0.30	61.79%
i.	0.60	72.57%
j.	1.10	86.43%

6.5

X_i	*Z* Score	Number of Students Above	Number of Students Below
60	−2.00	195	5
57	−2.50	199	1
55	−2.83	199	1
67	−0.83	159	41
70	−0.33	126	74
72	0.00	100	100
78	1.00	32	168
82	1.67	10	190
90	3.00	1	199
95	3.83	1	199

Note that the number of students has been rounded off to the nearest whole number.

6.7

	Z Score	Area
a.	−2.20	1.39%
b.	1.80	96.41%
c.	−0.20 & 1.80	54.34%
d.	0.80 & 2.80	20.93%
e.	−1.20	88.49%
f.	0.80	21.19%

6.9

	Z Score	Area
a.	−1.00 & 1.50	0.7745
b.	0.25 & 1.50	0.3345
c.	1.50	0.0668
d.	0.25 & −2.25	0.5865
e.	−1.00 & −2.25	0.1465
f.	−1.00	0.1587

6.11 Yes. The raw score of 110 translates into a *Z* score of +2.88; 99.80% of the area lies below this score, so this individual was in the top 1% on this test.

6.13 For the first event, the probability is 0.0919 and for the second, the probability is 0.0655. The first event is more likely.

Chapter 7

7.1 **a.** 5.2 ± 0.11 **b.** 100 ± 0.71
c. 20 ± 0.40 **d.** 1,020 ± 5.41
e. 7.3 ± 0.23 **f.** 33 ± 0.80

7.3

Confidence Level	Alpha	Area Beyond *Z*	*Z* Score
95%	0.05	0.0250	±1.96
94%	0.06	0.0300	±1.88
92%	0.08	0.0400	±1.75
97%	0.03	0.0150	±2.17
98%	0.02	0.0100	±2.33
99.9%	0.001	0.0005	±3.29

7.5 **a.** 2.30 ± 0.04
b. 2.10 ± 0.01, 0.78 ± .07
c. 6.00 ± 0.37

7.7 **a.** 378.23 ± 1.97 The estimate is that students spent between $376.26 and $380.20 on books.
b. 1.5 ± 0.04. The estimate is that students visited the clinic between 1.46 and 1.54 times on the average.
c. 2.8 ± 0.13
d. 3.5 ± 0.19

7.9 0.14 ± 0.07 The estimate is that between 7% and 21% of the population consists of unmarried couples living together.

7.11 **a.** P_s = (823/1,496) = 0.55; Confidence interval: 0.55 ± 0.02. Between 52% and 58% of the population agree with the statement.
b. P_s = (650/1,496) = 0.44; Confidence interval: 0.44 ± 0.02
c. P_s = (375/1,496) = 0.25; Confidence interval: 0.25 ± 0.02
d. P_s = (1,023/1,496) = 0.68; Confidence interval: 0.68 ± 0.02
e. P_s = (800/1.496) = 0.54; Confidence interval: 0.54 ± 0.02

7.13

Alpha (α)	Confidence Level	Confidence Interval
0.10	90%	100 ± 0.74
0.05	95%	100 ± 0.88
0.01	99%	100 ± 1.16
0.001	99.9%	100 ± 1.47

7.15 The confidence interval is 0.51 ± 0.05. The estimate would be that between 46% and 56% of the population prefer candidate A. The population parameter (P_u) is equally likely to be anywhere in the interval (that is, it's just as likely to be 46% as it is to be 56%), so a winner cannot be predicted.

7.17 The confidence interval is 0.23 ± 0.08. At the 95% confidence level, the estimate would be that between 240 (15%) and 496 (31%) of the 1,600 freshmen would be extremely interested. The estimated numbers are found by multiplying *N* (1,600) by the upper (0.31) and lower (0.15) limits of the interval.

Chapter 8

8.3 **a.** *Z*(obtained) = −41.00
b. *Z*(obtained) = 29.09

8.5 *Z*(obtained) = 6.04

8.7 **a.** *Z*(obtained) = −13.66
b. *Z*(obtained) = 25.50

8.9 *t*(obtained) = 4.50

8.11 *Z*(obtained) = 3.06

8.13 *Z*(obtained) = −1.48

8.15 **a.** *Z*(obtained) = −0.74
b. *Z*(obtained) = 2.19
c. *Z*(obtained) = −8.55
d. *Z*(obtained) = −18.07
e. *Z*(obtained) = 2.09
f. *Z*(obtained) = −53.33

8.17 *t*(obtained) = −1.14

Chapter 9

9.1 **a.** σ = 1.39, *Z*(obtained) = −2.52
b. σ = 1.61, *Z*(obtained) = 2.49

9.3 **a.** σ = 10.57, *Z*(obtained) = 1.70
b. σ = 11.28, *Z*(obtained) = −2.48

9.5 **a.** σ = 0.08, *Z*(obtained) = −11.25
b. σ = 0.12, *Z*(obtained) = −3.33
c. σ = 0.15, *Z*(obtained) = 20.00

9.7 These are small samples (combined *N*s of less than 100), so be sure to use Formulas 9.5 and 9.6 in step 4.
a. σ = 0.12, *t*(obtained) = −1.33
b. σ = 0.13, *t*(obtained) = 14.85

9.9 **a.** (France) σ = 0.0095, *Z*(obtained) = −31.58
b. (Nigeria) σ = 0.0075, *Z*(obtained) = −146.67
c. (China) σ = 0.0065, *Z*(obtained) = 76.92

d. (Mexico) $\sigma = 0.0107$, Z(obtained) $= -74.77$
e. (Japan) $\sigma = 0.0115$, Z(obtained) $= -43.48$

The large values for the Z scores indicate that the differences are significant at very low alpha levels (i.e., they are extremely unlikely to have been caused by random chance alone). Note that women are significantly happier than men in every nation except China, where men are significantly happier.

9.11 $P_u = 0.45$, $\sigma_p = 0.06$, Z(obtained) $= 0.67$

9.13 **a.** $P_u = 0.46$, $\sigma_p = 0.06$, Z(obtained) $= 2.17$
b. $P_u = 0.80$, $\sigma_p = 0.07$, Z(obtained) $= 1.43$
c. $P_u = 0.72$, $\sigma_p = 0.08$, Z(obtained) $= 0.75$

9.15 **a.** Z(obtained) $= 1.50$
b. Z(obtained) $= -1.00$
c. Z(obtained) $= 4.00$
d. $\sigma = 0.43$, Z(obtained) $= 1.86$
e. $\sigma = 0.14$, Z(obtained) $= -5.71$
f. $\sigma = 0.08$, Z(obtained) $= -5.50$

Chapter 10

10.1

Problem	Grand Mean	*SST*	*SSB*	*SSW*	*F* ratio
a.	12.17	231.67	173.17	58.5	13.32
b.	6.87	455.73	78.53	377.20	1.25
c.	31.65	8,362.55	5,053.35	3,309.2	8.14

10.3

Problem	Grand Mean	*SST*	*SSB*	*SSW*	*F* ratio
a.	4.39	86.28	45.78	40.50	8.48
b.	16.44	332.44	65.44	267.00	1.84

For Problem 10.3a, with alpha = 0.05 and $df = 2, 15$, the critical F ratio would be 3.68. We would reject the null hypothesis and conclude that decision making *does* vary significantly by type of relationship. By inspection of the group means, it seems that the "cohabitational" category accounts for most of the differences.

10.5

Grand Mean	*SST*	*SSB*	*SSW*	*F* ratio
9.28	213.61	2.11	211.50	0.08

10.7

Grand Mean	*SST*	*SSB*	*SSW*	*F* ratio
5.40	429.48	124.06	305.42	5.96

10.9

Nation	Grand Mean	*SST*	*SSB*	*SSW*	*F* ratio
Mexico	3.78	300.98	154.08	146.90	12.59
Canada	6.88	156.38	20.08	136.30	1.77
U.S.	5.13	286.38	135.28	151.10	10.74

At alpha = 0.05 and $df = 3, 36$, the critical F ratio is 2.92. There is a significant difference in support for suicide by class in Mexico and the United States, but not in Canada. The category means for Mexico suggest that the upper class accounts for most of the differences. For the United States, there is more variation across the category means, and the working class seems to account for most of the differences. Going beyond the ANOVA test and comparing the grand means, support is highest in Canada and lowest in Mexico.

Chapter 11

11.1 **a.** 1.11
b. 0.00
c. 1.52
d. 1.46

11.3 A computing table is highly recommended as a way of organizing the computations for chi square:

Computational Table for Problem 11.3

(1) f_o	(2) f_e	(3) $f_o - f_e$	(4) $(f_o - f_e)^2$	(5) $(f_o - f_e)^2 / f_e$
6	5	1	1	0.20
7	8	−1	1	0.13
4	5	−1	1	0.20
9	8	1	1	0.13
$N = 26$	$N = 26$	0		χ^2(obtained) = 0.65

There is 1 degree of freedom in a 2 × 2 table. With alpha set at 0.05, the critical value for the chi square would be 3.841. The obtained chi square is 0.65, so we fail to reject the null hypothesis of independence between the variables. There is no statistically significant relationship between race and services received.

11.5 a.

Computational Table for Problem 11.5

(1) f_o	(2) f_e	(3) $f_o - f_e$	(4) $(f_o - f_e)^2$	(5) $(f_o - f_e)^2/f_e$
21	17.5	3.5	12.25	0.70
29	32.5	−3.5	12.25	0.38
14	17.5	−3.5	12.25	0.70
36	32.5	3.5	12.25	0.38
$N = 100$	$N = 100.0$	0		χ^2(obtained) = 2.15

With 1 degree of freedom and alpha set at 0.05, the critical region will begin at 3.841. The obtained chi square of 2.15 does not fall within this area, so the null hypothesis cannot be rejected. There is no statistically significant relationship between unionization and salary.

b. Column percentages:

	Status	
Salary	Union	Non-Union
High	60.00%	44.60%
Low	40.00%	55.40%
Totals	100.00%	100.00%

Although the relationship is not significant, unionized fire departments tend to have higher salary levels.

11.7 The obtained chi square is 5.12, which is significant ($df = 1$, alpha = 0.05). The column percentages show that more affluent communities have higher quality schools.

11.9 The obtained chi square is 6.67, which is significant ($df = 2$, alpha = 0.05). The column percentages show that shorter marriages have higher satisfaction.

11.11 The obtained chi square is 12.63, which is significant ($df = 4$, alpha = 0.05). The column percentages show that proportionally more of the students living "off campus with roommates" are in the high-GPA category.

11.13 The obtained chi square is 19.33, which is significant ($df = 3$, alpha = 0.05). Legalization was not favored by a majority of any region, but the column percentages show that the West was most in favor.

11.15

Problem	Chi Square	Significant at $\alpha = 0.05$?	Column Percentages
a.	25.19	Yes	The oldest age group was most opposed
b.	1.80	No	The great majority (72%–75%) of all three age groups were in favor of capital punishment.
c.	5.23	Yes	The oldest age group was most likely to say yes. Although significant, the differences in column percentages are small.
d.	28.43	Yes	The youngest age group was most likely to support legalization.
e.	14.17	Yes	The oldest age group was least likely to support suicide.

Chapter 12

12.1

	Authoritarianism	
Efficiency	Low	High
Low	37.04%	70.59%
High	62.96%	29.41%
Totals	100.00%	100.00%

The conditional distributions change, so there is a relationship between the variables. The change from column to column is quite large, and the maximum difference is (70.59 − 37.04) = 33.55. Using Table 12.5 as a guideline, we can say that this relationship is strong. This is confirmed by the measures of association. Phi is 0.33 and lambda is 0.32.

Since both variables are ordinal, we can discuss the direction of the relationship. From inspection of the percentages, we can see that efficiency decreases as authoritarianism increases—workers with dictatorial bosses are less productive (or, maybe, bosses become more dictatorial when workers are inefficient), so this relationship is negative in direction.

12.3

	2000 Election	
2004 Election	Democrat	Republican
Democrat	87.31%	11.44%
Republican	12.69%	88.56%
Totals	100.00%	100.00%

The maximum difference for this table is (87.31 − 11.44) or 75.87. Phi is 0.75, and lambda is 0.71. This is a very strong relationship. People are very consistent in their voting habits.

12.5

	Director Experienced?	
Turnover	No	Yes
Low	14.29%	40.91%
Moderate	32.14%	36.36%
High	53.57%	22.73%
Totals	100.00%	100.00%

Maximum difference = 53.57 − 22.73 = 30.84
V = 0.36; lambda = 0.13
This is a strong relationship (disregard lambda).

12.7

	Living Arrangement	
Sense of Isolation	Housing Development	Integrated Neighborhood
Low	80.00%	20.00%
High	20.00%	80.00%
Totals	100.00%	100.00%

Maximum difference = 60
Phi = 0.59; lambda = 0.55
This is a strong relationship.

12.9 a.

	Gender	
Right to Abortion?	Male	Female
Yes	41.78%	40.35%
No	58.22%	59.65%
Totals	100.00%	100.00%

Maximum difference = 1.43
Phi = 0.01; lambda = 0.00
This is a very weak relationship.

b.

	Gender	
Capital Punishment?	Male	Female
Yes	78.68%	69.07%
No	21.32%	30.93%
Totals	100.00%	100.00%

Maximum difference = 9.61
Phi = 0.11; lambda = 0.00
This is a weak relationship.

c.

	Gender	
Right to Suicide?	Male	Female
Yes	68.05%	60.44%
No	31.95%	39.56%
Totals	100.00%	100.00%

Maximum difference = 7.61
Phi = 0.08; lambda = 0.00
This is a weak relationship.

d.

	Gender	
Sex Education?	Male	Female
Yes	87.04%	87.04%
No	12.96%	12.96%
Totals	100.00%	100.00%

Maximum difference = 0.00
Phi = 0.00; lambda = 0.00
There is no relationship between these variables.

e.

	Gender	
Women Should Take Care of Running Their Homes and Leave Running the Country to Men	Male	Female
Yes	14.78%	15.94 %
No	85.22%	84.06%
Totals	100.00%	100.00%

Maximum difference = 1.17
Phi = 0.01; lambda = 0.00
There is no relationship between these variables.

Chapter 13

13.1 a. $G = 0.71$ **b.** $G = 0.69$ **c.** $G = -0.88$

These relationships are strong. Facility in English and income increase with length of residence (+0.71). Be sure to compute the column percentages and use them to help interpret the direction of a relationship. In the first table, 80% of the "newcomers" were "Low" in English facility and 60% of the "oldtimers" were "High." In this relationship, low scores on one variable are associated with low scores on the other, and scores increase together (as one increases, the other increases), so this is a positive relationship. In contrast, contact with the old country decreases with length of residence (−0.88). Most newcomers have higher levels of contact, and most oldtimers have lower levels.

13.3 $G = 0.27$. Be careful interpreting the direction of this relationship. The positive sign of gamma means that cases tend to fall along the diagonal from upper left to lower right. In this case, white-collar families are more associated with organized sports and blue-collar families with sandlot sports. Computing percentages will help you identify the direction of the relationship.

13.5 $G = 0.22$

13.7 $G = -0.11$

13.9 a. $G = -0.14$
b. $G = -0.17$
c. $G = -0.14$
d. $G = -0.13$
e. $G = 0.39$

Income has weak negative relationships with the first four dependent variables. Be careful in interpreting direction for these tables and remember that a negative gamma means that cases tend to be clustered along the diagonal from lower left to upper right. Computing percentages will clarify direction. For example, for the first table, low income is associated with opposition to abortion (63% of the people in this column said "*No*") and high income is associated with support (48% of the people in this column said "*Yes,*" and this is the highest percentage of support across the three income groups).

Income has a moderate positive relationship with support for traditional gender roles. Note the way in which the dependent variable is coded: "agree" means support for traditional gender roles. A positive relationship means that cases tend to fall along the diagonal from upper left to lower right. In this case, agreement is greater for low income and declines as income increases. Is this truly a "positive" relationship? As always, percentages will help clarify the direction of the relationship.

13.11 $r_s = 0.78$

13.13 $r_s = 0.62$

Chapter 14

14.1 (*HINT: When finding the slope, remember that* Turnout *is the dependent or* Y *variable.*)

	For Turnout (*Y*) and		
	Unemployment	Education	Neg. Campaigning
Slope (*b*)	3.00	12.67	−0.90
Y-intercept (*a*)	39.00	−94.73	114.01
Reg. Eq.	$Y = (39) + (3)X$	$Y = (-94.73) + (12.67)X$	$Y = (114.01) + (-0.90)X$
r	0.95	0.98	−0.87
r^2	0.90	0.97	0.76

14.3 (*HINT: When finding the slope, remember that* Number of visitors *is the dependent, or Y,* variable.)

Slope (*b*)	−0.37
Y-intercept (*a*)	13.42
r	−0.31
r^2	0.09

14.5

Dependent Variables		Independent Variables: Growth	Density	Mobility
Homicide	*a*	2.98	3.74	21.30
	b	0.24	0.01	−0.20
	r	0.61	0.31	−0.22
	r^2	0.37	0.10	0.05
Robbery	*a*	99.98	66.06	0.55
	b	2.33	0.32	1.38
	r	0.24	0.69	0.07
	r^2	0.06	0.48	0.01
Car Theft	*a*	147.30	472.02	6,763.17
	b	36.80	−0.41	−75.83
	r	0.74	−0.17	−0.69
	r^2	0.55	0.03	0.48

c. For a growth rate of −1, the predicted homicide rate would be 2.74. For a population density of 250, the predicted robbery rate would be 146.06.

For a state with a mobility score of 90, the predicted rate of auto theft would be −61.53. Since negative crime rates are impossible, the predicted rate of car theft should be phrased as "zero" or "very low."

14.7 $b = 0.05$, $a = 53.18$, $r = 0.40$, $r^2 = 0.16$

14.9

		Prestige	No. children	Church attendance	Hours of TV per day
Age					
	r	−0.30	0.67	−0.30	0.16
	r^2	0.09	0.45	0.09	0.03

		Prestige	No. children	Church attendance	Hours of TV per day
Sex					
	r	−0.28	0.19	0.11	−0.29
	r^2	0.08	0.04	0.01	0.08

Chapter 15

15.1 a. For turnout (Y) and unemployment (X) while controlling for negative advertising (Z), $r_{yx.z} = 0.95$. The relationship between X and Y is not affected by the control variable Z.

b. For turnout (Y) and negative advertising (X) while controlling for unemployment (Z), $r_{yx.z} = -0.89$. The bivariate relationship is not affected by the control variable.

c. Turnout (Y) = 70.25 + (2.09) unemployment (X_1) + (−0.43) negative advertising (X_2). For unemployment (X_1) = 10 and negative advertising (X_2) = 75, turnout (Y) = 58.90.

d. For unemployment (X_1): $b_1^* = 0.66$. For negative advertising (X_2): $b_2^* = -0.41$. Unemployment has a stronger effect on turnout than negative advertising. Note that the independent variables' effect on turnout is in opposite directions.

e. $R^2 = 0.98$

15.3 a. For strife (Y) and unemployment (X), controlling for urbanization (Z), $r_{yx.z} = 0.79$.

b. For strife (Y) and urbanization (X), controlling for unemployment (Z), $r_{yx.z} = 0.20$.

c. Strife (Y) = (−14.60) + (4.94) unemployment (X_1) + (0.16) urbanization (X_2). With unemployment = 10 and urbanization = 90, strife (Y') would be 49.19.

d. For unemployment (X_1): $b_1^* = 0.78$. For urbanization (X_2): $b_2^* = 0.13$.

e. $R^2 = 0.65$

15.5 a. Turnout (Y) = 83.80 + (−1.16) Democrat (X_1) + (2.89) minority (X_2).

b. For $X_1 = 0$ and $X_2 = 5$, $Y' = 98.25$

c. $Z_y = (-1.27)Z_1 + (.84)Z_2$

d. $R^2 = 0.51$

15.7 a. $Z_y = (0.35)$ grads $(Z_1) = (-0.74)$ rank (Z_2)

b. $R^2 = 0.85$

Glossary

Alpha (α). The probability of error or the probability that a confidence interval does not contain the population value. Alpha levels are usually set at 0.10, 0.05, 0.01, or 0.001. Chapter 7

Alpha level (α). The proportion of area under the sampling distribution that contains unlikely sample outcomes, given that the null hypothesis is true. Also, the probability of Type I error. Chapter 8

Analysis of variance. A test of significance appropriate for situations in which we are concerned with the differences among more than two sample means. Chapter 10

ANOVA. See Analysis of variance. Chapter 10

Association. The relationship between two (or more) variables. Two variables are said to be associated if the distribution of one variable changes for the various categories or scores of the other variable. Chapter 12

Bar chart. A graphic display device for nominal or ordinal variables with few categories. Categories are represented by bars of equal width, the height of each corresponding to the number (or percentage) of cases in the category. Chapter 3

Beta-weights (b^*). Standardized partial slopes. Chapter 15

Bias. A criterion used to select sample statistics as estimators. A statistic is unbiased if the mean of its sampling distribution is equal to the population value of interest. Chapter 7

Bivariate table. A table that displays the joint frequency distributions of two variables. Chapter 11

Cells. The cross-classification categories of the variables in a bivariate table. Chapter 11

Central limit theorem. A theorem that specifies the mean, standard deviation, and shape of the sampling distribution, given that the sample is large. Chapter 7

Chi square test. A nonparametric test of hypothesis for variables that have been organized into a bivariate table. Chapter 11

Coefficient of determination (r^2). The proportion of all variation in Y that is explained by X. Found by squaring the value of Pearson's r. Chapter 14

Coefficient of multiple determination (R^2). A statistic that equals the total variation explained in the dependent variable by all independent variables combined. Chapter 15

Column. The vertical dimension of a bivariate table. By convention, each column represents a score on the independent variable. Chapter 11

Column percentages. Percentages calculated within each column of a bivariate table. Chapter 11

Conditional distribution of *Y*. The distribution of scores on the dependent variable for a specific score or category of the independent variable when the variables have been organized into table format. Chapter 12

Conditional means of *Y*. The mean of all scores on Y for each value of X. Chapter 14

Confidence interval. An estimate of a population value in which a range of values is specified. Chapter 7

Confidence level. A frequently used alternate way of expressing alpha, the probability that an interval estimate will not contain the population value. Confidence levels of 90%, 95%, 99%, and 99.9% correspond to alphas of 0.10, 0.05, 0.01, and 0.001, respectively. Chapter 7

Control variable. A third or Z variable. A control variable that is introduced in a statistical analysis to see if it affects the original bivariate relationship. Chapter 15

Correlation matrix. A table showing the correlations between all possible combinations of variables. Chapter 14

Cramer's *V*. A chi square–based measure of association. Appropriate for nominally measured variables that have been organized into a bivariate table of any number of rows and columns. Chapter 12

Critical region (region of rejection). The area under the sampling distribution that, in advance of the test itself, is defined as including unlikely sample outcomes, given that the null hypothesis is true. Chapter 8

Cumulative frequency. An optional column in a frequency distribution that displays the number of cases within an interval and all preceding intervals. Chapter 2

Cumulative percentage. An optional column in a frequency distribution that displays the percentage of cases within an interval and all preceding intervals. Chapter 2

Data. Any information collected as part of a research project and expressed as numbers. Chapter 1

Data reduction. Summarizing many scores with a few statistics. A major goal of descriptive statistics. Chapter 1

Dependent variable. A variable that is identified as an effect, result, or outcome variable. The dependent variable is thought to be caused by the independent variable. Chapter 1

Descriptive statistics. The branch of statistics concerned with (1) summarizing the distribution of a single variable or (2) measuring the relationship between two or more variables. Chapter 1

Deviations. The distances between the scores and the mean. Chapter 5

Direct relationship. A multivariate relationship in which the control variable has no effect on the bivariate relationship. Chapter 15

Dispersion. The amount of variety or heterogeneity in a distribution of scores. Chapter 5

Dummy variable. A nominal level variable that has been recoded into exactly two categories (zero and one) for inclusion in regression equations. Chapter 14

Efficiency. The extent to which the sample outcomes are clustered around the mean of the sampling distribution. Chapter 7

EPSEM. The **E**qual **P**robability of **SE**lection **M**ethod for selecting samples. Every element or case in the population must have an equal probability of selection for the sample. Chapter 7

Expected frequency (f_e). The cell frequencies that would be expected in a bivariate table if the variables were independent. Chapter 11

Explained variation. The proportion of all variation in Y that is attributed to the effect of X. Chapter 14

***F* ratio.** The test statistic computed in Step 4 of the ANOVA test. Chapter 10

Five-step model. A step-by-step guideline for conducting tests of hypotheses. A framework that organizes decisions and computations for all tests of significance. Chapter 8

Frequency distribution. A table that displays the number of cases in each category of a variable. Chapter 2

Frequency polygon. A type of graph appropriate for interval-ratio variables. Class intervals are represented by dots placed over the midpoints, the height of each corresponding to the number (or percentage) of cases in the interval. All dots are connected by straight lines. Same as a line chart. Chapter 3

Gamma (G). A measure of association appropriate for variables measured with "collapsed" ordinal scales that have been organized into table format; G is the symbol for gamma. Chapter 13

Histogram. A type of graph appropriate for interval-ratio variables. Class intervals are represented by contiguous bars of equal width (equal to the class limits), the height of each corresponding to the number (or percentage) of cases in the interval. Chapter 3

Hypothesis. A statement about the relationship between variables that is derived from a theory. Hypotheses are more specific than theories, and all terms and concepts are fully defined. Chapter 1

Hypothesis testing. Statistical tests that estimate the probability of sample outcomes if assumptions about the population (the null hypothesis) are true. Chapter 8

Independence. The null hypothesis in the chi square test. Two variables are independent if, for all cases, the classification of a case on one variable has no effect on the probability that the case will be classified in any particular category of the second variable. Chapter 11

Independent random samples. Random samples gathered in such a way that the selection of a particular case for one sample has no effect on the probability that any other particular case will be selected for the other samples. Chapter 9

Independent variable. A variable that is identified as a causal variable. The independent variable is thought to cause the dependent variable. Chapter 1

Inferential statistics. The branch of statistics concerned with making generalizations from samples to populations. Chapter 1

Interaction. A multivariate relationship in which a bivariate relationship changes substantially across the categories of the control variable. Chapter 15

Interquartile range (Q). The distance from the third quartile to the first quartile. Chapter 5

Intervening relationship. A multivariate relationship in which the dependent and independent variables are linked through the control variable. Once the third variable is controlled, the relationship becomes substantially weaker. Chapter 15

Lambda. A proportional reduction in error (PRE) measure of association for variables measured at the nominal level that have been organized into a bivariate table. Chapter 12

Least-squares principle. This principle states that the mean is a good measure of central tendency because it is the point of minimized variation of the scores, as measured by the squared differences between the mean and all the scores. Chapter 4

Level of measurement. The mathematical characteristic of a variable and the major criterion for selecting statistical techniques. Variables can be measured at any of three levels, each permitting certain mathematical operations and statistical techniques. The characteristics of the three levels are summarized in Table 1.2. Chapter 1

Line chart. See Frequency polygon. Chapter 3

Linear relationship. A relationship between two variables in which the observation points (dots) in the scattergram can be approximated with a straight line. Chapter 14

Marginals. The row and column subtotals in a bivariate table. Chapter 11

Maximum difference. A way to assess the strength of an association between variables that have been organized into a bivariate table. The maximum difference is the largest difference between column percentages for any row of the table. Chapter 12

Mean. The arithmetic average of the scores. $\overline{X}$ represents the mean of a sample, and μ, the mean of a population. Chapter 4

Mean square estimate. An estimate of the variance calculated by dividing the sum of squares within (*SSW*) or the sum of squares between (*SSB*) by the proper degrees of freedom. Chapter 10

μ. The mean of a population. Chapter 7

μ_p. The mean of a sampling distribution of sample proportions. Chapter 7

$\mu_{\overline{x}}$. The mean of a sampling distribution of sample means. Chapter 7

Measures of association. Statistics that quantify the strength and, for ordinal and interval-ratio level variables, direction of the association between variables. Chapter 1

Measures of central tendency. Statistics that summarize a distribution of scores by reporting the most typical or representative value of the distribution. Chapter 4

Measures of dispersion. Statistics that indicate the amount of variety or heterogeneity in a distribution of scores. Chapter 5

Median (*Md*). The point in a distribution of scores above and below which exactly half of the cases fall. Chapter 4

Midpoint. The point exactly halfway between the upper and lower limits of a class interval. Chapter 2

Mode. The most common value in a distribution or the largest category of a variable. Chapter 4

Multiple correlation. A multivariate technique for examining the combined effects of more than one independent variable on a dependent variable. Chapter 15

Multiple correlation coefficient (*R*). A statistic that indicates the strength of the correlation between a dependent variable and two or more independent variables. Chapter 15

Multiple regression. A multivariate technique that breaks down the separate effects of the independent variables on the dependent variable; used to make predictions of the dependent variable. Chapter 15

N_d. The number of pairs of cases ranked in different order on two variables. Chapter 13

Negative association. A bivariate relationship where the variables vary in opposite directions. As one variable increases, the other decreases, and high scores on one variable are associated with low scores on the other. Chapter 12

Nonparametric. A "distribution-free" test. These tests do not assume that the sampling distribution is normal in shape. Chapter 11

Normal curve. A theoretical distribution of scores that is symmetrical, unimodal, and bell shaped. The standard normal curve always has a mean of 0 and a standard deviation of 1. Chapter 6

Normal curve table. A detailed description of the area between a *Z* score and the mean of any standardized normal distribution. See Appendix A. Chapter 6

N_s. The number of pairs of cases ranked in the same order on two variables. Chapter 13

Null hypothesis (H_0). A statement of "no difference." In the context of single-sample tests of significance, the population from which the sample was drawn is assumed to have a certain characteristic or value. Chapter 8

Observed frequency (f_o). The cell frequencies actually observed in a bivariate table. Chapter 11

One-tailed test. A type of hypothesis test used when (1) the direction of the difference can be predicted or (2) concern focuses on outcomes in only one tail of the sampling distribution. Chapter 8

One-way analysis of variance. Applications of ANOVA in which the effect of a single independent variable on a dependent variable is observed. Chapter 10

Parameter. A characteristic of a population. Chapter 7

Partial correlation. A multivariate technique for examining a bivariate relationship while controlling for other variables. Chapter 15

Partial correlation coefficient. A statistic that shows the relationship between two variables while controlling for other variables; $r_{yx.z}$ is the symbol for the partial correlation coefficient when controlling for one variable. Chapter 15

Partial slopes. In a multiple regression equation, the slope of the relationship between a particular independent variable and the dependent variable while controlling for all other independents in the equation. Chapter 15

Pearson's *r* (*r*). A measure of association for variables that have been measured at the interval-ratio level. Chapter 14

Percentage. The number of cases in a category of a variable divided by the number of cases in all categories of the variable, the entire quantity multiplied by 100. Chapter 2

Percentage change. A statistic that expresses the magnitude of change in a variable from time 1 to time 2. Chapter 2

Phi (ϕ). A chi square–based measure of association. Appropriate for nominally measured variables that have been organized into a 2×2 bivariate table. Chapter 12

Pie chart. A graphic display device especially for nominal or ordinal variables with few categories. A circle (the pie) is divided into segments proportional in size to the percentage of cases in each category of the variable. Chapter 3

Pooled estimate. An estimate of the standard deviation of the sampling distribution of the difference in sample means based on the standard deviations of both samples. Chapter 9

Population. The total collection of all cases in which the researcher is interested. Chapter 1

Population pyramid. A graph used to display the age-sex distribution of a population. This type of graph can be used to display other variables as well. Chapter 5

Positive association. A bivariate relationship where the variables vary in the same direction. As one variable increases, the other also increases, and high scores on one variable are associated with high scores on the other. Chapter 12

Probability. The likelihood that a defined event will occur. Chapter 6

Proportion. The number of cases in one category of a variable divided by the number of cases in all categories of the variable. Chapter 2

Proportional reduction in error (PRE). The logic that underlies the definition and computation of lambda and gamma. The statistic compares the number of errors made when predicting the dependent variable while ignoring the independent variable with the number of errors made while taking the independent variable into account. Chapter 12 and 13

P_s (*P*-sub-*s*). Any sample proportion. Chapter 7

P_u (*P*-sub-*u*). Any population proportion. Chapter 7

Quantitative research. Research based on the analysis of numerical information or data. Chapter 1

Range (*R*). The highest score minus the lowest score. Chapter 5

Rate. The number of actual occurrences of some phenomenon or trait divided by the number of possible occurrences per some unit of time. Chapter 2

Ratio. The number of cases in one category divided by the number of cases in some other category. Chapter 2

Regression line. The single, best-fitting straight line that summarizes the relationship between two variables. Regression lines are fitted to the data points by the least-squares criterion, whereby the line touches all conditional means of Y or comes as close to doing so as possible. Chapter 14

Representative. The quality a sample is said to have if it reproduces the major characteristics of the population from which it was drawn. Chapter 7

Research. Any process of gathering information systematically and carefully to answer questions or test theories. Statistics are useful for research projects in which the information is represented in numerical form or as data. Chapter 1

Research hypothesis (H_1). A statement that contradicts the null hypothesis. In the context of single-sample tests of significance, the research hypothesis says that the population from which the sample was drawn does not have a certain characteristic or value. Chapter 8

Row. The horizontal dimension of a bivariate table, conventionally representing a score on the dependent variable. Chapter 11

Sample. A carefully chosen subset of a population. In inferential statistics, information is gathered from a sample and then generalized to a population. Chapter 1

Sampling distribution. The distribution of a statistic for all possible sample outcomes of a certain size. Under conditions specified in two theorems, the sampling distribution will be normal in shape with a mean equal to the population value and a standard deviation equal to the population standard deviation divided by the square root of *N*. Chapter 7

Scattergram. A type of graph that depicts the relationship between two variables. Chapter 14

Significance testing. See Hypothesis testing. Chapter 8

Simple random sample. A method for choosing cases from a population by which every case and every combination of cases has an equal chance of being included. Chapter 7

Skew. The extent to which a distribution of scores has a few scores that are extremely high (positive skew) or extremely low (negative skew). Chapter 4

Slope (*b*). The amount of change in one variable per unit change in the other; *b* is the symbol for the slope of a regression line. Chapter 14

Spearman's rho (r_s). A measure of association appropriate for ordinally measured variables that are "continuous" in form; r_s is the symbol for Spearman's rho. Chapter 13

Spurious relationship. A multivariate relationship in which there is no actual causal relationship between the dependent and independent variables. Both are caused by some other variable. Once the third variable is controlled, the relationship becomes substantially weaker. Chapter 15

Standard deviation. The square root of the squared deviations of the scores around the mean, divided by *N*. The most important and useful descriptive measure of dispersion; *s* represents the standard deviation of a sample and σ the standard deviation of a population. Chapter 5

$\sigma_{\bar{x}-\bar{x}}$. Symbol for the standard deviation of the sampling distribution of the differences in sample means. Chapter 9

σ_{p-p}. Symbol for the standard deviation of the sampling distribution of the differences in sample proportions. Chapter 9

Standard error of the mean. The standard deviation of a sampling distribution of sample means. Chapter 7

Standardized partial slopes (beta-weights). The slope of the relationship between a particular independent variable and the dependent variable when all scores have been normalized. Chapter 15

Statistics. A set of mathematical techniques for organizing and analyzing data. Chapter 1

Student's *t* distribution. A distribution used to find the critical region for tests of sample means when σ is unknown and sample size is small. Chapter 8

Sum of squares between (*SSB*). The sum of the squared deviations of the sample means from the overall mean, weighted by sample size. Chapter 10

Sum of squares within (*SSW*). The sum of the squared deviations of scores from the category means. Chapter 10

***t* (critical).** The *t* score that marks the beginning of the critical region of a *t* distribution**.** Chapter 8

***t* (obtained).** The test statistic computed in Step 4 of the five-step model. The sample outcome expressed as a *t* score. Chapter 8

Test statistic. The value computed in Step 4 of the five-step model that converts the sample outcome into either a *t* score or a *Z* score. Chapter 8

Theory. A generalized explanation of the relationship between two or more variables. Chapter 1

Total sum of squares (*SST*). The sum of the squared deviations of the scores from the overall mean. Chapter 10

Total variation. The spread of the *Y* scores around the mean of *Y*. Chapter 14

Two-tailed test. A type of hypothesis test used when (1) the direction of the difference cannot be predicted or (2) concern focuses on outcomes in both tails of the sampling distribution.

Type I error (alpha error). The probability of rejecting a null hypothesis that is, in fact, true. Chapter 8

Type II error (beta error). The probability of failing to reject a null hypothesis that is, in fact, false. Chapter 8

Unexplained variation. The proportion of the total variation in *Y* that is not accounted for by *X*. Chapter 14

Variable. Any trait that can change values from case to case. Chapter 1

Variance. The squared deviations of the scores around the mean divided by *N*. A measure of dispersion used primarily in inferential statistics and also in correlation and regression techniques; s^2 represents the variance of a sample, and σ^2 the variance of a population. Chapter 5

***X*.** Symbol used for any independent variable. Chapter 12

X_i ("*X* sub *i*"). Any score in a distribution. Chapter 4

***Y* intercept (*a*).** The point where the regression line crosses the *Y* axis. Chapter 14

***Y′*.** Symbol for predicted score on *Y*. Chapter 14

***Y*.** Symbol used for any dependent variable. Chapter 12

***Z*(critical).** The *Z* score that marks the beginnings of the critical region on a *Z* distribution**.** Chapter 8

***Z*(obtained).** The test statistic computed in Step 4 of the five-step model for certain tests of significance. The sample outcomes expressed as a *Z* score. Chapter 8

***Z* scores.** Standard scores; the way scores are expressed after they have been standardized to the theoretical normal curve. Chapter 6

Zero-order correlations. Correlation coefficients for bivariate relationships. Chapter 15

χ^2(critical). The score on the sampling distribution of all possible sample chi squares that marks the beginning of the critical region. Chapter 11

χ^2(obtained). The test statistic as computed from sample results. Chapter 11

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FREQUENTLY USED FORMULAS

CHAPTER 11

Chi square

$$\chi^2(\text{obtained}) = \Sigma\frac{(f_o - f_e)^2}{f_e}$$

CHAPTER 12

Phi

$$\phi = \sqrt{\frac{\chi^2}{N}}$$

Cramer's *V*

$$V = \sqrt{\frac{\chi^2}{(N)(\text{Minimum of } r-1,\ c-1)}}$$

Lambda

$$\lambda = \frac{E_1 - E_2}{E_1}$$

CHAPTER 13

Gamma

$$G = \frac{N_s - N_d}{N_s + N_d}$$

Spearman's rho

$$r_s = 1 - \frac{6\Sigma D^2}{N(N^2 - 1)}$$

CHAPTER 14

Least-squares regression line

$$Y = a + bX$$

Slope

$$b = \frac{\Sigma(X - \overline{X})(Y - \overline{Y})}{\Sigma(X - \overline{X})^2}$$

Y intercept

$$a = \overline{Y} - b\overline{X}$$

Pearson's *r*

$$r = \frac{\Sigma(X - \overline{X})(Y - \overline{Y})}{\sqrt{[\Sigma(X - \overline{X})^2][\Sigma(Y - \overline{Y})^2]}}$$

CHAPTER 15

Partial correlation coefficient

$$r_{yx.z} = -\frac{r_{yx} - (r_{yz})(r_{xz})}{\sqrt{1 - r_{yz}^2}\sqrt{1 - r_{xz}^2}}$$

Least-squares multiple regression line

$$Y = a + b_1X_1 + b_2X_2$$

Partial slope for X_1

$$b_1 = \left(\frac{s_y}{s_1}\right)\left(\frac{r_{y1} - r_{y2}r_{12}}{1 - r_{12}^2}\right)$$

Partial slope for X_2

$$b_2 = \left(\frac{s_y}{s_2}\right)\left(\frac{r_{y2} - r_{y1}r_{12}}{1 - r_{12}^2}\right)$$

Y intercept

$$a = \overline{Y} + b_1\overline{X}_1 - b_2\overline{X}_2$$

Beta-weight for X_1

$$b_1^* = b_1\left(\frac{s_1}{s_y}\right)$$

Beta-weight for X_2

$$b_2^* = b_2\left(\frac{s_2}{s_y}\right)$$

Standardized least-squares regression line

$$Z_y = b_1^*Z_1 + b_2^*Z_2$$

Coefficient of multiple determination

$$R^2 = r_{y1}^2 + r_{y2.1}^2(1 - r_{y1}^2)$$