Financial Derivatives
Chapter #7
Swaps

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January–March 2011

1 Overview

- Definition and preliminaries
- Diversity of swap contracts
- How are swaps
  - designed?
  - used?
  - valued?
- Advantages of swaps
- We will focus on the most popular swaps (i.e. interest rate and currency) and also mention more exotic/complex swap deals.

2 Preliminaries

- *Treasury rates* are the rates at which a government can borrow funds in its own currency. Because the chance of a government defaulting on its repaying obligations is virtually zero, Treasury rates are often termed risk–free rates.
- *London Interbank rates* are the rates at which large international banks are prepared to pay (*LIBID*—London Interbank Bid Rate) or require as payment (*LIBOR*—London Interbank Offer Rate) between them for deposits of 1, 3, 6 and 12 months in all of the world's major currencies. LIBOR is a widely–used reference rate. LIBOR rates are determined through trading between banks and are generally higher than the corresponding Treasury rate since they are not free of default risk.

- *Repo rates* or repurchase agreement rates are the rates applicable to a dealer who finances an investment by agreeing to sell securities he/she owns now and buy them back later for a higher price. The price difference is the interest earned, i.e. the repo rate. Repo agreements involve minimal credit risk since if any of the parties defaults, the other party keeps either the securities or the cash. The most common type of repo is an overnight repo, which is renegotiated every day.
- n–*year zero rate* (or n–year zero–coupon rate or n–*year spot rate*) is the interest earned on an investment that lasts for n years when the whole return is realised at the end of n years (there are no intermediate payments)

3 Day count conventions

- In reality, interest rate securities earn interest over the periods of time that are held; these periods are rarely as convenient as "1 or 2 years" as used in textbooks but number of days over a reference period
- Day count conventions are usually expressed as

$$\frac{\text{Number of days between loan dates}}{\text{Number of days in reference period or Basis}} \times \text{ Interest earned in basis period}$$

- Day count conventions are different between countries and interest rate markets. In the USA

$\frac{\text{Actual}-2}{\text{Actual}}$	Treasury bonds
$\frac{30}{360}$	Corporate and municipal bonds
$\frac{\text{Actual}-2}{360}$	T–bills and money market

while in the UK

$\frac{\text{Actual}}{\text{Actual}}$	Government bonds
$\frac{30}{365}$	Corporate bonds
$\frac{\text{Actual}}{365}$	Money market

Other countries have similar conventions. For example, JP¥ interest rate markets follow the USA convention.

4 Forward rate agreements (FRAs)

- An FRA is an over–the–counter agreement made at time t to exchange $LIBOR$ rate (denote L) for a fixed rate K on a loan of principal A, for the period T to $T+m$

$$+L_{T,T+m} \times \frac{\text{days}}{\text{Basis}} \times A$$

$$\downarrow$$

$$t\_\_\_\_\_\_\_T\_\_\_\_\_\_\_\_ \qquad T+m$$

$$\uparrow$$

$$-K \times \frac{\text{days}}{\text{Basis}} \times A$$

- Since $L_{T,T+m}$ becomes known at T, an FRA is usually settled in cash then; the payoff at T is

$$FRA_T = \frac{A\left(L_{T,T+m} - K\right)\frac{\text{days}}{\text{Basis}}}{1 + L_{T,T+m}\frac{\text{days}}{\text{Basis}}}$$

- Being long (short) an FRA means receive (pay) $LIBOR$ and pay (receive) the fixed rate K.

EXAMPLE (3): A company buys a 12–month FRA for the 3–month $LIBOR$ on a principal of \$5 million at a rate of 5.5%. The payoff of this FRA is positive if in 12–months the 3–month \$ $LIBOR$ exceeds 5.5%. If this turns out to be 6%, the payoff at $T = 12$ months is

$$\$5,000,000 \times \frac{(0.06 - 0.055)\,\frac{90}{360}}{1 + 0.06\frac{90}{360}} = \$6,157.64$$

5 T–bill futures

- These contracts are futures contracts (traded in exchanges) with an underlying asset of a Treasury bill with $A = \$1$ million face value. The contract on the 3–month T–bill is the most popular.
- The price of a T–bills futures contract is quoted in terms of an index which depends on the discount rate for the 3–month T–bill

$$\text{Index} = 100 - \text{Discount Rate (\%)}$$

  If the index is 97.30, then the discount rate is 2.70%.
- The futures price is

$$A \times \left[1 - \frac{\text{Discount Rate}}{100} \times \frac{\text{Actual} - 2}{360}\right]$$

In our example,

$$\$1,000,000 \times \left[1 - \frac{2.70}{100} \times \frac{90}{360}\right] = \$992,025$$

- One point change in the index implies a \$25 change in the T–bill futures price.

6 Definitions and mechanics

- A swap is an agreement between two companies to exchange cash flows in the future. The agreement defines the exact dates when the cash flows are to be paid and the way in which they are to be calculated.
- Introduced in the early '80s, traded over–the–counter (OTC). Phenomenal growth recently in the swap market.
- A swap contract can be thought as a sequence of forward contracts. Unlike forwards that lead to the exchange of cash flows on just one future date, swaps typically involve cash flow exchanges on several future dates.

7 Diversity of swap contracts

- In addition to the size of the OTC swap, the diversity of contracts that are structured in the market bears some attention.
- Swaps can be structured on different underlying instruments with different maturity dates. The following table summarises the basic types of swaps.

Swap Type		Term	Remarks
Int. rate	Plain–vanilla	2–10 yrs	Same currency, one party pays fixed and the other floating
	Basis swap	2–10 yrs	Same currency, parties pay floating flows keyed to different indices
Currency	Fixed to fixed	2–10 yrs	Different currencies, both parties pay fixed interest payments
	Fixed to floating	2–10 yrs	Different currencies, one party pays fixed and the other floating
	Floating to floating	2–10 yrs	Different currencies, both parties pay floating interest payments

8 Interest rate swap

- A "plain vanilla" interest rate swap involves exchange of cash flows every six months, for a predetermined number of years.
- One party agrees to make a fixed interest payment on a predetermined notional principal amount.
- The other party agrees to pay cash flows equal to interest at a floating rate on the same notional amount.
- The floating rate in many interest rate swaps is the $LIBOR$ (typically the 6–month rate).
- At the end of the swap contract, the principal amount itself is not exchanged, since it will not change the nature of the deal.
- Very useful contracts for "backing out" the yield curve for long maturities (for which government bonds are not traded).

$$
\begin{array}{cccccccc}
 & & \pm L_{T_1,T_1+m}\times\frac{\text{days(s)}}{\text{Basis}}\times A & & \pm L_{T_2,T_2+m}\times\frac{\text{days(s)}}{\text{Basis}}\times A & & \pm L_{T_T,T_T+m}\times\frac{\text{days(s)}}{\text{Basis}}\times A \\
 & & \downarrow & & \downarrow & & \downarrow \\
T_d\text{-----}T_0\text{-----} & & T_1 & \text{-----} & T_2 & \text{-----} & T_T \\
 & & \uparrow & & \uparrow & & \uparrow \\
 & & \mp K\times\frac{\text{days(s)}}{\text{Basis}}\times A & & \mp K\times\frac{\text{days(s)}}{\text{Basis}}\times A & & \mp K\times\frac{\text{days(s)}}{\text{Basis}}\times A
\end{array}
$$

Trade date	27-February-2003
Effective date	5-March-2003
Business day convention (all dates)	Following business day
Holiday calendar	U.S.
Termination date	5-March-2006
Fixed Amounts	
Fixed-rate payer	Microsoft
Fixed-rate notional principal	USD 100 million
Fixed rate	5% per annum
Fixed-rate day count convention	Actual/365
Fixed rate payment dates	Each 5-March and 5-September commencing 5-September-2003, up to and including 5-March-2006
Floating Amounts	
Floating-rate payer	Intel
Floating-rate notional principal	USD 100 million
Floating rate	USD 6-month LIBOR
Floating-rate day count convention	Actual/360
Floating-rate payment dates	Each 5-March and 5-September commencing 5-September-2003, up to and including 5-March-2006

9 Standard features

- Notional principal*, A
  - Used as a basis for calculating fixed and floating payments
  - It never changes hands
- Fixed rate, R_F
  - Rate quoted daily by market participants for executing ("selling") swaps
  - Usually quoted as $\frac{\text{Actual}}{365}$ or $\frac{30E}{360}$
  - Since $LIBOR$ is quoted as $\frac{\text{Actual}}{360}$ in some countries (e.g. EURO, USD), to make it comparable to a fixed rate quoted as $\frac{\text{Actual}}{365}$, multiply with $\frac{360}{365}$.

*Also known as currency amount, notional amount, notional quantity or calculation amount.

- Trade date, T_d
  - Date parties contract to enter into swap
- Effective date or Value date, T_0
  - Date when first fixed and floating payments start to accrue
- Generally, no payments take place on trade or effective dates
- Payment dates, T_t
  - Annually
  - Semi–annually
  - Quarterly
- Maturity date, T_T
  - Last payment date
  - Not adjusted for business day conventions (see below)

- Payment date conventions
  - “Modified Following” or “Modified” Business Day (default)
    * If a payment date falls on a weekend or a bank holiday, payment is made on the next “good” business day if in the same calendar month,
    * Otherwise, payment made on first preceding day that is a business day.
  - “Following” Business Day
    * Payment made on next good business day, even if in a different month.
  - “Preceding” Business Day
    * Closest previous good business day.
  - End of Month
    * Last good business day in each month.

- Fixed Rate Payments, F_{T_t}
  - Referred to as *Fixed Leg* of the swap
  - If T_t is the time of the prior fixed rate payment, and $T_{t+1}$is the next, then over the

    $$\text{Calculation Period} = T_{t+1} - T_t$$

    the fixed rate payment is

    $$F_{T_{t+1}} = A \times R_F \times \text{Day Count Fraction} \tag{1}$$

- Day Count Fraction
  - Defines method of calculating number of days between payments
  - $T_{t+1} - T_t =$ Number of days, from to and including T_t, until, but excluding T_{t+1}

– Standard conventions

1. Actual/360 (e.g. EURO, USD)

$$\text{Day Count Fraction} = \frac{T_{t+1} - T_t}{360}$$

2. Actual/365 (e.g. GBP, JPY)

$$\text{Day Count Fraction} = \frac{T_{t+1} - T_t}{365}$$

3. Actual/Actual

$$\begin{array}{cc} \text{If Leap year} & \text{Day Count Fraction} = \frac{T_{t+1}-T_t}{366} \\ \text{Otherwise} & \text{Day Count Fraction} = \frac{T_{t+1}-T_t}{365} \end{array}$$

4. 30E/360: The year is assumed to have 360 days, i.e. 12 months with 30 days each.

- Floating Rate Payments, L_{T_t}
  - Referred to as the *Floating Leg* of the swap
  - Set in advance
  - Paid in arrears
  - Payment

$$L_{T_1} = A \times \left(L_{T_1, T_1+m} \pm \text{ LIBOR margin} \right) \times \text{Day Count Fraction} \quad (2)$$

- Reset Calendar: follows holiday calendar of the city where rate is quoted
- Payment Calendar: is a joint holiday calendar of index source and location of counterparts.

10 Worked example (7Y USD Swap)

- Fixed Leg

t	Day of week	Payment date, T_t	F_{T_t}	$T_t - T_{t-1}$
0	Mon	Aug-01-94		
1	Tue	Aug-01-95	$3,715,902.78^{\dagger}$	365
2	Thu	Aug-01-96	$3,726,083.33$	366
3	Fri	Aug-01-97	$3,715,902.78$	365
4	Mon*	Aug-03-98	$3,736,263.89$	367
5	Mon*	Aug-02-99	$3,705,722.22$	364
6	Tue	Aug-01-00	$3,715,902.78$	365
7	Wed	Aug-01-01	$3,715,902.78$	365

†Verify that

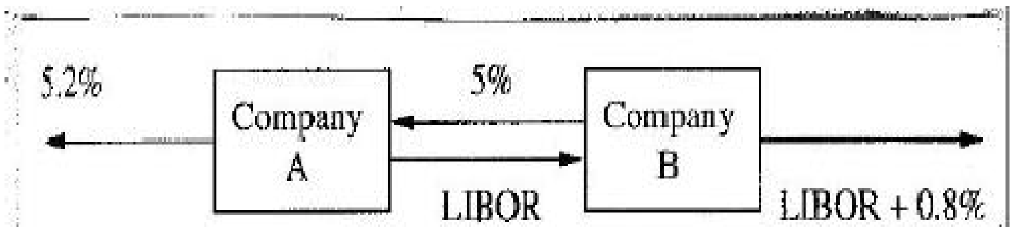
$$F_{T_1} = A \times R_F \times \frac{T_1 - T_0}{360} = \$50,000,000 \times 0.0733 \times \frac{365}{360} = \$3,715,902.78$$

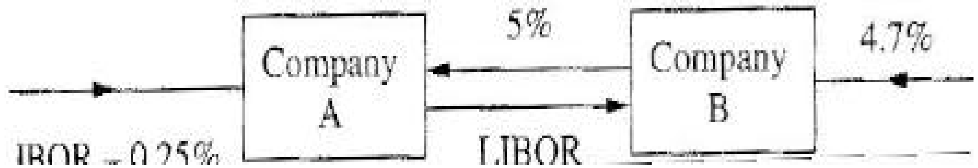
11 Advantages of interest rate swaps

- Transform a liability
  - Suppose company A (B) has issued bonds promising coupon payments of 5.2% ($LIBOR + 0.8\%$) every six months.
  - Companies could alter the nature of their liabilities by entering a 5%–to–$LIBOR$ swap deal.
  - The deal leaves firm A having to make payments $LIBOR + 20$ basis points[‡], and firm B paying 5.8%.

[‡]Note that $0.1\% = 10$ basis points.

- Transform an asset
  - Suppose company A (B) receives payments based on $LIBOR - 25$ basis points (4.7%) through holdings in bonds or some alternative investment.
  - Companies could alter the nature of their liabilities by entering a 5%–to–$LIBOR$ swap deal.
  - The deal leaves firm A receiving payments based on 4.75%, and firm B receives $LIBOR - 0.3\%$.


5.2%
Company A
5%
LIBOR
Company B
LIBOR + 0.8%

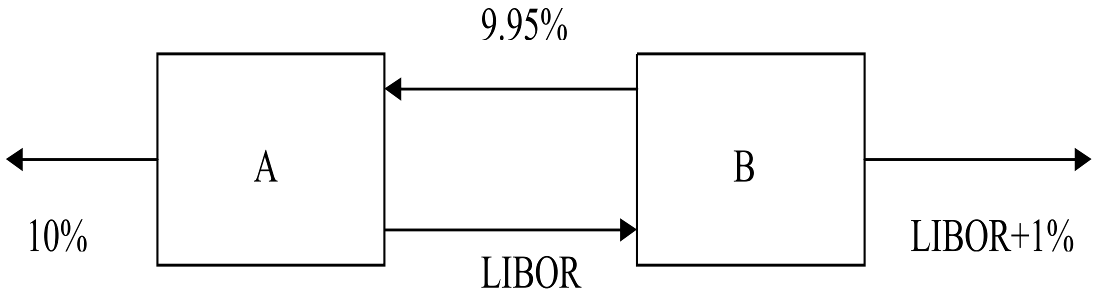

5%
4.7%
Company A
Company B
LIBOR

- The comparative–advantage argument
  - An explanation commonly put forward to explain the popularity of swaps concerns comparative advantages.
  - It is argued that some companies have a comparative advantage when borrowing in fixed–rate markets, while other firms in floating–rate markets.
  - Consider the following examples, with companies A and B being quoted different rates for borrowing funds:

	Fixed	Floating
Company A	10.0%	$6mLIBOR + 0.3\%$
Company B	11.2%	$6mLIBOR + 1.0\%$

  - Company A wants to borrow floating, while B fixed.

– They can both do better, if A borrows at a fixed rate of 10%, B borrows at 6$mLIBOR + $1.0% and they enter a 9.95%–to–$LIBOR$ swap deal.



– Company A ends up paying $LIBOR +$ 0.05%, while company B ends up borrowing at 10.95%. Those rates are better than those offered to firms in the market.

12 Currency swaps

- In its simplest form involves exchanging interest payments and *principal* in one currency for interest payments and *principal* in another currency.
- Also used to transform the nature of assets or liabilities.
- Ideal long–term protection for foreign exchange risk.
- Most of the standard features described earlier still apply.

Description of the cash flows exchanged as part of a "fixed–to–fixed" currency swap

$$
\begin{array}{ccccccc}
 & \pm Kc_1 \times \frac{\text{days(s)}}{\text{Basis}} \times Ac_1 & & \pm Kc_1 \times \frac{\text{days(s)}}{\text{Basis}} \times Ac_1 & & \pm\left(1 + Kc_1\right) \times \frac{\text{days(s)}}{\text{Basis}} \times Ac_1 \\
 & \downarrow & & \downarrow & & \downarrow \\
T_d\text{-----}T_0\text{-----} & T_1 & \text{-----} & T_2 & \text{-----} & T_T \\
 & \uparrow & & \uparrow & & \uparrow \\
 & \mp Kc_2 \times \frac{\text{days(s)}}{\text{Basis}} \times Ac_2 & & \mp Kc_2 \times \frac{\text{days(s)}}{\text{Basis}} \times Ac_2 & & \mp\left(1 + Kc_2\right) \times \frac{\text{days(s)}}{\text{Basis}} \times Ac_2
\end{array}
$$

13 Valuation

- Interest rate swap: FRA method

$FRA\,(1)$ $\quad T_d$-----T_0----- $\quad T_1$

$$\pm L_{T_1,T_1+m}\times\frac{\text{days(s)}}{\text{Basis}}\times A \;\downarrow\; T_1 \;\uparrow\; \mp K\times\frac{\text{days(s)}}{\text{Basis}}\times A$$

$FRA\,(2)$ $\quad T_d$-----T_0----------T_1---------- $\quad T_2$

$$\pm L_{T_2,T_2+m}\times\frac{\text{days(s)}}{\text{Basis}}\times A \;\downarrow\; T_2 \;\uparrow\; \mp K\times\frac{\text{days(s)}}{\text{Basis}}\times A$$

$FRA\,(3)$ $\quad T_d$-----T_0----------T_1---------------T_2-------- $\quad T_3$

$$\pm L_{T_2,T_2+m}\times\frac{\text{days(s)}}{\text{Basis}}\times A \;\downarrow\; T_3 \;\uparrow\; \mp K\times\frac{\text{days(s)}}{\text{Basis}}\times A$$

Thus, the value of a "plain vanilla" interest rate swap is *equal to the sum of a series of (non–discounted)* $FRAs$.

$$V_{SW} = \sum_{i=1}^{N} FRA_i$$

- Interest rate swap: Principal or bond–equivalent method

$$\begin{aligned} V_{SW} &= B_{fl} - B_{fix} \\ &\text{or} \\ V_{SW} &= B_{fix} - B_{fl} \end{aligned}$$

where B_{fl} and B_{fix} the values of the fixed–rate and the floating rate bonds underlying the swap.

The values of the bonds underlying the swap deal can be calculated by

$$B_{fix} = \sum_{i=1}^{T} F_{T_i} e^{-L_{0,T_i} T_i} + A e^{-L_{0,T_T} T_T}$$

where A the notional principal, T_i the time when the i^{th} payment is exchanged $(1 \leq i \leq T)$, L_{0,T_i} the LIBOR zero rate today (at the value date) for a maturity T_i and F_{T_i} the fixed payment made on each date (see equation (1)).

$$B_{fl} = \left(A + L_{T_1}\right) e^{-L_{0,T_1} T_1}$$

where L_{T_1} is the next floating rate payment (which is already known at T_0) defined in (2)

- EXAMPLE

Financial institution pays 6m LIBOR flat and receives 8% per annum (with semi–annual compounding) on notional principal of \$100 million. Continuously compounded LIBOR for 3m, 9m and 15m are 10%, 10.5% and 11% respectively. The 6m LIBOR on last payment date was 10.2%. What is the value of the swap with remaining life 1.25 years?

$$\begin{aligned} F_{T_i} &= A \times R_F \times \text{Day Count Fraction} \\ &= 100M \times 8\% \times 0.5 = \$4M \end{aligned}$$

$$\begin{aligned} L_{T_{t+1}} &= A \times L_{0,6M} \times \text{Day Count Fraction} \\ &= 100M \times 10.2\% \times 0.5 = \$5.1M \end{aligned}$$

Day Count Fraction[§]

$$\begin{aligned} B_{fix} &= F_{T_{3m}} e^{-L_{0,3M}\frac{3}{12}} + F_{T_{9m}} e^{-L_{0,9M}\frac{9}{12}} + \left(F_{T_{15m}} + A\right) e^{-L_{0,15M}\frac{15}{12}} \\ &= 4e^{-0.10\times 0.25} + 4e^{-0.105\times 0.75} + 104e^{-0.11\times 1.25} \\ &= \$98.24M \end{aligned}$$

$$\begin{aligned} B_{fl} &= \left(A + L_{T_{3m}}\right) e^{-L_{0,3M}\frac{3}{12}} \\ &= (100 + 5.1)\, e^{-0.1\times 0.25} = \$102.51M \end{aligned}$$

$$V_{SW} = 98.24 - 102.51 = -\$4.27M$$

[§]A more precise calculation would require accounting for the day count convention, e.g. Actual/360.

- Currency swaps

$$V_{SW} = B_D - S_0 B_F$$

or

$$V_{SW} = S_0 B_F - B_D$$

where B_F the value, measured in the foreign currency, of the foreign–denominated bond underlying the swap, B_D the value, measured in the domestic currency, of the domestic–denominated bond underlying the swap, and S_0 is the current spot exchange rate (no. of domestic–currency units per unit of foreign currency).

14 Other swaps

- Different floating reference rate
  - 3m LIBOR, Treasury bill rate, etc.
- Amortizing swap
  - The principal reduces through the life of swap in a predetermined way.
- Step–up swap
  - The principal increases through the life of swap in a predetermined way.
- Compounding swap
  - Only one cash flow exchange, at the end of the life of the swap. Intermediate payments are calculated and compounded forward to the maturity date.

- Equity swap
  - One party promises to pay the return on an equity index on a notional principal.
- Forward–starting swap
- Currency coupon swap
  - Exchange fixed in one currency for floating in another currency
- Cancelable swap
  - One side has the option to terminate one or more payment dates
- Commodity swap, Volatility/Variance/Correlation swaps, Swaptions, etc.

15 Reading

- Hull [1], chapter 6.
- Jarrow and Turnbull [2], chapter 14.
- Stulz [3], chapter 16.
- Sundaresan [4], chapter 16 (pp. 559–594)
- Hull [1], chapter 25 (advanced reading).

References

[1] J. C. Hull. *Options, Futures and other Derivatives*. Prentice Hall Inc., Upper Saddle River, New Jersey, 5th edition, 2003.

[2] R. A. Jarrow and S. Turnbull. *Derivative Securities*. South–Western College Publishing, Thomson Learning, 2nd edition, 2000.

[3] R. M. Stulz. *Risk Management and Derivatives*. South–Western College Publishing, Thomson Learning, 2003.

[4] S. Sundaresan. *Fixed Income Markets and their Derivatives*. South–Western College Publishing, Thomson Learning, 2nd edition, 2002.