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ΕΠΙΒΛΕΠΩΝ ΚΑΘΗΓΗΤΗΣ **ΥΠΟΓΡΑΦΗ**

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ΒΕΒΑΙΩΣΗ ΕΚΠΟΝΗΣΗΣ ΔΙΠΛΩΜΑΤΙΚΗΣ ΕΡΓΑΣΙΑΣ

«Δηλώνω υπεύθυνα ότι η συγκεκριμένη πτυχιακή εργασία για τη λήψη του Μεταπτυχιακού Διπλώματος Ειδίκευσης στη Λογιστική και Χρηματοοικονομική έχει συγγραφεί από εμένα προσωπικά και δεν έχει υποβληθεί ούτε έχει εγκριθεί στο πλαίσιο κάποιου άλλου μεταπτυχιακού ή προπτυχιακού τίτλου σπουδών, στην Ελλάδα ή στο εξωτερικό. Η εργασία αυτή έχοντας εκπονηθεί από εμένα, αντιπροσωπεύει τις προσωπικές μου απόψεις επί του θέματος. Οι πηγές στις οποίες ανέτρεξα για την εκπόνηση της συγκεκριμένης διπλωματικής αναφέρονται στο σύνολό τους, δίνοντας πλήρεις αναφορές στους συγγραφείς, συμπεριλαμβανομένων και των πηγών που ενδεχομένως χρησιμοποιήθηκαν από το διαδίκτυο».

ΟΝΟΜΑΤΕΠΩΝΥΜΟ ΦΟΙΤΗΤΗ ΥΠΟΓΡΑΦΗ

ΧΡΗΣΤΟΣ ΚΑΛΑΤΗΣ

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ABSTRACT

According to one-factor option pricing models, the option price is monotonically increasing or decreasing and perfectly correlated with the underlying asset price. In this thesis, I empirically test the validity of these two properties using daily data from the S&P 500 index option for the period between 2004 and 2008. I found that option prices systematically deviate from being monotonically increasing or decreasing and perfectly correlated with the underlying asset price. I attempt to explain these violations. One possible explanation is that the underlying asset price follows a two-dimensional diffusion process, where the second process is stochastic volatility. In that case the monotonicity property can be violated. Since option price changes also depend on the changes in the spot variance. The second explanation lies on the liquidity of the option market. In that case, violations occur because option market is illiquid.

1. Introduction

The option pricing models presented in most textbooks are one-factor formulas with the price of the underlying asset being the only source of uncertainty. Some examples of them are the Black and Scholes (1973), Merton (1973), Cox and Ross (1976) and Rubinstein (1994) models. Since they assume that the underlying asset price follows a one-dimensional diffusion process, they share two basic properties. The first one is the monotonicity property, according to which call prices and put prices are increasing/decreasing monotonically in the underlying asset price. The second one is the perfect correlation property, according to which call and put prices are perfectly correlated with the underlying asset price, since it is the only stochastic process of the model. In so doing, this thesis aims to empirically examine the monotonicity property. I specifically address the following question: Do call prices move on the same direction and put prices on the opposite direction with the underlying asset? If no, how often does this happen? Are the violations rates different across moneyness and maturity categories?

For my study I use data for the period from 2004 to 2008 from the S&P 500 option index which is one of the most active worldwide, taking the midpoint of the bid-ask spread using daily observations. I use daily data for two main reasons. The first one is to minimize the impact of time decay which is inevitable in option pricing. The second one is that since I want to test option prices movements of different sign than that predicted by one-factor models, I want to minimize the interval between two observations.

For the purpose of testing the monotonicity and perfect correlation properties I use four type of errors like the ones introduced by Bakshi,Cao and Chen (2000). The first one, type I error, occurs when the call price moves on the opposite direction with the underlying asset price, and the put price moves on the same direction. The violation rate for the calls for the whole period is 12.75%, ranging from 11.96% to 14.98% across years, while the equivalent rate for puts is 10.08% for the whole period, ranging from 5.98% to 12.78% across years. These violation rates differ for the different types of moneyess and maturity. Of a given maturity, out-of-the-money (OTM) calls present the highest violation frequency, followed by at-the-money (ATM) calls while the lowest rates are observed for in-the-money (ITM) calls. When puts are used instead of calls, we end up with same conclusion. In terms of maturity, short-term calls present the highest violation frequency, followed by medium-term, while long-term calls present the lowest frequency. When puts are used, short-term are the most frequently violated followed by long-term puts, and medium-term puts present the lowest violation frequency. I also test the magnitude of the changes of the options and the index when these violations occur, and I found that they are both statistically significant and higher than the minimum tick size for all categories of moneyness and maturity. Finally, I test how often call and put prices move on the same direction, and I found that for the whole period the rate was 8.21% ranging from 7.59% to 8.84% across years. What is more interesting is that given that call and put prices move together, they are more likely to go down together than up together.

The second error, type II, occurs when the option price does not change when the underlying asset price changes. This error type is quite rare, since it occurs 1.72% in any sample for calls while the equivalent rate for puts is 1.69%. It is more frequent in OTM options mostly because of their low delta that makes them less sensitive to underlying asset price changes. The third error, type III, occurs when option price changes while the index does not change. But, the index remained the same only once in the sample period, so there is no point to further address this error type.

The last error, type IV, occurs when option prices overadlust to the underlying asset price changes. Specifically, this error occurs when the magnitude of an option price change is higher than the magnitude of the underlying asset price change, given that there is no type I error. This type of error violates the bounds of an option delta, which cannot be higher than 1 for calls and lower than -1 for puts. Calls present a type IV violation frequency of 6.66% for the whole sample period, ranging from 3.10% to 9.96% across years, while the equivalent rate for puts is 6.42%, ranging from 4.92% to 10.33%. When type IV error is tested across moneyness, ITM options are more likely to present this error, mostly because of their higher delta values which are close to one, followed by ATM and OTM options. In terms of maturity, short-term options are more likely to present a type IV error.

In my attempt to explain the violation, I introduce another stochastic variable to the one-factor option pricing model. The second variable is volatility, which is negatively correlated with the price and could possibly explain why calls move on the opposite and puts on the same direction with the underlying asset. My measure of volatility is the implied volatility (IV) of the ATM options, calculated by solving Black and Scholes (1973) formula with respect to volatility. Next, I calculated the sensitivity of the option prices to the underlying asset price changes and volatility changes, using the delta and vega from Black and Scholes (1973) model. Comparing the theoretical sign change with the observed sign change of the option price, I tested if stochastic volatility can explain the violations. It actually explains 42.46% of calls and 35.58% of puts type I errors. The explanatory power of stochastic volatility is higher for OTM options, while it explains better long-term calls and short-term puts. For type IV errors, I tested if the magnitude of the theoretical change of the option is higher than the underlying asset price change. Stochastic volatility explains 23.73% of calls and 40.71% of puts type IV errors. Its explanation validity is higher in OTM calls and ATM puts, while in terms of maturity it explains better shot-term calls, and its explanatory power appears to increase with the maturity of puts. I also tested if the violations can be explained by liquidity factors of the contract such as the trading volume, the open interest and the bid-ask spread. These factors seem to have a greater impact on type IV and II errors compared to type I, while the spread seems to be the most significant one.

The rest of this thesis is organized as follows. Section 2 offers a literature review on this topic. In section 3, I briefly address the basic theoretical properties of the options under a one-dimensional stochastic process. Section 4 describes the data while section 5 presents the empirical results of this study. Section 6 introduces a two-dimensional stochastic model used to price options. In section 7 I test if the violations can be explained by liquidity factors. Section 8 concludes this thesis.

2. Literature Review

Bakshi,Cao and Chen (2000) conducted one of the first studies to test empirically the monotonicity property implied by one-dimensional diffusion processes, calls(puts) move on the same(different) direction with the underlying asset. Besides the monotonicity property, they also tested other two well-known properties of the options: the perfect correlation between options and the underlying asset and the redundancy property, according to which a combination of a risk-free asset and the underlying asset can replicate the option. They used intraday data of the S&P 500 index option for the period March 1, 1994 to August 31, 1994 to minimize the effect of time decay and used the mid-point of the bid-ask spread. They found that calls present a type I violation frequency between 7.2% and 16.3%. The results are similar for puts. When interday data were used the violation rate decreased and they attributed it to time decay. While there is a connection between violation rates and maturity, there is no connection between moneyness and violation rates. They also found that regardless of the change in the underlying asset, calls and puts with the same strike and maturity tend to go down together more often than up together. Type II violation rates range from 3.5% to 35.6%, type III error is quite rare while type IV error frequency is at 11.7% for calls and 13.7% for puts. They suggested four possible explanations of the reported violations. First, market microstructure factors which can explain type IV and II violations but not type I. Second, put-call parity violations (but only 3% of the data that show any type of violation, violate the parity too). Third, the impact of time decay, which is larger for interday data compared to intraday ones, but even in the interday ones it is not large enough to explain the magnitude of the violations. Fourth, they introduced a two-factor stochastic process for the underlying asset price. The second variable should not be perfectly correlated with the price of the underlying asset, so that the discrepancies in the monotonicity property could be explained. They used the stochastic volatility (SV) model (see Heston (1993)) because volatility is negatively correlated with the price, when volatility increases(decreases) the price of the underlying asset decreases(increases) but also volatility has a different impact on the option price than the price of the underlying asset. The results from the simulations they conducted with the stochastic volatility (SV) model, showed that 11% of the calls move on the opposite direction with the underlying asset changes. The SV model explained 47% of type I violations but it was sort in type II and IV and it could not explain precisely the movements of the observed option prices. They finally examine the implications of these notations on hedging and showed that hedging can have different results from the ones expected and known in the textbooks because losses can be doubled if the call(put) and the underlying asset move on the opposite(same) direction. They also tested if the hedging error decreases with the increase of the hedging frequency, without considering transaction costs, and although it decreases initially, after a certain point of frequency it actually increases.

Norden (2001) used daily equity American style data from the Swedish option market and the Stockholm Stock Exchange from July 1, 1995 to February 1, 1996 to test the basic properties of American options as presented in the textbooks. This study differs from the others because it uses equity options instead of index options. In equity options the underlying asset is actually traded, whereas in index options the underlying is a non-traded asset. The implication from this difference is that it is easier to replicate the equity option using the underlying and a risk-free asset, but also in terms of hedging, the equity option is a more suitable hedging instrument compared to the index option. Another difference is that the liquidity in the Swedish market is far from close to the equivalent US market or any other highly liquid market, so differences in the violation frequency are expected compared to the highly liquid markets. If Bakshi et al. (2000) findings about liquidity are correct, with this lack of liquidity, higher violation frequencies are expected. Norden (2001) found that calls(puts) move on the opposite(same) direction with the underlying asset 8.75%(9.18%) of the time. The results for type II and type III violations for calls(puts) are 4.09%(5.34%) and 6.16%(5.95%), respectively. Type IV violation frequency was 12.05% for calls and 11.22% for puts, but he observed that as the moneyness of the option increases the type IV violation frequency also increases, peaking in ITM options. Unlike the initial expectations, his results from an exchange with limited liquidity, are quite similar to the ones obtained by Bakshi et al. (2000) from a highly liquid market. The only difference is in type III violation, where Bakshi et al. (2000) results are not significant because the S&P 500 index rarely remains unchanged on a daily basis. Norden (2001) also tested the results of delta-neutral and delta-vega-neutral strategies and the results were not those expected. His results can be explained by type I violations, in which case a trader's losses could be doubled instead of being neutralized. The performance of the delta-vega-neutral strategy was superior to the delta-neutral, as expected but yet far from a perfect hedge.

Perignon (2006) tested the monotonicity property of options that is implied by one-dimensional diffusion models. He used option data from five different indices, the European (DJ EURO STOXX-50), British (FTSE 100), French (CAC 40), German (DAX) and the Swiss (SMI) stock indices for the year 2002. Consequently, his sample is not uniform since the five indices differ with each other in terms of trading activity and liquidity. His intention was to test empirically the validity of the two main arguments that are supposed to explain the violations in the monotonicity property. He tested the stochastic volatility and the microstructure biases as possible explanatory factors and furthermore if rational trading tactics can be attributed as the causes of the violations. Unlike previous studies, Perignon (2006) did not use mid-point prices of the bid-ask spread, but the observed transaction prices instead, in order to address explicitly the impact of microstructure biases to the violations. As in other studies for different equity indices, Perignon (2006) found strong evidence against the monotonicity property. He addressed explicitly type I violation and his results indicate that depending on the sampling interval and the specific index, calls move on the opposite direction with the underlying asset 7-32% of the time. The results for puts are quite similar, since they move on the same direction with the underlying asset 6-35% of the time. He also found that the violation rate is more likely to decrease when the sampling interval increases and the trading volume of an option contract increases. His findings about the sampling interval have direct impact on the ideal hedging frequency, as they contradict the theoretical implications of the continuous-time models i.e., the more frequent the hedge, the better the performance of the strategy. When it comes to the explanations of the violations, his first candidate was a change in the fundamentals values of the underlying asset, and more specifically volatility. His findings confirmed his initial expectations, since a great part of the violations can be attributed to the stochastic volatility. In terms of microstructure effects, he examined whether a trade was buyer initiated or seller initiated, and the results support his hypothesis that bid-ask bounces can generate a great part of the monotonicity property violations. Last, he found that violation rates are higher at the close of a trading day and on Friday compared to the other days of the week.

Buraschi and Jiltsov (2006) tested the redundancy property of the options i.e., that they can be replicated by a position in the underlying asset and in a risk-free asset. If the redundancy property holds, traders would be indifferent about holding options, but this is far from true in the real world as the authors state, since the trading volume of options had been increasing in the last fifteen years. Their main focus is to connect the open interest of options with the heterogeneity in beliefs about the fundamental values of the underlying asset, using a model that imposes restrictions on the option price and the open interest. In their model, they allow the dividend growth rate to be stochastic and they consider rational agents with identical preferences but heterogeneous and incomplete information. Traders with different beliefs are expected to form different optimal portfolios affecting both the volatility smile and the risk premium. The optimistic traders will take a position in the OTM calls while the pessimistic ones will take a position in the OTM puts, as the authors suggest. They conducted their study using daily data from the S&P 500 index option from October 1986 to August 1996. They built a difference in beliefs index using the standard deviation from the Survey of Professional Forecasters and the Consumer Confidence Survey and they addressed the following five questions. First, to what extent open interest and option trading volume can be explained by the difference in beliefs among traders about the fundamental values of the underlying asset. Second, they compared the hedging performance of their model against the performance of one-factor, Black and Scholes (1973), and two-factor, Heston (1993), hedging strategies. Third, they tested the impact of the difference in beliefs on the implied volatility smile. Fourth, they examined the forecasting dynamic of difference in beliefs on the future volatility of the underlying asset. Fifth, they tested if the differences in beliefs can explain the violations of option pricing recorded by the empirical test of Bakshi et al. (2000). For the purpose of this thesis, I only address their results about the empirical test of Bakshi et al. (2000). Their results about violation rates were quite similar to those of Bakshi et al. (2000) since they found that call(puts) present a type I violation frequency 17%-24%(15%-22%) of the time, while type IV rate is between 4%-11% and 2%-4% for calls and puts, respectively. The results from the simulations they conducted using their own model, were pretty close to the observed type I violation frequency except from the long term ITM calls, where the model violation frequency was quite lower, 9.45% compared to 16.50%. But their model failed to replicate type IV violations frequency, probably because this type can be attributed to tick size effects as Bakshi et al. (2000) suggest. Finally, they tested their model performance after controlling for stochastic volatility but the significance of the difference in beliefs remained quite high.

Dennis and Mayhew (2009) examined how empirical tests of option pricing models are affected by noise in option prices attributed to microstructural factors. More specifically they examined the impact of microstructural noise on the monotonicity property of options as in Bakshi et al.(2000), the test of transition density diffusion of Ait-Sahalia (2002) the speed of convergence test of Carr and Wu (2003) and implied risk-neutral moment estimators of Bakshi, Kapadia and Madan (2003). For the purpose of this thesis, I emphasize on their findings concerning the impact of microstructural noise on the test of Bakshi et al. (2000). As they state in their study, low-priced stocks are expected to be more severely affected by noise compared to the high-priced stocks or the indices because the tick size is relatively large to the stock price. Unlike other studies, they did not use observed option prices to conduct their one study. The reason behind that is to eliminate the effect of microstructural noise. In order to test the impact of noise on the empirical tests of option pricing models, they obtained option data through simulations, using the Black and Scholes (1973) model, and added microstructural frictions in order to resemble real-world data. Using the option prices from the simulations, they conducted several regressions to test the sensitivity of type I. II and III violations to five parameters: the noise level, the maturity of the option, the stock volatility, the option moneyness and the interval between the observations of the sample. In terms of noise levels , the type I violation is highly sensitive and increases as the noise level increases. The results for type II(III) violation frequency indicate a negative(positive) relation with the noise level. They also found that the sensitivity to maturity decreases as the noise level increases and consequently type I and II violation frequency decreases. For type III violations, the results indicate that the sensitivity to maturity is not statistically significant. Stochastic volatility, seems to have a negative impact on the frequency of the three types of violations. The results for option moneyness indicate that as options move out of the money, the frequency of type I and II violations increases, while for type III it decreases. Finally, the magnitude of the sampling interval is negatively related with all three types of violation.

Fahlenbrach and Sandas (2009) tested the perfect correlation property between option prices and the underlying asset implied by the one-dimensional diffusion processes used to price options. Their work is based on the same methodology as that of Bakshi,Cao and Chen (2000), and they introduced two microstructure effects, namely trades and signed orders, that could potentially be used to explain the discrepancies of perfect correlation between movements of the option prices and movements of the underlying asset. The data they used is from mid-point prices of the bid-ask spread of European-style options and Futures on the Financial Times Stock Exchange 100 stock index (FTSE 100) for the period between August 1,2001 to July 30,2002. They conducted several simulations under an empirical stochastic volatility model, and the results showed that there is a strongly negative relation between trading volume and the magnitude of changes in the underlying asset price, but the magnitude of volatility changes should be strongly related with the trading volume. The hypothesis of negative relation between trades and innovations in the index contradict the results from the data. Despite that, the stochastic volatility model explains one-third of the sample that show violations from perfect correlation between options and the index. They tried to explain the other two-third of the violations of perfect correlation incorporating micro-structure effects. Using intraday data they showed that the relative appearance of violations of type I and IV in calls is double in trade intervals than no-trade intervals. The results for puts are quite similar for type I violation but they are tripled for type IV. They also tested what happened before and after a trade using a 30-minute window divided in 30 1-minute sub-windows. The minute before and after the trade the violation rate is actually quadruple than the other 28 intervals for both type I and IV for calls and puts. They attribute this discrepancy to aggressive and stale quotes. An aggressive sell limit order, probably because of liquidity problems of the seller, can reduce the best ask price by six ticks on average, causing the mid-point to reduce too, despite the increase in the best bid price. Same results can be produced by stale quotes, if the futures quotes change by several ticks but the option quotes do not change also, it is possible that options quotes overadjust (causing a type IV violation), or even move in the opposite(same) direction for calls(puts) with the change in the futures quotes(causing a type I violation).

Lin, Chen and Tsai (2011) tested the monotonicity of options based on the assumption that the underlying asset price follows a one-dimensional diffusion process. They used intraday data from the Taiwan Futures Exchange (TAIFEX) from July 1, 2006 to December 1, 2006. TAIFEX is one of the most active index options in the world based on the amount of contracts that is traded every day. Their study intends to answer the questions of how often calls and puts violate the monotonicity property in the TAIFEX and to what extent these violations can be attributed to microstructure effects. Except from microstructure factors they test the validity of rational trading tactics and stochastic volatility as possible candidates to explain the violations. The findings of their study show that the rate of type I violation is 22.11% for calls and 18.74% for puts, while option prices are more likely to decrease rather than increase when violations occur and a put-call pair with the same strike and maturity is more likely to go down together than up together regardless the change in the underlying asset. The TAIFEX options present a type II violation of 4.17%(4.68%) of the time for calls(puts). Two possible explanations given by Lin,Chen and Tsai (2011) are that the magnitude of the underlying price change might not be enough to trigger a move in the option or that stale quotes do not allow the option index to change as fast as the underlying. Type III violation frequency is quite small because the spot index is highly liquid and as a result rarely remains unchanged. Option prices overadjust (type IV violation) 19.68% of the time for calls and 16.7% for puts. Comparing their results to the ones from Bakshi et al. (2000), type I violation is less frequent for the TAIFEX than the S&P 500, and they did not find any connection between the maturity or the moneyness of the options and the violation rates. Their results also support the hypothesis that stochastic volatility can explain a big part of the violations. Their results also suggest that violations can be attributed to microstructure factors. More specifically, the violation rates are highest during the middle of the day from 10:45 a.m. to 11:45 a.m.. Finally, their results indicate that the violations are more likely to occur right before the option market closes or on Fridays. Also they showed that the relation between the violation frequency and the trading volume is actually negative.

Hilliard (2013) tested the observed price changes of options using the Greeks (delta, gamma and theta) as regressors. In order to minimize the effect of gamma and theta she used 1-hour intervals with observations from the S&P500 from January 1998 to December 2006. In order to calculate the Greeks, she used the American version of the binomial model (ABM). Besides traditional forms of volatility such as historical and implied volatility, she incorporated another measure of volatility, price change implied volatility. It is close to implied volatility but instead of focusing on price levels it focus on price changes. Her results are robust even when Black (1976) pricing model for futures was used to estimate the Greeks. She used S&P 500 futures as the underlying asset for the following reasons. First, they are highly liquid, second they are traded directly, third they do not suffer from strict arbitrage considerations or stale quotes, and finally they reflect the traders' believes concerning future dividends. She attributed a lower than 100% R^2 found in her regression as an indication of the existence of a type I violation. She attributes these violations to stochastic volatility, nonsynchronous trading or stale quotes, the segmentation between the underlying asset and the derivative market and other omitted variables from the regression model. When the price change implied volatility is included as a regressor, R^2 coefficient is maximized. Despite however the high R^2s from the regressions there is sufficient evidence to reject the ABM model because the coefficients of delta for calls were below one while those of puts were above one, which comes in contradiction with the null hypothesis that the coefficients of delta should be one both for calls and puts. The frequency of type I violations was close to that reported by Bakshi et al. (2000). She also observed that by increasing moneyness, the violation rate tends to decrease, which seems to hold for all maturity classes. In terms of maturity, type I violation frequency is lower for long-term options, with short-term ones exhibiting the highest violation frequency.

Pan, Shiu and Wu (2014) examined the violation of monotonicity property under the assumption that the option price follows a one-dimensional diffusion model. They used option data from the Taiwan Stock Exchange Capitalization Weight Stock Index (TAIEX), which are European-style options traded on the TAIFEX, from 2005 to 2013. Their main goal was to test to what extent these violations can be attributed to stochastic volatility and demand pressure effects into option pricing as introduced by Garleanu, Pedersen and Poteshman (2009). Their measure of stochastic volatility was the average implied volatility calculated from the ATM options, while their measure for demand pressure is the one introduced by Bollen and Whaley (2004) because the TAIFEX is an order driven rather than a quote driven market. They tested only for type I violation and they found that the frequency of this violation was significantly high. Specifically, they found that for both calls and puts the violation rate is close to 34%. Their hypothesis about the possible factors that can explain the violations, namely stochastic volatility and demand pressure, were confirmed by the results of their study. Specifically stochastic volatility explains 54% of the violations for calls and 52% for puts. These findings suggest that a multi-factor model is probably more suitable for option pricing than a one-dimensional diffusion model, and the second factor that should be considered in the model is stochastic volatility. When demand pressure is considered as a possible factor, the explanatory ratio is even higher, since 62% of the violations can be explained by demand pressure. The percentage of the violations that cannot be explained by stochastic volatility or demand pressure is 18%. They conclude that these two factors are the dominant ones when it comes to explaining the violation. Finally, they tried to connect the moneyness of the options and the explanatory superiority of each factor. According to their findings, stochastic volatility is better at explaining the violation of ATM options, while demand pressure explanatory ratio is superior in the OTM and ITM options. OTM options violations are expected to be driven by demand pressure effects, since individual traders mostly see them as buying lotteries and they are willing to buy them regardless the movement in the price of the underlying price.

Sim, Ryu and Yang (2015) investigated the violations of monotonicity and perfect correlation properties according to a one-dimensional diffusion option pricing models using data from Korea Stock Exchange 200 (KOSPI 200) index option, an emerging market which shows high liquidity and participation of many individual traders. They found evidence against both monotonicity and perfect correlation. Some of the violations can be explained by individual traders, which implies that they are connected with demand pressure effects and limits to arbitrage in the options market. They suggest that another variable should be taken into account that is not perfectly correlated with the underlying price, namely stochastic volatility. According to them other parameters besides stochastic volatility such as the absence of continuous trading, transaction costs as well as jumps in the underlying prices make perfect hedge impossible. They also make the hypothesis that option prices can be influenced by demand pressure apart from the changes in their fundamental values. Consequently, they reexamine the violations as proposed by Bakshi et al. (2000) and examine whether they can be attributed to individual traders who are noisy and less informed than institutional ones. Their findings can be summarized as follows. First, in a highly liquid market such as KOSPI 200, the violation rates of monotonicity and correlation properties are close to that reported in previous studies. Using mid-point bid-ask prices for 5-minute intervals they found that calls(puts) present a type I violation 18.78%(17.79%) of the time. As far as type II and IV violations their findings for calls(puts) are 22.58%(24.05%) and 11.61%(10.89%) respectively. Second, they found that the violation frequency is connected to the moneyness of the option which comes in contrast with the findings of Bakshi et al. (2000). Specifically, ITM options are more vulnerable than OTM options to type I violation. They also found that OTM options do not change so often when the underlying price has changed compared to ITM options (type II violation), while ITM options overadjust more often than OTM (type IV violation). Third, OTM options that are heavily traded by individual traders present the most frequent type I violation frequency. They explain it by demand pressure effects, because those uninformed traders see the OTM options as buying lotteries. Finally, they provide evidence that KOSPI 200 index options seem to be more vulnerable to stale quotes than S&P 500 index options

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3. <u>Basic Properties of Options</u>

In this section I briefly introduce the basic properties of options assuming that the underlying price follows a one-dimensional stochastic process.

3.1. Stochastic Process

The term stochastic process can be attributed to any variable whose value can change over time in a way that cannot be predicted with absolute accuracy, for example the value of a stock the next day. Stochastic processes can be classified according to time and variable. Discrete time are the ones that are allowed to change only is specific time intervals, while a continuous time process may change at any time. In terms of variable, a process is continuous when it can take any price within a range, while discrete variables are the ones that are allowed to take only discrete values.

In strict terms, stock values are neither continuous time nor continuous variable stochastic processes. First of all, trading in an organized exchange takes place only during certain time of the day, usually in the morning hours, while stock prices are observed up to the second decimal digit. But with certain simplifications, stock prices can be characterized as continuous time and continuous variable processes. During a trading interval, accounted as a trading day, stock prices can change at any time since anyone can place his order to buy or sell a stock whenever wanted to. While traders are not obliged to place discrete prices for their orders, they can buy or sell at any price in the two decimal digits range, so stock prices can be characterized as continuous time processes.

Under the assumption of weak-form market efficiency, we can say that stock prices follow a Markov process. More specifically, a market is called weak-form efficient when current prices reflect all the available information contained in past prices. This is quite reasonable, because there are many traders occupied with technical analysis, that if there was a specific pattern in a stock price they would have probably taken advantage of it and eliminate it. Besides that, there is no evidence of abnormal returns from trading on the information from past prices. On the other hand, a Markov process is a stochastic process where future values can be predicted only by using current values. Consequently, Markov process is quite close to the weak-form of market efficiency.

A specific type of Markov processes is used to describe the stock price movements known as Wiener process or Brownian motion, the term used to describe the process is physics. A variable z that follows a Wiener process, has the following two properties:

1. For a small period of time, Δt, the change, Δz, of the variable is

$$\Delta z = \varepsilon\sqrt{\Delta t} \tag{1}$$

where $\varepsilon \sim N(0,1)$

2. For any short time interval, Δt, the changes of the variable, Δz, are independent with each other.

The first property of the Wiener process implies that a variable has a mean change of zero and a variance rate of 1.0, while the second property ensures that the variable follows a Markov process.

The process described in equation (1) implies that the underlying variable has a drift rate of zero and a variance rate of 1.0. These values of the underlying variable are far from the ones that can be used to describe a stock. We know that a stock has a drift rate different from zero and a variance rate different from one. Consequently, the process described in equation (1) cannot be used to describe the movements of a stock price. So another equation needs to be introduced for the movement of the stock price which is known as generalized Wiener process or arithmetic Brownian motion and is the following one:

$$\Delta x = a\Delta t + b\Delta z \tag{2}$$

where a and b are constants or

$$\Delta x = a(x,t)\Delta t + b(x,t)\Delta z \tag{3}$$

where a and b are functions of the underlying price and the time. Equation (3) is also known as Ito's process. In both equations (2) and (3) Δz is a Wiener process as described in (1).

The problem with the arithmetic Brownian motion, model (2), is that both the drift and the variance rates do not depend on the price of the underlying variable. That comes in contradiction with reality. More specifically, a stock with a higher price is expected to have higher drift and variance rates, in absolute terms, compared to a stock with lower price. But the arithmetic Brownian motion fails to incorporate the relative size of the rates according to the underlying stock price. Moreover according to the arithmetic Brownian motion, the underlying price can even have negative values.

So, a new model was developed in order to describe the movements of stock prices. It is quite similar to the ones described in equations (2) and (3) allowing the drift and the variance rate to depend on the level of the stock price. Consequently, under this model stock values are non-negative. This model is known as geometric Brownian motion and is described algebraically below,

$$\Delta S = \mu S \Delta t + \sigma S \Delta z \qquad (4)$$

where S is the underlying stock price, ΔS is the change of the stock price, the parameters μ,σ are the expected values for the rate of return and volatility of the stock respectively and Δz is a Wiener process as described in (1).

Equation (4) can be written as

$$\Delta S = \mu(S,t) S \Delta t + \sigma(S,t) S \Delta z \qquad (5)$$

allowing the expected rates of the return and volatility to be functions of the underlying price and the time.

3.2. Option pricing

The payoff from a European call(put) option contract is the positive difference between the price of the underlying asset(strike price) at the expiry date of the contract and the strike price(the price of the underlying asset). Figure 1 and 2 present the payoff function for a European call and put respectively. Under the no arbitrage principle, the discounted expected payoff gives the current price of the options. Algebraically, the payoff function is the following:

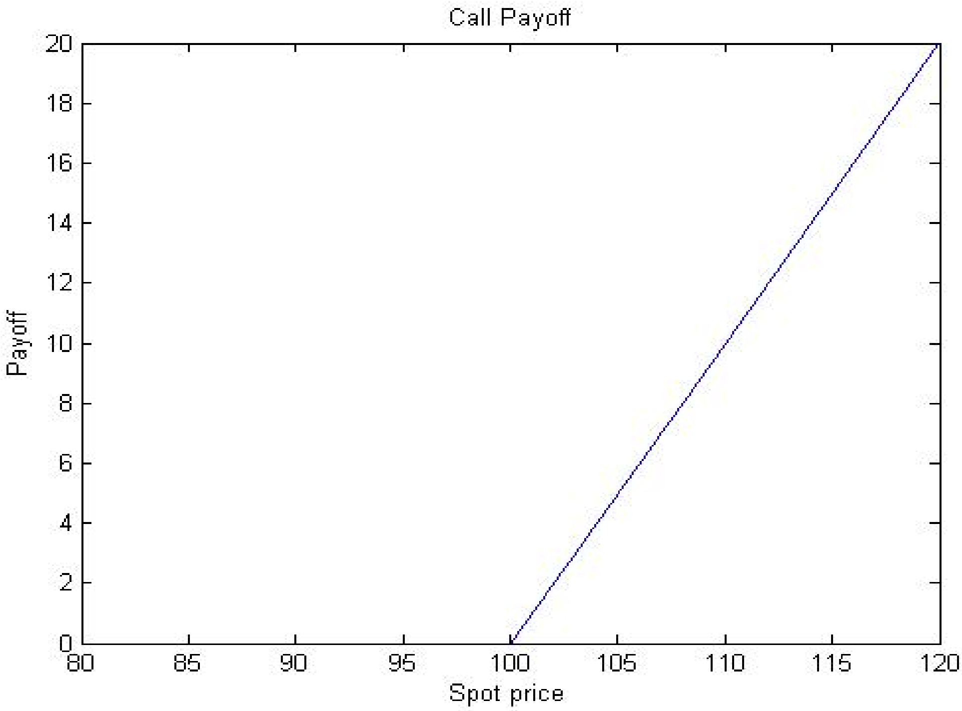
$$C=e^{-rT}E[\max(S_T-K,0)], \text{ for calls and} \quad (6)$$

$$P=e^{-rT}E[\max(K-S_T,0)], \text{ for puts} \quad (7)$$

where C(P) is the price of a call(put) option, r is the discount rate, T is the time to maturity, S_T is the price of the underlying asset at the expiry date and K is the strike price of the option.

Figure 1

Call payoff

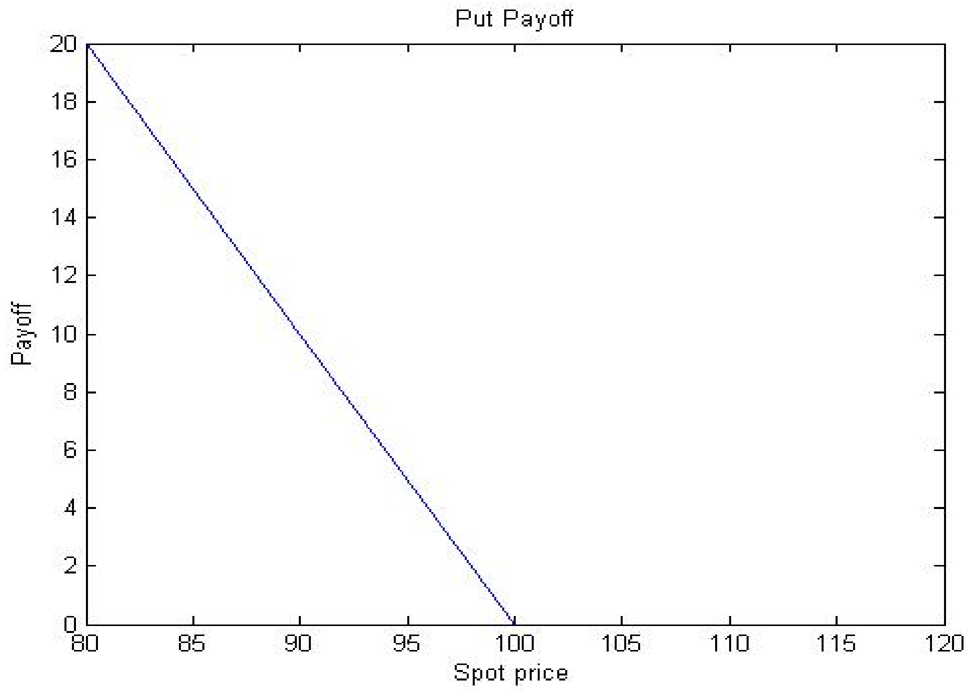


Payoff of a call option at maturity with strike price=100.

While it seems straightforward to price an option based on equations (6) and (7) this is not the case. Certain parameters might be known and observed such as the strike price and the time to maturity. The future price of the underlying stock might be neither observed nor known at the present, but someone can conduct several numbers of simulation for the stock price based on equations (4) and (5), using the fundamental parameters of the stock (average return and volatility) and obtain an approximation of the price level at expiration. After having found the approximate price at expiration, we know what is the payoff for the option, but not in present values. The problem is the rate that should be used as a discount rate.

Figure 2

Put payoff



Payoff of a put option at maturity with strike price=100.

Black and Scholes (1973) and Merton (1973) with their pioneering work gave a solution to that problem. They constructed a portfolio with a position in the option and the underlying stock. Specifically they proposed a long position in the underlying asset and a short(long) position in the call(put), what is known today as hedging. This portfolio is not vulnerable to the move either of the stock or the option. Any losses on the stock(option) would be offsetted by the gains from the position in the option(stock).

What they constructed was a riskless portfolio, which helped them to solve the problem of the discount rate in equations (6) and (7). In the absence of arbitrage opportunities, the return of this portfolio would be the risk-free rate. Consequently, they could use as the discount rate the risk-free rate itself. So, under the assumption that the underlying stock follows a geometric Brownian motion as described in equations (4) and (5), but with the difference that the drift rate is not the average return of the stock but the risk-free rate as described below,

$$\Delta S = rS\Delta t + \sigma S\Delta z \qquad (8)$$

and incorporating Ito's lemma (1951) they derived the following formulas for European call and put option prices.

$$C = S_0e^{-qT}N(d_1) – Ke^{-rT}N(d_2) \quad \text{and} \qquad (9)$$

$$P = Ke^{-rT}N(-d_2) - S_0e^{-qT}N(-d_1) \qquad (10)$$

where $d_1 =(\ln(S_0/K) + (r - q + \sigma^2/2)T) \div \sigma\sqrt{T}$

and $d_2 =d_1- \sigma\sqrt{T}$

S_0 is the price of the stock at T=0, q is the dividend yield and N(x) is the cumulative probability distribution function.

3.3. Options price changes with respect to underlying price changes

In order to calculate the rate of change of the option price to the price of the underlying asset, one can differentiate equations (9) and (10) with respect to the stock price, $\frac{\partial C}{\partial S}$ and $\frac{\partial P}{\partial S}$, and the results are as follows, for calls and puts, respectively:

$$C_s = e^{-qT}N(d_1) \qquad (11)$$

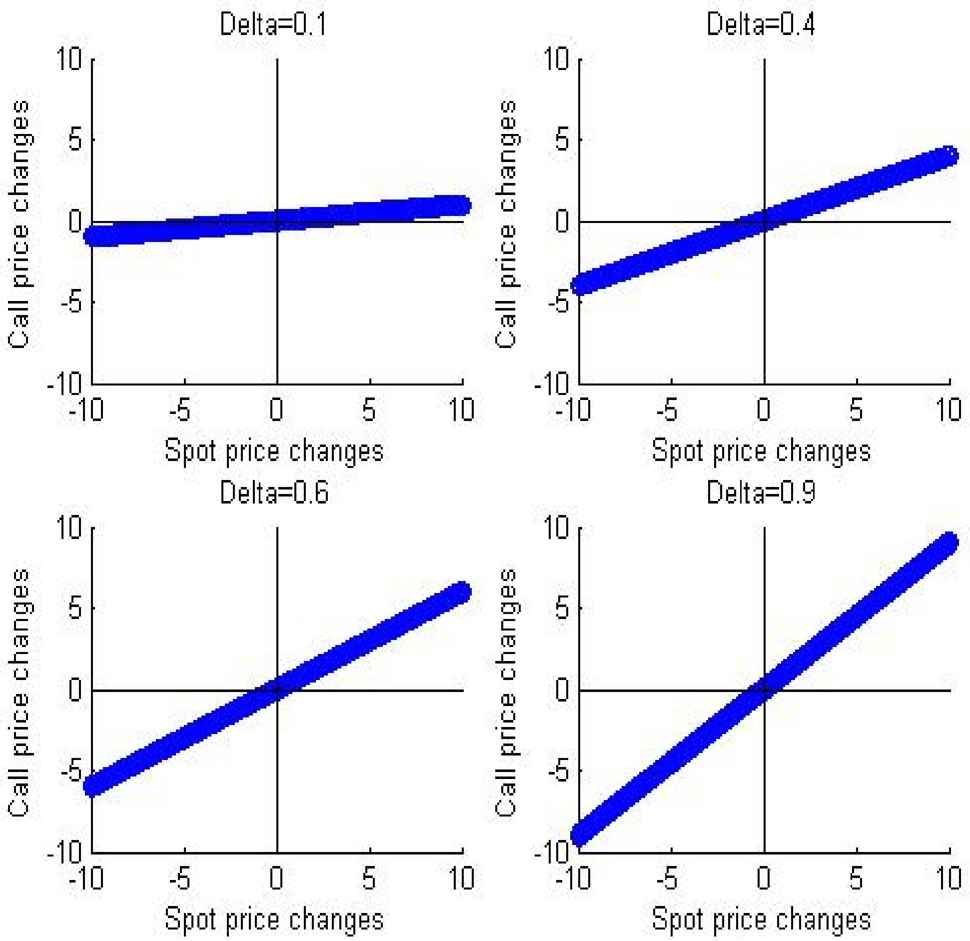
$$P_s = e^{-qT} (N(d_1) -1) \qquad (12)$$

where C_s and P_s are the first partial derivatives for calls and puts, respectively, with respect to the stock price, known as the delta of the option.

From equations (11) and (12), specific upper and lower bounds can be set for the deltas of the call and the put options that follow a one-dimensional diffusion process where only the underlying stock price is allowed to be a stochastic process, as the one proposed by Black and Scholes (1973) and Merton (1973). Since the values of the cumulative probability distribution function are strictly between zero and one, $N(x)\epsilon[0,1]$, the following bounds are set to the option delta,

Figure 3

Call price changes with respect to stock price changes



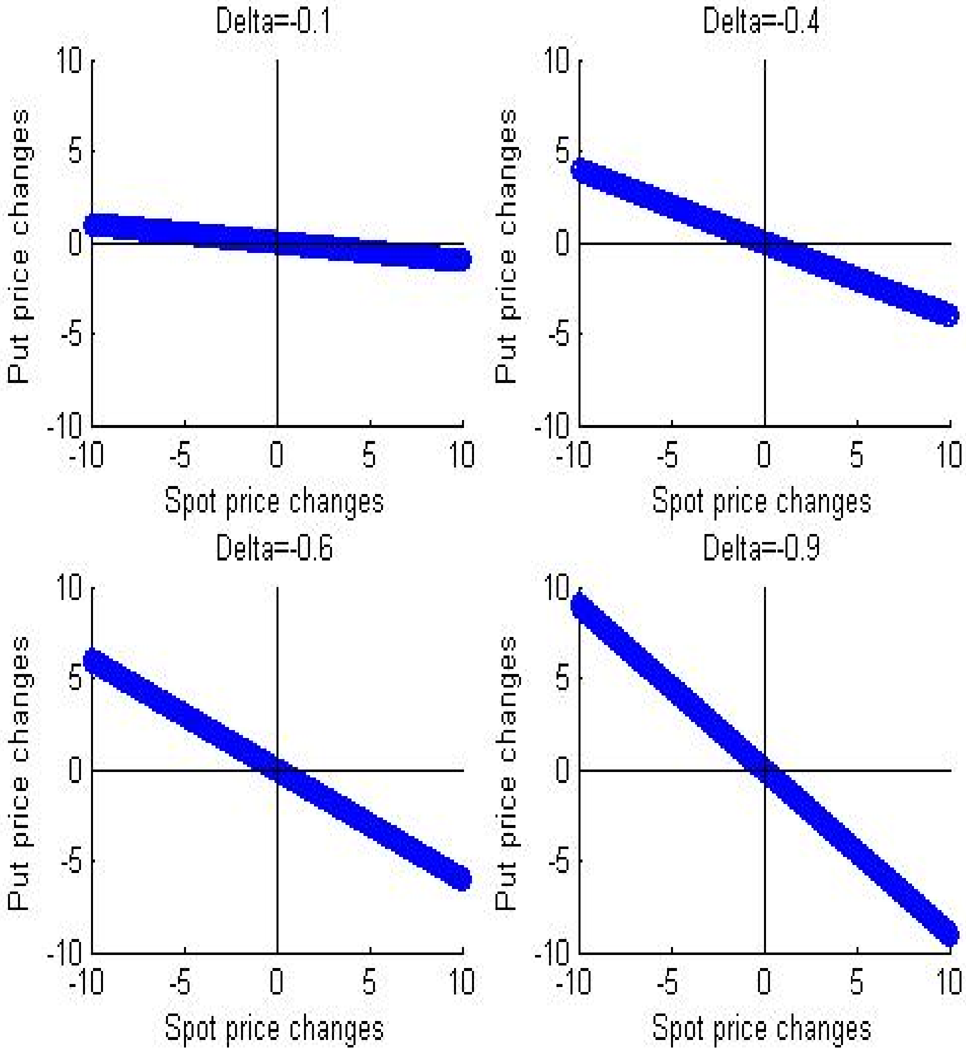
For the stock price changes, the call price changes have been plotted based on the elasticity of the call to the stock price, the Delta. I have used 4 different deltas from 0.1 to 0.9.

$$0 \leq C_s \leq 1 \tag{13}$$

and $$-1 \leq P_s \leq 0 \tag{14}$$

Figure 4

Put price changes with respect to stock price changes



For the stock price changes, the put price changes have been plotted based on the elasticity of the put to the stock price, the Delta. I have used 4 different deltas from -0.9 to -0.1.

From equations (13) and (14) we conclude that the price of a call(put) option is a non-decreasing(non-increasing) function of the underlying asset's price. This is known as the monotonicity property of the options that follow a one-factor model. Consequently, call prices are expected to move in the same direction with the movement of the underlying price, while put prices are expected to move in the opposite direction. In figures 3 and 4 we see the changes of call and put prices, respectively, with respect to the underlying price. The higher the sensitivity of the option (option's delta), the higher the expected change of the option.

There are other two properties for the options based on the assumption of one-dimensional diffusion process. The second is known as the correlation property and implies that since the underlying price is the only stochastic process of the model, option prices and the underlying price should be perfectly correlated.

The third property is known as the redundancy property of the options. In a world with no arbitrage opportunities, a trader can replicate an option contract by holding two positions, one in the underlying and the other one in a risk-free asset.

3.4. Monotonicity & perfect correlation test

The first two properties of the options, monotonicity and perfect correlation, can be tested empirically using observed data from the market. Based on these properties and equations (13) and (14) these are the predictions for the changes in the option prices with respect to changes in the underlying price:

1) For any time interval, the changes in the put price should be of the opposite sign with the changes in the underlying price, $\Delta P \Delta S \leq 0$, where ΔP is the change in the put option.
2) For any time interval, the changes in the call price should be of the same sign with the changes in the underlying price, $\Delta C \Delta S \geq 0$, where ΔC is the change in the call option.
3) For any time interval, the absolute changes of the put option should not exceed the absolute changes of the underlying, $\Delta P/\Delta S \geq -1$.
4) For any time interval, the changes of the call option should not exceed the changes of the underlying, $\Delta C/\Delta S \geq 1$,and

5) For any time interval, the changes of the call option and the changes of the put option should be of different sign, $\Delta P \Delta C \leq 0$.

3.5. Implied Volatility

In their formula Black and Scholes (1973) and Merton (1973) take into account six parameters in order to price an option. Five of them, the underlying price, strike price, risk-free rate, time to maturity and the dividend yield are either specified on the terms of the contract or can be observed from market data. The only value of the underlying asset that is not known and cannot be observed directly is volatility.

So, it is fair enough to say that when traders attempt to price an option, what they actually price is its volatility. Since the other five factors are known and consequently publicly available, any discrepancies in the model predictions are probably attributed to the different estimations of the volatility.

If we solve equations (9) and (10) with respect to volatility, we can get the implied volatility (IV) which algebraically is given as:

$$\sigma^2 = 2\,(C_t + qC + K(r-q)C_k)/(K^2 C_{KK}) \qquad (15)$$

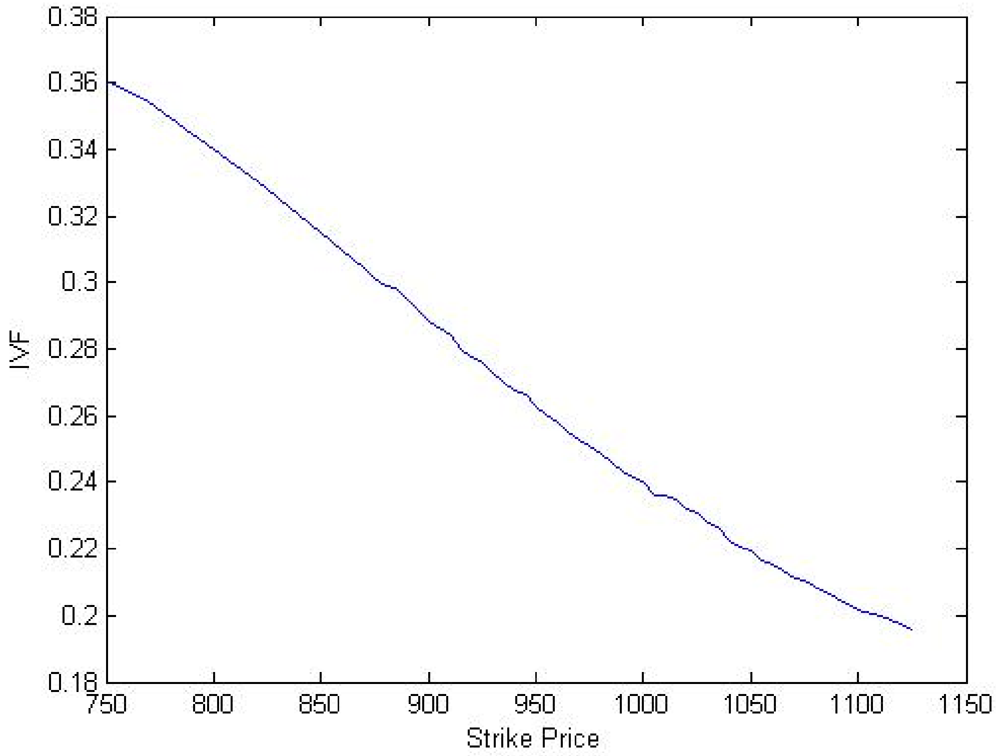
where C_t is the first partial derivative of a call option with respect to time(t), and $C_K(C_{KK})$ is the first(second) partial derivative of a call option with respect to strike price (K).

Nowadays, volatility measures are very popular and the CBOE publishes indices based on the implied volatilities, the most popular of which is the VIX which is based on a wide range of S&P 500 index options.

Equation (15) indicates that this IV estimate is a function of moneyness and maturity. Figure 5 shows the IVs in a typical day for all range of moneyness of option contracts with the same maturity period. While someone would expect the IV to be a straight line parallel to the horizontal axis, this is far from true. This phenomenon is known as volatility smirk. When the IV is about to be used in practice, usually is calculated from ATM options because it is a better approximation of the real world volatility than the one obtained from OTM or ITM options.

Figure 5

Implied Volatility function



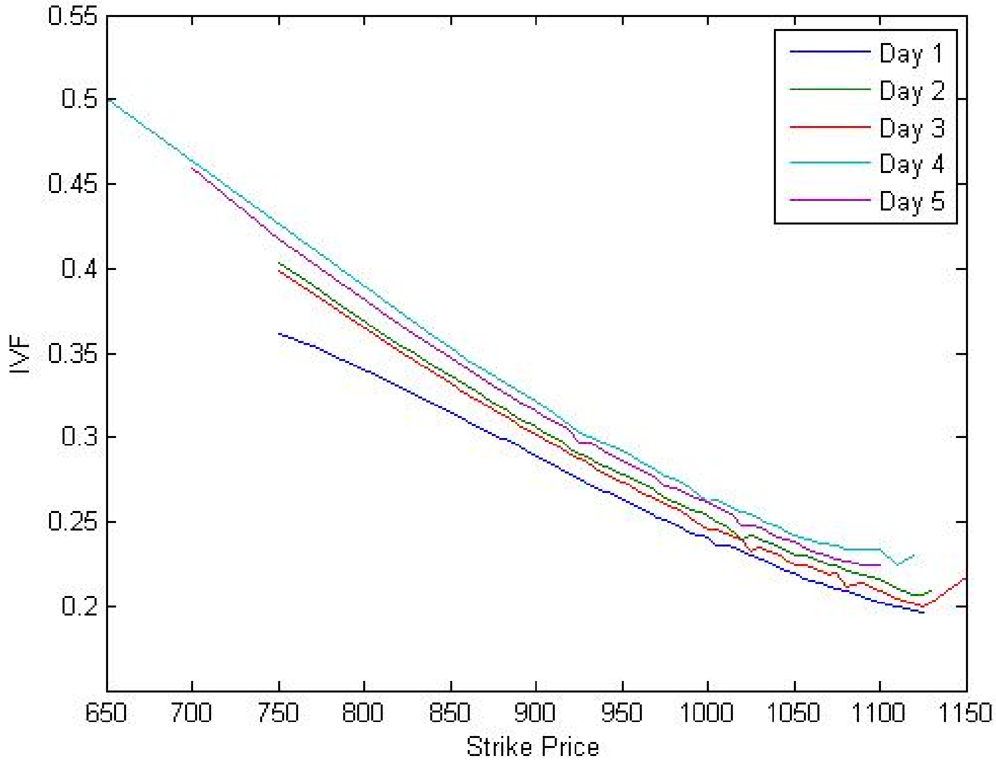
For one day using all the call options with the same maturity period, the IV was calculated for all the range of moneyness using the formula described in equation (15). On the vertical axis is presented the IV, while on the horizontal the strike prices.

3.6. Option price changes with respect to volatility changes

The option pricing formula of Black and Scholes (1973) and Merton (1973) uses a constant rate of volatility as a parameter to the model. But this is far from true in the real world, where volatility is actually a stochastic process just like the price of the asset. In fact from Figure 6, we can observe that there is a different IV for every trading day, which supports the hypothesis of stochastic volatility, Besides this simple illustration, there is a number of studies that indicate that volatility is actually a stochastic process (see Heston (1993)). It is also widely known that volatility is negatively related with the stock price. Specifically, an increase(decrease) in volatility is expected to be followed by a decrease(increase) in the price of the stock.

Figure 6

IV for 5 trading days



For 5 days using all the call options with the same maturity period, the IV was calculated for all the range of moneyness using the formula described in equation (15). On the vertical axis is presented the IV, while on the horizontal the strike prices.

On the other hand, option prices are positively related with volatility. Unlike stocks, an option holder does not have to buy or sell the underlying asset unless its price guarantees him a profit. Consequently, a call(put) holder does not care if the price of the underlying asset ends up at the expiration date 5,10 or 20$ below(above) the strike price because all he is going to lose is his initial investment.

So a new measure needs to be introduced, that will allow us to calculate the expected change of the option price with respect to volatility changes. More formally, what we look for is the first partial derivative of the option price with respect to volatility, known as Vega of the option, $\frac{\partial P}{\partial \sigma}$ and $\frac{\partial C}{\partial \sigma}$. It can be calculated by differentiating equations (9) and (10) with respect to volatility and the result is:

$$C_{\sigma}=S_0\sqrt{T}\ N'(d_1) \tag{16}$$

where N'(x) is the density probability function for a standard normal distribution.

Equation (16) is the same for both calls and puts, since the volatility has the same impact on both of them. A high value of Vega implies that the option is highly sensitive to volatility changes. On the other hand, a low Vega value suggests that the option is not quite sensitive to volatility changes.

4. Data

The data used in this thesis are from the S&P 500 index, traded on the New York Security Exchange (NYSE), one of the most active and liquid indices in the world. The S&P 500 index option quotes are based on the observed data from the Chicago Board Options Exchange (CBOE) a highly liquid market and one of the most active index options on the world.

The option prices are retrieved from OptionMetrics and presented in the form of the best bid and the best ask price available, and I used the mid-point of the bid-ask spread of the observed option price. My sample is from January 2004 to December 2008 with a number of total observations of 997,475 both for calls and puts. The risk-free rate that is used is the 3-month USD LIBOR downloaded from Datastream, and the dividend yield of the S&P 500 was downloaded from Bloomberg database.

Option data are filtered as follows:

- Any option contract with a remaining time to maturity less than 10 days was omitted from my sample, in order to avoid the effect of the biases that come from reaching the expiration.
- I considered in my sample only option contracts that were actually traded, so any contract with zero trading volume was omitted.
- I considered in my sample only options with positive implied volatility.
- Most of the contracts expire on the third Friday of the expiring months, but not all of them. Because of the relative big size of the sample, I decided to remove any contract that does not expire on the third Friday.
- A usual problem with option studies are the effects of the tick size, the minimum allowed change in the option price. So, I included in my sample only mid-points of the bid-ask spread that are equal or higher than 3/8$.
- Finally, any option with zero open interest was omitted from my sample.

After applying those filters to the initial data, my sample was reduced to 260,380 observations for calls and puts. Since, my test focuses on the movement of the contract the next trading day, I included in my sample only contracts for which, after the previous filters were applied, there were observations for two consecutive days. My final sample includes 187,386 observations, 78,535 for calls and 108,851 for puts respectively. This is a huge reduction from the initial data, only 18.79% are included in the final sample, but there is still a quite large number of observations and the final data are of high quality compared to the initial ones.

Next, I categorized the data in nine classes based on two criteria, the time to maturity of the option and the moneyness, the relative size of its strike price to the underlying spot price. With respect to the time to maturity, I made three categories of options. The first category is the short-term options that includes options with time to maturity less than two months. The second category, medium-term, includes the options with time to maturity more than two months, but less than six months and the last one, long-term, includes the options with more than six months to expiration.

Table 1

Number of observations on the daily S&P 500 option sample

	OTM	ATM	ITM	Total
	Calls			
Short-Term	17321	20921	5427	43669
Medium-Term	12049	8144	2154	22347
Long-Term	7456	4165	898	12519
Total	36826	33230	8479	78535
	Puts			
Short-Term	32196	20976	3915	57087
Medium-Term	18749	9562	2404	30715
Long-Term	13155	5962	1932	21049
Total	64100	36500	8251	108851

This table reports the total number of observations for calls and puts and the observations in each category of moneyness and maturity from January 2004 to December 2008. Short-term are the options with less than two months, medium-term those with two to six months and long-term those with more than six months to expiration. ITM calls(puts) are the ones with S/K(K/S) ratio more than 1.03, ATM those with ratio 0.97 to 1.03 and OTM those with ratio less than 0.97.

Also, I classified the options in three categories based on their moneyness. For call options I calculated the ratio of the underlying price (S) with the strike price (K), S/K, and I categorized them according to this ratio. The first category, OTM calls, are the ones with S/K ratio less than 0.97. The second one, (ATM) options were the ones with S/K ratio between 0.97 and 1.03, and the last one, ITM, the ones with ratio more than 1.03. For put options, I used the same classification, but instead of the S/K ratio I used the inverse one, K/S, and the same three categories were applied.

I have reported in Table 1 the total number of observation for calls and puts, as well as the number of option contracts in each of the nine categories based on the option maturity and moneyness. Short-term calls are the most actively traded, they account for more than half of the observations, with long-term calls being the least actively trade. In terms of puts, the same pattern seems to hold.

When it comes to moneyness, ITM options are by far the least active, with OTM options being the most active both for calls and puts. The only difference is that the number of observations of OTM puts exceeds by far the number of ATM puts. In contrast the difference between OTM and ATM calls is not that large. Long-term ITM options are the least active category for both calls and puts. The most active for calls are the short-term ATM options, while for puts are the short-term OTM options.

The market structure of the Chicago Board Options Exchange is the following. The trading hours for equity options are from 8:30 a.m.to 3:00 p.m. Central Time, while for index options it depends on the specific index. The trading hours for the SPX specifically are from 8:30 a.m. to 3:15 a.m. Central Time. All the options listed on the CBOE are issued exclusively by the Options Clearing Corporation (OCC), which is also responsible for clearing every transaction in the CBOE. Traders can place their orders only through a broker or the online trading network. Depending on the type of order (market, limit e.t.c.) it will be executed immediately or when the price limit is met. For contracts with no activity, market makers are responsible to provide liquidity making bid and ask quotes up to ten contracts out of their own capital.

Last I would like to mention that the data handling was conducted in MATLAB and the programming codes are available if requested.

5. Empirical Findings

According to the predictions of the monotonicity and the correlation properties as described above, for the one-dimensional diffusion models, there are four types of violations:

- **Type I violation:** $\Delta S \Delta C < 0$, that is $\Delta S > 0$ and $\Delta C < 0$ or $\Delta S < 0$ and $\Delta C > 0$ for calls. On the other hand for puts, $\Delta S \Delta P > 0$, that is $\Delta S > 0$ and $\Delta P > 0$ or $\Delta S < 0$ and $\Delta P < 0$.
- **Type II violation:** $\Delta C = 0$ but $\Delta S \neq 0$ for calls and $\Delta P = 0$ but $\Delta S \neq 0$ for puts.
- **Type III violation:** $\Delta C \neq 0$ but $\Delta S = 0$ for calls and $\Delta P \neq 0$ but $\Delta S = 0$ for puts.
- **Type IV violation:** $\Delta C/\Delta S > 1$, $\Delta S \neq 0$ for calls and $\Delta P/\Delta S < -1$, $\Delta S \neq 0$ for puts.

5.1. Option price violations

In this section I analyze if the observed data violate the monotonicity and correlation properties of one-dimensional diffusion processes. If yes, how often does this happen? Specifically, Table 2 reports the violation frequencies of type I-IV errors. The violation rates are reported as a percentage of the total observations of the whole period but also for each year exclusively both for calls and puts.

The first pattern observed from Table 2 is that quite frequently, call prices go up(down) and put prices go down(up) when the underlying index goes down(up). This phenomenon comes in contradiction with both the monotonicity and correlation properties of one-dimensional diffusion processes. For the whole sample, the violation frequency for calls is 12.75% and for puts is 10.08%. The phenomenon appears to be slightly more persistent for calls compared to puts, but still that rate is significant for the latter.

Next I examined the violation rates for each year exclusively to test if the phenomenon is present in the whole sample period or its occurrence is restricted only in some sub-periods. The results both for calls and puts indicate that its occurrence is not isolated in any sub-period but the violation rates are significant and stable for all years. For calls the type I violation rates are quite similar for the five years, ranging from 14.98% in ‘04 to 11.96% in ‘05, while the rates for ‘06,’07 and ’08 are 12.21%, 12.92% and 12.26% respectively. On the other hand, violations rates for puts spread in a wider range across the years. The highest rate was in ’04 when 12,78% of the time puts moved on the same direction with the index, while the lowest was in ’08 with 5.98% of puts moving on the same direction. The rates for the years ’05, ’06 and ’07 are 9.63%, 11.51% and 12.46% respectively.

Table 2

Violation Frequency 2004-2008

Year	Observations	Type I	Type II	Type III	Type IV	Overall
		calls				
2004	11687	14,98%	1,94%	0,00%	8,81%	24,24%
2005	12511	11,96%	1,93%	0,00%	5,93%	18,99%
2006	14195	12,21%	2,16%	0,00%	7,72%	20,98%
2007	16481	12,92%	1,40%	0,00%	9,96%	22,86%
2008	23661	12,26%	1,45%	0,38%	3,10%	16,77%
2004-2008	78535	12,75%	1,72%	0,11%	6,66%	20,27%
		puts				
2004	16814	12,78%	1,87%	0,00%	10,33%	23,46%
2005	16351	9,63%	2,07%	0,00%	6,81%	17,71%
2006	19895	11,51%	2,40%	0,00%	4,92%	18,15%
2007	25031	12,46%	1,71%	0,00%	5,46%	18,86%
2008	30760	5,98%	0,90%	0,40%	5,89%	12,77%
2004-2008	108851	10,08%	1,69%	0,11%	6,42%	17,55%

This table reports the violation frequencies of each error type as a percentage of the total observations for the period January 1, 2004 to December 31,2008 from S&P 500 index options data. Type I is $\Delta S \Delta C < 0$ or $\Delta S \Delta P > 0$, type II is $\Delta C = 0$ or $\Delta P = 0$ but $\Delta S \neq 0$, Type III is $\Delta C \neq 0$ or $\Delta P \neq 0$ but $\Delta S = 0$ and type IV is $\Delta C/\Delta S > 1$ or $\Delta P/\Delta S < -1$ for $\Delta S \neq 0$.

Type II violation does not occur as frequently as type I. In type II violation, option price has not changed even if the underlying index price has changed. The rates are quite low for the whole period, only 1.72% of the time calls do not change when the underlying asset has changed, while the rate for puts is almost the same 1.69%. Testing for each year exclusively, the rates for both calls and puts are not quite different than the ones for the whole period. The range of occurrence rates is quite small for calls with the highest frequency occurring in ‘06 when 2.16% of the calls did not change when the underlying asset has changed, and the lowest in ‘07 when 1.40% of the calls presented a type II violation. The rates for ’04, ’05 and ’08 are 1.94%, 1.93% and 1.45%, respectively. The range of type II violations rates for puts is slightly higher. The highest rate occurred in ’06 for puts also, when 2.40% of the puts did not change when the underlying index had changed, while the lowest type II violation rate occurred in ’08 with 0.90% occurrence rate, The occurrence rates in ’04, ’05 and ’07 are 1.87%, 2.07% and 1,71%, respectively. Type II error can be attributed to two possible causes:

- The changes in the underlying in index are quite small to trigger a change in the call and put prices given the fact of the minimum tick size implied by the CBOE.
- Option prices may not change as fast as the index price, meaning that option markets are slower in adjusting to new information.

Type III violation rates are very low for both calls and puts. Type III error occurs when the call or the put prices change but the underlying has not changed. The actual rate is the same for calls and puts for the whole period, that is 0.11%. The reason for this very low occurrence rate is that S&P 500 cash index presents very high liquidity. Consequently it rarely remains unchanged in the daily interval. Specifically, in the whole period between 2004 and 2008 the index did not change in the daily interval just once, in early 2008. It is interesting though, that the day the cash index did not change, all call and put prices in the sample actually changed.

The last violation is type IV, which occurs when the (absolute)magnitude of the (put)call price changes exceeds the change of the underlying index. The violation rates for calls and puts are quite similar for the whole period being close to 6.5%. The range of occurrence rates for every year is not quite similar for calls and puts. Specifically, the highest violation rate for calls occurred in ’07 when 9.96% of the call prices appeared to overadjust to the index changes and the lowest occurred in ’08 when the violation rate was 3.10% of the time. The violation rates in ‘04, ‘05 and ’06 are 8.81%, 5.93% and 7.72%, respectively. The highest violation rate for puts occurred in ’04 when 10.33% of the time put prices overadjusted while the lowest rate occurred in 2006 with a violation rate of 4.92%. The violation rates for puts in ’05,

’07 and ’08 were 6.81%, 5.46% and 5.89% respectively. This type of error can also be connected with microstructure effects such as the minimum tick size. If the magnitude of the price change of the underlying index is smaller than the tick size, then option prices might overadjust. If it does not overadjust, then a type II error will occur.

The empirical findings of violation rates can be summarized as follows. Type I violation rates are the most persistent among the four error types, 12.75% of the calls move on the opposite direction with the underlying asset while 10.08% of the puts move on the same direction with the index. Type II violation rates are quite low, since only 1.72% of the calls and 1.69% of the puts do not change when the underlying index has changed. Type III violations rates are very rare since the S&P 500 cash index remained unchanged only once in the whole period. But that day all the prices of the calls and the puts changed despite the fact that the index did not change. Finally, type IV violation is the second most frequent error type, since 6.53% of the time calls overadjust to index changes while the respective rate for puts is quite similar, 6.30%. When all type of errors are considered together, the violation rate for calls is 20.27% while that for puts is 17.55% for the period 2004-2008.

Type II and IV can be attributed to microstructure factors such as the minimum tick size. If the change in the cash index is small relative to the tick size, then either the option price will not change (type II error) or it will increase/decrease by one tick size (type IV error). Also type II error can be attributed to option market staleness which may lead to type IV errors later. But type I error, which is the most frequent one, is a strong evidence against one-dimensional diffusion processes since the properties of monotonicity and perfect correlation of option prices and the underlying price that come under the assumption of one factor models seem not to hold. In the rest of the empirical section, I will emphasize on type I, II and IV violations since they are the most frequent ones.

5.2. Call and put price changes of the same direction

From the observed data, there is strong evidence that call(put) prices can move on the opposite(same) direction with the underlying index. Consequently, we expect that there are pairs of calls and puts with the same strike price and time to maturity that move on the same direction. In this section I will address the following question: How often call and put prices move on the same direction? Given that, a second question arises: It is more likely to go up or down together? From one-dimensional diffusion processes, we expect that calls and puts always move on the opposite direction, so we can consider the same direction movements as violation. Specifically for call and put options with the same strike price and expiration date, we can classify the violations in four categories as follows:

- **Type A violation:** $\Delta C>0$ and $\Delta P>0$ but $\Delta S>0$.
- **Type B violation:** $\Delta C<0$ and $\Delta P<0$ but $\Delta S>0$.
- **Type C violation:** $\Delta C>0$ and $\Delta P>0$ but $\Delta S<0$.
- **Type D violation:** $\Delta C<0$ and $\Delta P<0$ but $\Delta S<0$.

Table 3

Type A-D violation frequency

	Type A	Type B	Type C	Type D	
2004	0,93%	3,56%	0,61%	3,72%	8,82%
2005	0,91%	3,72%	0,34%	2,93%	7,90%
2006	1,27%	2,97%	0,42%	2,94%	7,59%
2007	0,75%	2,89%	0,74%	4,46%	8,84%
2008	0,69%	4,02%	0,89%	2,35%	7,96%
2004-2008	0,88%	3,47%	0,64%	3,22%	8,21%

This table reports type A-D violation frequencies for the S&P 500 index options from 2004 to 2008. Type A is $\Delta C>0$ and $\Delta P>0$ but $\Delta S>0$, type B is $\Delta C<0$ and $\Delta P<0$ but $\Delta S>0$, type C is $\Delta C>0$ and $\Delta P>0$ but $\Delta S<0$ and type D is $\Delta C<0$ and $\Delta P<0$ but $\Delta S<0$.

In table 3 I have reported the violation frequencies of type A-D errors for the whole period between 2004 and 2008, and for each year exclusively. Type A violation is rare to occur since for the whole period only 0.88% of the time calls and puts went up when the index also increased. If every year is examined separately, the results are close to those reported for the whole sample period. The highest rate occurred in ’06 when 1.27% of the options had a type A violation, and the lowest occurred in ’08 when 0.69% of the time call and put prices increased together with the underlying index. For the years ’04, ’05 and ’07 the type A violation rates are 0.93%, 0.91% and 0.75%, respectively.

Type B error frequency is much higher than that of type A error. In fact its occurrence rate is 4 times higher than that of type A violation, and generally it is the most frequent error type observed in the sample. Specifically, 3.47% of the time call and put prices go down together when the index goes up. Testing for each year exclusively, there are not high discrepancies from the total rate, with the highest rate occurring in ’08 when 4.02% of the time call and put prices went down together when the index increased and the lowest was in ’07 when the violation rate was 2.89% of the time. For the years ’04, ’05 and ’06 type B violation rates were 3.56%, 3.72% and 2.97%, respectively.

Type C error is quite rare, less than one fifth of type B frequency, while it is the least frequent error type in the sample. Specifically, only 0.64% of the time call and put prices go up together when the underlying index goes down for the whole period. Observing the violation rates of each year separately, there are not any high discrepancies from the whole period violation rates, with the highest one occurring in ’08 when 0.89% of the time call and put prices went up together while the index went down and the lowest one in ’05 when 0.34% of the options presented a type C error. For the years ’04, ’06 and ’07 the type C violation rates are 0.61%, 0.42% and 0.74%, respectively.

The last error, type D, is about five times more frequent than type C violation and is actually the second most frequent error after type B. Specifically, 3.22% of the time call and put prices go down together when the underlying index goes down for the whole period. Examining each year exclusively, it is the error with the highest range across years but even so there are no high discrepancies from the whole period rates. The highest rate occurred in ’07 when 4.46% of the time call and put prices went down together when the index went down and the lowest was in ’08 with type D violation rate of 2.35%. For the years ’04, ’05 and ’06 the type D violation rates are 3.72%, 2.93% and 2.94%, respectively.

In the previous part of this section I analyzed the violations rates of type A-D errors for the whole period as well as for each year exclusively. This is how I covered the first question of how often call and put prices go up or down together. Now I will address the second question of whether call and put prices are more likely to go up or down together.

Observing Table 3, we can see a pattern in the violation rates since type B and D errors are more frequent than type A and C errors. Since in both type B and D errors call and put prices go down together, we can conclude that it is more likely for call and put prices to go down together than up together regardless of the underlying index price change. The above conclusion holds for the whole sample period and for each year separately.

There are two possible explanations to this pattern. The first one is the inevitable negative impact of time decay in option prices. When options come closer to expiration their value decreases all other being equal. Consequently, in case of small changes in the underlying price, the impact of the price change might be smaller than the impact of time decay causing the option to move on the opposite direction. The second explanatory factor goes beyond the one-dimensional diffusion process, allowing volatility to be stochastic. We know that volatility is negatively related to stock prices and positively related to option prices. So if the underlying price increases then probably the volatility has decreased causing option prices to decrease. But the explanatory dynamic of stochastic volatility is limited, since it can explain only why type B is more frequent than type A error but not why type D is more frequent than type C.

Summarizing the empirical results of Table 3, we see that when the underlying index goes up, call and put prices go up together (type A error) 0.88% of the time and go down together (type B error) 3.47% of the time. On the other hand, when the underlying index goes down call and put prices go up together (type C error) 0.64% of the time and down together (type D error) 3.22% of the time. When all types of error are considered together, call and put prices move together 8.21% of the time for the whole period, while the violation rates are robust across the years examined. Of a given violation, call and put prices are more likely to go down together than up together since type B and D rates are higher than type A and C rates.

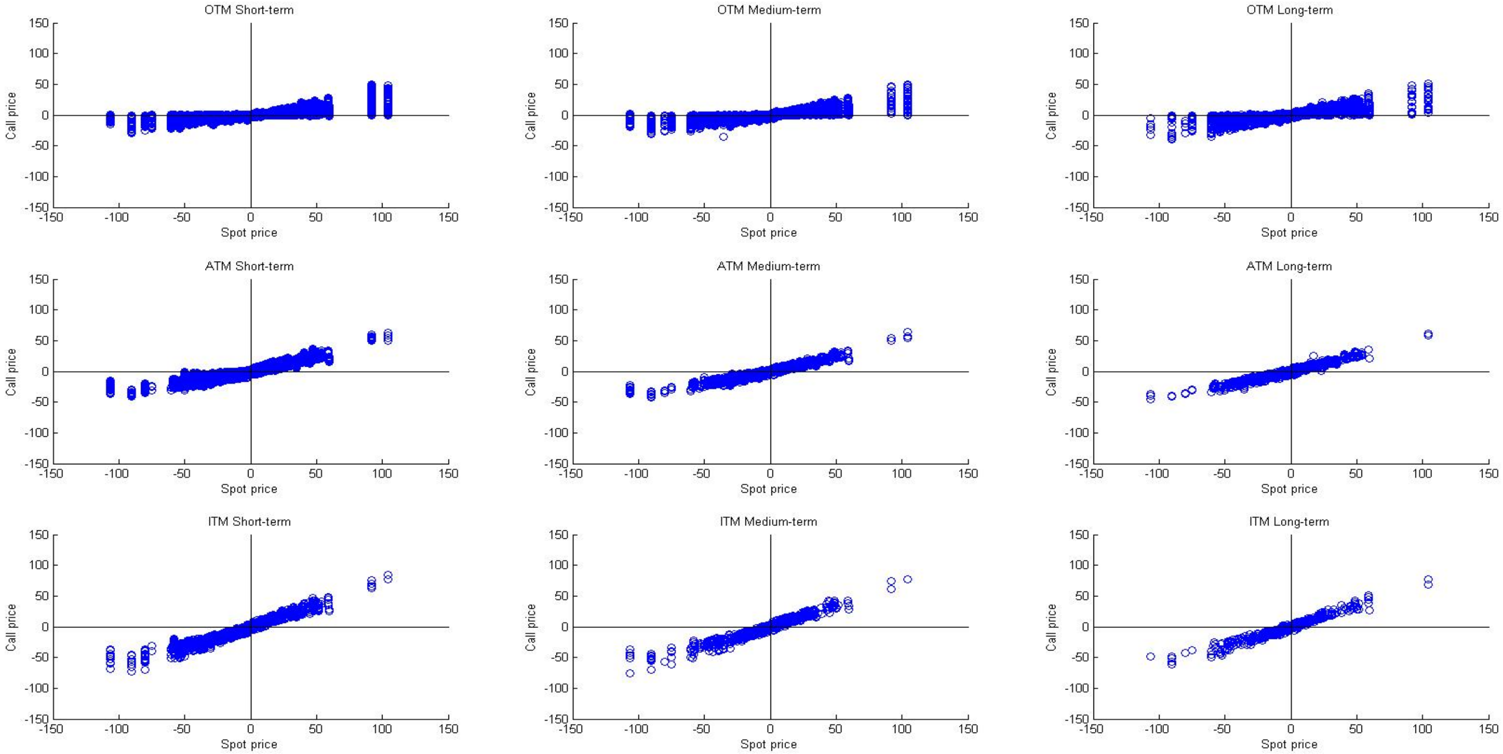
5.3. Violation rates across moneyness and maturity

In this section I examine the occurrence rate of type I, II and IV errors across moneyness and maturity. As a first check, we can observe the patterns in Figures 7 and 8 where the changes of the call and put prices, respectively, are plotted against the changes of the underlying index. If the monotonicity and the perfect correlation properties were valid there should be no observations in the second and fourth quadrants for calls and no observations at the first and third quadrants for puts. A simple look at the figures reveals that there are observations in the “forbidden” quadrants both for calls and puts which is a strong evidence of violation of monotonicity and perfect correlation properties.

Another interesting pattern drawn from figures 7 and 8 is the sensitivity of the moneyness categories to the index changes. More specifically, the more in the money an option is the more sensitive it is to underlying price changes. This observation stands for both calls and puts. More formally, ITM options appear to have the highest delta values with OTM options having the lowest ones. Now, one can connect this pattern with type II and IV violations. Since OTM options are the least sensitive to index changes, we would expect to present the highest frequency of type II error and the lowest of type IV error. Reversely, ITM options with the highest sensitivity are expected to have the lowest type II error rate and the highest type IV rate.

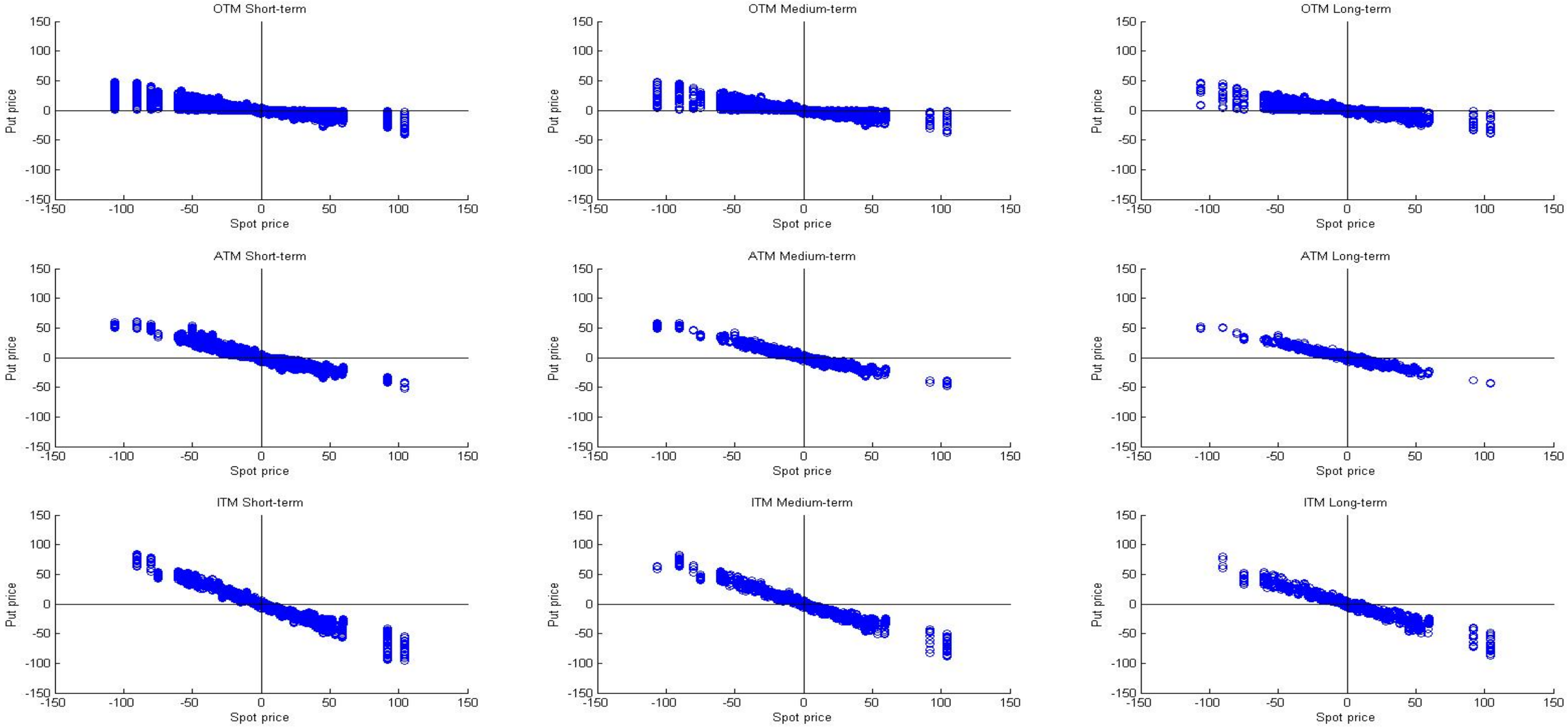
Table 4 reports the frequencies of occurrence of type I, II and IV errors for calls for different categories of moneyness and maturity. Table 5 reports the same results for puts.For the three error types, in terms of moneyness, ITM calls present the highest violation rate with a frequency of 27.23% of the time. ATM calls have the lowest violations frequency with a rate of 17.95% with OTM calls present a 20.41% violation frequency. In the put option categories, ITM puts are the ones with the highest overall violation frequency with a rate of 23.32% while OTM puts are the ones with the lowest violation rate of 16.70%. ATM puts present a 17.39% violation rate. In terms of maturity, the violation frequency for calls decreases with the increase of maturity. Short-term calls present a 21.99% violation rate, while the rates for medium-term and long-term are 18.42% and 16.89%, respectively. In contrast this pattern does not hold for puts. While short-term puts present the highest violation rate also, with a rate of 18.47%, medium-term puts present the lowest rate with 15.77% while long-term puts have an overall violation rate of 17.04%. The highest violation rate of 30.00% for calls is observed for short-term ITM calls,

Figure 7



Changes of call prices with respect to changes of the underlying index price for different categories of maturity and moneyness. Short-term are the options with less than two months, medium-term those with two to six months and long-term those with more than six months to expiration. ITM calls are the ones with S/K ratio more than 1.03, ATM those with ratio 0.97 to 1.03 and OTM those with ratio less than 0.97.

Figure 8



Changes of put prices with respect to changes of the underlying index price for different categories of maturity and moneyness. . Short-term are the options with less than two months, medium-term those with two to six months and long-term those with more than six months to expiration. ITM puts are the ones with K/S ratio more than 1.03, ATM those with ratio 0.97 to 1.03 and OTM those with ratio less than 0.97.

while the lowest rate, 16.22%, comes from medium-term ATM calls. The highest rate, 27.13%, for puts comes from short-term ITM puts also, while the lowest one, 15.00%, is observed for medium-term OTM puts.

Type I error violation rates tend to decrease with respect to the moneyness of call options, with violation rates of 16.08%, 10.47% and 7.22% for OTM, ATM and ITM call options, respectively. The same pattern holds for puts also, with OTM options presenting the highest rate, 12.08% while the rates for ATM and ITM puts are 7.95% and 3.96%, respectively. In terms of maturity, type I violation rates appear to decrease with the increase of the maturity with short-term calls presenting the highest violation rate, 14.04% with medium-term and short-term following with 11.72% and 10.07%, respectively. This pattern does not hold for puts since the highest rate, 10.70%, is observed for short-term puts also, but the rates for medium-term and long-term puts are 8.88% and 10.15%, respectively. The highest violation rate, 19.35%, for calls is observed for short-term OTM calls while the lowest one, 6.92%, is observed for medium-term ITM calls. Short-term OTM puts are also the ones with the highest type I violation rate, 13.41%, while short-term ITM puts being the ones with the lowest, 3.14%. Finally, the results indicate that in any type I violation, call and put prices are more likely to go down than up, except for long-term ITM puts.

Type II error rates appear to decrease the more in the money an option is, confirming the initial hypothesis that we made by observing Figures 7 and 8, which holds both for calls and puts and also for any maturity category. The violation rates for OTM, ATM and ITM calls are 2.58%, 1.04% and 0.60%, respectively while the rates for puts are 2.29%, 0.91% and 0.42%, respectively. In terms of maturity, there is not an obvious pattern either for calls or puts. The violation rates for short-term, medium-term and long-tern calls are 1.65%, 1.90% and 1.61%, respectively and the equivalent rates for puts are 1.60%, 1.66% and 1.95%, respectively. Although with a first look it seems that the type II violation rates increase with the increase of maturity for puts, the differences between the maturity categories are very small. The highest type II violation rate, 2.71%, for calls is observed for medium-term OTM calls while the lowest one, 0.50%, for short-term ITM calls. The highest rate, 2.56%, for puts is observed for long-term OTM puts and the lowest one, 0.41%, for short-term and long-term ITM puts.

Table 4

Violation frequency across moneyness and maturity for calls

	Calls				
	Error type	OTM	ATM	ITM	Total
Short	Type I	19,35%	11,38%	7,35%	14,04%
	Type I ΔC<0	16,03%	9,45%	4,83%	11,48%
	Type I ΔC>0	3,33%	1,93%	2,52%	2,56%
	Type II	2,70%	1,09%	0,50%	1,65%
	Type IV	1,60%	7,26%	24,04%	7,46%
	Type I,II and IV	23,30%	18,82%	30,00%	21,99%
Medium	Type I	14,37%	9,07%	6,92%	11,72%
	Type I ΔC<0	11,20%	6,79%	3,99%	8,90%
	Type I ΔC>0	3,17%	2,28%	2,92%	2,82%
	Type II	2,71%	1,01%	0,79%	1,90%
	Type IV	2,08%	6,83%	18,26%	5,55%
	Type I,II and IV	18,81%	16,22%	24,56%	18,42%
Long	Type I	11,24%	8,62%	7,13%	10,07%
	Type I ΔC<0	7,74%	5,52%	3,79%	6,72%
	Type I ΔC>0	3,50%	3,10%	3,34%	3,35%
	Type II	2,11%	0,89%	0,89%	1,61%
	Type IV	3,41%	8,22%	14,87%	5,91%
	Type I,II and IV	16,30%	16,95%	21,71%	16,89%
Total	Type I	16,08%	10,47%	7,22%	12,75%
	Type I ΔC<0	12,77%	8,30%	4,51%	9,99%
	Type I ΔC>0	3,31%	2,16%	2,71%	2,76%
	Type II	2,58%	1,04%	0,60%	1,72%
	Type IV	2,15%	7,28%	21,60%	6,66%
	Type I,II and IV	20,41%	17,95%	27,73%	20,16%

This table reports the violation rates across moneyness and maturity for call options of the S&P 500 for the period 2004-2008. Short-term are the options with less than two months, medium-term those with two to six months and long-term those with more than six months to expiration. ITM calls are the ones with S/K ratio more than 1.03, ATM those with ratio 0.97 to 1.03 and OTM those with ratio less than 0.97. Type I is $\Delta S \Delta C < 0$, type II is $\Delta C = 0$ but $\Delta S \neq 0$ and type IV is $\Delta C/\Delta S > 1$ for $\Delta S \neq 0$.

Table 5

Violation frequency across moneyness and maturity for puts

		Puts			
	Error type	OTM	ATM	ITM	Total
Short	Type I	13,41%	7,94%	3,14%	10,70%
	Type I ΔP<0	11,36%	6,14%	1,92%	8,80%
	Type I ΔP>0	2,04%	1,80%	1,23%	1,90%
	Type II	2,18%	0,94%	0,41%	1,60%
	Type IV	2,24%	10,42%	24,44%	7,04%
	Type I,II and IV	17,48%	18,38%	27,13%	18,47%
Medium	Type I	10,19%	7,54%	4,08%	8,88%
	Type I ΔP<0	6,92%	4,95%	2,62%	5,97%
	Type I ΔP>0	3,26%	2,59%	1,46%	2,91%
	Type II	2,29%	0,71%	0,46%	1,66%
	Type IV	2,88%	8,28%	17,73%	5,84%
	Type I,II and IV	15,00%	15,84%	21,46%	15,77%
Long	Type I	11,51%	8,65%	5,49%	10,15%
	Type I ΔP<0	6,50%	4,39%	2,54%	5,54%
	Type I ΔP>0	5,01%	4,26%	2,95%	4,61%
	Type II	2,56%	1,11%	0,41%	1,95%
	Type IV	3,66%	7,32%	12,76%	5,62%
	Type I,II and IV	17,22%	16,37%	17,91%	17,04%
Total	Type I	12,08%	7,95%	3,96%	10,08%
	Type I ΔP<0	9,07%	5,54%	2,27%	7,37%
	Type I ΔP>0	3,01%	2,41%	1,70%	2,71%
	Type II	2,29%	0,91%	0,42%	1,69%
	Type IV	2,73%	9,35%	19,80%	6,42%
	Type I,II and IV	16,70%	17,39%	23,32%	17,43%

This table reports the violation rates across moneyness and maturity for put options of the S&P 500 for the period 2004-2008. Short-term are the options with less than two months, medium-term those with two to six months and long-term those with more than six months to expiration. ITM puts are the ones with K/S ratio more than 1.03, ATM those with ratio 0.97 to 1.03 and OTM those with ratio less than 0.97. Type I is $\Delta S \Delta P > 0$, type II is $\Delta P = 0$ but $\Delta S \neq 0$ and type IV is $\Delta P/\Delta S < -1$ for $\Delta S \neq 0$.

Type IV error rates appear to increase the more in the money an option is, confirming the initial hypothesis that we made by observing Figures 7 and 8, which holds both for calls and puts and also for any maturity category. The violation rates for OTM, ATM and ITM calls are 2.15%, 7.28% and 21.60%, respectively. The rates for puts are 2.73%, 9.35% and 19.80%, respectively. In terms of maturity, no clear pattern seems to appear either for calls or puts, with type IV violation rates for short-term, medium-term and long-term calls to be equal to 7.46%, 5.55% and 5.91%, respectively. The type IV violation rates for short-term, medium-term and long-term puts are 7.04%, 5.84% and 5.62%, respectively. The highest type IV violation rate, 24.04%, for calls is observed for short-term ITM calls and the lowest one, 1.60%, for short-term OTM calls. For puts, the highest type IV violation rate, 27.13%, is observed for short-term ITM puts and the lowest one, 2.24%, for short-term OTM puts.

Summarizing the empirical results of the three types of violations across moneyness and maturity, we observe that ITM options present the highest violation frequency mostly because they are exposed to a high type IV violation rate. In terms of maturity, short-term options present the highest overall violation rate. Also four patterns can be observed in the results of tables 4 and 5. The first concerns type I error. Both for calls and puts, the more in the money an option is the lower is the type I violation frequency. The second one also concerns type I error but holds only for calls and implies that the frequency of the violation decreases as the maturity of the call increases. The third pattern is about type II error and implies that the more in the money an option is the lower the type II violation rate. The fourth pattern is about type IV error and implies that the more in the money an option is the higher the type IV violation frequency. Finally, given a type I violation call and put prices are more likely to go up rather than down.

5.4. Magnitude of changes given a violation

In this section I have calculated the mean change for the underlying index as well as the mean change of the option and its t-statistic given a specific type of violation. Tables 6 and 7 report the mean changes for calls and puts, respectively, given a type I violation. Table 8 reports the mean changes of the index given a type II violation for both calls and puts. To avoid the impact of the opposite sign changes in the mean values, I have separated the violation in which $\Delta S>0$ but $\Delta C<0(\Delta P>0)$ from those in which $\Delta S<0$ but $\Delta C>0(\Delta P<0)$ for type I violations. Similarly for type II violations I have separated those in which $\Delta S>0$ but $\Delta C=0(\Delta P=0)$ from those in which $\Delta S<0$ but $\Delta C=0(\Delta P=0)$.

For type I violations the mean changes of the options are statistically significant as implied by their t-statistic values. All the changes are higher than the minimum tick size of the CBOE, both for calls and for puts across moneyness and maturity. An interesting conclusion that is implied by the results of Tables 6 and 7 is that the mean changes of the index given an OTM option violation are significantly higher than the mean changes of the index for other moneyness categories. Since the changes of the options and the underlying index given a type I violation are statistically significant and higher than the minimum tick size, we conclude that they are also economically significant.

Table 6

Magnitude of call price changes given a type I violation

		OTM	ATM	ITM	Total	OTM	ATM	ITM	Total
					Calls				
		$\Delta S<0$ $\Delta C>0$				$\Delta S>0$ $\Delta C<0$			
Short	ΔS	-9,95	-1,17	-1,06	-5,68	5,84	1,74	1,21	3,98
	ΔC	0,25	0,53	0,85	0,42	-0,32	-0,70	-1,02	-0,51
	T-stat	22,49	22,17	13,50	29,79	-40,25	-44,34	-17,49	-57,97
Medium	ΔS	-4,98	-1,25	-1,18	-3,50	4,63	1,46	1,37	3,61
	ΔC	0,33	0,56	0,85	0,45	-0,42	-0,81	-1,00	-0,55
	T-stat	21,50	18,18	10,12	26,79	-24,76	-20,84	-7,93	-32,02
Long	ΔS	-3,57	-1,28	-1,70	-2,73	3,49	1,62	1,63	2,90
	ΔC	0,55	0,80	0,81	0,64	-0,63	-0,96	-1,24	-0,74
	T-stat	14,55	17,20	6,29	21,86	-19,13	-14,69	-5,00	-23,97
Total	ΔS	-7,03	-1,21	-1,18	-4,48	5,20	1,67	1,29	3,77
	ΔC	0,34	0,59	0,84	0,47	-0,39	-0,74	-1,03	-0,54
	T-stat	30,21	32,41	18,01	44,52	-48,48	-50,07	-19,27	-69,04

This table reports the magnitude of price changes for calls and the index given a type I violation. Type I is $\Delta S\Delta C<0$. Short-term are the options with less than two months, medium-term those with two to six months and long-term those with more than six months to expiration. ITM calls are the ones with S/K ratio more than 1.03, ATM those with ratio 0.97 to 1.03 and OTM those with ratio less than 0.97.

Table 7

Magnitude of put price changes given a type I violation

		OTM	ATM	ITM	Total	OTM	ATM	ITM	Total
				Puts					
		$\Delta S>0$ $\Delta P>0$				$\Delta S<0$ $\Delta P<0$			
Short	ΔS	3,00	1,45	1,43	2,39	-3,46	-1,62	-1,60	-2,96
	ΔP	0,31	0,84	1,63	0,55	-0,37	-0,88	-2,07	-0,53
	T-stat	12,75	14,58	6,27	18,85	-31,91	-27,81	-10,13	-41,20
Medium	ΔS	2,88	1,61	1,25	2,46	-2,52	-1,48	-1,51	-2,22
	ΔP	0,36	0,74	1,76	0,52	-0,48	-1,00	-1,95	-0,66
	T-stat	15,38	12,17	5,82	18,60	-22,48	-19,08	-10,36	-29,39
Long	ΔS	2,58	1,70	1,93	2,31	-2,50	-1,61	-1,51	-2,26
	ΔP	0,50	0,82	0,98	0,61	-0,58	-0,96	-1,32	-0,70
	T-stat	18,09	15,02	7,96	24,27	-19,94	-14,96	-7,37	-25,37
Total	ΔS	2,82	1,57	1,59	2,39	-3,11	-1,58	-1,55	-2,69
	ΔP	0,39	0,81	1,40	0,56	-0,43	-0,92	-1,83	-0,58
	T-stat	26,56	23,72	10,78	35,08	-43,57	-36,72	-15,87	-56,25

This table reports the magnitude of price changes for puts and the index given a type I violation. Type I is $\Delta S \Delta P > 0$. Short-term are the options with less than two months, medium-term those with two to six months and long-term those with more than six months to expiration. ITM puts are the ones with K/S ratio more than 1.03, ATM those with ratio 0.97 to 1.03 and OTM those with ratio less than 0.97.

For type II violations we can also see that the mean changes of the index are statistically significant as implied by their t-statistic values. All the changes are higher than the minimum tick size. The observation made for the magnitude of the mean change of the index for OTM options with type I violation seems to hold also for the mean change of the index given an OTM option with type II violation. Again the mean changes of the index are higher than any other moneyness category both for calls and puts. We can also conclude that type II violations are economically significant, since they are statistically significant and higher than the minimum tick size.

Table 8

Magnitude of price changes given a type II violation

		OTM	ATM	ITM	Total	OTM	ATM	ITM	Total
		Calls							
		ΔS>0				ΔS<0			
Short	ΔS	6,16	2,31	1,81	4,87	-9,23	-1,60	-1,29	-6,42
	T-stat	15,69	13,95	4,10	17,40	-5,47	-11,04	-4,03	-5,83
Medium	ΔS	4,28	2,15	1,69	3,81	-9,63	-1,85	-1,88	-7,82
	T-stat	14,28	12,77	6,99	15,63	-4,75	-6,13	-5,21	-4,93
Long	ΔS	4,43	2,07	2,22	3,99	-4,99	-1,35	-1,15	-4,00
	T-stat	6,75	6,97	3,80	7,37	-3,71	-6,66	-3,46	-4,01
Total	ΔS	5,26	2,25	1,81	4,42	-8,44	-1,62	-1,36	-6,34
	T-stat	21,35	18,40	7,29	23,65	-7,97	-13,87	-5,56	-8,42
		Puts							
Short	ΔS	2,82	1,28	1,12	2,48	-3,90	-1,94	-1,90	-3,47
	T-stat	14,04	9,55	2,68	15,07	-18,10	-14,04	-5,08	-19,90
Medium	ΔS	2,42	1,77	1,96	2,32	-2,97	-1,67	-	-2,81
	T-stat	14,36	7,87	3,79	15,76	-16,27	-9,75	-	-17,21
Long	ΔS	2,72	1,59	1,64	2,51	-3,07	-1,84	-2,03	-2,88
	T-stat	14,00	11,36	5,07	15,40	-10,90	-9,74	-17,46	-11,94
Total	ΔS	2,67	1,48	1,54	2,45	-3,48	-1,87	-1,94	-3,19
	T-stat	24,02	16,28	4,69	26,16	-25,53	-18,56	-6,25	-27,81

This table reports the magnitude of price changes for the options and the index given a type II violation. Type II is $\Delta C = 0$ or $\Delta P = 0$ but $\Delta S \neq 0$. Short-term are the options with less than two months, medium-term those with two to six months and long-term those with more than six months to expiration. ITM calls(puts) are the ones with S/K(K/S) ratio more than 1.03, ATM those with ratio 0.97 to 1.03 and OTM those with ratio less than 0.97.

6. Beyond one-factor model

In this section I introduce another stochastic variable, namely stochastic volatility, in the option pricing formula of Black and Scholes (1973) and Merton (1973) to test if it can explain any of the violations observed in option prices. The reason I choose volatility as the second stochastic variable in the model is the negative correlation empirically observed between the price of an asset and its volatility. Consequently, even if the price of the underlying index increases, the effect of volatility on call(put) prices may cause them to decrease(increase). Similarly, if the price of the underlying index decreases the effect of volatility may cause them to increase (decrease). So stochastic volatility seems to be a possible explanatory factor especially for type I violation.

As a measure of volatility, I calculated the implied volatility of the ATM options as introduced in section 3.4 using equation (15). Because of the volatility smirk, the implied volatility is not parallel with the horizontal axis. In that case the IV of ATM options can be considered as a good approximation of the true volatility. For each trading day I calculated the implied volatility of all the ATM calls and puts, and the mean of all these IVs is my estimate of the volatility for every trading day.

My next step was to see how volatility changes can influence the changes of the option prices. By Ito's lemma, the changes of option prices are as follows:

$$dC=\{C_t + \frac{1}{2}\sigma^2 S^2 C_{ss}\}dt + C_s\, dS \qquad (17)$$

where the subscripts on C stand for the partial derivatives of C.

In equations (17) I introduce another variable which is actually the effect of stochastic volatility on the option price and is as follows:

$$dC=\mu_c\, dt + C_s\, dS + C_\sigma\, d\sigma \qquad (18)$$

The first partial derivatives of the option price with respect to price and volatility are delta and vega, respectively. I calculated them using equations (13), (14) and (16) as described in sections 3.2 and 3.5.

For the option quotes that present any type of violation, I calculated the partial derivatives and based on equation (18) I calculated the change of the option price that the model implies. Next I calculated the observed option price changes as it comes from the real-world data and I compared them with ones from my model.

Table 9

Type I and Type IV violations explained by stochastic volatility

		OTM	ATM	ITM	Total
		Calls			
Short	Type I	39,44%	31,55%	12,53%	34,63%
	TypeIV	56,48%	18,12%	13,64%	19,18%
Medium	Type I	57,04%	43,84%	20,81%	51,26%
	TypeIV	50,48%	31,60%	21,49%	31,84%
Long	Type I	71,60%	45,68%	32,81%	62,25%
	TypeIV	38,18%	24,52%	26,83%	29,56%
Total	Type I	49,14%	35,62%	16,67%	42,46%
	TypeIV	48,29%	22,20%	16,29%	23,73%
		Puts			
Short	Type I	33,66%	20,17%	26,83%	29,84%
	TypeIV	26,44%	34,14%	25,89%	30,65%
Medium	Type I	46,28%	38,97%	40,82%	44,16%
	TypeIV	45,45%	53,03%	48,16%	49,56%
Long	Type I	42,93%	37,02%	33,96%	41,06%
	TypeIV	53,14%	66,75%	65,95%	61,15%
Total	Type I	38,59%	27,83%	33,33%	35,58%
	TypeIV	39,84%	42,67%	37,64%	40,71%

This table reports type I and IV violations explained by stochastic volatility calculated from the implied volatility of the ATM options. Type I is $\Delta S \Delta C < 0$ or $\Delta S \Delta P > 0$ and type IV is $\Delta C/\Delta S > 1$ or $\Delta P/\Delta S < -1$ for $\Delta S \neq 0$. Short-term are the options with less than two months, medium-term those with two to six months and long-term those with more than six months to expiration. ITM calls(puts) are the ones with S/K(K/S) ratio more than 1.03, ATM those with ratio 0.97 to 1.03 and OTM those with ratio less than 0.97.

For type I violation I tested whether the change suggested by the model is of the same sign with the observed changes. If the changes share the same sign, then stochastic volatility can explain the violation for the specific option, otherwise the violation cannot be explained by stochastic volatility.

For type IV violation I tested whether the absolute magnitude of the changes suggested by the model are higher than the changes of the index, $\frac{\Delta C}{\Delta S}$>1 for calls and $\frac{\Delta P}{\Delta S}$<-1 for puts. If the changes suggested by the model overadjust, then stochastic volatility can explain the type IV violation for the specific option, otherwise the violation for this option cannot be explained by stochastic volatility.

The results of the above tests are reported in Table 9 where for each moneyness and maturity category there are the percentages of type I and IV violations that are explained by stochastic volatility. I tested for type II and III violations, but the explanatory power of stochastic volatility was trivial in both error types.

Stochastic volatility seems to explain better type I violation the more OTM a call is, with explanation percentages ranging from 49.14%, 35.62% to 16.67% for OTM, ATM and ITM calls, respectively. This pattern does not hold exactly for puts, since the explanation percentages are equal to 38.59%, 27.83% and 33.33% for OTM, ATM and ITM puts, respectively. In terms of maturity, the explanation validity of stochastic volatility appears to increase with the maturity of the call, with explanation percentages of 34.63%, 51.26% and 62.25% for short-term, medium-term and long-term calls, respectively. The highest explanation percentage, 71.60%, of stochastic volatility for type I violation for calls is observed for long-term OTM calls and the lowest rate, 12.53%, for short-term ITM calls. Medium-term OTM puts present the highest explanation rate, 46.28%, while the lowest rate, 20.17%, is observed for short-term ATM puts.

For type IV violation, the explanation rate of stochastic volatility seems again to increase the more OTM a call is. Stochastic volatility explains 49.14%, 35,62% and 16,67% of the violations for OTM, ATM and ITM calls, respectively. The explanation rates of stochastic volatility for type IV violation for OTM, ATM and ITM puts are 39.84%, 42.67% and 37.64%, respectively. The explanation rates of stochastic volatility for type IV violation in short-term, medium-term and long-term calls are 19.18%, 31.84% and 29.56%, respectively. For puts, stochastic volatility explains type IV violations in short-term, medium-term and long-term options 30.65%, 49.56%

and 61.15% of the time, respectively. The highest explanation rate, 56.48%, of type IV violation for calls is observed for short-term OTM options while the lowest one, 13.64%, is observed for short-term ITM calls. For puts the highest explanation rate, 66.75%, of type IV violation is observed for long-term ATM options, while the lowest one, 25.89%, is observed for short-term ITM options.

Table 10

Quantitative fit of option price changes

	β_0	β_1	β_2	Adj R^2	P-value
			Calls		
One-factor	-0,05	0,78	-	0,91	(0.000)
	(0.000)	(0.000)	-		
Wald-Test	-	0.000	-		
Two-factor	-0,04	0,86	0,29	0,93	(0.000)
	(0.000)	(0.000)	(0.000)		
Wald-Test	-	0.000	0.000		
			Puts		
One-factor	0,09	1,14	-	0,94	(0.000)
	(0.000)	(0.000)	-		
Wald-Test	-	0.000	-		
Two-factor	0,09	1,05	0,33	0,95	(0.000)
	(0.000)	(0.000)	(0.000)		
Wald-Test	-	0.000	0.000		

The results are based on the regressions below:

$\Delta C(t, \tau_i, K_i) = \beta_0 + \beta_1 [C_s \Delta S] + \varepsilon((t, \tau_i, K_i)$

$\Delta C(t, \tau_i, K_i) = \beta_0 + \beta_1 [C_s \Delta S] + \beta_2 [C_\sigma \Delta\sigma] + \varepsilon((t, \tau_i, K_i)$

where i stands for the ith option in the sample and C_s , C_σ are the first partial derivatives with respect to price and volatility, respectively. ΔS and $\Delta\sigma$ are the daily changes of price and volatility, respectively. In parenthesis are the P-values of the coefficients. I conducted a Wald-Test to test if $\beta_1 = 1$ in (19) and $\beta_1 = 1$, $\beta_2 = 1$ in (20) and I report their p-values. The last column reports the P-values of the F-test.

Summarizing the explanation rates of stochastic volatility, we can conclude that it explains a significant part of type I violation, 42.46% for calls and 35.58% for puts. The explanation rate of stochastic volatility for type IV violation for puts remain high, which is not however the case for calls. Since stochastic volatility explains a great part of the violations, these results indicate that we can reject one-dimensional diffusion processes for option pricing and adopt a two-factor process with volatility being the second stochastic variable.

In order to test the superiority of the two-factor model against one-factor, I run the following regressions for calls and puts across the whole sample:

$$\Delta C(t, \tau_\iota, K_\iota) = \beta_0 + \beta_1 [C_s \Delta S] + + \varepsilon((t, \tau_\iota, K_\iota) \quad (19)$$

$$\Delta C(t, \tau_\iota, K_\iota) = \beta_0 + \beta_1 [C_s \Delta S] + \beta_2 [C_\sigma \Delta\sigma] + \varepsilon((t, \tau_\iota, K_\iota) \quad (20)$$

where i stands for the ith option in the sample and C_s , C_σ are the first partial derivatives with respect to price and volatility which I calculated using equations (13), (14) and (16). ΔS and $\Delta\sigma$ are the daily changes of the price and the volatility, respectively. I also conducted a Wald-Test in (19) and (20) to test if $\beta_1 = 1$ and $\beta_1 = 1$, $\beta_2 = 1$, respectively. The results of the regression are reported in table 10.

From the results in Table 10 we can see that the adjusted R^2 of the two-factor model is higher than the adjusted R^2 of the one-factor model. Also all the coefficients are statistically significant. The intercept is different from zero while β_1 for calls (puts) is significantly lower (higher) than 1 both for the one-factor and the two-factor model. However, the inclusion of stochastic volatility makes coefficient β_1 to come closer to its theoretical value of 1.

7. Liquidity Factors

In this section I test if the violations can be attributed to the liquidity factors of the option contract. To do so, I run the following logit regression:

$$V_i = \beta_0 + \beta_1 * Volume_i + \beta_2 * [Open\ Interest]_i + \beta_3 * Spread_i + \varepsilon_i \qquad (21)$$

where V_i is a discrete variable equal to one if the i-th option price change presents an error type and zero otherwise, Volume is the number of contracts traded each day, Open Interest is the number of contracts outstanding and Spread is the bid-ask spread divided by the mid-point of the bid and ask prices. The trading volume and the open interest are measured in 10K contracts. The results of (21) are presented in Table 11.

According to the results in Table 11, the possibility of type I violation for calls and puts is positively related with the trading volume and the bid-ask spread. For calls the open interest is also positively related to the occurrence of a type I error, at the 5% significance level, but the impact of open interest for puts is not statistically significant. So we conclude that is more possible for a type I error to occur when there is a high bid-ask spread and a high trading volume. For calls only it is also more possible to observe a type I violation when there is high open interest.

The possibility of a type II error is positively related to the bid-ask spread both for calls and puts while volume is negatively related to type II error, at 10% significance level for calls, and at 1% significance level for puts. The impact of open interest is not statistically significant for calls, while for puts it is positive and significant at the 5% significance level. According to the results, the possibility of type II error increases with the increase of the bid-ask spread and the decrease of the trading volume. So we conclude that type II error is more likely to occur in option contracts with limited liquidity.

The possibility of a type IV error is negatively related with the open interest and the bid-ask spread at the 1% significance level both for calls and puts, while trading volume is not statistical significant either for calls or puts. So, it is more likely to observe a type IV violation for option contracts with low bid-ask spread and open interest.

Table 11

Regression coefficients of liquidity factors

	Volume	Open Interest	Spread	R^2	P-Value
		Calls			
Type I	0,010	0,001	0,159	0,019	0.0000
	(0.000)	(0.029)	(0.000)		
Type II	-0,003	0,000	0,028	0,040	0.0000
	(0.090)	(0.707)	(0.000)		
Type IV	-0,004	-0,001	-0,568	0,071	0.0000
	(0.159)	(0.001)	(0.000)		
		Puts			
Type I	0,011	0,000	0,155	0,015	0.0000
	(0.000)	(0.624)	(0.000)		
Type II	-0,004	0,000	0,035	0,029	0.0000
	(0.000)	(0.012)	(0.000)		
Type IV	0,002	-0,002	-0,525	0,062	0.0000
	(0.242)	(0.000)	(0.000)		

This table reports the average marginal effects of the logit regression:

$$V_i = \beta_0 + \beta_1 * Volume_i + \beta_2 * [Open\ Interest]_i + \beta_3 * Spread_i + \varepsilon_i$$

where V_i is a discrete variable equal to one if the i-th option price change presents an error type and zero otherwise. Volume is the number of contracts traded each day, Open Interest is the number of contracts outstanding and Spread is the bid-ask spread divided by the mid-point of the bid and ask prices. The trading volume and the open interest are measured in 10K contracts. Type I is $\Delta S \Delta C < 0$ or $\Delta S \Delta P > 0$, type II is $\Delta C = 0$ or $\Delta P = 0$ but $\Delta S \neq 0$ and type IV is $\Delta C/\Delta S > 1$ or $\Delta P/\Delta S < -1$ for $\Delta S \neq 0$. P-values are in parenthesis. The last column reports the P-values of the F-test.

Summarizing the regression results for the possibility of error occurrence, we conclude that if liquidity factors can explain type IV and type II violations, then these factors are not exactly the same. In particular, for a type II violation the probability increases with respect to the bid-ask spread and trading volume. In contrast, for type IV violation the results indicate that its probability is related to open interest.

8. Conclusion

According to one-factor models, the option price is perfectly correlated and monotonically increasing or decreasing with the underlying asset price. But, according to the results of this thesis based on index option data from the S&P 500 index, one of the most active indices, quite often calls move on the opposite direction and puts on the same direction with the underlying asset. The violation rates for calls and puts are significant, since more than 10% of the options violate the monotonicity and correlation properties. Also option prices often overadjust, as the option price change is higher than the index price change, and sometimes options do not change even if the underlying index has changed.

I introduced a second factor, namely stochastic volatility, in the option pricing formula and tested whether the violations can be explained by changes in volatility. According to my results a significant percentage of the “wrong” sign changes are explained by stochastic volatility and the explanatory ratio is higher for calls than for puts. Stochastic volatility can explain many of the overreaction violations also, with its explanatory power being higher for puts than for calls. Violations can also be explained by liquidity factors, the most important of which is the bid-ask spread. The explanatory power of liquidity factors is higher in type IV and II errors for which spread is negatively and positively related, respectively, with the possibility of error occurrence.

There are other explanatory factors that can be tested except from stochastic volatility and the liquidity factors. These factors are demand pressure and the sign of the order, that is whether it is a buyer or seller initiated trade. In order to test these factors, intraday data should be used, which is beyond the scope of this thesis.

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