Financing Investment

By Joao F. Gomes*

We examine investment behavior when firms face costs in the access to external funds. We find that despite the existence of liquidity constraints, standard investment regressions predict that cash flow is an important determinant of investment only if one ignores q. Conversely, we also obtain significant cash flow effects even in the absence of financial frictions. These findings provide support to the argument that the success of cash-flow-augmented investment regressions is probably due to a combination of measurement error in q and identification problems. (*JEL* E22, E44, G31)

A recurrent puzzle in the investment literature is that measures of Tobin's q and of the cost of capital appear to have a negligible impact on investment. A recent strand of empirical work has sought to investigate this puzzle by studying the interaction between investment and financing decisions at the plant and firm level. The findings of these studies seem to suggest that financial constraints play an important role in shaping corporate investment: (i) cash flow is highly significant in investment regressions; and (ii) small firms appear more liquidity constrained than large ones. These micro data sets have also allowed researchers to uncover a wealth of new empirical regularities about firms and their investment decisions: (i) investment is lumpy, periods of low investment are followed by large investment spikes; (ii) small firms grow faster and invest more than large firms; and (iii) entry and exit rates are very large.

We seek to understand these facts using a model of investment behavior where heterogeneous firms face costly external finance. In this environment firms seek to maximize their value by making three interrelated decisions: (i) they must choose whether to participate or not in the market; if they decide to participate they must (ii) choose how much to invest; and (iii) how to finance their investment. This model successfully replicates all the stylized facts described above, thus providing a useful laboratory to investigate the role of financing constraints and their implications for the performance of empirical investment equations.

Using the model's stationary distribution of firms to run standard investment regressions, we obtain four main findings. First, the existence of financial constraints is not sufficient to establish cash flow as a significant regressor in standard investment equations, beyond q. In the context of a fully specified model, the effect of financial constraints should be already included in the market value of the firm and thus should also be captured by q. Second, financing constraints are also not necessary to obtain significant cash-flow effects. We show that it is possible to construct examples where cash flow adds some predictive power to investment equations, even in the absence of financial frictions. Third, as Thomas J. Sargent (1980) and Matthew D. Shapiro (1986) documented, we find that, in the context of these general-equilibrium models, a spurious correlation between investment, cash flow, and output is likely if one neglects the effects of underlying shocks. Clearly this implies that focusing on these reduced-form investment equations is quite problematic. Finally, we study the effects of alternative sources of measurement error in the model. As expected, we find that using incorrect measures of fundamentals can lead an econometrician to assign a much larger role to cash flow in the reduced-form investment regressions. These

* The Wharton School, University of Pennsylvania, 3620 Locust Walk, Philadelphia, PA 19104, and CEPR. (e-mail: gomesj@wharton.upenn.edu). I would like to thank Andy Abel, Rui Albuquerque, Marty Eichenbaum, Francisco Gomes, Jeremy Greenwood, George Hall, Hugo Hopenhayn, Per Krusell, Sergio Rebelo, Richard Rogerson, Amir Yaron, two anonymous referees, as well as numerous seminar participants for all the valuable comments. Financial support from JNICT and the Bank of Portugal is gratefully acknowledged. Any errors are mine.

findings highlight the enormous difficulties in using standard investment regressions in practice, and they cast serious doubt on the common interpretation of cash-flow effects in the data as evidence of financing constraints.

This paper is organized as follows. Section I provides a brief overview of recent empirical findings about investment behavior and its relation to financial variables. Section II describes our economic environment. Section III details both the calibration methods and characterizes the competitive equilibrium generated by the model. The theoretical implications for the empirical investment equations are then examined in Section IV. Section V summarizes our findings.

I. Investment and Finance

A. *Investment Behavior*

Investment is a central macroeconomic variable. Its fluctuations account for a large fraction of the cyclical volatility of output and income, and most economists link high rates of investment to long-run economic growth. Unfortunately, understanding investment behavior has proved to be a very difficult task (see Andrew B. Abel, 1980; Abel and Olivier J. Blanchard, 1986). Earlier work focused on representative agent frameworks with convex costs that smooth investment over time (Dale W. Jorgenson, 1963; James Tobin, 1969; Robert E. Lucas, Jr. and Edward C. Prescott, 1971; Fumio Hayashi, 1982, among others). Empirical work built directly on these theories to estimate empirical investment equations, using both aggregate and firm-level data (George von Furstenburg, 1977; Abel, 1980; Lawrence H. Summers, 1981).

The empirical performance of these early models was disappointing, however, and attention shifted recently towards alternative theories emphasizing fixed costs and irreversibilities (for example, Sargent, 1980; Abel and Janice C. Eberly, 1994; Avinash K. Dixit and Robert S. Pindyck, 1994). A significant implication of this new theoretical approach is that investment should be "lumpy": significant amounts of investment, or disinvestment, take place in a relatively short period of time, while most periods are characterized by only minor changes to the existing capital

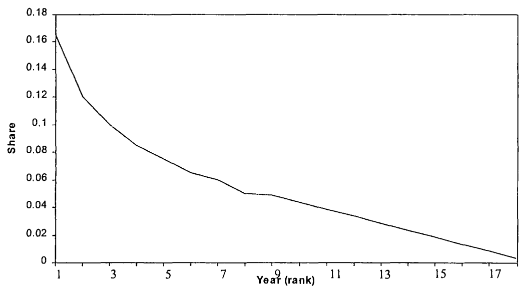

FIGURE 1. FIRM-LEVEL INVESTMENT SHARES

stock. Empirical evidence provides support for this approach. Figure 1, adapted from Mark Doms and Timothy Dunne (1998), ranks the average rates of investment for over 12,000 plants for the period 1972 to 1988. The concentration of plant-level investment is clear: about 25 percent of investment spending takes place in one year and on average about half of a plant's total investment takes place during a three-year period.

A consequence of these models is that simple aggregation theorems no longer hold. The large nonlinearities in investment behavior imply that the cross-sectional distribution of firm/plant-level investment may affect aggregate variables, which requires modeling firm heterogeneity explicitly (Ricardo J. Caballero and Eduardo Engel, 1994; Caballero et al., 1995; Abel and Eberly, 1996; Marcelo Veracierto, 1997; Russell Cooper et al., 1999).

B. *Finance*

In contrast with the predictions of the Franco Modigliani and Merton H. Miller (1958) theorem, most firms seem to prefer internal sources to finance investment. According to Stephen A. Ross et al. (1993), about 80 percent of all financing is done with internally generated funds. Explanations for this behavior usually highlight the role of information asymmetries (Stewart C. Meyers and Nicholas S. Majluf, 1984) and agency issues (Michael C. Jensen and William H. Meckling, 1976) in raising the costs of external funds.

In a detailed study, Stephen M. Fazzari et al. (1988) find that low-dividend-paying firms (a priori more likely to be financially constrained) are generally smaller, invest more, and grow faster than high-dividend firms. They also have higher cash flows and higher average q's.

In addition, reduced-form investment equations find evidence of highly significant cash-flow effects while, on the other hand, Tobin's q appears to have only a marginal impact on investment. A number of authors have interpreted this as evidence supporting the existence of significant finance constraints, particularly for small firms.

Several other studies cast doubt on this interpretation however. From a theoretical standpoint, work by Sargent (1980), Shapiro (1986), and, in a somewhat different context, Peter M. Garber and Robert G. King (1983), suggest a number of potential problems with the estimation of reduced-form equations from structural models of firm and investment behavior. In particular, they show that when the underlying shocks to output and cash flow are positively autocorrelated it is possible to obtain strong, but quite spurious, correlations between these variables and investment when one ignores more structural information.

On the empirical side, work by Simon Gilchrist and Charles P. Himmelberg (1995), Jason G. Cummins et al. (1998), and Timothy Erickson and Toni M. Whited (2000) argue that, at least in some cases, the observed cash-flow effect in earlier studies merely reflects the fact that cash flow might contain information about the firm's investment opportunities, otherwise not reflected in the measure of Tobin's q being used. Although their methodologies and results differ somewhat, two common findings emerge.[1] First, it is clear that serious consideration of measurement issues assigns a much larger role to fundamentals in the estimated equations. Second, both in Cummins et al. (1998), Erickson and Whited (2000), and even in some subsamples in Gilchrist and Himmelberg (1995), the cash-flow effect actually disappears: cash flow adds no significant predictive power to the investment equation.

This debate about the significance of cash-flow augmented investment regressions is very difficult to understand, however, due to the absence of a solid theoretical structure behind these reduced-form regressions. Our main goal here is to provide a theoretical background against which one can better understand these results.

II. Economic Environment

A pattern that emerges from the evidence discussed is one of substantial firm heterogeneity across a number of different characteristics such as firm size, growth, investment, and financing patterns. It seems, then, important that our framework be consistent with this and thus able to produce a well-defined distribution of firms that provides a reasonable description of the data.

The environment is an equilibrium variant of the standard neoclassical model of investment augmented to include financing constraints at the firm level as in William A. Brock and Blake LeBaron (1990) and entry and exit in the tradition of Boyan Jovanovic (1982) and Hugo A. Hopenhayn (1992). Financial intermediation is not modeled explicitly however. Instead we proceed in the spirit of work by S. Rao Aiyagari and Mark Gertler (1991) and Javier Diaz-Gimenez et al. (1992), and summarize this costly activity with a simple functional form that captures the basic notion that external funds seem to be more costly than internal ones. This is sufficient to make internal funds "valuable" for firms.

The economy consists of three sectors: households, financial intermediaries, and producers. Households are represented by a single agent maximizing lifetime utility. Income comes from wages and dividends on the shares that the household holds in every firm. Firms produce a single output that can be transformed in either capital or consumption goods, combining labor services with capital goods. Firms pay dividends directly to the consumer but require

[1] Briefly, Gilchrist and Himmelberg (1995) adopt a procedure similar to that developed in Abel and Blanchard (1986) to construct an alternative measure of fundamentals that is then used in standard investment regressions instead of average q. Cummins et al. (1998) use earnings forecasts from the I/B/E/S data set to construct an alternative measure of fundamentals. This is then used in both the standard investment equations and also in some semiparametric regressions. Finally, Erickson and Whited (2000) develop a very sophisticated econometric methodology based on high-order Generalized Methods of Moments (GMM) estimators, specifically designed to handle measurement error. Minor differences also exist in the definition of variables and sample construction.

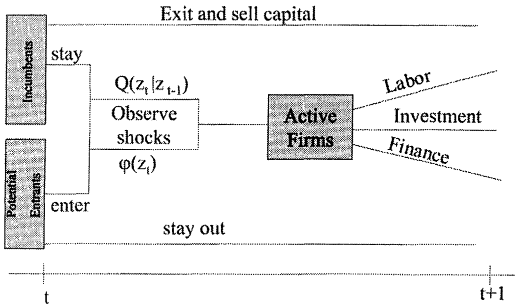

FIGURE 2. TIMING OF EVENTS

the services of financial intermediaries to obtain additional funds. Financial intermediaries are also summarized in a single agent that provides these services at some cost.

A. *Firm Behavior*

Production is carried out in all periods by a continuum of firms. Figure 2 provides a graphical description of the timing of the decisions made by firms in this economy. In any given period each incumbent chooses (i) whether to exit the market or continue producing, and, if it stays; (ii) how much to invest; and (iii) how to finance the investment (internal or external funds). Incumbent firms choose to exit at the start of the period before any current variables are observed and before they make any other decisions. Firms that exit sell all their assets, hire no labor, and earn zero profits.

In every period there is also a continuum of potential entrants that decide whether or not to enter the market. Entrants choose to enter also at the beginning of the period before any current variables are observed.

Production requires two inputs: capital, k, and labor, l, and is subject to an individual technology shock, z. Firms hire labor at the market wage rate $w > 0$ and discount future cash flows by a factor of $\beta \in [0, 1)$. The space of inputs is a subset of the space of (nonnegative) real numbers, $\mathcal{K} \times \mathcal{L} \subseteq \Re^2_+$. The stochastic process for this shock has bounded support $Z = [\underline{z}, \bar{z}]$, $-\infty < \underline{z} < \bar{z} < \infty$. Also define $\Im_z$ and $\Im_k$ as, respectively, the minimal sigma-fields generated by Z and $\mathcal{K}$. Production of the single good is carried out by each firm according to the production function $F: \mathcal{K} \times \mathcal{L} \times Z \to \Re_+$. Assumption 1 summarizes the conditions imposed on F.

ASSUMPTION 1: *The production function* F: $\mathcal{K} \times \mathcal{L} \times Z \to \Re_+$ *has the following properties:* (*i*) *strictly increasing, strictly concave, twice continuously differentiable, homogeneous, and satisfies the Inada conditions;* (*ii*) $\forall h > 0$, $F(hk, hl, z) < hF(k, l, z)$.

Without decreasing returns to scale [part (ii)] the individual decision rules and the distribution of firms would not be well defined. We also assume that there is a fixed cost of production, $f \geq 0$. This cost must be paid every period the firm remains in the market.

Assumption 2 below concerns the idiosyncratic technology shocks. We use z' to denote the value of next period level of technology.

ASSUMPTION 2: (*a*) *Incumbents' shocks* (*i*) *are uncorrelated across firms, and* (*ii*) *have a common stationary and monotone* (*increasing*) *Markov transition function* $Q(z', z)$: $Z \times \Im_z \to [0, 1]$ *that satisfies the Feller property;* (*b*) *entrants draw their initial level of technology independently from a common distribution* $\varphi(z)$.

Without idiosyncratic shocks the production sector could be consolidated into a single producer, as all firms would not only have the same decision rules, but would also make the same choices. For simplicity it is assumed that the stochastic process for each firm follows a common Markov process. Monotonicity of the transition function guarantees that profits are increasing in the current shock, a result that will prove useful later. Assumption 2 also determines the distribution of shocks for new firms. These are assumed to be drawn *after* the entry decision is made, potentially implying substantial heterogeneity in productivity across all new entrants, a fact consistent with the evidence in Dunne (1994).

We can now summarize the static decisions of the firm, thus simplifying the dynamic decision problem below.

- Profits

$$(1) \quad \pi(k, z; w) = \max_{l \geq 0}\{F(k, l; z) - wl - f\}.$$

• Labor Demand

(2)

$$l(k, z; w) = \underset{l \geq 0}{\operatorname{argmax}}\{F(k, l; z) - wl - f\}.$$

• Supply of Output

$$(3) \quad y(k, z; w) = F(k, l(k, z; w); z).$$

Assumption 1 above guarantees that all these functions are well defined and unique. Together with standard profit-maximization conditions they also guarantee that the profit function $\pi(k, z; w)$ is (i) continuous, strictly increasing, and strictly concave in $k \in \mathcal{K}$; (ii) continuous and strictly increasing in $z \in \mathcal{Z}$; and (iii) continuous and strictly decreasing in the wage rate $w > 0$.

The cost of obtaining a stock of capital k' for next period, given that the current stock is k, will be represented by the traditional investment function $i(k, k')$: $\mathcal{K} \times \mathcal{K} \rightarrow \mathcal{R}$:

$$(4) \quad i(k, k') = k' - (1 - \delta)k, \quad \delta \in [0, 1].$$

The final aspect concerns the finance costs. This is a key assumption for our analysis. Ideally we would prefer to model financial intermediation endogenously (as in Douglas W. Diamond, 1984, for example). This approach, however, would demand a far more complex model than the one necessary to study the effect of intermediation costs on investment behavior at the firm level. Therefore we choose to summarize this costly activity with a simple functional form capturing the basic notion that external funds are costly.

Clearly a firm will only choose to use external sources as a last resource when current investment opportunities justify the additional cost. Alternatively, it is never optimal to issue new securities while paying dividends. Thus the financing cost function has the form

$$(5) \quad \lambda = \lambda(i(k, k') - \pi(k, z; w)) = \lambda(k, k', z; w).$$

We make only very general assumptions about the nature of λ. In particular we assume that these costs are positive and increasing if the firm uses external funds. If no external finance is required, these costs are zero. This

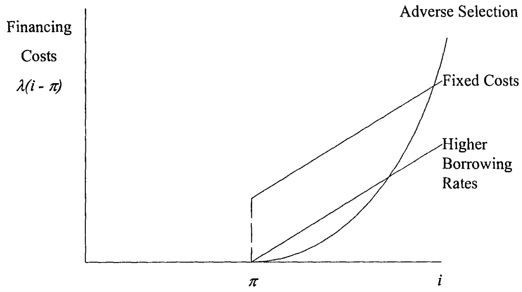

FIGURE 3. FINANCING COSTS

assumption can accommodate several specifications of the cost function, possibly motivated by alternative interpretations of the nature of the financial imperfection.

ASSUMPTION 3: *The financing cost function* λ: $\mathcal{R} \rightarrow \mathcal{R}_+$ *satisfies: (i)* $\forall a \leq 0$, $\lambda(a) = 0$; *(ii)* $\forall a > 0$, $\lambda(a) > 0$, $\lambda'(a) > 0$.

Figure 3 depicts alternative representations of this function consistent with Assumption 3.

The dynamic nature of each firm's problem should be clear from the interaction of the investment decision with the Markov structure for the productivity shock. Given the recursive structure of this environment it is simpler to define each firm's problem using dynamic programming. Let $p(k, z; w)$ denote the market value of a firm that has a capital stock of k and a productivity z, while facing a wage rate equal to w. The dynamic problem facing each firm is

$$(6) \quad p(k, z; w) = \max_{k' \geq 0}\left\{\pi(k, z; w) - i(k, k') - \lambda(k, k', z; w) + \beta \max\left((1 - \delta)k', \int p(k', z'; w) \times Q(dz'|z)\right)\right\}.$$

The right-hand side of (6) specifies the decisions faced by this firm. The first three terms reflect the current dividends: profits minus investment spending and financing costs. The last term is the expected continuation value, allowing for the exit decision. Firms will exit when expected profitability is below the market value of its assets:

$$(7) \quad \int p(k', z'; w)Q(dz'|z) < (1 - \delta)k'.$$

In what follows we will focus solely on stationary equilibria, where all prices and aggregate quantities as well as the distribution of firms across states are constant. Below we show that such equilibrium indeed exists and is unique. Thus we will assume that $w = w'$.

The fact that the exit decision is incorporated in the solution of the dynamic problem of each firm together with the finance cost function complicates the dynamic program (6) considerably. Nonetheless one can show that a solution to this problem exists and establish some useful properties for the value function $p(k, z; w)$.

PROPOSITION 1: *There is a unique function* $p(k, z; w)$ *that satisfies* (6).

PROOF:
See Gomes (2000).

PROPOSITION 2: *The value function* $p(k, z; w)$ *is* (*i*) *continuous and increasing in* (k, z), *and* (*ii*) *continuous and strictly decreasing in* w.

PROOF:
See Gomes (2000).

Proposition 1 guarantees the existence of a value function $p(k, z; w)$ that solves (6), while Proposition 2 establishes some of its properties. Associated with this solution there are two decision rules concerning capital accumulation and the exit decision, denoted $k(k, z; w)$ and $x(k, z; w)$ respectively. Both are computed in the process of solving the dynamic problem of the firm. Capital accumulation is described by the condition

$$(8) \quad k(k, z; w) = \min\left\{\arg\max_{k' \geq 0}\left\{\pi(k, z; w) - i(k, k') - \lambda(k, k', z; w) + \beta \max\left((1-\delta)k', \int p(k', z'; w) \times Q(dz', z)\right)\right\}\right\}.$$

Given the structure of the problem, the maximizer on the right-hand side of (6) need not be unique. Equation (8) states that a firm chooses the value that does not require external finance.[2] Intuitively, if borrowing does not (strictly) raise the value of the firm it is not used.

Since exit takes place *before* the shock is observed, firms know in advance whether they will choose to exit or not in the next period. Exit is therefore completely determined by the current state and can be summarized by a threshold value for the idiosyncratic productivity level. The exit decision can be described as follows

$$(9) \quad x(k, z; w) = \begin{cases} 1 & z > z^* \quad \text{(stay)} \\ 0 & z \leq z^* \quad \text{(exit)} \end{cases}$$

where

$$(10) \quad z^*(k, z; w) = \min\left\{\inf\left\{z: \int p(k', z'; w) \times Q(dz'|z) \geq (1-\delta)k'\right\}, \bar{z}\right\}.$$

Intuitively, conditions (9) and (10) formalize the idea that firms choose to stay only if their future prospects (as measured by their expected value next period) are good enough

[2] Only two solutions can exist for each state, one with and one without external finance. Clearly if the value of the firm is the same in both cases and the value function is increasing in capital, then the optimal investment choice must be higher with external finance. Technically, (8) provides a measurable selection of the optimal policy correspondence.

to justify the fixed cost of operating the firm next period. Because shocks are positively correlated, this is only true if the current level of productivity is above some threshold z^*.

Propositions 1 and 2 characterize the solution to the individual problem of each firm for a given wage rate w. The equilibrium level of w is uniquely determined by the free-entry condition:

$$\int p(0, z; w)\varphi(dz) \leq 0. \tag{11}$$

This condition must hold with equality if entry is positive. In this case Proposition 2 guarantees that there is a unique value of w that solves (11). The exact level of entry, denoted by B, is then determined by the market-clearing conditions below.

B. *Aggregation*

With the description of individual firm behavior complete we can now characterize the aggregate variables for this economy. Since each firm can be described by its current individual state (k, z), we can summarize the aggregate distribution of firms with a measure defined over this state space.

Formally we define the measure μ such that $\forall(k, z) \in \mathcal{K} \times Z$, μ(k, z) denotes the mass of firms in the state (k, z). For any set $\Theta = (\mathcal{K}, \mathfrak{z}) \in \mathfrak{I}_k \times \mathfrak{I}_z$, the law of motion for this measure μ is given by

$$\mu'(\Theta) = \begin{cases} T(\Theta, (k, z))\mu(k, z), & \text{if } k > 0 \\ T(\Theta, (0, z))\mu(0, z) \\ \quad + B \int_Z \int_{\mathfrak{z}} \chi(\mathcal{K}) v(dz) Q(dz'|z), & \text{if } k = 0 \end{cases} \tag{12}$$

with

$$T(\Theta, (k, z)) = \int_{\mathfrak{z}} \chi(\mathcal{K}) x(k, z; w) Q(dz'|z), \tag{13}$$

and

$$\chi(\mathcal{K}) = \begin{cases} 1 & \text{if } k(k, z; w) \in \mathcal{K}, \\ 0 & \text{if } k(k, z; w) \notin \mathcal{K}. \end{cases} \tag{14}$$

Condition (12) specifies the law of motion for the aggregate measure of firms: next period's measure is determined directly from combining the surviving firms with entrants (which have zero capital at the moment of entry). Condition (13) describes the law of motion for the individual state of surviving firms by combining the optimal decision rules concerning capital accumulation and exit. It computes all the conditional transition probabilities in the product space $\mathcal{K} \times Z$, where the conditioning event is that the firm survives until next period. Naturally in a stationary equilibrium we expect that $\mu' = \mu = \mu^*$. The invariant distribution μ^* summarizes then the distribution of firms in a stationary industry equilibrium.

With this definition at hand it is straightforward to characterize the aggregate quantities that will be used below to state the market-clearing conditions. These are the (net) aggregate supply of final goods, the aggregate labor demand, total profits, and the aggregate investment, defined respectively as

$$Y(\mu, B; w) = \int (y(k, z; w) - f) \times x(k, z; w)\mu(dk, dz) - Bf, \tag{15}$$

$$L(\mu, B; w) = \int l(k, z; w)x(k, z; w)\mu(dk, dz) + B \int l(0, z; w)\varphi(dz), \tag{16}$$

$$(17)\quad \Pi(\mu, B; w) = \int \pi(k, z; w) x(k, z; w) \mu(dk, dz) + B \int \pi(0, z; w) \varphi(dz),$$

$$(18)\quad I(\mu, B; w) = \int i(k(k, z; w), k) x(k, z; w) \mu(dk, dz) + B \int k(0, z; w) \varphi(dz).$$

Similarly, total intermediation costs for this economy are given by the function

$$(19)\quad \Lambda(\mu, B; w) = \int \lambda(k, k(k, z; w), z; w) \times x(k, z; w) \mu(dk, dz) + B \int \lambda(0, k(0, z; w), z; w) \varphi(dz).$$

Proposition 3 establishes that aggregate quantities are jointly linear homogeneous in the level of entry B and the measure of firms μ. This property will be useful later when we compute the competitive equilibrium of the model.

PROPOSITION 3: *All aggregate quantities* $Y(\mu, B; w)$, $L(\mu, B; w)$, $\Pi(\mu, B; w)$, $I(\mu, B; w)$, *and* $\Lambda(\mu, B; w)$ *defined above are jointly homogeneous of degree one in* B *and* μ.

PROOF:

See Gomes (2000).

C. *Households*

The model is completed with a description of household behavior. Households are summarized by a single representative consumer who derives utility from work and consumption. Household income comes from wages and dividends on the shares of existing firms. The household problem can be written as

$$(20)\quad \max_{c, l, s_t(k_t, z_t) \geq 0} E_0 \left[\sum_{t=0}^{\infty} \tilde{\beta}^t U(c_t, 1 - l_t) \right]$$

$$\text{s.t.}\quad c_t + \int (\tilde{p}_t(k, z) - d_t(k, z)) s_t(k, z) \mu(dk, dz) = \int \max\{\tilde{p}_t(k, z), \phi k\} s_{t-1}(k, z) \times \mu(dk, dz) + w_t l_t,$$

where c is consumption, and $\tilde{p}_t(k, z)$, $d_t(k, z)$, and $s_t(k, z)$ denote the price, dividends, and the fraction of shares owned by the household, respectively. For convenience we assume that dividends are paid just after shares are bought. Note that, since we are restricting ourselves to a stationary measure of firms μ, the assumption of a stationary equilibrium is implicit in this formulation of the household problem. We summarize our (standard) assumptions about preferences in Assumption 4.

ASSUMPTION 4: *The preference relation satisfies* (*i*) $0 < \tilde{\beta} < 1$, and (*ii*) $U(c, 1 - l)$: $\mathfrak{R}_+ \times \mathcal{L} \rightarrow \mathfrak{R}$ *is strictly increasing, strictly concave, and twice continuously differentiable.*

Under these assumptions it is easy to establish that in a stationary equilibrium the discount rates for consumers and firms are the same, and the price of a share, $\tilde{p}(k, z)$, equals the value of the firm, $p(k, z)$.

PROPOSITION 4: *In equilibrium* $\tilde{p}(k, z) = p(k, z)$ *and* $\tilde{\beta} = \beta$.

PROOF:
See Gomes (2000).

Since all aggregate quantities and prices are constant, the consumer problem can then be further simplified into a static problem of the form

$$\max_{c,l \geq 0} U(c, 1 - l) \tag{21}$$

$$\text{s.t.} \quad c = wl + \Pi(\mu, B; w),$$

which is a standard concave problem with interior solutions given by two optimal decision rules for consumption and labor supply, denoted respectively by $C(w, \Pi(\mu, B; w))$ and $L^s(w, \Pi(\mu, B; w))$.

D. *Stationary Competitive Equilibrium*

With the model complete we are now ready to state the conditions required to characterize a stationary competitive equilibrium for this economy.

Definition 1: A stationary competitive equilibrium is a set of (i) allocation rules $L^s(w, \Pi(\mu, B; w))$ and $C(w, \Pi(\mu, B; w))$ for the representative household and $k(k, z; w)$, $l(k, z; w)$ and $x(k, z; w)$ as well as a value function $p(k, z; w)$ for each firm; (ii) aggregate quantities $Y(\mu, B; w)$, $L(\mu, B; w)$, $\Pi(\mu, B; w)$, $I(\mu, B; w)$, and $\Lambda(\mu, B; w)$; (iii) a wage rate w; and (iv) a measure μ of firms and a level of entry B, such that:

- the consumer decision rules solve (21);
- the firm decision rules and value function solve (6) for each firm;
- the free-entry condition (11) is satisfied;
- markets clear:

$$L^s(w, \Pi(\mu, B; w)) = L(\mu, B; w); \tag{22}$$

$$C(w, \Pi(\mu, B; w)) = Y(\mu, B; w) - I(\mu, B; w) - \Lambda(\mu, B; w); \tag{23}$$

- consistency: conditions (15)–(19) are satisfied and the distribution μ obeys the law of motion (12), with $\mu = \mu'$.

PROPOSITION 5: *A stationary competitive equilibrium with positive entry exists.*

PROOF:
See Gomes (2000).

III. Stationary Equilibrium

Computing the competitive equilibrium in Definition 1 involves three steps. First, we need to restrict the model by specifying the functional forms assumed by the general functions in the model. Then, as many parameters as possible must be determined either by matching properties of the model to U.S. data or by using prior empirical evidence. Second, because exact analytical solutions are impossible to obtain in this environment, we need to develop a numerical algorithm capable of approximating the competitive equilibrium up to an arbitrarily small error. Gomes (2000) provides the details on this procedure. The final step is to implement the numerical algorithm and compute the approximate stationary competitive equilibrium.

A. *Calibration*

We need to specify a functional form for technology, one for the utility function and another for the intermediation cost function. We assume that a time period in the model corresponds to one year, since the available data has yearly or lower frequency.

Preferences.—Since preferences are not crucial to our analysis we will take a minimalist approach and summarize household behavior with a momentary utility function similar to that in Gary D. Hansen (1985):

$$U(c, l) = \log c + H(1 - l). \tag{24}$$

This function has the convenient property that labor-supply decisions are independent from the level of wealth and from the real interest rate. Given this specification we only need to assign parameter values to β, the intertemporal discount factor, and H, which is determined by the fraction of workers in the population. In the steady state the real interest rate equals $r = 1/\beta - 1$. Over the last century this value has averaged about 6.5 percent per year, thus we set $\beta = 1/1.065$. Since the share of employed workers in the labor force equals about 60 percent, we choose this value for the parameter H.

Technology.—To completely specify the production technology of this economy we need to choose a functional form for the production function and the stochastic process for the productivity shocks. We also need to assign parameter values for the investment function, the sunk cost of entrants and the fixed cost of production.

We start by assuming that production takes place in each firm according to a decreasing returns Cobb-Douglas production function

$$(25) \quad y = A\varepsilon^{z}k^{\alpha_k}l^{\alpha_l} \quad 0 < \alpha_l + \alpha_k < 1.$$

With this specification we need only to assign values for α_k and α_l, the two output elasticities.[3] To do this we need first to determinate the degree of decreasing returns in the economy and then one of the two elasticities. Using disaggregated data for manufacturing industries, Craig Burnside (1996) estimates returns to scale to be just under 1. Therefore we set $\alpha_l + \alpha_k = 0.95$, a value that does not depart substantially from the standard CRS assumption. Since the labor share of income has averaged about 0.65 over the postwar period, we set $\alpha_l = 0.65$, implying a value of $\alpha_k = 0.3$.

The stochastic process for the level of technology for incumbents is assumed to be described by the relation

$$(26) \qquad z' = \rho z + \varepsilon',$$

where ε is assumed to follow a (truncated) normal distribution with 0 mean, standard deviation of σ and finite support $[-4\sigma, 4\sigma]$. The initial level of technology for entrants is assumed to follow a uniform distribution over $Z = [-4\sigma/\sqrt{1 - \rho^2}, 4\sigma/\sqrt{1 - \rho^2}]$.[4]

The parameters ρ and σ have implications for the degree of persistence and dispersion in the size distribution of firms. We thus restrict their values so that the model is able to (approximately) replicate the second moments of the distribution of investment rates, as obtained from Compustat (Standard & Poor's, 1999a).[5] In our benchmark model this implies choosing values of $\rho = 0.62$ and $\sigma = 0.15$.

Similarly, the rate of depreciation in the capital stock is set to equal 0.12 so that the model matches the average investment to capital ratio found in the data.

Finally, we are left with the fixed cost of production f. Given the degree of returns to scale, the nature of the financing costs and the structure for the idiosyncratic shocks, the fixed cost is closely connected to the level of firm turnover in the model. This is an important element in the analysis as we want to address the potential selection bias issues raised by the fact that the Compustat data set is itself an unbalanced panel with significant firm turnover. We will quantify f to match this fact closely in each simulation.

Financing Costs.—Modeling the costs of external finance is a somewhat more complicated issue and requires some caution. Financial models usually focus on two types of costs associated with this activity: informational costs and transaction costs. Informational costs are related to the extra premium that is associated with the bad signal that a firm may transmit to the market when trying to raise funds as well as the deterioration in balance sheet. These costs are very hard to quantify and the number of empirical studies that address the issue is very limited. Transaction costs are generally associated with the compensation to intermediaries plus all the legal, accounting, and other bureaucratic costs. Constructing quantified measures of these costs have been the subject of much research. Given the availability of data we will focus solely on transaction costs as a source of imperfection in financial markets.

Clifford W. Smith, Jr. (1977) provides de-

[3] The parameter A is introduced only to scale the capital stock. In each simulation we recalibrate A so that the cross-sectional average of the capital stock is identical to that in the data.

[4] These conditions are imposed to satisfy Assumption 2.

[5] For a description of the data see the Appendix.

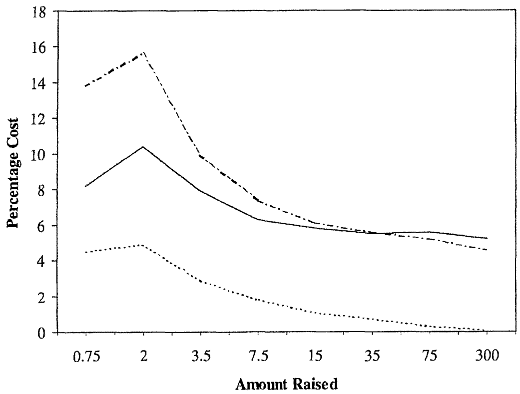

FIGURE 4. COSTS OF ISSUING SECURITIES

Note: The top line signifies underwriting; the middle line signifies rights with standby; the bottom line signifies rights.

tailed evidence on the flotation costs associated with issuing new equity. His data covers all equity issues registered between 1971 and 1975. Figure 4 is based on his work and depicts these costs, as a fraction of the amount raised, for the different methods of finance. Regardless of the method used, the significance of economies of scale in this process is clear: external funding can be extremely expensive for very small operations and this is reflected in the declining average costs of financing.

To obtain a quantitative measure of these costs we fit a linear cost function of the type

$$\lambda = \lambda_0 + \lambda_1 \times \text{NewIssues} \tag{27}$$

to this data and obtain the following results for the total costs of raising new equity (units in millions of U.S. dollars):

$$\lambda = 0.48 + 0.028 \times \text{NewIssues}. \tag{28}$$

Since this regression fits the data quite well (this should be expected given the pattern of average costs depicted on Figure 4) we choose to specify this functional form for the unit cost function and use our regression results to set $\lambda_1 = 0.028$. Since the intercept term is sensitive to the unit of measurement, a better measure for λ_0 can be obtained by looking at the average cost for very small issues. These average about 0.108, implying an intercept term of $\lambda_0 = 0.08$, a value that we adopt. We will examine the

TABLE 1—CALIBRATION

Parameter	Benchmark value	Empirical restriction
Technology		
α_k	0.3	Degree of returns to scale
α_l	0.65	Labor share
δ	0.145	Investment to capital ratio
Technology Shock		
ρ	0.762	Persistence in investment
σ	0.0352	Cross-section variance of investment
Financing Costs		
λ_0	0.08	Fixed flotation costs
λ_1	0.028	Unit flotation costs
Preferences		
β	1/1.065	Interest rate
H	0.6	Employment share

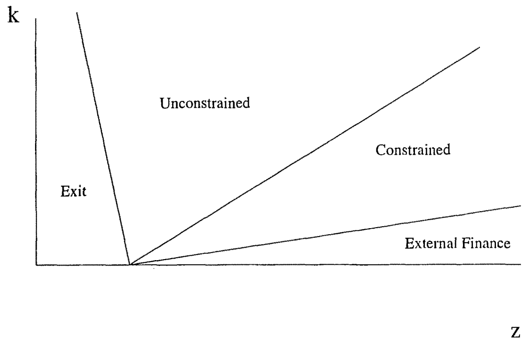

FIGURE 5. FIRM BEHAVIOR

consequences of adopting alternative assumptions on the nature and the magnitude of these costs later.

Table 1 summarizes our benchmark calibration procedure.

B. *Results*

Optimal Firm Decisions.—Before characterizing the equilibrium and the full cross-sectional distribution of firms in this economy, it is helpful to develop some intuition about optimal firm behavior. Figure 5 provides a very simple qualitative illustration of the combined finance-investment decisions. This representation highlights a few of the interesting features of this economy. First, only firms with relatively low levels of productivity leave the market in any given period. Second, external financing is used only by those firms who are either very small or

TABLE 2—AGGREGATE RESULTS

Variable	Model
Investment share (I/Y)	0.21
Share of financing costs (Λ/Y)	0.0062
External finance/total costs	0.17
Flotation costs/external finance	0.39

TABLE 3—CROSS-SECTIONAL RESULTS

Variable	Data	Model
Matched quantities		
Average size (capital)	80.89	80.89
Investment rate I/K	0.145	0.145
Standard deviation I/K	0.139	0.160
Autocorrelation I/K	0.239	0.191
Other quantities		
Mean q	1.56	1.12
Growth rate (sales)	0.036	0.031
Average CF/K	0.292	0.221
Standard deviation CF/K	0.214	0.091
Negative investment (fraction)	0.08	0.13

Note: CF denotes cash flow

very productive (the "borrowing" firms). The explanation is simple: in the absence of frictions, productive firms would invest, anticipating future levels of high productivity. The marginal product of capital must be very high, or alternatively, they must be very far away from their desired capital stock. In this case it pays to take the extra costs of raising external finance and invest.

For firms either somewhat larger or somewhat less productive, the difference between the actual and desired level is not quite as large (or the marginal productivity of capital is not quite as high), and it is not profitable to accept the financing costs. Since their investment decision is constrained by the fact that they face costly access to external funds, we define these firms as "constrained." The remaining firms are defined as "unconstrained" in the sense that they invest less than their available funds. Notice that we actually observe which firms are constrained by current cash flows, unlike much of the empirical literature.

Aggregate Quantities.—Table 2 provides some summary statistics about the aggregate quantities generated by the stationary equilibrium of the model. These are reasonably consistent with U.S. data with one exception: the model implies that the financing costs are only a very small fraction of total GDP. This is, of course, a consequence of the stylized, and limited, role of the financial sector and is not altogether surprising. Table 2 also shows the fraction of investment that is financed by external funds. The fact that these funds are very expensive produces two effects: (i) a direct "cost" effect; firms use external finance infrequently (about once in every seven years); and (ii) an indirect "size" effect; if external funds are required, they are raised in very large quantities.

Firm Behavior.—Table 3 focuses on the microeconomic implications of the model by examining some summary statistics from the cross-sectional distribution of firms. The first part of the table contains those variables that the model was calibrated to approximately match in the numerical exercise. Unfortunately, the high degree of nonlinearities in the solution makes it nearly impossible to match these moments exactly. Nevertheless we see that our approximation appears quite reasonable along those dimensions.

The second half of the table concerns the model's ability to replicate other interesting features of the data and thus provides a more interesting measure of the empirical success of the model. Consistent with the early findings of Eric B. Lindenberg and Ross (1981), as well as several other more recent studies, we find that the model implies average values of q consistently above 1 on average. Here this is a consequence of industry selection: the surviving firms in the sample are the most profitable ones. In addition, the model does a reasonably good job of replicating some features regarding the behavior of cash flow. Given the role that q and cash flow play in the investment regressions below, this is an important finding.

The Role of Financing Constraints.—Table 4 examines the role of financial constraints. It is easy to see that they are clearly nontrivial in the benchmark economy: over half of all the firms are constrained according to our definition above. Although these firms are somewhat smaller than the average unconstrained firm, it appears that the significance of financing con-

TABLE 4—FINANCE

Variable	External finance	Constrained	Unconstrained
Firms	0.07	0.63	0.30
Share of investment	0.79	0.74	−0.53
Size (capital)	55.96	171.93	298.57
Mean I/K	1.20	0.188	−0.086
Tobin's q	1.34	1.14	1.08
Marginal product of capital	0.24	0.22	0.19

straints is essentially determined by the individual productivity (z) and not the size of the firm (k). This becomes clear from examining the next rows: constrained firms have a higher marginal productivity of capital and invest more than those unconstrained. They also have a higher value of Tobin's q on average.

Marginal productivities provide an additional insight into the model. The marginal productivity of capital for constrained firms is actually higher than the rental price of capital ($r + \delta = 0.21$), thus they do not satisfy a standard Euler equation. Unconstrained firms on the other hand are relatively unproductive. On average, however, they do not satisfy the Euler equation either: they overaccumulate capital, as evidenced by the low marginal product.[6,7] A third group is made by those firms who use external finance. These are clearly the most productive of firms as they must incur the financing costs. They are also the smallest on average.

A potential problem with the analysis above is that, in practice, we can not determine exactly which firms are financially constrained. A common empirical proxy for this is firm size. We examine the results of applying this procedure to our model in Table 5. We separate firms in two categories: "small" firms, with a capital stock below the sample average, and "large" firms, those whose size is above the market average.

TABLE 5—FINANCE AND SIZE

Variable	Small firms	Large firms
Fraction of firms	0.72	0.28
Share of investment	0.85	0.15
Size (capital)	139.80	359.79
Mean I/K	0.185	0.035
Fraction constrained	0.61	0.39
Exit rate	0.12	0.01
Tobin's q	1.12	1.15
Marginal product of capital	0.21	0.22

An interesting feature of this model is its ability to replicate the negative correlation between firm size and firm growth, extensively documented by many authors (for example, David S. Evans, 1987; Bronwyn H. Hall, 1987). Small firms are growing faster as evidenced by the larger investment to capital ratio. Remember that in our model, size depends on profitability: firms become large only because they had a very good history of technology shocks. Because technology levels are mean reverting they must, at least on average, grow at a slower rate than small firms. Small firms on the other hand appear more risky with very large annual exit rates. Despite the fact that more small firms face financing constraints, the size of the firm appears as a very imperfect proxy for the existence of financing constraints. There are two reasons for this: (i) the persistence in the idiosyncratic shocks renders productivity largely uncorrelated with size and, as we have seen above, (ii) several small firms are actually using external funds, and thus are not financially constrained.[8]

[6] This subgroup includes a large fraction of the exiting firms which explains the large average drop in investment in this sample.

[7] This result however is not robust. Here capital accumulation is the only way firms can insure against paying financing costs in the future. Relaxing this assumption, however, does not change our main results about the absence of cash-flow effects. Results for a version of the model with cash holdings are available upon request.

[8] This fact is also documented by some Euler-equation-based tests of financing constraints (see Whited, 1992, for example).

IV. Empirical Implications

A. *Investment Equations*

The equilibrium distribution generated by the solution to the model is suitable to address the issue of estimating reduced-form investment equations. Section I noted the emphasis placed by several studies on these econometric procedures to determine the relevance of finance constraints. The empirical methodology is to specify a functional form for investment of the type

$$(29)\quad \frac{i_{i,t}}{k_{i,t-1}} = b_0 + b_1 q_{i,t-1} + b_2 \frac{\pi_{i,t-1}}{k_{i,t-1}} + f_t + d_i + \varepsilon_{i,t}$$

where

$$(30)\quad q_{i,t} = \frac{p_{i,t}}{k_{i,t}}$$

and p is the market value of the firm defined in (6). Hence we are using beginning of period average q and beginning of period cash flow in these regressions; f_t is a year effect and d_i denotes a firm-specific fixed effect. The year effects are introduced to eliminate the effects of business cycles in the regression.

Empirical Results.—The basic source for our empirical analysis is the Compustat data set. We use the combined annual, full coverage, and research 1998 files. We start our sample in 1979 since the coverage of over-the-counter firms increased substantially after this year, giving us a much larger cross section of firms.

Substantial changes in reporting and accounting methods lead us to stop the sample in 1988 (for details, see Ben S. Bernanke et al., 1990). The total number of firms in this sample for these years is 9,761. This number is then substantially reduced to eliminate the effects of unreliable data, outliers, and events such as mergers, in the sample period described. After these procedures are applied we are left with an unbalanced panel of 12,321 firm-year observations.

The second column of Tables 6 and 7 documents the results of estimating equation (29) in first differences with and without imposing $b_2 = 0$.[9]

TABLE 6—STANDARD INVESTMENT EQUATIONS

Coefficient	Data	Benchmark model	No financing constraints
b_1	0.06	2.82	8.07
	(0.01)	(0.08)	(0.10)
$\bar{R}^2$	0.12	0.53	0.84

Note: Standard errors are in parentheses.

TABLE 7—CASH-FLOW-AUGMENTED EQUATIONS

Coefficient	Data	Benchmark model	No financing constraints
b_1	0.06	4.13	11.52
	(0.01)	(0.39)	(0.05)
b_2	0.14	−2.67	−10.19
	(0.04)	(0.77)	(0.10)
$\bar{R}^2$	0.25	0.53	0.98

Note: Standard errors are in parentheses.

As others before us we also find a "cash-flow effect": the cash-flow regressor appears to substantially improve the predictive power of the regression as the adjusted R^2 also improves substantially when this variable is included.

Theoretical Results.—We now turn to the results of estimating equation (29) using the artificial data generated by our model. To make these results comparable we simulate all of our economies below over a period of ten years using the transition function (13). We then scale the stationary distribution of firms (12) (which is a measure) so that we have, in every period, the average number of firms in the Compustat sample (a little over 1,200). This will then be our artificial counterpart to the Compustat data set. We then apply the same econometric procedures to estimate (29) for this data.[10] Finally we repeat this procedure 1,000 times and report the sample means for both coefficients, the standard errors, and the adjusted R^2.

[9] This procedure is commonly employed as possible correction of measurement error in several studies.

[10] Year effects are not necessary since we are only looking at a nonfluctuating economy. Nevertheless this is verified for the artificial data.

The results for our benchmark model are shown in the third column of Tables 6 and 7. We obtain three conclusions from this exercise. First, relative to the standard neoclassical model we find that our benchmark calibration delivers a much lower correlation between q and investment. This is not very surprising given that we have departed substantially from the strong homogeneity assumptions often imposed in the literature. As a result, the proper characterization of the investment decision of each firm will not, in general, resemble a simple linear function of average q. Given the empirical failure of simple q models, this is probably a good result. Second, the coefficient on q is much higher than in the data. This result was also documented by Sargent's (1980) early study and should not be very surprising as the optimal investment decisions in this model follow some type of (s, S) behavior and involve some dramatic, although infrequent, changes in investment rates at some trigger points. It is well known that one can easily eliminate this problem by introducing some additional convex adjustment costs on investment. Since our goal is to understand the "cash-flow effect" and not to replicate individual coefficients we abstract from these for simplicity. Third, and more importantly, we do not find evidence of a cash-flow effect. Unlike our empirical results, the cash-flow-augmented regressions add no significant explanatory power to the investment equation. Although financial constraints are very important for many firms, as we have seen in Table 4, the independent informational content of cash flow appears quite small.[11]

The last column is the most convincing: it shows the results of estimating (29) for a model with *no financing constraints at all.* That is, for this model we have set the function $\lambda(\cdot)$ equal to zero everywhere. Here however we do find a cash-flow effect. The implication of these results is quite apparent: not only is the existence of financial constraints not sufficient to establish cash flow as a significant regressor beyond average q, but it also appears not to be necessary. This is an important result. It is natural that we may ask whether it survives alterations in our benchmark model.

Alternative Samples.—To better understand our results it is instructive, however, to first replicate regression (29) for alternative subsamples of firms. In particular we consider six common types of subsamples: "balanced panel" of firms, "small" versus "large" firms, and "constrained," "unconstrained," and "borrowing" (those who use external funds) firms. Clearly several of these subsets overlap. We emphasize again that the exact identification of "constrained," "unconstrained," and "borrowing" firms is only possible in the model.

Tables 8 and 9 illustrate two main results.[12] First, we find no evidence of strong cash-flow effects, with the possible exception of the "unconstrained" and "small" samples. The result for unconstrained firms is very curious: cash flow is significant for firms who are clearly identified as not suffering from financial constraint. Second, cash-flow effects do not exist for the subsample of "constrained" firms. Again this is a very strong result. Taken together these two results are again very suggestive of the (lack of) statistical power of these regressions as a useful means of identifying financially constrained firms.

Why does cash flow add any explanatory power at all in some of these regressions? Simply because of specification error: the right equation for investment behavior is the policy rule (8). For some firms this function is highly nonlinear and a linear function of q does a very poor job. In this case cash flow may improve the quality of the linear approximation. As we have seen, however, this has nothing to do with financing constraints.

[11] This is true despite the large and highly significant cash-flow coefficients in the regression. There are at least three reasons for this. First, as we have noted above, since we abstracted from convex adjustment costs in this model, the regression coefficients will be generally quite large. Second, as we will see below, with technology shocks cash flow becomes highly correlated with q. Third, the equation is badly misspecified due to the (generally) highly nonlinear nature of investment decisions. All these render the estimated coefficients on these linear equations rather uninformative. Instead we choose to focus on the adjusted R^2 as a better indicator of additional informative content of the cash-flow regressor.

[12] Qualitatively, these results are not always independent of model specification. Nevertheless, they survive most of the extensions and provide a clear example of the problems in interpreting the results of these regressions.

TABLE 8—STANDARD INVESTMENT EQUATIONS: SUBSAMPLES

Coefficient	Data	Balanced panel	Constrained	Unconstrained	External finance	Large	Small
b_1	0.06	2.12	0.60	1.83	4.79	3.19	1.99
	(0.01)	(0.78)	(0.02)	(0.14)	(0.30)	(0.07)	(0.21)
$\bar{R}^2$	0.12	0.58	0.99	0.34	0.98	0.74	0.71

Note: Standard errors are in parentheses.

TABLE 9—CASH-FLOW-AUGMENTED EQUATIONS: SUBSAMPLES

Coefficient	Data	Balanced panel	Constrained	Unconstrained	External finance	Large	Small
b_1	0.06	8.09	0.04	15.59	12.01	6.15	18.60
	(0.01)	(0.34)	(0.12)	(0.94)	(0.45)	(0.34)	(2.20)
b_2	0.14	−12.57	1.13	−22.48	−17.13	−5.82	−37.60
	(0.04)	(0.70)	(0.24)	(1.54)	(1.00)	(0.66)	(4.98)
$\bar{R}^2$	0.25	0.61	0.99	0.52	0.99	0.77	0.83

Note: Standard errors are in parentheses.

B. *Alternative Specifications*

We now examine the robustness of our findings by extending the benchmark model along several dimensions. Although the quantitative details change across different specifications, the strong qualitative message remains: information about financing constraints should be captured by the (appropriately measured) q regressor and cash flow is essentially redundant and uninformative.

Alternative Financing Constraints.—The calibration of the function $\lambda(\cdot)$ that describes the costs of raising external finance was based on data about the transaction costs of issuing new equity. There are a number of reasons why we may want to expand upon this initial characterization. First, from an economic viewpoint one may argue that equity finance accounts for only a small fraction of total external finance. A bank loan, say, may well have much lower costs than those we have assumed above. Second, from a mathematical perspective one may also be interested in examining whether our results are sensitive to changes in the functional form of $\lambda(\cdot)$.

In this section we consider two alternative specifications of the financing costs function $\lambda(\cdot)$. While these are not exhaustive they cover some of the obvious extensions one would like to examine and provide a very good indication of the robustness of our initial findings. In all experiments we adjust the calibration of the technology shocks to still match the investment distribution (the first part of Table 3) closely.

Our first experiment is a nice blend of our two considerations above. Motivated by the relatively low costs of bank finance that most firms use, we interpret $\lambda(\cdot)$ as the difference between a "borrowing" rate $(r + \lambda)$, at which external funds can be obtained from banks, and a "lending" rate (r), implicit in the opportunity cost of internal funds to firms. Formally we impose

$$\lambda = 0.02 \times \text{Borrowing}. \tag{31}$$

The interest rate spread is then set at 2 percent, the average spread between six-month CDs (lending rate) and the prime rate (borrowing rate) for the period 1968–1997.[13]

An alternative experience is to focus on the role of the fixed costs in the function $\lambda(\cdot)$. To do so let

$$\lambda = 0.15. \tag{32}$$

[13] An alternative would be to use the rate of commercial paper instead of the prime rate. This would imply an interest rate spread of around 0.5 percent. However, many firms do not have access to commercial paper. In addition, a model with such a small interest rate spread is very similar to the model without financing constraints discussed above.

TABLE 10—STANDARD INVESTMENT EQUATIONS: ALTERNATIVE SPECIFICATIONS

		Financing costs		Marginal
Coefficient	Data	Variable only	Fixed only	q
b_1	0.06	2.92	2.82	−0.86
	(0.01)	(0.07)	(0.11)	(1.03)
$\bar{R}^2$	0.12	0.61	0.32	0.06

Note: Standard errors are in parentheses.

TABLE 11—CASH-FLOW-AUGMENTED EQUATIONS: ALTERNATIVE SPECIFICATIONS

		Financing costs		Marginal
Coefficient	Data	Variable only	Fixed only	q
b_1	0.06	4.88	6.25	−0.22
	(0.01)	(0.29)	(0.50)	(0.45)
b_2	0.14	−4.11	−7.00	5.41
	(0.04)	(0.59)	(1.00)	(0.16)
$\bar{R}^2$	0.25	0.63	0.35	0.50

Note: Standard errors are in parentheses.

This specification is somewhat similar to those used in the investment with fixed cost literature. Note that the fixed cost is about 50 percent higher than in the initial calibration.

The results of estimating equation (29) for these alternative models are displayed in columns 3 and 4 of Tables 10 and 11. We find that the linear equation is a poorer fit when fixed costs are very high. Again this is because high fixed costs give rise to very discontinuous investment decisions. Eliminating the fixed costs, on the other hand, improves the overall quality of the fit. Regardless, the adjusted R^2 still changes very little with the inclusion of cash flow in the regressions. Again we see no evidence of a strong cash-flow effect in the artificial data.

Marginal q.—The final column of Tables 10 and 11 looks at the performance of an alternative specification of equation (29) that uses marginal, instead of average, q as a measure of fundamentals for the benchmark model. We define marginal q as the right-hand derivative of the value function $p(k, z)$.

$$\text{(33)} \qquad \lim_{h \to 0^+} \frac{p(k+h, z) - p(k, z)}{h}.$$

We find that, as first documented by Caballero and John V. Leahy (1996), average q can actually perform much better than marginal q in these reduced-form equations. This is a consequence of the discontinuity in the investment decision of the firm in the presence of fixed costs. Caballero and Leahy (1996) show that in this case, investment is not even a monotonic function of marginal q. In the context of our benchmark calibration we find that it is actually possible to obtain an overall negative correlation between the two variables.

The weak performance of marginal q explains the success of cash flow in this regression. Again, however, we argue that this is so only because we have a very poor measure of fundamentals.

C. *Productivity Shocks, Cash Flow, and Investment*

The robustness of the previous results is striking. Despite the important role of financing constraints in the investment decision of the firm we find that the cash-flow regressor is, in general, only marginally significant. Perhaps even more surprising, we find that even when cash flow has a strong role in the augmented investment equations, this gives very little information on the importance of finance constraints.

Tables 12 and 13 provide us with some of the intuition behind these results. They depict the cross correlations between the variables of interest for our full sample of firms, for both the simple benchmark model and the version without any financing constraints.[14] In both cases it is apparent the strong collinearity between q, cash flow, and also sales, here just equal to output. Moreover all of these variables are also strongly correlated with the technology shock as well. As Sargent (1980) pointed out, this is important because in the context of a fully specified general-equilibrium model with productivity shocks the standard investment regressions are generally not justified on theoretical grounds as both the

[14] With the exception of cross correlations with investment, which are usually a little higher, the results are quantitatively very similar for the other subsamples that we have studied.

TABLE 12—CROSS CORRELATIONS: BENCHMARK MODEL

Variable	Investment	Tobin's q	Cash flow	Sales	exp(z)
Investment	1.00	0.53	0.57	0.56	0.53
Tobin's q		1.00	0.92	0.93	0.92
Cash flow			1.00	0.99	0.96
Sales				1.00	0.95
exp(z)					1.00

TABLE 13—CROSS CORRELATIONS: NO FINANCING CONSTRAINTS

Variable	Investment	Tobin's q	Cash flow	Sales	exp(z)
Investment	1.00	0.45	0.45	0.48	0.42
Tobin's q		1.00	0.95	0.97	0.95
Cash flow			1.00	0.99	0.98
Sales				1.00	0.97
exp(z)					1.00

right- and left-hand-side variables are endogenous. Indeed previous work by Shapiro (1986) shows how, by ignoring the effect of the underlying exogenous shocks, we can generally expect to obtain a spurious correlation between investment and output (or cash flow) that can not be interpreted as evidence for an accelerator type model or, in our context, as evidence of financing constraints. As a result, estimation of a reduced-form equation like (29) is clearly inappropriate, not only because it does not describe the correct decision rule of the firm (6), but also because it will, in general, be unable to incorporate the effects of the technology shock, z, on the endogenous variables.

As a related point Tables 12 and 13 also suggest that, since all variables are strongly correlated amongst themselves, one can expect that measurement error in one of them (q, for example) may lead an econometrician to assign a larger role for the others (cash flow and sales usually) in the regression. As our exercises demonstrate however, this is, essentially, without economic significance.

D. *Measurement Error*

The results above are important in two ways. First, they make it clear that the existence of financial constraints is not sufficient to establish cash flow as a significant regressor, beyond q. In the context of a fully specified model, the effect of financial constraints must be *already included* in the market value of the firm and should also be captured by q. Second, the collinearity between cash flow and q suggests that any sizable measurement error in the construction of q can reduce the overall correlation between q and investment and perhaps generate a cash-flow effect.

In this section we explore these ideas by analyzing the effects of measurement error on our theoretical regressions. First, we explore the effects of measurement error in the stock of capital, certainly one of the variables that requires more assumptions in its empirical construction. A simple way to illustrate this point is to make the price of capital goods unobservable to the econometrician. This measurement error in the construction of the capital stock is only one of several identified by researchers in empirical studies.

To consider this we provide yet another extension of the benchmark model with stochastic prices for investment goods. Specifically we assume that investment goods can be transformed into consumption goods at the relative price ϕ. This rate of transformation is stochastic and firm specific, reflecting idiosyncratic shocks to the value of the firm's capital stock. The shock to the value of investment goods was calibrated using data from Jeremy Greenwood et al. (1997) on the behavior of the relative price of investment goods. They estimate that the (detrended) price of investment goods follows the process

$$(34)\quad \phi' = \bar{\phi}(1 - 0.64) + 0.64\phi + \varepsilon, \qquad \sigma_{\varepsilon} = 0.035,$$

where $\bar{\phi}$, the average value of capital goods in units of consumption goods, is normalized to 1. We assume that these values hold for our model as well and restrict the distribution of the innovations to be normal for incumbents and uniform for new entrants. While this calibration procedure is not theoretically correct it is nevertheless used to provide a simple illustration of the effects of measurement error in cross-sectional investment regressions.

TABLE 14—STANDARD INVESTMENT EQUATIONS: MEASUREMENT ERROR

Coefficient	Data	Measurement error Capital stock	Measurement error Classical	Measurement error No constraints
b_1	0.06	2.08	1.59	6.71
	(0.01)	(0.24)	(0.08)	(0.12)
$\bar{R}^2$	0.12	0.29	0.18	0.73

Note: Standard errors are in parentheses.

TABLE 15—CASH-FLOW-AUGMENTED EQUATIONS: MEASUREMENT ERROR

Coefficient	Data	Measurement error Capital stock	Measurement error Classical	Measurement error No constraints
b_1	0.06	0.65	0.45	8.19
	(0.01)	(0.61)	(0.09)	(0.14)
b_2	0.14	1.25	4.72	−5.35
	(0.04)	(0.40)	(0.22)	(0.32)
$\bar{R}^2$	0.25	0.39	0.46	0.81

Note: Standard errors are in parentheses.

To introduce measurement error in the capital stock we assume that the econometrician can not observe the actual price of the investment goods, ϕ, and simply estimates this to be constant across firms and equal to its average value of 1.

The results of this procedure are documented in the third column of Tables 14 and 15. While all correlations are now lower, we observe that cash flow does increase the overall explanatory power of the regression. While the improvement is not dramatic here it illustrates the potential of the measurement-error argument. In this case the success of the cash-flow regressor is clearly identified from our theoretical construction. It is only due to the problems associated with the construction of the q variable that we obtain significant cash-flow effects.

A more direct alternative is perhaps to examine the effects of introducing classical measurement error in average q. As a simple illustration suppose that there is some normally distributed noise that prevents the econometrician from having an exact measure of average q. Specifically, suppose that the econometrician observes only

$$(35) \qquad \tilde{q} = q + \xi, \qquad \xi \sim N(0, \sigma_\xi^2).$$

We then set σ_ξ^2 equal to 1/10 of the variance of q, implying a signal to noise ratio at 10. This may or may not be conservative but again this calculation is intended as merely suggestive. The results in column 4 are those one would expect with classic measurement error: lower coefficients on q and lower $\bar{R}^2$ as well. Interestingly the cash-flow effect also seems stronger in this case.

Finally we also show the results of introducing classic measurement error for the model without financing constraints. Again these confirm that financing constraints are not necessary to observe cash-flow effects, in the sense that the adjusted R^2 increases significantly.

The conclusion from these experiences seems, by now, quite clear: regardless of the model the existence of financial constraints is neither sufficient nor necessary to establish cash flow as a significant regressor, beyond q. The cash-flow effects are either a combination of the artificial correlation induced by the technology shocks and the measurement error in q or a consequence of the nonlinearities in investment and the poor fit of (29).

V. Conclusions

Macroeconomists often emphasize the role of financial constraints as an important source of propagation of shocks across time and firms. Moreover, recent work by Bernanke and Alan S. Blinder (1988), Anil K Kashyap et al. (1993), Bernanke et al. (1997), and Thomas F. Cooley and Vicenzo Quadrini (1998) suggests that in the presence of these constraints, monetary policy can have powerful effects on individual firm decisions and aggregate conditions. Much of the evidence on the role of financing constraints at the firm level relies on the results of estimating cash-flow-augmented investment equations like (29).

This work questions the conclusion that one can safely attribute the empirical success of cash flow to the importance of financing constraints in investment decisions by firms. We do so by addressing this issue from a different, more structural, perspective. We begin by fully specifying a model of investment under financial constraints consistent with several empirical regularities about firm behavior observed in the data.

We obtain three main findings from this artificial panel of firms. First, despite the presence of liquidity constraints, it is hard to find evidence that cash flow adds significant explanatory power to the investment regressions. Thus the existence of financial frictions is not sufficient to obtain significant cash-flow effects. We argue that, in the context of a fully specified model, the effect of financial constraints should be included in the market value of the firm and thus captured by a good measure of q. Second, financing constraints are also not necessary to obtain these cash-flow effects in the model. It is possible to construct simple examples where cash flow adds some predictive power to investment equations, even in the absence of financial frictions. Third, as Sargent (1980) and Shapiro (1986) documented, we find that, in the context of these general-equilibrium models, the correlation between investment, cash flow, and sales is quite artificial and a reflection of the underlying technology shocks. Clearly this implies that the focus on reduced-form investment equations can be quite problematic. In a related point, we also find that it is also possible to observe cash-flow effects solely due to the misspecification induced by fitting a linear equation to a nonlinear decision rule.

These results suggest that the presence of measurement error in one of these variables may lead an econometrician to assign a larger role to others. We formally confirm this conjecture by explicitly examining the effects of measurement error in investment regressions.

These findings, however, do not question the existence or the importance of these constraints for investment decisions and/or the propagation of monetary policy shocks. It may well be that these constraints are relevant in practice. In fact, this approach to modeling equilibrium investment and financing behavior, can be extended to account for aggregate business cycles and monetary policy shocks thus providing an ideal laboratory to study the importance of these effects.

Nevertheless our results highlight the enormous difficulties in using standard investment regressions in practice, and cast serious doubt on the common interpretation of cash-flow effects as evidence in favor of financing constraints.

APPENDIX

This Appendix provides a brief description of the sources and methods used to generate the stylized facts analyzed in the text.[15]

A. *Variables*

Investment.—Investment expenditures, $I_{i,t}$, is spending on plant, property, and equipment minus capital retirements.

Capital Stock.—We use the perpetual inventory method described in Michael A. Salinger and Summers (1983) to convert the book value of the gross capital stock into its replacement value:

$$\text{(A1)} \quad K_{i,t} = \left[K_{i,t-1} \left(\frac{P_t^k}{P_{t-1}^k} \right) + I_{i,t} \right] (1 - 2/L_t),$$

where the recursion is started with the reported value of the capital stock in the first year the firm is in the Compustat files, P_t^k denotes the price of capital goods, taken to be the deflator for nonresidential fixed investment from DRI (Standard & Poor's, 1999b), and L_t is the average life of capital goods computed using the double declining balance method as

$$\text{(A2)} \qquad L_{i,t} = \frac{K_{i,t-1}^r + I_{i,t}}{DEP_{i,t}},$$

[15] See Whited (1992) for a more detailed description.

with K_t^r denoting the reported value of the capital stock at period t.

Market Value.—Market value of a firm, $V_{i,t}$, is constructed as

$$V_{i,t} = D_{i,t} + E_{i,t} - INV_{i,t}, \tag{A3}$$

where $E_{i,t}$ is the market value of equity, D_t is the value of short- plus long-term debt, and INV_t is the end-of-period value of inventories. The market value of equity is the sum of common equity (number of shares outstanding times the end-of-period market price) plus preferred equity (firm preferred pay-out divided by S&P's preferred dividend yield, from the DRI data set).

Average Q.—Tobin's q (beginning of period) is computed as

$$Q_{i,t} = \frac{V_{i,t-1}}{K_{i,t-1}}. \tag{A4}$$

Cash Flow.—Cash flow is defined as the sum of operating income and depreciation for the period.

B. *Sample Selection*

Most previous studies document a number of irregularities in the sample period described, such as mergers, reporting, and/or coding errors etc.[16] To maintain comparability with the existing literature we use the following procedure to eliminate extreme observations:

- The capital stock must be positive in all periods;
- Investment can not exceed beginning of period capital stock;
- Tobin's q must be positive and can not exceed ten;
- Cash flow (in absolute value) can not exceed five times the capital stock;
- The capital accumulation equation must be satisfied.

[16] Gilchrist and Himmelberg (1995), Abel and Eberly (1996), and Cummins et al. (1998), for example.

REFERENCES

Abel, Andrew B. "Empirical Investment Equations: An Integrative Framework." *Carnegie-Rochester Conference Series on Economic Policy,* Spring 1980, *12*(6), pp. 39–91.

Abel, Andrew B. and Blanchard, Olivier J. "The Present Value of Profits and Cyclical Movements in Investment." *Econometrica*, March 1986, *54*(2), pp. 249–73.

Abel, Andrew B. and Eberly, Janice C. "A Unified Model of Investment under Uncertainty." *American Economic Review*, December 1994, *84*(5), pp. 1369–84.

______. "Investment and q with Fixed Costs: An Empirical Investigation." Working paper, University of Pennsylvania, 1996.

Aiyagari, S. Rao and Gertler, Mark. "Asset Returns with Transactions Costs and Uninsured Individual Risk." *Journal of Monetary Economics*, June 1991, *27*(3), pp. 311–31.

Bernanke, Ben S. and Blinder, Alan S. "Credit, Money, and Aggregate Demand." *American Economic Review*, May 1988 (*Papers and Proceedings*), *78*(2), pp. 435–39.

Bernanke, Ben S.; Campbell, John Y. and Whited, Toni M. "U.S. Corporate Leverage: Developments in 1987 and 1988." *Brookings Papers on Economic Activity*, 1990, (1), pp. 255–78.

Bernanke, Ben S.; Gertler, Mark and Gilchrist, Simon. "The Financial Accelerator in a Quantitative Business Cycle Framework." Working paper, New York University, 1997.

Brock, William A. and LeBaron, Blake. "Liquidity Constraints in Production-Based Asset-Pricing Models," in R. Glenn Hubbard, ed., *Asymmetric information, corporate finance, and investment.* Chicago: University of Chicago Press, 1990, pp. 231–55.

Burnside, Craig. "Production Function Regressions, Returns to Scale, and Externalities." *Journal of Monetary Economics*, April 1996, *37*(2), pp. 177–201.

Caballero, Ricardo J. and Engel, Eduardo. "Explaining Investment Dynamics in U.S. Manufacturing: A Generalized (S, s) Approach." Working Paper No. 94-32, Massachusetts Institute of Technology, 1994.

Caballero, Ricardo J.; Engel, Eduardo M. R. A. and Haltiwanger, John C. "Plant-Level Adjustment and Aggregate Investment Dynamics." *Brookings Papers on Economic Activity*, 1995, (2), pp. 1–39.

Caballero, Ricardo J. and Leahy, John V. "Fixed Costs: The Demise of Marginal *q*." National Bureau of Economic Research (Cambridge, MA) Working Paper No. 5508, 1996.

Cooley, Thomas F. and Quadrini, Vicenzo. "Monetary Policy and the Financial Decisions of Firms." Working paper, University of Rochester, 1998.

Cooper, Russell; Haltiwanger, John and Power, Laura. "Machine Replacement and the Business Cycle: Lumps and Bumps." *American Economic Review*, September 1999, *89*(4), pp. 921–46.

Cummins, Jason G.; Hassett, Kevin A. and Oliner, Stephen D. "Investment Behavior, Observable Expectations and Internal Funds." Working paper, New York University, 1998.

Diamond, Douglas W. "Financial Intermediation and Delegated Monitoring." *Review of Economic Studies*, July 1984, *51*(3), pp. 393–414.

Diaz-Gimenez, Javier; Prescott, Edward C.; Fitzgerald, Terry and Alvarez, Fernando. "Banking in Computable General Equilibrium Models." *Journal of Economic Dynamics and Control*, July–October 1992, *16*(3–4), pp. 533–59.

Dixit, Avinash K. and Pindyck, Robert S. *Investment under uncertainty*. Princeton, NJ: Princeton University Press, 1994.

Doms, Mark and Dunne, Timothy. "Capital Adjustment Patterns in Manufacturing Plants." *Review of Economic Dynamics*, April 1998, *1*(2), pp. 409–29.

Dunne, Timothy. "Plant Age and Technology Use in U.S. Manufacturing Industries." *Rand Journal of Economics*, Autumn 1994, *25*(3), pp. 488–99.

Erickson, Timothy and Whited, Toni M. "Measurement Error and the Relationship between Investment and *q*." *Journal of Political Economy*, October 2000, *108*(5), pp. 1027–57.

Evans, David S. "The Relationship between Firm Growth, Size, and Age: Estimates for 100 Manufacturing Industries." *Journal of Industrial Economics*, June 1987, *35*(4), pp. 567–81.

Fazzari, Stephen M.; Hubbard, R. Glenn and Peterson, Bruce C. "Financing Constraints and Corporate Investment." *Brookings Papers on Economic Activity*, 1988, (1), pp. 141–95.

Garber, Peter M. and King, Robert G. "Deep Structural Excavation? A Critique of Euler Equation Methods." National Bureau of Economic Research (Cambridge, MA) Technical Paper No. 31, 1983.

Gilchrist, Simon and Himmelberg, Charles P. "Evidence on the Role of Cash Flow for Investment." *Journal of Monetary Economics*, December 1995, *36*(3), pp. 541–72.

Gomes, Joao F. "Financing Investment: Technical Appendix." Working paper, University of Pennsylvania, 2000.

Greenwood, Jeremy; Hercowitz, Zvi and Krusell, Per. "Long-Run Implications of Investment-Specific Technological Change." *American Economic Review*, June 1997, *87*(3), pp. 342–62.

Hall, Bronwyn H. "The Relationship between Firm Size and Firm Growth in the U.S. Manufacturing Sector." *Journal of Industrial Economics*, June 1987, *35*(4), pp. 583–606.

Hansen, Gary D. "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics*, November 1985, *16*(3), pp. 309–27.

Hayashi, Fumio. "Tobin's Marginal *q* and Average *q*: A Neoclassical Interpretation." *Econometrica*, January 1982, *50*(1), pp. 213–24.

Hopenhayn, Hugo A. "Entry, Exit, and Firm Dynamics in Long Run Equilibrium." *Econometrica*, September 1992, *60*(5), pp. 1127–50.

Jensen, Michael C. and Meckling, William H. "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure." *Journal of Financial Economics*, October 1976, *3*(4), pp. 305–60.

Jorgenson, Dale W. "Capital Theory and Investment Behavior." *American Economic Review*, May 1963 (*Papers and Proceedings*), *53*(2), pp. 247–59.

Jovanovic, Boyan. "Selection and the Evolution of Industry." *Econometrica*, May 1982, *50*(3), pp. 649–70.

Kashyap, Anil K; Stein, Jeremy C. and Wilcox, David W. "Monetary Policy and Credit Conditions: Evidence from the Composition of External Finance." *American Economic Review*, March 1993, *83*(1), pp. 78–98.

Lindenberg, Eric B. and Ross, Stephen A. "Tobin's *q* Ratio and Industrial Organization." *Journal of Business*, January 1981, *54*(1), pp. 1–32.

Lucas, Robert E., Jr. and Prescott, Edward C.

"Investment Under Uncertainty." *Econometrica*, September 1971, *39*(5), pp. 659–81.

Meyers, Stewart C. and Majluf, Nicholas S. "Corporate Financing and Investment Decision When Firms Have Information that Investors Do Not Have." *Journal of Financial Economics*, June 1984, *13*(2), pp. 187–221.

Modigliani, Franco and Miller, Merton H. "The Cost of Capital Corporation Finance and the Theory of Investment." *American Economic Review*, June 1958, *48*(3), pp. 261–97.

Ross, Stephen A.; Westerfield, Randolph W. and Jordan, Bradford D. *Fundamentals of corporate finance*, 2nd Ed. Homewood, IL: Irwin Press, 1993.

Salinger, Michael A. and Summers, Lawrence H. "Tax Reform and Corporate Investment: A Microeconomic Simulation Study," in Martin S. Feldstein, ed., *Behavioral simulation methods in tax policy analysis*. Chicago: University of Chicago Press, 1983, pp. 247–87.

Sargent, Thomas J. " 'Tobin's *q*' and the Rate of Investment in General Equilibrium." *Carnegie-Rochester Conference Series on Economic Policy*, Spring 1980, *12*(6), pp. 107–54.

Shapiro, Matthew D. "Investment, Output, and the Cost of Capital." *Brookings Papers on Economic Activity*, 1986, (1), pp. 111–52.

Smith, Clifford W., Jr. "Alternative Methods for Raising Capital: Rights versus Underwritten Offerings." *Journal of Financial Economics*, December 1977, *5*(3), pp. 273–307.

Standard & Poor's. Compustat. New York: Standard & Poor's, 1999a.

______. *DRI*. New York: Standard & Poor's, 1999b.

Summers, Lawrence H. "Taxation and Corporate Investment: A *q*-Theory Approach." *Brookings Papers on Economic Activity*, 1981, (1), pp. 67–127.

Tobin, James. "A General Equilibrium Approach to Monetary Theory." *Journal of Money, Credit, and Banking*, February 1969, *1*(1), pp. 15–29.

Veracierto, Marcelo. "Plant Level Irreversible Investment and Equilibrium Business Cycles." Working paper, Cornell University, 1997.

von Furstenburg, George. "Corporate Investment: Does Market Valuation Matter in the Aggregate?" *Brookings Papers on Economic Activity*, 1977, (2), pp. 347–97.

Whited, Toni M. "Debt, Liquidity Constraints, and Corporate Investment: Evidence From Panel Data." *Journal of Finance*, September 1992, *47*(4), pp. 1425–60.