Άσκηση 6: Δημιουργήστε ένα νέο αρχείο word με τίτλο “Άσκηση_6” και γράψτε το ακόλουθο κείμενο, καθώς και τις μαθηματικές παραστάσεις.

Ορισμός: Για οποιοδήποτε φυσικό αριθμό $n \in N$ ορίζουμε τον $\mathbb{R}^n$ ως το σύνολο των (διατεταγμένων) n-άδων πραγματικών αριθμών $\mathbb{R}^n := \left\{ x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \middle| x_1, \cdots, x_n \in \mathbb{R} \right\}$

(κατά σύμβαση γράφουμε αυτές τις n-άδες (συντεταγμένες του x) ως στήλες).

$$\lim_{x \to \infty} \frac{x^2 - 3x + 2}{2x^3 - 2x} = \lim_{x \to \infty} \frac{x^2}{x^3} \frac{1 - 3\frac{1}{x} + 2\frac{1}{x^2}}{2 - 2\frac{1}{x^2}} = \lim_{x \to \infty} \frac{1}{x} \frac{1 - 3\frac{1}{x} + 2\frac{1}{x^2}}{2 - 2\frac{1}{x^2}}$$

$$= \lim_{x \to \infty} \frac{1}{x} \lim_{x \to \infty} \frac{1 - 3\frac{1}{x} + 2\frac{1}{x^2}}{2 - 2\frac{1}{x^2}} = \lim_{x \to \infty} \frac{1}{x} \frac{\lim\limits_{x \to \infty} 1 - 3 \lim\limits_{x \to \infty} \frac{1}{x} + 2 \lim\limits_{x \to \infty} \frac{1}{x^2}}{\lim\limits_{x \to \infty} 2 - 2 \lim\limits_{x \to \infty} \frac{1}{x^2}}$$

$$= 0 \cdot \frac{1 - 3 \cdot 0 + 2 \cdot 0}{2 - 2 \cdot 0} = 0 .$$

Η πλαισιωμένη Εσσιανή μήτρα διαστάσεων (n+m)x(n+m):

$$\overline{H} = \begin{pmatrix} 0 & 0 & \cdots & 0 & g_{1x_1} & g_{1x_2} & \cdots & g_{1x_n} \\ 0 & 0 & \cdots & 0 & g_{2x_1} & g_{2x_2} & \cdots & g_{2x_n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & g_{mx_1} & g_{mx_2} & \cdots & g_{mx_n} \\ g_{1x_1} & g_{2x_1} & \cdots & g_{mx_1} & L_{x_1x_1} & L_{x_1x_2} & \cdots & L_{x_1x_n} \\ g_{1x_2} & g_{2x_2} & \cdots & g_{mx_2} & L_{x_2x_1} & L_{x_2x_2} & \cdots & L_{x_2x_n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{1x_n} & g_{2x_n} & \cdots & g_{mx_n} & L_{x_nx_1} & L_{x_nx_2} & \cdots & L_{x_nx_n} \end{pmatrix}.$$

Να υπολογιστεί το διπλό ολοκλήρωμα $\iint_R \frac{y^2}{1+x^2}dA$, όπου $R=[0,1]\times[0,1]$.

$$\iint_R \frac{y^2}{1+x^2}dA = \int_0^1 [\int_0^1 \frac{y^2}{1+x^2}dy]dx = \int_0^1 [\frac{y^3}{3(1+x^2)}\Big|_0^1]dx = \int_0^1 [\frac{1}{3(1+x^2 2)}]dx = \frac{1}{3}\arctan x\Big|_0^1 = \frac{\pi}{12}$$

Πηγή: Σημειώσεις του μαθήματος "Μαθηματικός Λογισμός σε Επιχειρησιακά και Οικονομικά Προβλήματα", Α. Τσεκρέκος.

Under the real probability measure P, the dynamics of the stochastic component of the spot price, its short-term mean and its variance are driven by the following SDEs:

$$dX_t = \left(\kappa\left(\upsilon_{1t} - X_t\right) + \lambda_X e^{\psi_t}\upsilon_{2t}\right)dt + e^{\psi_t}\sqrt{\upsilon_{2t}}\,dW_t^X \quad (1)$$

$$d\upsilon_{1t} = \left(\alpha_1\left(b_1 - \upsilon_{1t}\right) + \lambda_1\sigma_1 e^{\psi_t}\upsilon_{2t}\right)dt + \sigma_1 e^{\psi_t}\sqrt{\upsilon_{2t}}\,dW_t^{\upsilon_1} \quad (2)$$

$$d\upsilon_{2t} = \left(\alpha_2\left(b_2 - \upsilon_{2t}\right) + \lambda_2\sigma_2\upsilon_{2t}\right)dt + \sigma_2\sqrt{\upsilon_{2t}}\,dW_t^{\upsilon_2} \quad (3)$$

where the parameters $\alpha_2, b_2,$ and σ_2 are positive and λ_2 is the market price of volatility risk (see, for instance, Heston 1993). ρ_{12} (respectively $\rho_{13};\rho_{23}$) denotes the correlation coefficient between the Brownian motions W_t^X and $W_t^{\upsilon_1}$ (respectively, W_t^X and $W_t^{\upsilon_2}$; $W_t^{\upsilon_1}$ and $W_t^{\upsilon_2}$). All parameters are supposed to be time independent.

Because we have at all times five to seven liquid futures contracts compared to three sources of randomness, the situation of completeness still prevails, and we can state that under the unique pricing measure Q, the dynamics of the stochastic component of the spot price, its short-term mean and its variance are driven by the following stochastic differential equations:

$$dX_t = \kappa\left(\upsilon_{1t} - X_t\right)dt + e^{\psi_t}\sqrt{\upsilon_{2t}}\,d\hat{W}_t^X \quad (4)$$

$$d\upsilon_{1t} = \alpha_1\left(b_1 - \upsilon_{1t}\right)dt + \sigma_1 e^{\psi_t}\sqrt{\upsilon_{2t}}\,d\hat{W}_t^{\upsilon_1} \quad (5)$$

$$d\upsilon_{2t} = \alpha_2\left(b_2 - \upsilon_{2t}\right)dt + \sigma_2\sqrt{\upsilon_{2t}}\,d\hat{W}_t^{\upsilon_2} \quad (6)$$

where b_2 and σ_2 are, respectively, the long-term mean and the volatility of the variance of the log price.

Πηγή: Geman, H. and Nguyen, V.-N. (2005). Soybean inventory and forward curve dynamics. Management Science, 51:1076–1091.