

ΣΕΥΓΑ ΔΙΑΧΕΙΡΙΣΗ Τ.Μ.

x_1	$P(x_1)$
x_2	$P(x_2)$
\vdots	\vdots
x_m	$P(x_m)$

	y_1	y_2	\dots	y_j	\dots	y_k
x_1						
x_2						
\vdots						
x_i						
\vdots						
x_m						

$P_{ij} \triangleq P(X=x_i, Y=y_j)$

1	1	2	3	4	5	6	$P_X(x)$
1	1/36	1/36	1/36	1/36	1/36	1/36	1/6
2	1/36	1/36	1/36	1/36	1/36	1/36	1/6
3	1/36	1/36	1/36	1/36	1/36	1/36	1/6
4	1/36	1/36	1/36	1/36	1/36	1/36	1/6
5	1/36	1/36	1/36	1/36	1/36	1/36	1/6
6	1/36	1/36	1/36	1/36	1/36	1/36	1/6
$P_Y(y)$	1/6	1/6	1/6	1/6	1/6	1/6	

ΠΑΡΑΔΕΙΓΜΑ 2

2 ΖΑΡΙΑ \rightarrow ΔΙΜΑΙΑ
 \rightarrow ΑΝΕΞΑΡΤΗΤΑ
 $X = \text{ΖΑΡΙΑ 1} \quad S_X = \{1, 2, \dots, 6\}$
 $Y = \text{ΖΑΡΙΑ 2} \quad S_Y = \{1, 2, \dots, 6\}$

$A, B \text{ ΑΝΕΞΑΡΤΗΤΑ}$
 $P(A \cap B) = P(A)P(B)$

$P(X=3, Y=4) \stackrel{\text{ΑΝΕΞΑΡ.}}{=} P(X=3)P(Y=4)$
 $\stackrel{\text{ΔΙΜΑΙΑ}}{=} \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

ΑΠΟ ΚΩΝΟΥΡ
 ΜΑΖΑ
 ΠΙΘΑΝΟΤΗΤΕΣ

	x	1	2	3	4	5	6	$P_Y(y)$
y								
1		1/36	1/36	1/36	1/36	1/36	1/36	1/6
2		1/36	1/36	1/36	1/36	1/36	1/36	1/6
3		1/36	1/36	1/36	1/36	1/36	1/36	1/6
4		1/36	1/36	1/36	1/36	1/36	1/36	1/6
5		1/36	1/36	1/36	1/36	1/36	1/36	1/6
6		1/36	1/36	1/36	1/36	1/36	1/36	1/6
$P_X(x)$		1/6	1/6	1/6	1/6	1/6	1/6	

ΠΑΡΑΔΕΙΓΜΑ 2

2 ΖΑΡΙΑ \rightarrow ΔΙΜΑΙΑ
 \rightarrow ΙΣΑ
 $P(X=3, Y=4) = 0$
 $P(X=3, Y=3) = P(X=3) = \frac{1}{6}$

	x	1	2	3	4	5	6	$P_Y(y)$
y								
1								
2								
3								
4								
5								
6								
$P_X(x)$								

	x	1	2	3	4	5	6	
y								$P_Y(y)$
1		1/6	0	0	0	0	0	1/6
2		0	1/6	0	0	0	0	1/6
3		0	0	1/6	0	0	0	1/6
4		0	0	0	1/6	0	0	1/6
5		0	0	0	0	1/6	0	1/6
6		0	0	0	0	0	1/6	1/6
	$P_X(x)$	1/6	1/6	1/6	1/6	1/6	1/6	

STADIUM 3

x \ y	1	2	3	4	5	6	
1	1	1	1	1	1	1	1/6
2	1	2	0	1	1	1	1/6
3	1	0	2	1	1	1	1/6
4	1	1	1	1	1	1	1/6
5	1	1	1	1	1	1	1/6
6	1	1	1	1	1	1	1/6
	1/6	1/6	1/6	1/6	1/6	1/6	1/36

$P(X=4, Y=2) = 0$

$P(X=4, Y=3) = \frac{2}{36}$

STADIUM 4

X, Y ΔΙΦΑΝΤΑ ΚΑΙ ΜΕ=ΑΡΤΗΤΑ

$W = \max(X, Y)$

$Z = \min(X, Y)$

W \ Z	1	2	3	4	5	6	
1	1/36	0	0	0	0	0	1/36
2	2/36	1/36	0	0	0	0	2/36
3	2/36	2/36	1/36	0	0	0	5/36
4	2/36	2/36	2/36	1/36	0	0	7/36
5	2/36	2/36	2/36	2/36	1/36	0	8/36
6	2/36	2/36	2/36	2/36	2/36	1/36	11/36
	1/36	2/36	3/36	4/36	5/36	6/36	1/36

$P(W=w, Z=z)$

$W=1, \dots, 6$

$Z=1, \dots, 6$

$P(W=5, Z=4) = P(X=5, Y=4) + P(X=4, Y=5)$

$= \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = \frac{2}{36} = \frac{1}{18}$

$P(W=1, Z=1) = P(X=1, Y=1) = \frac{1}{36}$

$P(W=1, Z=4) = 0$

A1: 2 MAXNPE

X = # ΔΡΡΑ ΤΟΥ Α1

A2: 2 MAXNPE

Y = # ΔΡΡΑ ΤΟΥ Α2

ΑΠΟ ΚΑΝΟΝ ΜΑΖΑ ΠΡΟΒΑΒΗΤΗΤΑ ΤΩΝ X, Y

$$S_X = \{0, 1, 2\} = S_Y$$

$\frac{41}{45}$

X \ Y	0	1	2	
0	$\frac{1}{3}$	$\frac{4}{15}$	$\frac{1}{45}$	$\frac{28}{45}$
1	$\frac{4}{15}$	$\frac{4}{45}$	0	$\frac{16}{45}$
2	$\frac{1}{45}$	0	0	$\frac{1}{45}$
	$\frac{28}{45}$	$\frac{16}{45}$	$\frac{1}{45}$	$\frac{2}{3}$

$$|S| = \binom{10}{2} \cdot \binom{8}{2} = \binom{10}{2, 2, 6}$$

45 · 28

$$P(X=0, Y=0) = \frac{28}{45} \cdot \frac{7}{9} \cdot \frac{6}{8} \cdot \frac{4}{4} = \frac{1}{3}$$

$(P(A_1, A_2, \dots, A_n) = P(A_1) P(A_2|A_1) P(A_3|A_1, A_2) \dots P(A_n|A_1, A_2, \dots, A_{n-1}))$

$$P(X=0, Y=0) = \frac{\binom{8}{4}}{\binom{10}{4}} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4} \cdot \frac{1}{10 \cdot 9 \cdot 8 \cdot 7} = \frac{1}{3}$$

$$P(X=0, Y=0) = \frac{\binom{8}{2} \binom{6}{2}}{\binom{10}{2} \binom{8}{2}} = \frac{\frac{2}{3} \cdot \frac{5}{2}}{\frac{10 \cdot 9}{2}} = \frac{1}{3}$$

$$P(X=1, Y=1) = \frac{2 \cdot 8 \cdot 1 \cdot 7}{\binom{10}{2} \binom{8}{2}} = \frac{4}{45}$$

$$P(X=1, Y=0) = \frac{2 \cdot 8 \cdot \binom{7}{2}}{\binom{10}{2} \binom{8}{2}} = \frac{4}{15}$$

$$P(X=2, Y=0) = \frac{1}{\binom{10}{2}} = \frac{1}{45}$$

$$1 \cdot \frac{\binom{2}{2} \cdot \binom{8}{2}}{\dots}$$

01	12	23
02	13	;
03	14)
04	;	
05	;	
06	;	
07	;	
08	18	
09		
} 45		

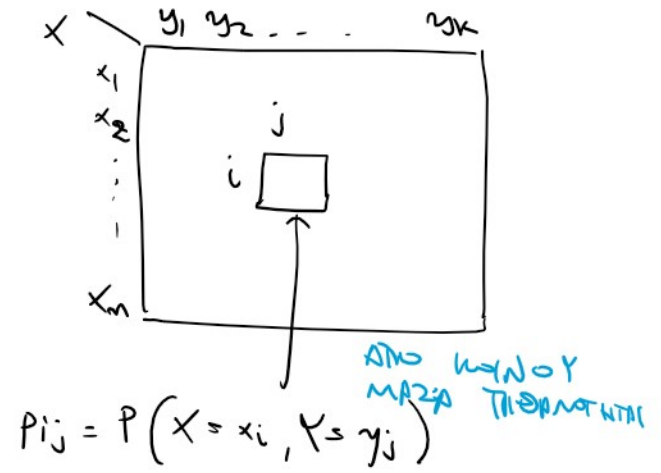
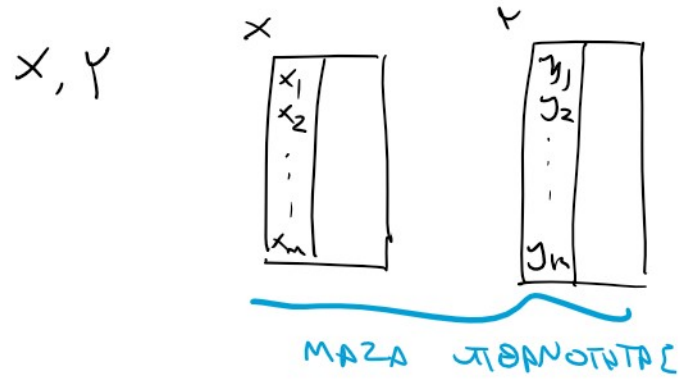
$$= \frac{\binom{2}{2} \cdot \binom{8}{2}}{\binom{10}{2} \binom{8}{2}}$$

$$= \frac{2}{10} \cdot \frac{1}{9} = \frac{1}{45}$$

y \ x	-5	-2	2	5	$P_Y(y)$
0	0.01	0.01	0.01		
1	0.05		0.05	0	0.18
2	0.07	0.07		0.07	
3	0.15		0	0.15	
$P_X(x)$		0.23	0.15		

y \ x	0	1	2	3	$P_Y(y)$
0	1/6	1/12	1/12	1/12	15/36
1	1/12	1/18	1/36	1/36	7/36
2	1/12	1/36	1/18	1/36	7/36
3	1/12	1/36	1/36	1/18	7/36
$P_X(x)$	15/36	7/36	7/36	7/36	

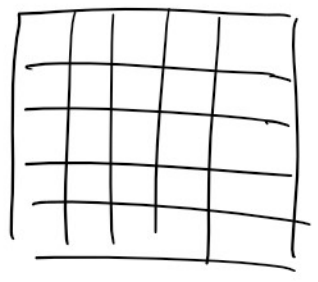
y \ x	0	1	2	3	4	5	6	7	8	$P_Y(y)$
0	1/18	1/18	1/18	1/27	1/27	1/27	0	0	0	5/18
1	1/18	1/18	1/18	1/27	1/27	1/27	1/18	1/18	1/18	8/18
2	0	0	0	1/27	1/27	1/27	1/18	1/18	1/18	5/18
$P_X(x)$	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9	



ΟΡΙΑΣΜΟΣ 6.1

X, Y S_X S_Y $P_{XY} : S_X \times S_Y \rightarrow [0, 1]$

$P_{XY}(x, y) \triangleq P(X = x, Y = y)$ ΣΥΝΑΡΤΗΣΗ ΜΑΖΑΣ ΠΙΘΑΝΟΤΗΤΑΣ
 (prob. MASS FUNCTION part.)

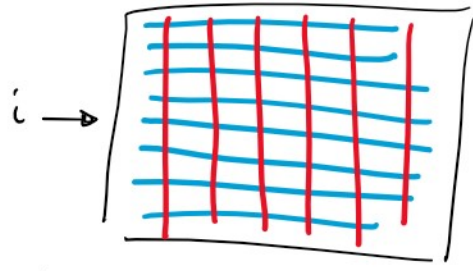


ΛΗΜΜΑ 6.1

1) $\sum_{x \in S_X, y \in S_Y} P_{XY}(x, y) = 1$



$x \in S_X, y \in S_Y$



```

sum = 0
for i = 1:m
    for j = 1:k
        sum = sum + P(X = x_i, Y = y_j)
    end
end
  
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2) $A \subseteq S_X \times S_Y$
 $P((X,Y) \in A) = \sum_{(x,y) \in A} P_{X,Y}(x,y)$

3) $P_X(x) = \sum_{y \in S_Y} P_{X,Y}(x,y) \quad | \quad P_Y(y) = \sum_{x \in S_X} P_{X,Y}(x,y)$

ΠΑΡΑΓΡΑΦΟΣ 6.2

X, Y ΠΑΡΑΔΕΙΓΜΑ 6.4

y	x	-5	-2	2	5	$P_Y(y)$
0		0.01	0.01	0.01	0.12	0.15
1		0.05	0.08	0.05	0	0.18
2		0.07	0.07	0.09	0.07	0.3
3		0.15	0.07	0	0.15	0.39
	$P_X(x)$	0.28	0.23	0.15	0.34	

$1 - 0.28 - 0.23 - 0.15$

ΠΑΡΑΔΕΙΓΜΑ 6.5

$S_X = \{0, 1, 3\}$
 $S_Y = \{0, 1, 2, 3\}$

y	x	0	1	2	3	$P_Y(y)$
0		1/6	1/12	1/12	1/12	15/36
1		1/12	1/18	1/36	1/36	7/36
2		1/12	1/36	1/18	1/36	7/36
3		1/12	1/36	1/36	1/18	7/36
	$P_X(x)$	15/36	7/36	7/36	7/36	

X : αριθμός ΑΓΑΝΤΩΡ
 Y : αριθμός ΧΡΗΣΤΗ

$Z = |X - Y|$

3	1/12	1/36	1/36	1/18	7/36
$p_X(x)$	15/36	7/36	7/36	7/36	

$$Z = |X - Y|$$

(THE APARTMENT)

$$\mathcal{S}_Z = \{0, 1, 2, 3\}$$

z	$P_Z(z)$
0	1/3
1	5/18
2	2/9
3	1/6

EKT
LCM

$$\frac{6}{18} + \frac{5}{18} + \frac{4}{18} + \frac{3}{18} = \frac{16}{18} = 1$$

$$P(Z=0) =$$

$$P(X=Y=0) + P(X=Y=1) + P(X=Y=2) + P(X=Y=3)$$

$$P(Z=1)$$

$$E(Z) = E(|X-Y|) = 0 \cdot \frac{1}{3} + 1 \cdot \frac{5}{18} + 2 \cdot \frac{2}{9} + 3 \cdot \frac{1}{6} = \frac{11}{9}$$

$$P(X=1|Y=0) = \frac{P(X=1, Y=0)}{P(Y=0)} = \frac{1/12}{15/36} = \frac{1}{5}$$

ΠΑΡΑΔΕΙΓΜΑ 6.6

$$P(Z=2) = \frac{1}{9}$$

$$P(Z=5) = \frac{5}{9}$$

$$P(Z=0) = \frac{1}{18}$$

$$P(Z=1) = \frac{1}{3}$$

x	0	1	2	3	4	5	6	7	8	$P_Y(y)$
0	1/18	1/18	1/18	1/27	1/27	1/27	0	0	0	5/18
1	1/18	1/18	1/18	1/27	1/27	1/27	1/18	1/18	1/18	8/18
2	0	0	0	1/27	1/27	1/27	1/18	1/18	1/18	5/18
$P_X(x)$	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9	

X = ΒΑΘΜΟΣ ΣΤΗΝ ΤΡΑΚΛΗ ΕΞΕΤΑΣΗ

$$\mathcal{S}_X = \{0, 1, \dots, 8\}$$

Y = ΒΑΘΜΟΣ ΣΤΗΝ ΓΡΟΟΔΟ

$$\mathcal{S}_Y = \{0, 1, 2\}$$

$$Z = X + Y$$

0	1/18
1	1/9
2	1/9
3	5/54
4	2/27
5	
6	
7	
8	
9	
10	1/18

$$P(Z \geq 5) = P(Z=5) + P(Z=6) + \dots + P(Z=10)$$

$$E(Z) = 0 \cdot \frac{1}{18} + 1 \cdot \frac{1}{9} + 2 \cdot \frac{1}{9} + \dots = 5$$

$$P(Z \geq 5 | Y=0) = \frac{P(Y=0 \text{ και } Z \geq 5)}{P(Y=0)}$$

8	
9	
10	1/18

$$P(Z \geq 5 | Y=0) = \frac{P(Z \geq 5, Y=0)}{P(Y=0)}$$

$$P(Y=0, X \geq 5) = \frac{1/27}{5/18} = \frac{2}{15}$$

ТАРАДЛИМА 6.7

K1: 50%

$$P(K1) = P(K2) = P(K3) = \frac{1}{3}$$

K2: 33%

K3: 25%

$$X = \begin{cases} 1/2 & \text{m.p. } 1/3 \\ 1/3 & \text{m.p. } 1/3 \\ 1/4 & \text{m.p. } 1/4 \end{cases}$$

x	P(x)
1/2	1/3
1/3	1/3
1/4	1/3

$Y \sim \text{ГЕРМ}(X)$

$$P(X = \frac{1}{2}, Y = 1) =$$

$$P(X = \frac{1}{2}) P(Y = 1 | X = \frac{1}{2}) = \frac{1}{3} \cdot \frac{1}{2}$$

$$P(X = \frac{1}{3}, Y = 1) =$$

$$P(X = \frac{1}{3}) P(Y = 1 | X = \frac{1}{3}) = \frac{1}{3} \cdot \frac{1}{3}$$

Y \ X	1/2	1/3	1/4
1	1/6	1/9	1/12
2	1/12	2/27	1/16
3	1/24	4/81	3/64
4

$P(X = \frac{1}{2}) = \frac{1}{3}$
 $P(X = \frac{1}{3}) = \frac{1}{3}$
 $P(X = \frac{1}{4}) = \frac{1}{3}$

$$P(X = \frac{1}{2}, Y = 2) =$$

$$P(X = \frac{1}{2}) P(Y = 2 | X = \frac{1}{2}) = \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{2}$$

$$P(X = \frac{1}{3}, Y = 2) = P(X = \frac{1}{3}) P(Y = 2 | X = \frac{1}{3}) = \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{3}$$