Πέμπτη 18-11-2021

Μάθημα 6ο:

Αρχές υπολογισμού ασφαλίστρου

- Net premium $\Pi(X) = E(X)$
  - Expected Value $\Pi(X) = (1+\theta)E(X)$

    θ premium loade factor
- Variance principle
- Standard deviation
- Zero utility $u(w) = E[u(w + \Pi(X) - X)]$
- Αρχή μεγίστης απώλειας

The Esscher Premium

$$\Pi(X) := \frac{E(Xe^{hX})}{E(e^{hX})} \qquad h > 0$$

[illegible]

$\tilde{X}$ $\qquad$ X אי' של. $(0, \infty)$ התפ. f.

$$\text{התפ } g(x) = \frac{e^{hx} f(x)}{\int_0^\infty e^{hy} f(y)\,dy}$$

$$M_{\tilde{X}}(t) = \frac{M_X(t+h)}{M_X(h)}$$

Πχ

$X \sim \exp(\lambda)$

$\tilde{X} \sim ?$

$h < \lambda$

Esscher transform

Λύση

$X \sim \exp(\lambda)$

$M_X(t) = \frac{\lambda}{\lambda - t}$

$X \sim \exp(\lambda)$

$$M_{\tilde{X}}(t) = \frac{M_X(t+h)}{M_X(h)} = \frac{\frac{\lambda}{\lambda - t - h}}{\frac{\lambda}{\lambda - h}} = \frac{\lambda - h}{\lambda - h - t}$$

$\tilde{X} \sim \exp(\lambda - h)$

$$P(X) = E(\tilde{X})$$

πχ

[illegible]

$\lambda = 1$

$\tilde{X} \sim \exp(1-h)$

$$P(X) = \frac{1}{1-h}$$

HW

$X \sim P(\lambda)$

$P(X)$

$\tilde{X} \sim j$

Risk Adjusted Premium

X μη αρνητική τ.μ. κατ. F

$$P(X) = \int_0^\infty [1 - F(x)]^{1/p} dx$$

$p \geq 1$

X^* $\overset{\delta.\kappa}{\sim} H$ όπου $1 - H(x) = [1 - F(x)]^{1/p}$

$E(X^*) = P(X)$

$\underline{\underline{\text{nx}}}$ $X \sim \exp(\lambda)$ $P(X)$

risk adjusted premium

$$1 - F(x) = e^{-\lambda x}$$

$$E(X) = \frac{1}{\lambda}$$

$$1 - H(x) = e^{-\frac{\lambda x}{\rho}}$$

$$X^* \sim \exp\left(\frac{\lambda}{\rho}\right)$$

$$E(X^*) = \frac{\rho}{\lambda}$$

$$\parallel$$

$$P(X)$$

HW

$X \sim \text{Pareto}(a, \lambda)$

$$F(x) = 1 - \left(\frac{\lambda}{\lambda + x}\right)^{a}$$

$P(x) = ;$

Ex $X \sim U(5, 15)$ $p : 1.2$

Ασκηση

Αποδειξη της iterativily της exponential αρχής

Λύση

Θέλω $\Pi(\Pi(X|Y)) = \Pi(X)$

$$\Pi(\Pi(X|Y)) = \frac{1}{\lambda}\log M_{\Pi(X|Y)}(\lambda)$$

$$= \frac{1}{\lambda}\log E[e^{\lambda \Pi(X|Y)}]$$

$$= \frac{1}{\lambda}\log E[e^{\cancel{\lambda}\frac{1}{\cancel{\lambda}}\log E(e^{\lambda X}|Y)}]$$

$$= \frac{1}{\lambda}\log E[E(e^{\lambda X}|Y)] = \frac{1}{\lambda}\log E(e^{\lambda X})$$

$$= \Pi(X)$$

Compound distributions

$\left(S = \sum_{i=1}^{N} X_i \right)$

$\Pi(X)$

$\Pi(S)$

Εαν Π προσθετική ① $(\Pi(X+Y) = \Pi(X) + \Pi(Y))$

Π iterative ② $(\Pi(\Pi(X|Y)) = \Pi(X))$

$$\Pi(S) \overset{②}{=} \Pi(\Pi(S|N)) \overset{①}{=} \Pi(N\,\Pi(X))$$

Π scale invariant $(\Pi(c \cdot X) = c\,\Pi(X))$

$c > 0$

$$\Pi(S) = \Pi(X)\,\Pi(N)$$