

Πέμπτη 18-11-2021

Μαθηματικά 6^ο:

Αρχές υπολογισμού ασφαλίσεων

- Net premium $\Pi(x) = E(x)$

- Expected Value $\Pi(x) = (1+\theta)E(x)$

θ premium loading factor

- Variance principle

→ Standard deviation

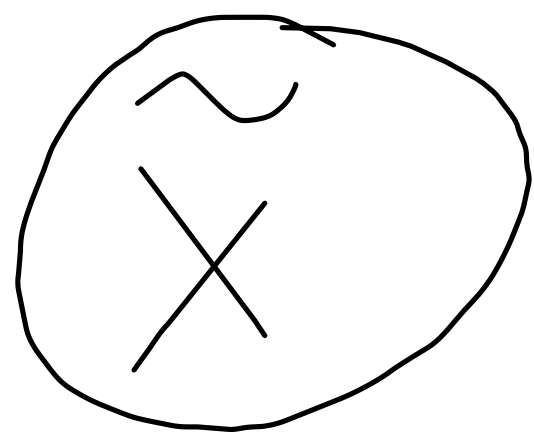
→ Zero utility $u(w) = E[u(w + \Pi(x) - X)]$

- Αρχι μέγιστος κέρδους

The Esscher Premium

$$\pi(x) := \frac{E(X e^{hx})}{E(e^{hx})} \quad h > 0$$

↑ to ∞



X
sw. v. $(0, \infty)$ f

sw. $g(x) = \frac{e^{hx} f(x)}{\int_0^{\infty} e^{hy} f(y) dy}$

$$M_{\tilde{X}}(t) = \frac{M_X(t+h)}{M_X(h)}$$

$$\frac{\pi_X}{X} \sim \exp(\lambda)$$

$$X \sim ?$$

$$h < \lambda$$

Esscher-Transform.

Lösung

$$X \sim \exp(\lambda)$$

$$M_X(t) = \frac{\lambda}{\lambda - t}$$

$X \sim \exp(\lambda)$

$$M_{X^h}(t) = \frac{M_X(t+h)}{M_X(h)}$$

$$= \frac{\frac{\lambda}{\lambda - t - h}}{\frac{\lambda}{\lambda - h}} = \frac{\lambda - h}{\lambda - h - t}$$

$$\sim X \sim \exp(\lambda - h)$$

$$P(X) = E(\tilde{X})$$

HW $\lambda = 1$ $X \sim \exp(1 - h)$
swdica

$$P(X) = \frac{1}{1-h}$$

HW

$$X \sim P(\lambda)$$

$P(X)$

$$X \sim j$$

Risk Adjusted Premium.

X με αποτίκην με κατα. F

$$P(X) = \int_0^{\infty} [1 - F(x)]^{1/p} dx$$

$$p \geq 1.$$

~~X^*~~

δ.κ
 H

οπov

$$1 - H(x) = [1 - F(x)]^{1/p}$$

$$E(X^*) = P(X)$$

$$\underline{\underline{\lambda x}} \quad X \sim \text{exp}(\lambda)$$

$$P(X)$$

risk adjusted premium

$$1 - F(x) = e^{-\lambda x}$$

$$E(X) = \frac{1}{\lambda}$$

$$1 - H(x) = e^{-\frac{\lambda x}{p}}$$

$$X^* \sim \text{exp}\left(\frac{\lambda}{p}\right)$$

$$E(X^*) = \frac{p}{\lambda}$$

$$\underline{\underline{P(X)}}$$

HW

$$X \sim \text{Pareto}(a, \lambda)$$

$$F(x) = 1 - \left(\frac{\lambda}{\lambda + x}\right)^a$$

$$P(x) = \lambda$$

Qx $X \sim U(S, 15)$

p: 1.2.

Άσκηση

Αποδείξτε την iterativity

της exponential αρχής

Λύση

$$\text{Θεώρημα } \pi(\pi(X|Y)) = \pi(X)$$

$$\pi(\pi(X|Y)) = \frac{1}{a} \log M_{\pi(X|Y)}(a)$$

$$= \frac{1}{a} \log E(e^{a\pi(X|Y)})$$

$$= \frac{1}{a} \log \left[e^{a \frac{1}{a} \log E(e^{ax}|Y)} \right]$$

$$= \frac{1}{a} \log E[E(e^{ax}|Y)] = \frac{1}{a} \log E(e^{ax})$$

$$= \pi(X)$$

Compound distributions

$$S = \sum_{i=1}^N X_i$$

$$\pi(x)$$

$$\pi(S)$$

Εα $\forall \pi$ προσθετική ① $(\pi(x+y) = \pi(x)\pi(y))$
 π iterative ② $(\pi(\pi(x/y)) = \pi(x))$

$$\pi(S) \stackrel{\text{②}}{=} \pi(\pi(S/N)) \stackrel{\text{①}}{=} \pi(N \pi(x))$$

π scale invariant

$$(\pi(c \cdot x) =$$

$$c \pi(x))$$

$$c > 0$$

$$\pi(S) = \pi(x) \pi(N)$$