

# Τυπολόγιο

$$\begin{aligned} \cos(y-x) &= \cos x \cos y + \sin x \sin y, & \sin\left(\frac{x}{2}\right) &= \pm \sqrt{\frac{1-\cos x}{2}} \\ \sin 2x &= 2 \sin x \cos x, & \cos 2x &= \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x \\ \sin x + \sin y &= 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right), & \cos x + \cos y &= 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) \\ \sin x - \sin y &= 2 \cos\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right), & \cos x - \cos y &= -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right) \\ \sin x \sin y &= \frac{1}{2} [\cos(x-y) - \cos(x+y)], & \cos x \cos y &= \frac{1}{2} [\cos(x+y) + \cos(x-y)] \\ \sin x \cos y &= \frac{1}{2} [\sin(x+y) + \sin(x-y)], & \cos x < \frac{\sin x}{x} < \frac{1}{\cos x}, & \cos\left(\frac{x}{2}\right) &= \pm \sqrt{\frac{1+\cos x}{2}} \end{aligned}$$

$$\begin{aligned} \forall \epsilon > 0 \exists \delta > 0 : 0 < |x-c| < \delta \Rightarrow |f(x) - L| < \epsilon. & \quad \forall \epsilon > 0 \exists \delta > 0 : c < x < c + \delta \Rightarrow |f(x) - L| < \epsilon. \\ \forall \epsilon > 0 \exists \delta > 0 : c - \delta < x < c \Rightarrow |f(x) - L| < \epsilon. & \quad \forall \epsilon > 0 \exists X \in \mathbb{R} : x > X \Rightarrow |f(x) - L| < \epsilon. \\ \forall \epsilon > 0 \exists X \in \mathbb{R} : x < X \Rightarrow |f(x) - L| < \epsilon. & \quad \forall M \in \mathbb{R} \exists \delta > 0 : 0 < |x-c| < \delta \Rightarrow f(x) > M. \\ \forall M \in \mathbb{R} \exists X \in \mathbb{R} : x > X \Rightarrow f(x) > M. & \quad \forall \epsilon > 0 \exists N \in \mathbb{N} : n > N \Rightarrow |a_n - L| < \epsilon. \end{aligned}$$

$$\begin{aligned} |f(x) - f(x_0)| &\leq C|x - x_0|, & |f(y) - f(x)| &\leq C|y - x| \\ (\arcsin y)' &= \frac{1}{\sqrt{1-y^2}}, & (\arccos y)' &= -\frac{1}{\sqrt{1-y^2}}, & (\arctan y)' &= \frac{1}{1+y^2} \\ f(\theta x_0 + (1-\theta)x_1) &< \theta f(x_0) + (1-\theta)f(x_1) \end{aligned}$$

$$\begin{aligned} L(f, P) &\triangleq \sum_{i=1}^n m_i(t_i - t_{i-1}), & U(f, P) &\triangleq \sum_{i=1}^n M_i(t_i - t_{i-1}), & \sum_{i=1}^n f(x_i)(t_i - t_{i-1}) \\ \left\{ \begin{array}{l} x = r \cos \theta, \\ y = r \sin \theta \end{array} \right\} &\Leftrightarrow \left\{ \begin{array}{l} r = \sqrt{x^2 + y^2}, \\ \cos \theta = \frac{x}{\sqrt{x^2 + y^2}}, \sin \theta = \frac{y}{\sqrt{x^2 + y^2}} \end{array} \right\} \\ \frac{1}{2} \int_a^b f^2(\theta) d\theta, & \pi \int_a^b f^2, & 2\pi \int_a^b x f(x) dx, & \int_a^b A(t) dt, & \int_a^b \sqrt{(f'(x))^2 + (g'(x))^2} dx \\ y' + P(x)y &= Q(x), & y(x) &= [S(x) + C] \exp[-R(x)] \end{aligned}$$

$$\begin{aligned} P_{n,a}(x) &\triangleq f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n \\ y(x) &= \left\{ y_0 + \int_{x_0}^x Q(u) \exp\left[\int_{x_0}^u P(t) dt\right] du \right\} \exp\left[-\int_{x_0}^x P(t) dt\right] \\ E_n(x) &= \frac{1}{n!} \int_a^{x_0} (x-t)^n f^{(n+1)}(t) dt, & |E_n(x)| &\leq M \frac{|x-a|^{n+1}}{(n+1)!} \\ s_n &= \sum_{k=1}^n f(k), & t_n &= \int_1^n f(x) dx \\ S &= \sum_{k=1}^{\infty} (-1)^{k-1} a_k, & s_n &= \sum_{k=1}^n (-1)^{k-1} a_k, & 0 < (-1)^n (S - s_n) &< a_{n+1} \\ A_{\parallel} &= \left[ \frac{A \cdot B}{\|B\|^2} \right] B, & A_{\perp} &= A - A_{\parallel}, & (x-x_0, y-y_0)(A, B) = 0 &\Leftrightarrow Ax + By = Ax_0 + By_0 \\ \begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} &= 0, & \begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix} &= 0, \\ A(x-x_0) + B(y-y_0) + C(z-z_0) &= 0 &\Leftrightarrow Ax + By + Cz = Ax_0 + By_0 + Cz_0 \\ y^2 = 4px, & \frac{x^2}{a^2} + \frac{y^2}{a^2(1-\epsilon^2)} = 1 &\Leftrightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, & \frac{x^2}{a^2} - \frac{y^2}{a^2(\epsilon^2-1)} = 1 &\Leftrightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \\ \begin{cases} u = (x-x_0) \cos \theta + (y-y_0) \sin \theta, \\ v = -(x-x_0) \sin \theta + (y-y_0) \cos \theta \end{cases} &\Leftrightarrow \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x-x_0 \\ y-y_0 \end{pmatrix} \\ \begin{cases} x = x_0 + u \cos \theta - v \sin \theta, \\ y = y_0 + u \sin \theta + v \cos \theta \end{cases} &\Leftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \\ \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} x-x_0 \\ y-y_0 \end{pmatrix} &\Leftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 + u \\ y_0 + v \end{pmatrix}, & \theta &= \frac{1}{2} \operatorname{arccot} \frac{A-C}{B} \\ \begin{vmatrix} x_0 & y_0 & z_0 \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} &= x_0 \begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix} - y_0 \begin{vmatrix} x_1 & z_1 \\ x_2 & z_2 \end{vmatrix} + z_0 \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}, & \begin{vmatrix} x_0 & y_0 \\ x_1 & y_1 \end{vmatrix} &= x_0 y_1 - x_1 y_0. \end{aligned}$$