Price Optimization Engine

Panos Repoussis, Sebastian Schenker

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Revenue Maximization of a Perishable Product with Fixed Capacity

This section describes a price optimization model that seeks to maximize revenues for the case of a single perishable product with fixed capacity. Typical examples are air tickets, hotel rooms, car rentals and theater tickets. We assume that customers purchase tickets or book hotel rooms at different points in time; however, all of them consume it at the same time. We also assume that price is flexible.

Let w be some future date and let q be a positive integer representing a lead time until w, e.g., the days before a stay at a hotel. The goal is to determine the optimal price level for each day during the lead time. Let T denote the set of q days, i.e, $T = \{1, \dots, q\}$. Moreover, let $P \subset \mathbb{N}$ be a given set of available prices. For each day $t \in T$ and for each price level $p \in P$ we are given a demand forecast (e.g., the number of expected bookings for w) which is denoted by D_p^t. Furthermore, we are given a maximum number of bookings C_w that can be accepted for the date w.

The above problem can be formulated as a mixed integer linear program (MILP). The main decision variable is to determine the price level p for each day t. For this purpose, a binary variable x_{pt} is introduced. Let also a non-negative continuous variable y_{tp} to denote the number of bookings that originate from day t and contribute to the revenues.

$$\max_{x,y} \sum_{t \in T} \sum_{p \in P} p y_{tp} x_{tp} \tag{1}$$

Subject to

$$\sum_{p \in P} x_{tp} = 1 \quad \forall t \in T \tag{2}$$

$$\sum_{t \in T} \sum_{p \in P} y_{tp} x_{tp} \leq C_w \tag{3}$$

$$y_{tp} \leq D_p^t \quad \forall t \in T, p \in P \tag{4}$$

Constraints (2) impose that a single price is selected per day. Constraint (3) ensures that the total number of bookings accepted during the lead time do not exceed the capacity of the target day. Constraints (4) ensure that the contributing daily bookings is not exceeding the demand for corresponding price.

1 Combinatorial perspective

Let us assume, without loss of generality, that $P = \{p_1, \ldots, p_n\}$ for some $n \in \mathbb{N}$. Let $t \in T$ be arbitrary but fixed. Order the products $D^t_{p1} \cdot p_1, \ldots, D^t_{pn} \cdot p_n$ in descending order. Mathematically, for $t \in T$ let $\Pi^t = \{\pi^t_1, \ldots, \pi^t_n\}$ be a permutation of P such that $D^t_{\pi^t_1} \cdot \pi^t_1 \geq \ldots \geq D^t_{\pi^t_n} \cdot \pi^t_n$ holds. We now consider two cases depending on the magnitude of C_w.

First case: $\sum_{t \in T} D^t_{\pi^t_1} \leq C_w$

In this case, the optimal solution is not restricted by the value C_w, only by the daily forecasting bounds D^t_p, i.e., the constraint (4). This implies an optimal solution is simply given by $x_{\pi^t_1} = 1$ and $x_{\pi^t_2} = 0, \ldots, x_{\pi^t_n} = 0$ and $y_{tp} = D^t_p$ for $t \in T$. This solution fulfills all constraints and maximises the objective as, for $t \in T$, π^t_1 yields the biggest benefit since $D^t_{\pi^t_1} \cdot \pi^t_1 \geq \ldots \geq D^t_{\pi^t_n} \cdot \pi^t_n$ holds.

Second case: $\sum_{t \in T} D^t_{\pi^t_1} > C_w$

In this case, C_w restricts the set of feasible solutions, e.g., the solution corresponding to the first case is not feasible anymore. However, we can find the optimal solution by selecting the *next best* permutations. Consider the list of all possible permutation tuples $(\pi^1_1, \ldots, \pi^q_1), (\pi^1_1, \ldots, \pi^{q-1}_1, \pi^q_2), \ldots, (\pi^1_n, \ldots, \pi^q_n)$.

Each tuple corresponds to a (not necessarily feasible) solution. We can order this list of tuples by their corresponding objective value and choose the first tuple that corresponds to a feasible solution.

2 Non-linear perspective

The objective function (1) contains the products of continuous with binary variables and we need to linearize them. Let a new non-negative continuous variable z, such that $z = x \times y$. To that end, we remove Constraints (3) and (4) and append to the mathematical model the following set of constraints:

$$z_{tp} \leq D_{tp} x_{tp} \quad \forall t \in T, p \in P \tag{5}$$

$$\sum_{t \in T} \sum_{p \in P} z_{tp} \leq C_w \tag{6}$$

Similarly, we replace the nonlinear terms from the objective function:

$$\max_{x,z} \sum_{t \in T} \sum_{p \in P} z_{tp} P_p \tag{7}$$

The discrete price optimization model (11), (6), (5) and (2) will find the optimal revenue and will set a price for each day during the lead time; however, it does not ensure that the contribution of the daily bookings is chronologically consistent until the capacity (e.g. number of available rooms) is fully consumed. In other words, as soon as a price is set and capacity exist, the corresponding daily predicted bookings should be consumed. The associated rule can be mathematically depicted as follows: if we considered less than the predicted number of bookings $\sum_{p \in P} z_{tp} < \sum_{p \in P} D_{tp} x_{tp}$ for a given day t , then for the next day $t{+}1$ the contribution of bookings should zero for all prices, i.e., $\sum_{p \in P} z_{tp} \leq 0$.

An additional binary variable a is introduced to model the above rule. In particular, let a be equal to 1 if capacity is reached on day t and thus $\sum_{p \in P} z_{tp} \leq \xi \sum_{p \in P} D_{tp} x_{tp}$, where ξ is the equality tolerance; 0, otherwise.

On the basis of the above, the following set of bigM constraints should be appended to the model:

$$-M_{1t} a_t \leq \sum_{p \in P} (z_{tp} - \xi x_{tp} D_{tp}) \quad \forall t \in T - \{q\}, \tag{8}$$

$$\sum_{p \in P} z_{(t+1)p} \leq M_{2t}(1 - a_t) \quad \forall t \in T - \{q\}, \tag{9}$$

$$a_t \geq \sum_{p \in P} (z_{tp} - x_{tp} D_{tp}) \quad \forall t \in T - \{q\}, \tag{10}$$

where $M_{1t} = \min\{C_w, \xi \max_p D_{tp}\}$ and $M_{2t} = \min\{C_w, \max_p D_{(t+1)p}\}$.

Markdown Optimization with No Replenishment and Salvage value

This section describes a markdown optimization model that seeks to maximize revenues for the case of a single product, with initial inventory I_0. The product perishes and has a low salvage value s once the season is over. Let a planning horizon of $t \in T$ days or weeks that represents the sales season. There is a given set of available prices P that consists of fixed float numbers with decreasing order. For each day $t \in T$ and price level $p \in P$ the forecast of the demand is given and it is denoted as D_p^t. The goal is to maximize the collected revenue of the sales season.

For perishable products, markdown is needed to manage the demand and maximize revenue. However, pricing is not flexible and only involves price reductions. The retailer initially sets high prices, and then "learns" the demand. Popular products sell quickly at high prices. Low-reservation-price products must be marked down. The willingness-to-pay (WTP) changes over the planning horizon. At the beginning, the WTP is often high (e.g. buy a bathing suit at the start of the summer) and then gradually reduces. During peak periods, the WTP often reduces, and consumers become more price-sensitive. Finally, we assume that every period the retailer re-runs the price optimization engine with updated info.

The above described markdown optimization problem can be formulated as a mixed integer linear program (MILP). The main decision variable is to determine the price level p for each period t. For this purpose, a binary variable $x_{t,p}$ is introduced. Let also a non-negative continuous variable $z_{t,p}$ to denote the estimated number of items that will be sold at period t for the selected price $x_{t,p}$. Finally, let non-negative continuous variable I_T to denote the ending inventory at period T.

$$\max_{x,z,I_T} \sum_{t\in T}\sum_{p\in P} z_{t,p}P_p + I_T s \tag{11}$$

Subject to

$$\sum_{t\in T}\sum_{p\in P} z_{t,p} + I_T = I_0 \tag{12}$$

$$z_{t,p} \leq x_{t,p}D_t^p \quad \forall t \in T, p \in P \tag{13}$$

$$\sum_{p\in P} x_{t,p} = 1 \quad \forall t \in T \tag{14}$$

$$x_{0,1} = 1 \tag{15}$$

$$\sum_{q=0}^{p} x_{t,q} - \sum_{p=0}^{p} x_{t+1,q} \geq 0 \quad \forall p \in P, t \in T - \{q\} \tag{16}$$

The objective function (11) calculates the total collected revenue for each period and the price level selected for each period. The total salvage value of the remaining inventory is subtracted from the revenue. Constraint (12) models the inventory constraint and limits the amount of inventory the retailer can sell to the initial value I_0. Constraint (13) ensures that the number of items to be sold at each period is not exceeding the demand. Constraint (14) enforces that one price is selected per period. Constraint (15) ensures that the full price will be selected for the first period. Finally, Constraints (16) stipulates that price markdowns are irreversible and can only be decreasing from period to period.

The above model (11) to (16) will work as expected for the case inventory is left at the end of the sales season. If this is not the case, then it does not ensure that the items sold is chronologically consistent until the initial inventory is fully consumed. In other words, as soon as a price is set and inventory exist, the corresponding demand should be consumed. The associated rule can be mathematically depicted as follows: if we considered less than the predicted demand $\sum_{p\in P} z_{t,p} < \sum_{p\in P} D_t^p x_{t,p}$ for a given period t , then for the next period t+1 the z-variables should be forced to zero for all prices, i.e., $\sum_{p\in P} z_{tp} \leq 0$. To that end, a new binary variable a is introduced to model the above rule and the block of constraints (8) to (10) must be appended to the model.