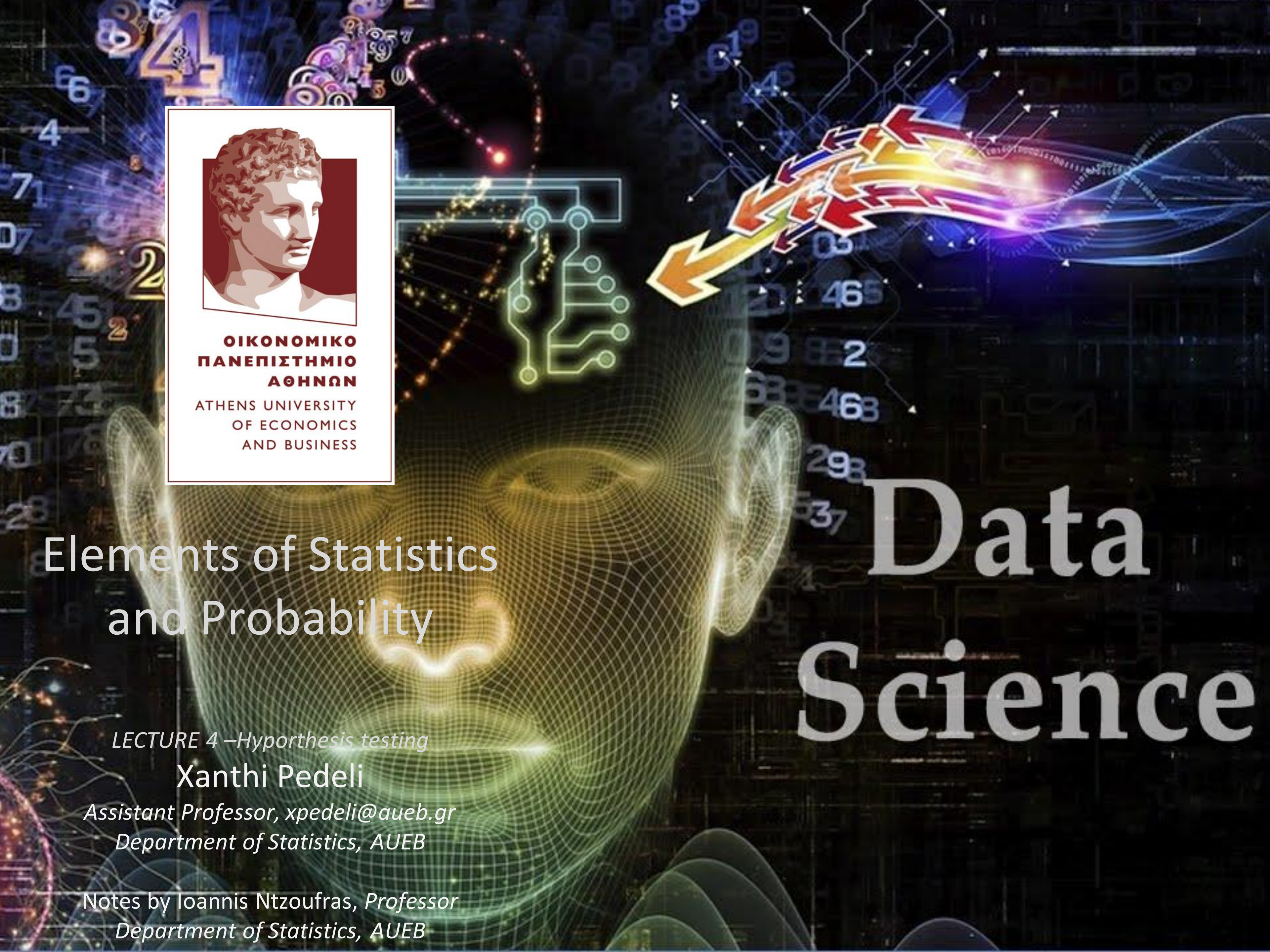

ΟΙΚΟΝΟΜΙΚΟ
ΠΑΝΕΠΙΣΤΗΜΙΟ
ΑΘΗΝΩΝ
ATHENS UNIVERSITY
OF ECONOMICS
AND BUSINESS

Elements of Statistics and Probability

LECTURE 4 –Hyporthesis testing

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4. *Hypothesis Tests & Pairwise associations*

In this module we will examine

- Exploratory data analysis
- Testing for normality
- General principles for hypothesis testing
- Standard errors and p-values
- Hypothesis tests for one sample (mean, variance, kurtosis, skewness)
- Association between one quantitative and one qualitative variable
- Association between two categorical variables
- Association between two ordinals
- Correlation between two quantitative variables

4.2. Testing for normality

- QQ plots
- PDF plots
- CDF Plots
- Testing for symmetry & skewness
- Testing for kurtosis
- Kolmogorov-Smirnov test
- Shapiro-Wilks test
- Other tests

4.2. Testing for normality

Salary dataset from Norusis

This is a data file containing information on 474 employees hired by a Midwestern bank between 1969 and 1971.

It was created for an Equal Employment Opportunity (EEO) court case involving wage discrimination.

4.2. Testing for normality

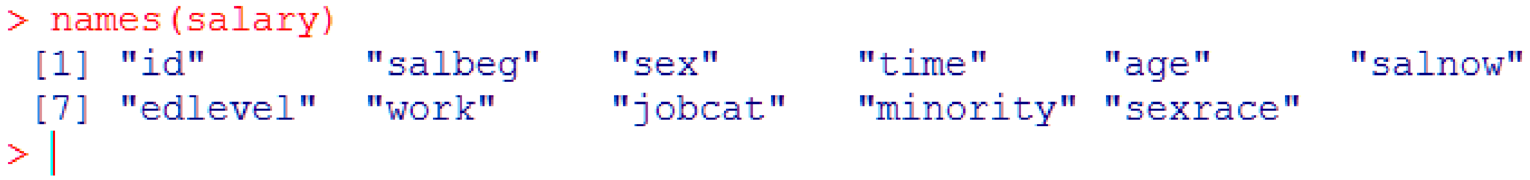
Salary dataset from Norusis

ID	Employee code
SALBEG	Beginning salary
SEX	Gender of employee (0=Female; 1=Male)
TIME	Job Seniority (months)
AGE	Age of Employee (years and fraction)
SALNOW	Current salary
EDLEVEL	Educational Level (years)
WORK	Work Experience (years x 100)
JOBCAT	Employment category (1 Clerical, 2 Office trainee, 3 Security officer, 4 College trainee, 5 Exempt employee, 6 MBA trainee 7 Technical)
MINORITY	Minority Classification (0=White; 1=Nonwhite)

4.2. Testing for normality

Dataset salary from Norusis

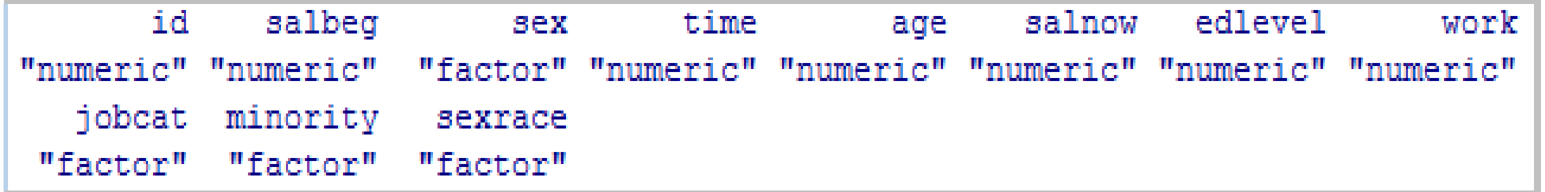
```
library(foreign)
salary<-read.spss('salary.sav', to.data.frame=T)
names(salary)
```



```
> names(salary)
 [1] "id"        "salbeg"    "sex"       "time"      "age"       "salnow"
 [7] "edlevel"   "work"      "jobcat"    "minority"  "sexrace"
> 
```

```
head(salary)
```

```
sapply(salary, class)
```

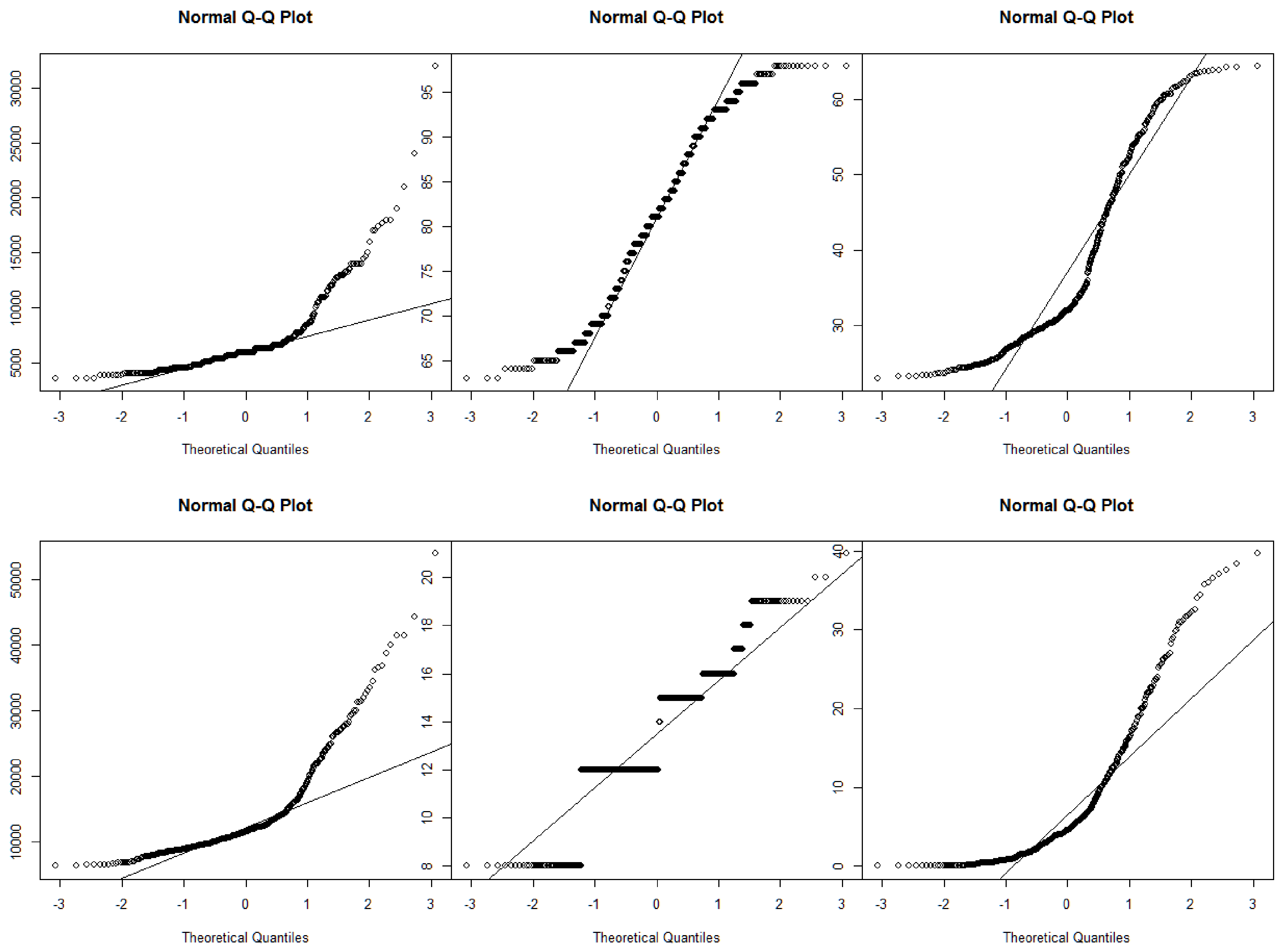


```
       id    salbeg       sex      time       age    salnow   edlevel      work 
"numeric" "numeric"  "factor" "numeric" "numeric" "numeric" "numeric" "numeric" 
   jobcat  minority   sexrace 
 "factor"  "factor"  "factor" 
```

```
sal.num <- salary[,which(sapply(salary, class)=="numeric")]
```

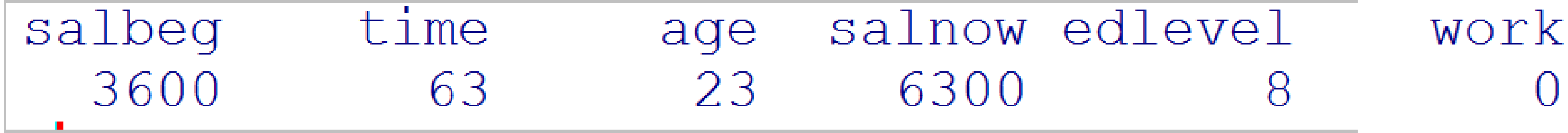
4.2. Testing for normality
QQ-plots

```
y<-sal.num[,-1]
p<-ncol(y)
par(mfrow=c(2,3))
for (i in 1:p){
        qqnorm(y[,i])
        qqline(y[,i])
}
```


Normal Q-Q Plot
5000
10000
15000
20000
25000
30000
Theoretical Quantiles
Normal Q-Q Plot
65
70
75
80
85
90
95
Theoretical Quantiles
Normal Q-Q Plot
30
40
50
60
Theoretical Quantiles
Normal Q-Q Plot
10000
20000
30000
40000
50000
Theoretical Quantiles
Normal Q-Q Plot
8
10
12
14
16
18
20
Theoretical Quantiles
Normal Q-Q Plot
0
10
20
30
40
Theoretical Quantiles
-3
-2
-1
0
1
2
3

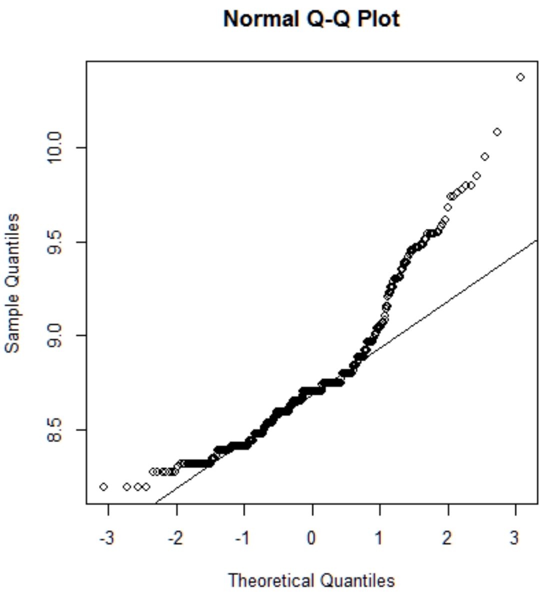
4.2. Testing for normality
QQ-plots

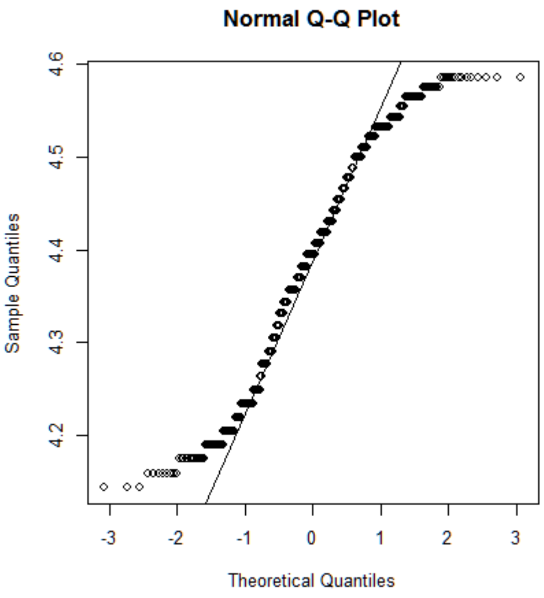
```
sapply(y,min)
```

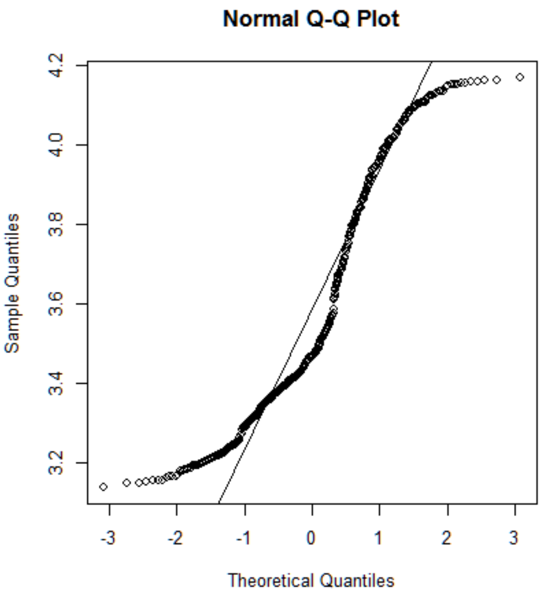


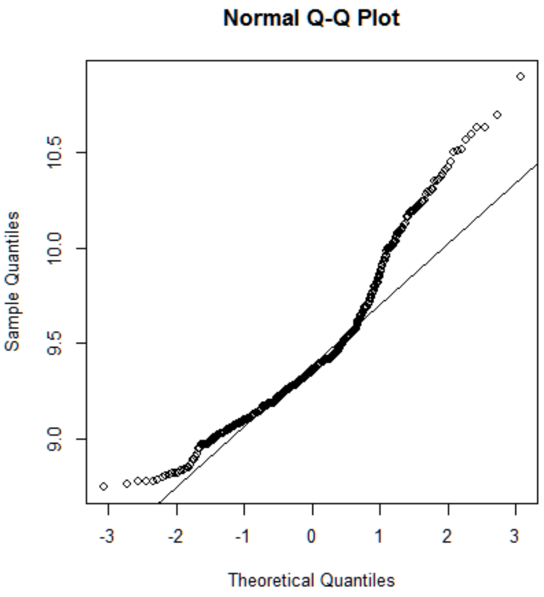
```
salbeg     time      age   salnow edlevel     work
  3600       63       23     6300       8        0
```

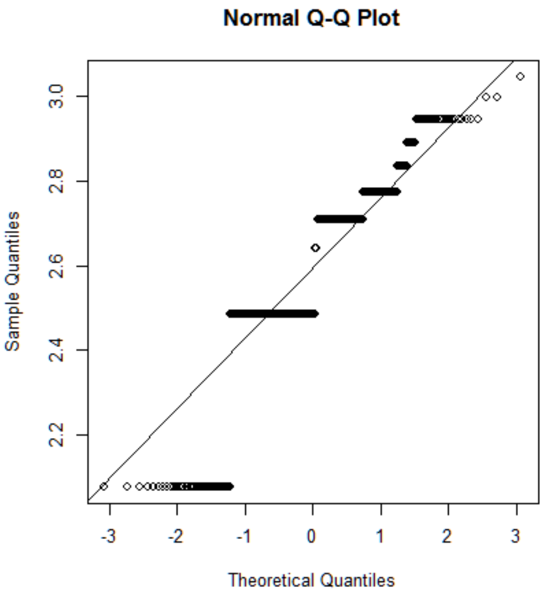
```
y[,6] <- y[,6]+0.5
y<-log(y)
p<-ncol(y)
par(mfrow=c(2,3))
for (i in 1:p){
        qqnorm(y[,i])
        qqline(y[,i])
}
```

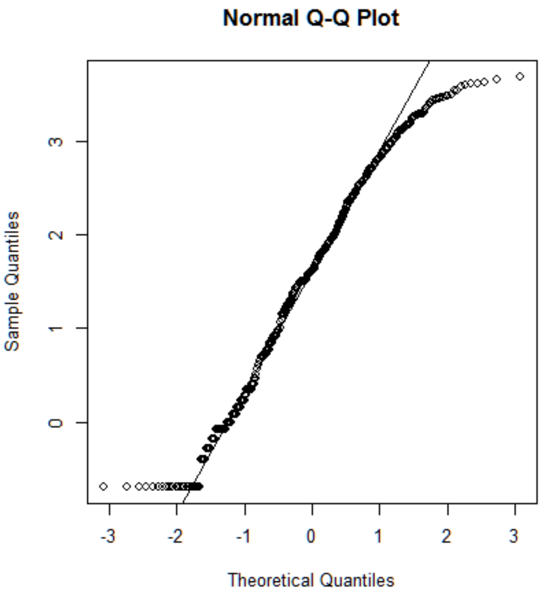

Normal Q-Q Plot
Sample Quantiles
8.5
9.0
9.5
10.0
Theoretical Quantiles
-3
-2
-1
0
1
2
3

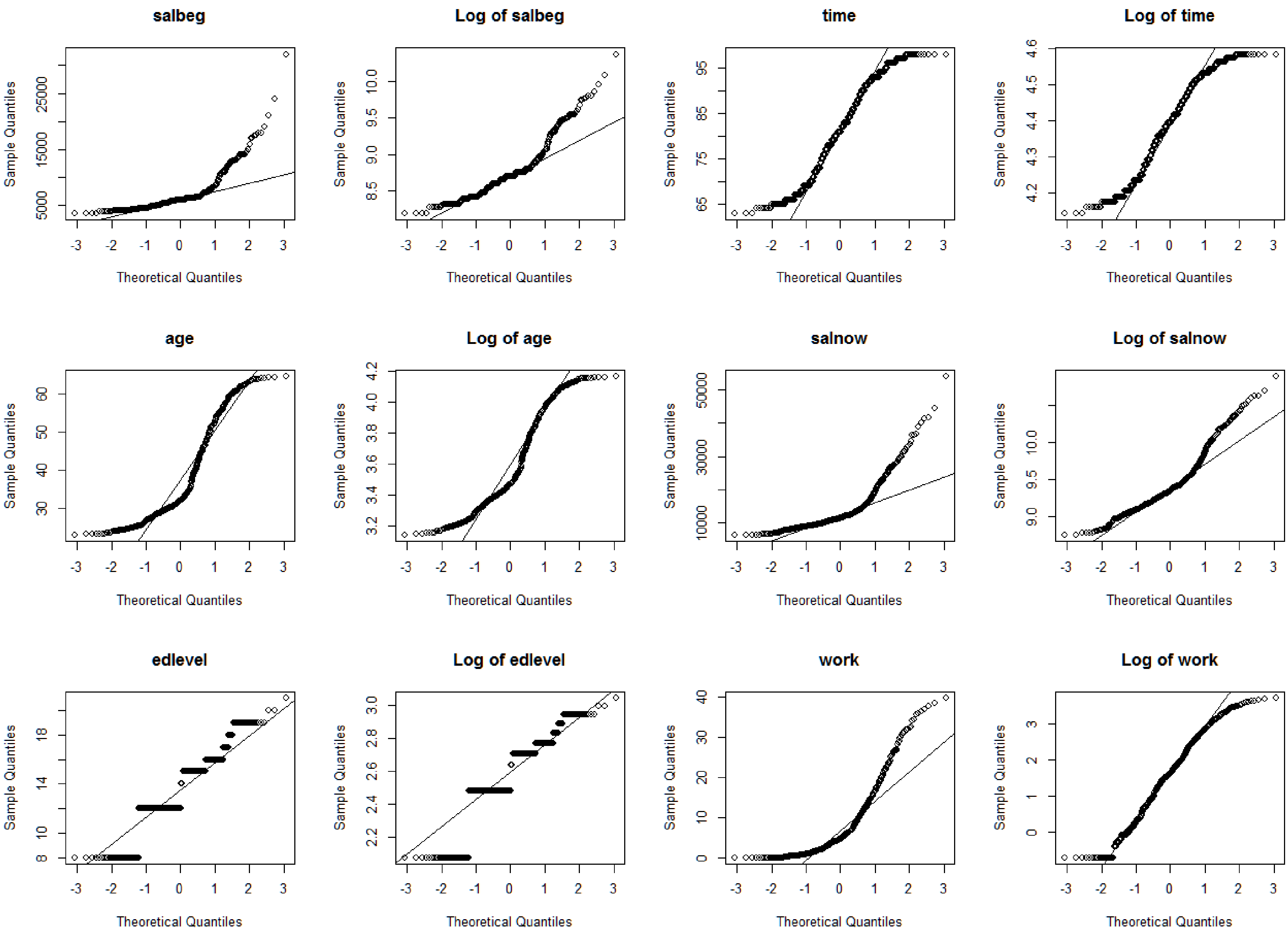

Normal Q-Q Plot
Sample Quantiles
4.2
4.3
4.4
4.5
4.6
Theoretical Quantiles
-3
-2
-1
0
1
2
3


Normal Q-Q Plot
Sample Quantiles
3.2
3.4
3.6
3.8
4.0
4.2
Theoretical Quantiles
-3
-2
-1
0
1
2
3


Normal Q-Q Plot
Sample Quantiles
9.0
9.5
10.0
10.5
Theoretical Quantiles
-3
-2
-1
0
1
2
3


Normal Q-Q Plot
Sample Quantiles
2.2
2.4
2.6
2.8
3.0
Theoretical Quantiles
-3
-2
-1
0
1
2
3


Normal Q-Q Plot
Sample Quantiles
0
1
2
3
Theoretical Quantiles
-3
-2
-1
0
1
2
3



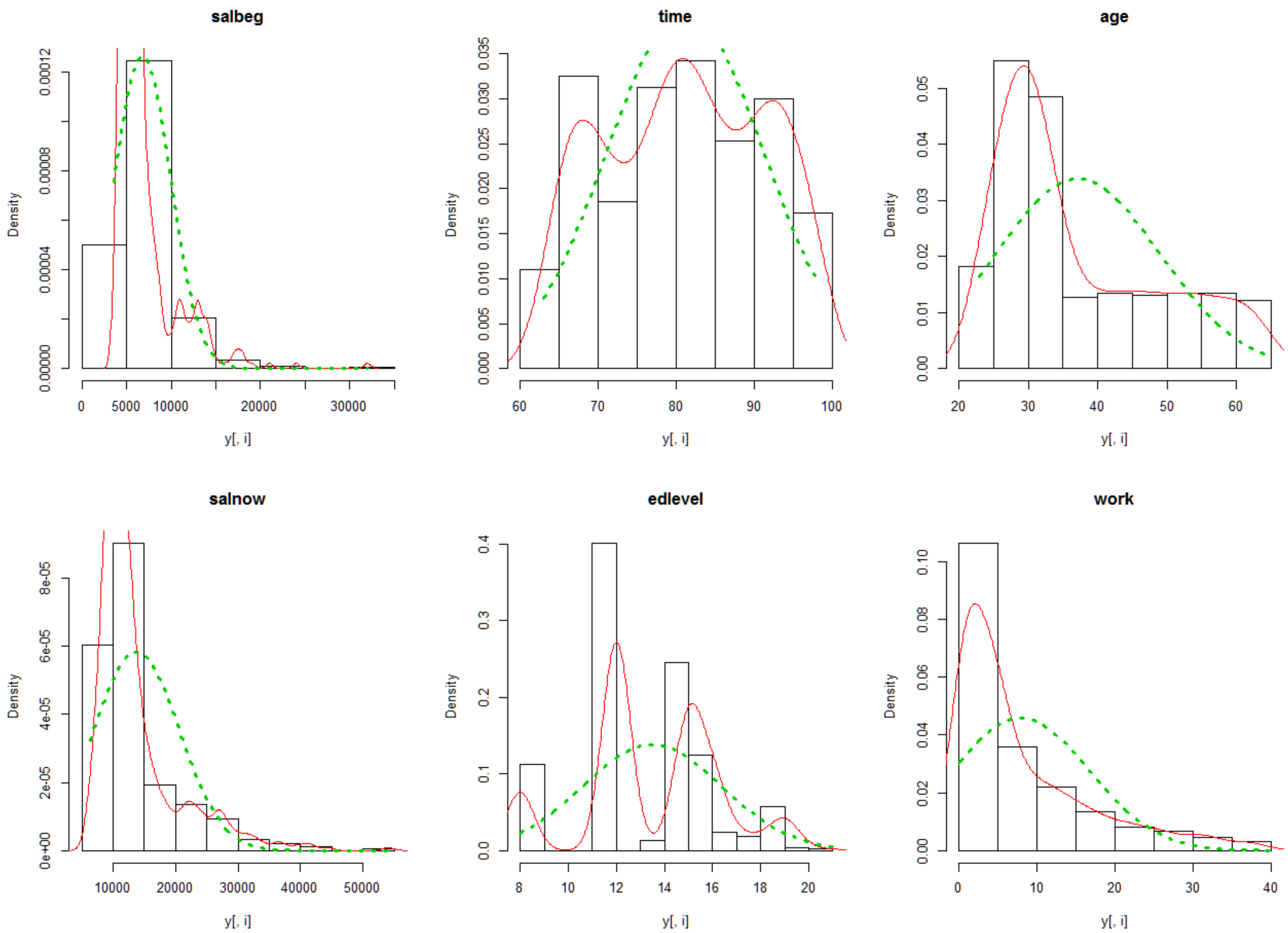
4.2. Testing for normality

Density plots

```
y<-sal.num[,-1]
p<-ncol(y)
par(mfrow=c(2,3))
for (i in 1:p){
        hist(y[,i], main=names(y)[i], probability=TRUE)
        lines(density(y[,i]), col=2)
        index <- seq( min(y[,i]), max(y[,i]),
        length.out=100)
        ynorm <- dnorm( index, mean=mean(y[,i]),
        sd(y[,i]) )
        lines( index, ynorm, col=3, lty=3, lwd=3 )
}
```

4.2. Testing for normality

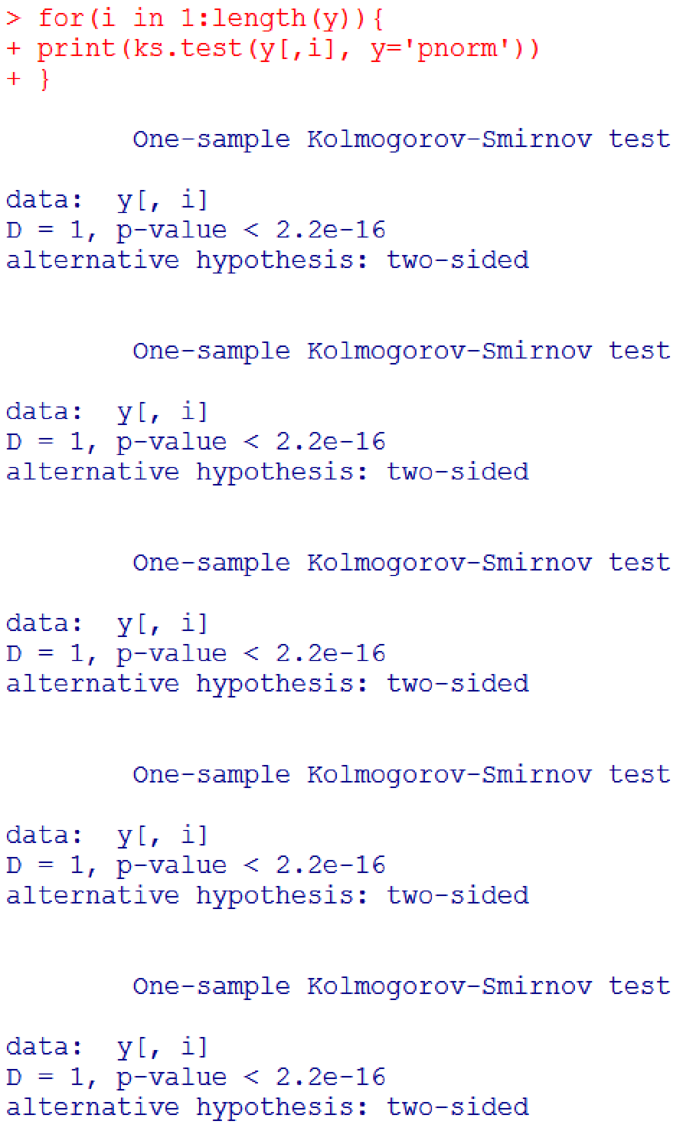
Density plots



4.2. Testing for normality

Kolmogorov-Smirnov test

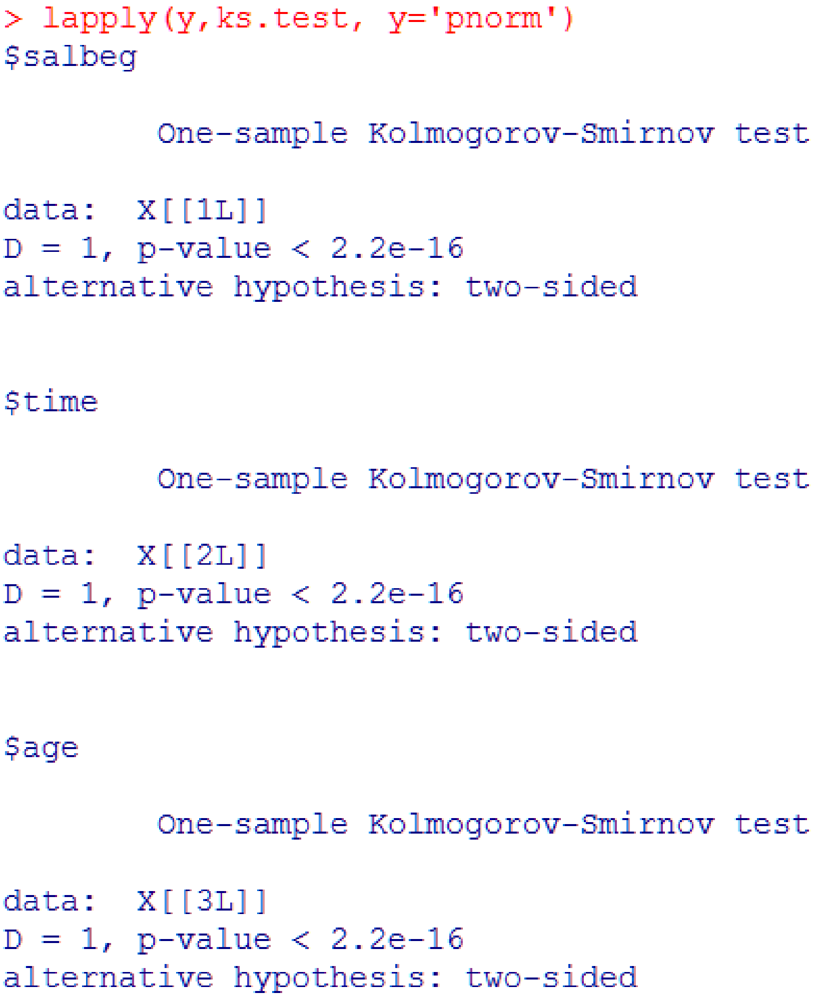
The KS test quantifies a distance between the empirical distribution function of the sample and the cumulative distribution function of the reference distribution. In the special case of testing for normality of the distribution, samples are standardized and compared with a standard normal distribution.



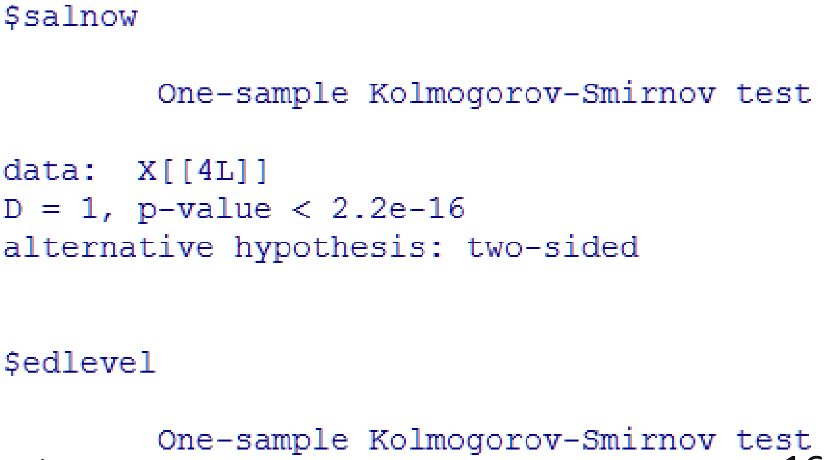
```
> for(i in 1:length(y)){
+ print(ks.test(y[,i], y='pnorm'))
+ }

        One-sample Kolmogorov-Smirnov test

data:  y[, i]
D = 1, p-value < 2.2e-16
alternative hypothesis: two-sided


        One-sample Kolmogorov-Smirnov test

data:  y[, i]
D = 1, p-value < 2.2e-16
alternative hypothesis: two-sided


        One-sample Kolmogorov-Smirnov test

data:  y[, i]
D = 1, p-value < 2.2e-16
alternative hypothesis: two-sided


        One-sample Kolmogorov-Smirnov test

data:  y[, i]
D = 1, p-value < 2.2e-16
alternative hypothesis: two-sided


        One-sample Kolmogorov-Smirnov test

data:  y[, i]
D = 1, p-value < 2.2e-16
alternative hypothesis: two-sided
```

4.2. Testing for normality

Kolmogorov-Smirnov test



```
> lapply(y,ks.test, y='pnorm')
$salbeg

        One-sample Kolmogorov-Smirnov test

data:  X[[1L]]
D = 1, p-value < 2.2e-16
alternative hypothesis: two-sided


$time

        One-sample Kolmogorov-Smirnov test

data:  X[[2L]]
D = 1, p-value < 2.2e-16
alternative hypothesis: two-sided


$age

        One-sample Kolmogorov-Smirnov test

data:  X[[3L]]
D = 1, p-value < 2.2e-16
alternative hypothesis: two-sided
```

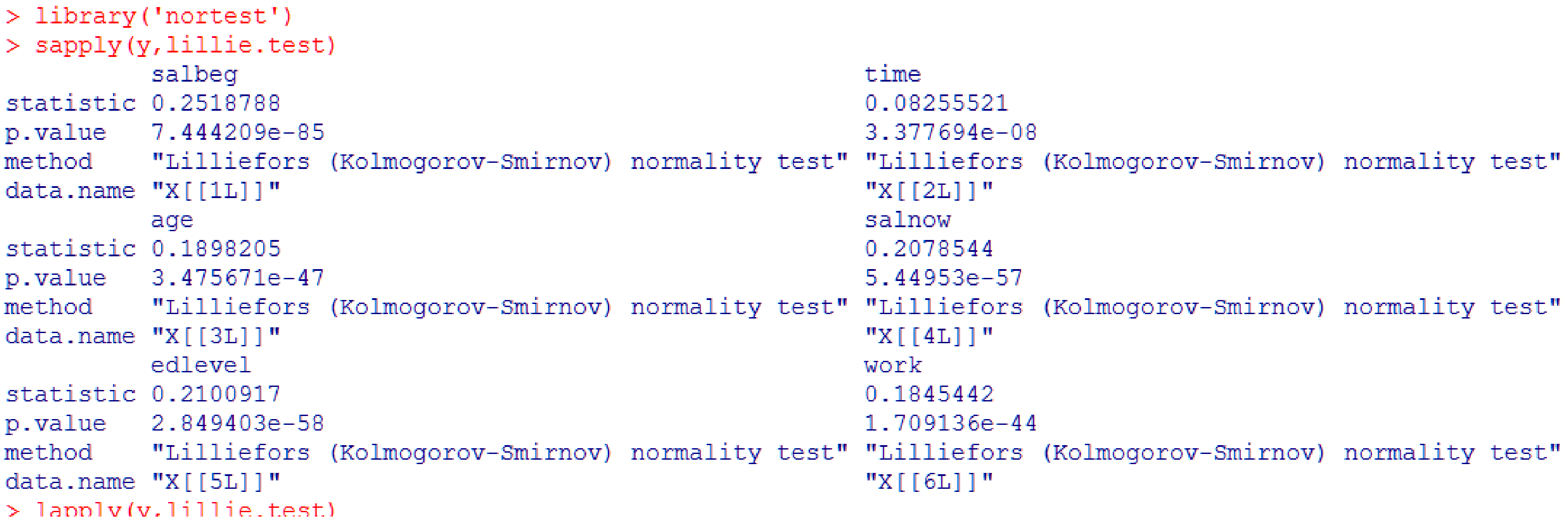


```
$salnow

        One-sample Kolmogorov-Smirnov test

data:  X[[4L]]
D = 1, p-value < 2.2e-16
alternative hypothesis: two-sided


$edlevel

        One-sample Kolmogorov-Smirnov test
```

4.2. Testing for normality

Lilliefors Kolmogorov-Smirnov test

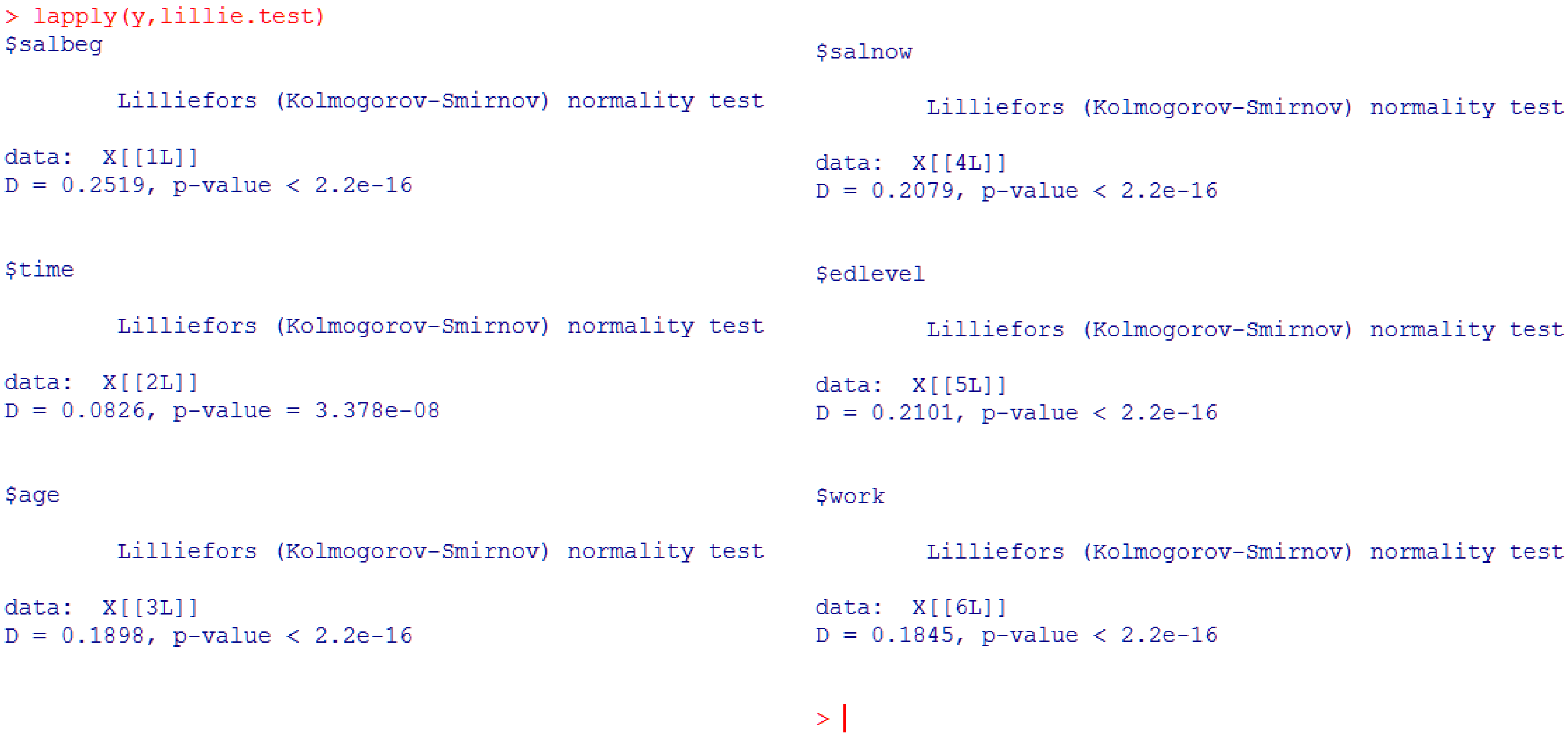
- Modification of the Kolmogorov-Smirnov test
- Less conservative



```
> library('nortest')
> sapply(y,lillie.test)
          salbeg                                          time
statistic 0.2518788                                       0.08255521
p.value   7.444209e-85                                    3.377694e-08
method    "Lilliefors (Kolmogorov-Smirnov) normality test" "Lilliefors (Kolmogorov-Smirnov) normality test"
data.name "X[[1L]]"                                       "X[[2L]]"
          age                                             salnow
statistic 0.1898205                                       0.2078544
p.value   3.475671e-47                                    5.44953e-57
method    "Lilliefors (Kolmogorov-Smirnov) normality test" "Lilliefors (Kolmogorov-Smirnov) normality test"
data.name "X[[3L]]"                                       "X[[4L]]"
          edlevel                                         work
statistic 0.2100917                                       0.1845442
p.value   2.849403e-58                                    1.709136e-44
method    "Lilliefors (Kolmogorov-Smirnov) normality test" "Lilliefors (Kolmogorov-Smirnov) normality test"
data.name "X[[5L]]"                                       "X[[6L]]"
> lapply(y,lillie.test)
```

4.2. Testing for normality

Lilliefors Kolmogorov-Smirnov test



```
> lapply(y,lillie.test)
$salbeg

        Lilliefors (Kolmogorov-Smirnov) normality test

data:  X[[1L]]
D = 0.2519, p-value < 2.2e-16


$time

        Lilliefors (Kolmogorov-Smirnov) normality test

data:  X[[2L]]
D = 0.0826, p-value = 3.378e-08


$age

        Lilliefors (Kolmogorov-Smirnov) normality test

data:  X[[3L]]
D = 0.1898, p-value < 2.2e-16


$salnow

        Lilliefors (Kolmogorov-Smirnov) normality test

data:  X[[4L]]
D = 0.2079, p-value < 2.2e-16


$edlevel

        Lilliefors (Kolmogorov-Smirnov) normality test

data:  X[[5L]]
D = 0.2101, p-value < 2.2e-16


$work

        Lilliefors (Kolmogorov-Smirnov) normality test

data:  X[[6L]]
D = 0.1845, p-value < 2.2e-16


> |
```

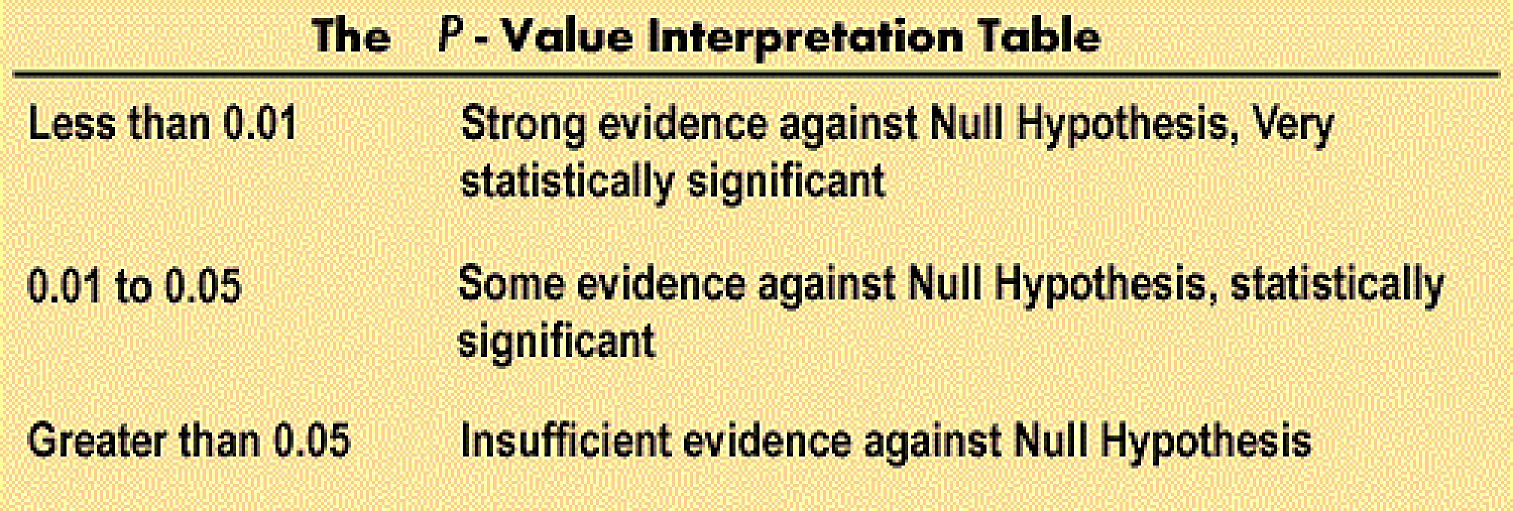
4.3. *General principles for hypothesis testing*

Hypothesis tests:

- Use a null hypothesis expressed with an equality concerning a set of parameters $H_0: \theta=\theta_0$
- The alternative hypothesis is usually the complementary statement e.g. $H_1: \theta\neq\theta_0$
- We use a Statistic which usually measures the distance between the observed data and the null hypothesis
- We use the distribution under the null hypothesis to find thresholds for rejecting the null
- So the reasoning is that if the Statistic is large i.e. the distance between the observed data and the null hypothesis is large, then something went wrong with our reasoning and probably this is the original hypothesis
- **We do reject the null hypothesis**
- **We do not accept the alternative**

4.3. *Standard errors & p values*

What is a p-value?



The P - Value Interpretation Table

Less than 0.01	Strong evidence against Null Hypothesis, Very statistically significant
0.01 to 0.05	Some evidence against Null Hypothesis, statistically significant
Greater than 0.05	Insufficient evidence against Null Hypothesis

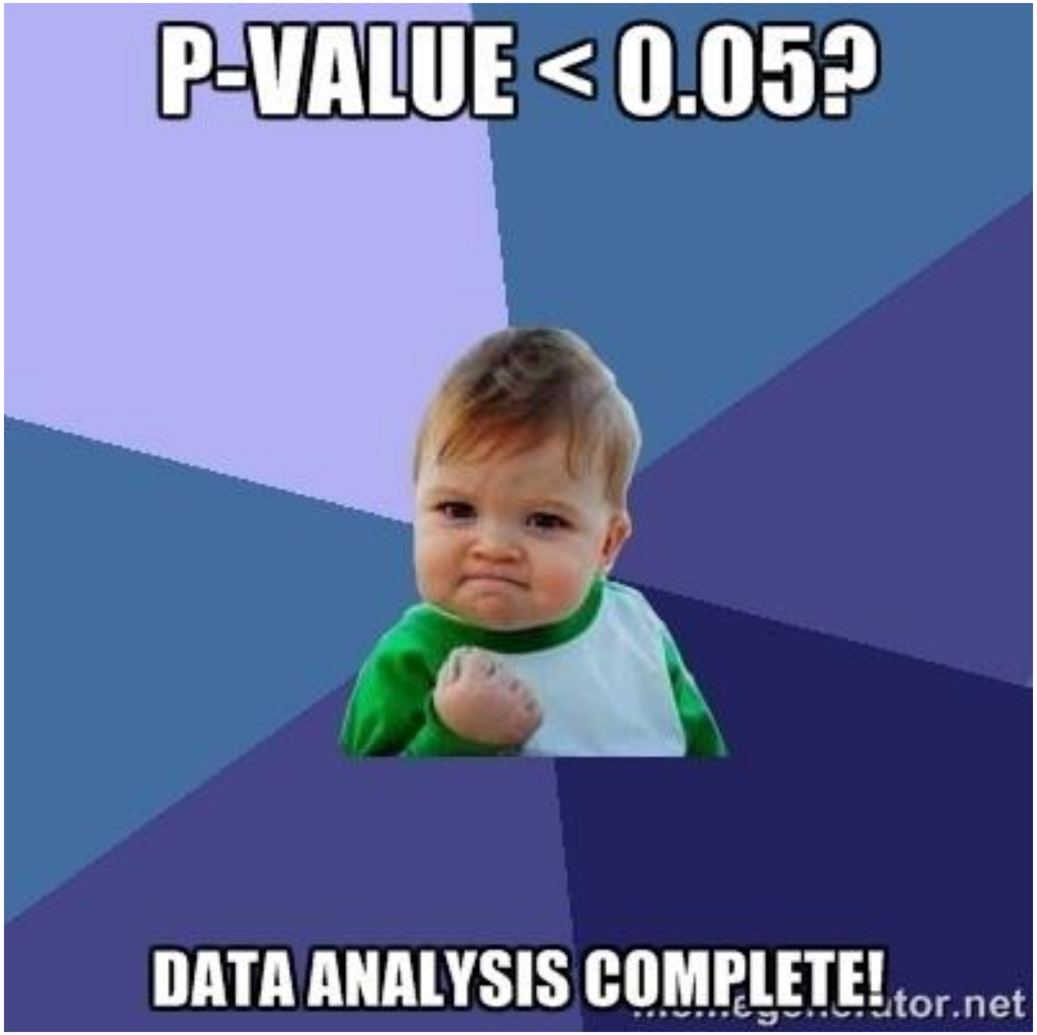
$0.05 < p < 0.10$ Not significant at α=5%

Significant at α=10%

Often reported as marginal significant result

4.3. *Standard errors & p values*

What is a p-value?



4.3. *Standard errors & p values*

What is a p-value?

https://www.youtube.com/watch?time_continue=17&v=9jW9G8MO4PQ

4.3. *Standard errors & p values*

What is a p-value?

- It is not the probability that the null hypothesis is not correct!
- It is the probability to observe a dataset with the same or more distant statistic than the one already observed if the null is true
- Hence if the p-value is small it means that not a lot of datasets are worse than the one you observed under the null
- So either the null is wrong and should be rejected or we are unlucky
- How much unlucky? The probability to reject the null when it is true is equal to α (usually equal to 5% or 1%)

4.3. *Standard errors & p values*

What is a standard error?

- Standard deviation or standard error
- Is it equal to $s/n^{1/2}$?
- What about estimators?

4.3. *Standard errors & p values*

What is a standard error?

- Estimators are functions of random variables (before observing the data)
- Estimates are the specific observed value for a specific sample
- Estimators are random variables. Therefore:
  - for each sample we have different value (estimate)
  - They have a mean
  - They have a variance

Standard error of an estimator is simply the standard deviation of an estimator

It is called error because it measures how close we are in the true value (i.e. the mean in the unbiased estimators)

4.3. *Standard errors & p values*

What is a standard error?

Good estimators

- They are unbiased

  (i.e. if we consider a lot of samples, the mean of the estimates will be equal to the true value)

- They have small standard error

  (i.e. the estimates are close to the mean/true value)

4.4. *Hypothesis tests for a single continuous variable*

Hypothesis test for the mean

- H_0: $\mu=\mu_0$ vs H_1: $\mu\neq\mu_0$
- Z-test for normal data with known variance
- T-test for normal data with unknown variance
- Assumption: Normality or large sample n>30 or >50

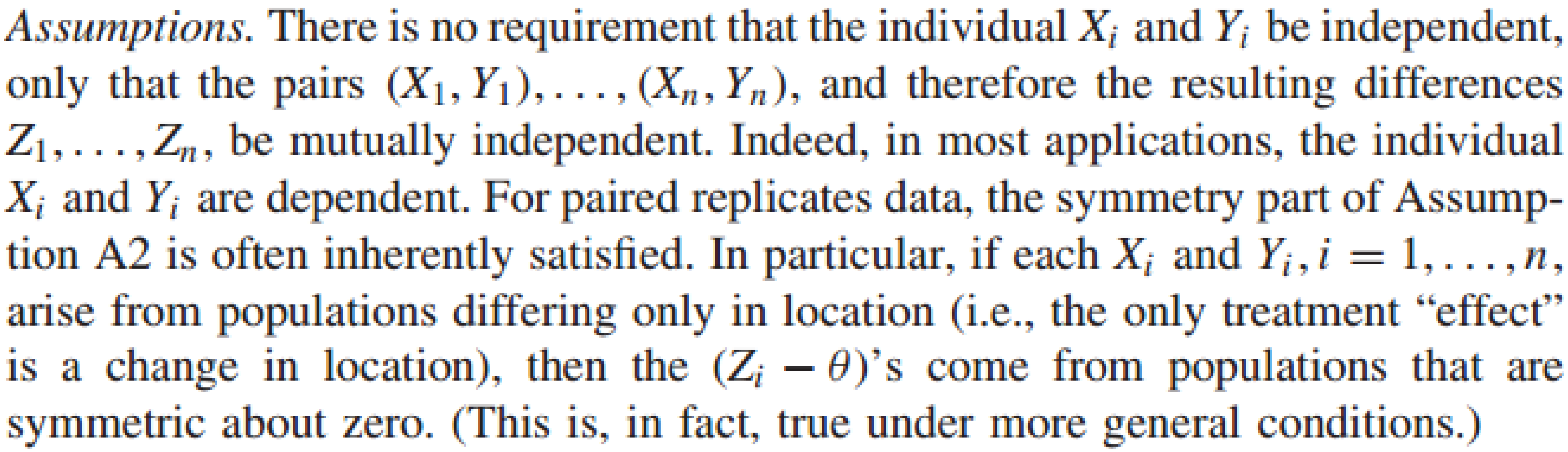
Hypothesis test for the median

- H_0: $M= M_0$ vs H_1: $M\neq M_0$
- Wilcoxon sign rank test

4.4. *Hypothesis tests for a single continuous variable*

Hypothesis test for the median

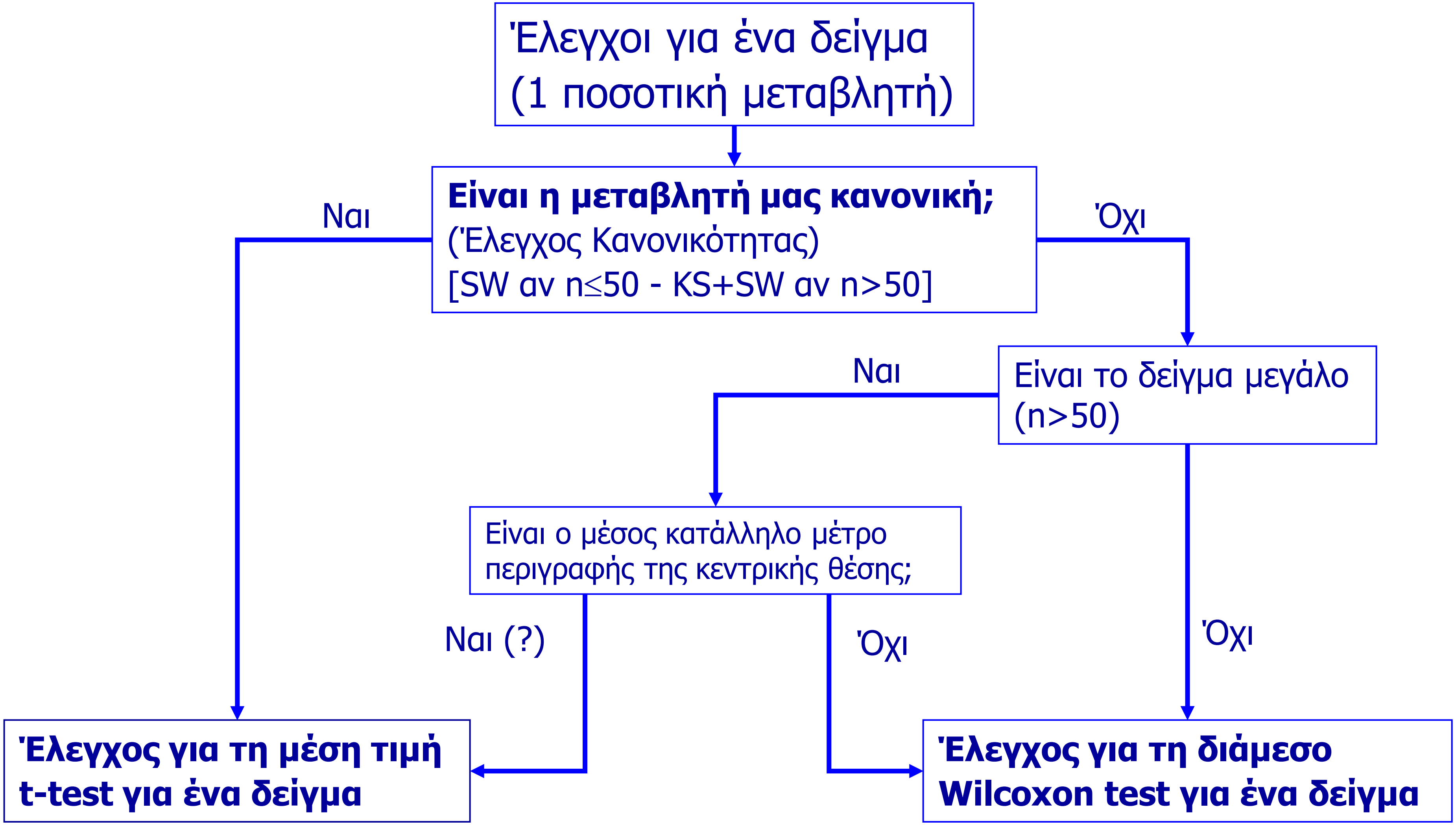
- H_0: $M = M_0$ vs H_1: $M \neq M_0$
- Wilcoxon sign rank test



Assumptions. There is no requirement that the individual X_i and Y_i be independent, only that the pairs $(X_1, Y_1), \ldots, (X_n, Y_n)$, and therefore the resulting differences $Z_1, \ldots, Z_n$, be mutually independent. Indeed, in most applications, the individual X_i and Y_i are dependent. For paired replicates data, the symmetry part of Assumption A2 is often inherently satisfied. In particular, if each X_i and $Y_i, i = 1, \ldots, n$, arise from populations differing only in location (i.e., the only treatment "effect" is a change in location), then the $(Z_i - \theta)$'s come from populations that are symmetric about zero. (This is, in fact, true under more general conditions.)

Source: Hollander, Wolfe & Chicken (2014). *Nonparametric Statistical Methods*. Wiley & sons.

4.4. *Hypothesis tests for a single continuous variable*



Έλεγχοι για ένα δείγμα
(1 ποσοτική μεταβλητή)

Είναι η μεταβλητή μας κανονική;
(Έλεγχος Κανονικότητας)
[SW αν n≤50 - KS+SW αν n>50]

Ναι

Όχι

Είναι το δείγμα μεγάλο
(n>50)

Ναι

Όχι

Είναι ο μέσος κατάλληλο μέτρο
περιγραφής της κεντρικής θέσης;

Ναι (?)

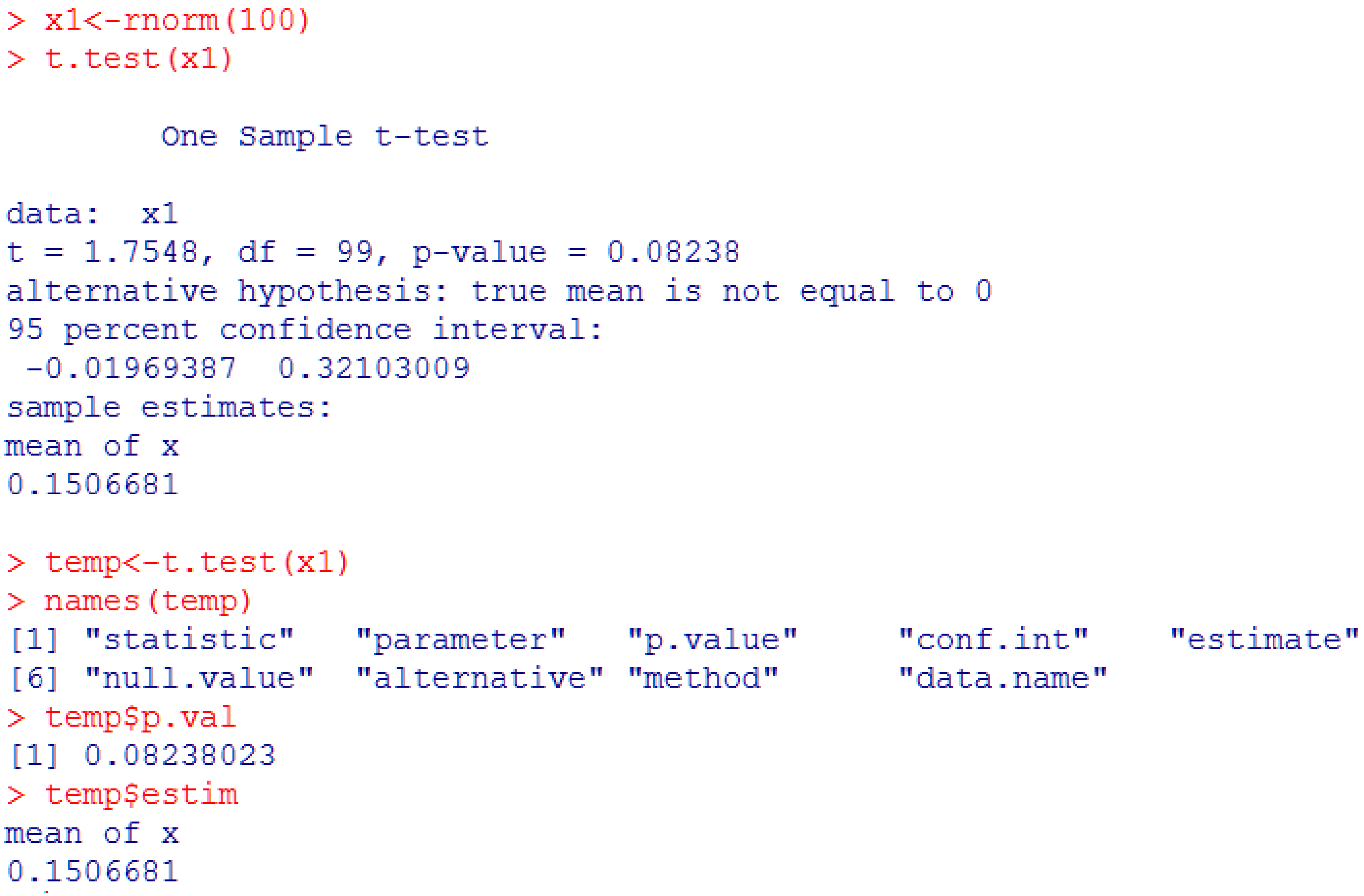
Όχι

Έλεγχος για τη μέση τιμή
t-test για ένα δείγμα

Έλεγχος για τη διάμεσο
Wilcoxon test για ένα δείγμα

4.4. *Hypothesis tests for a single continuous variable*

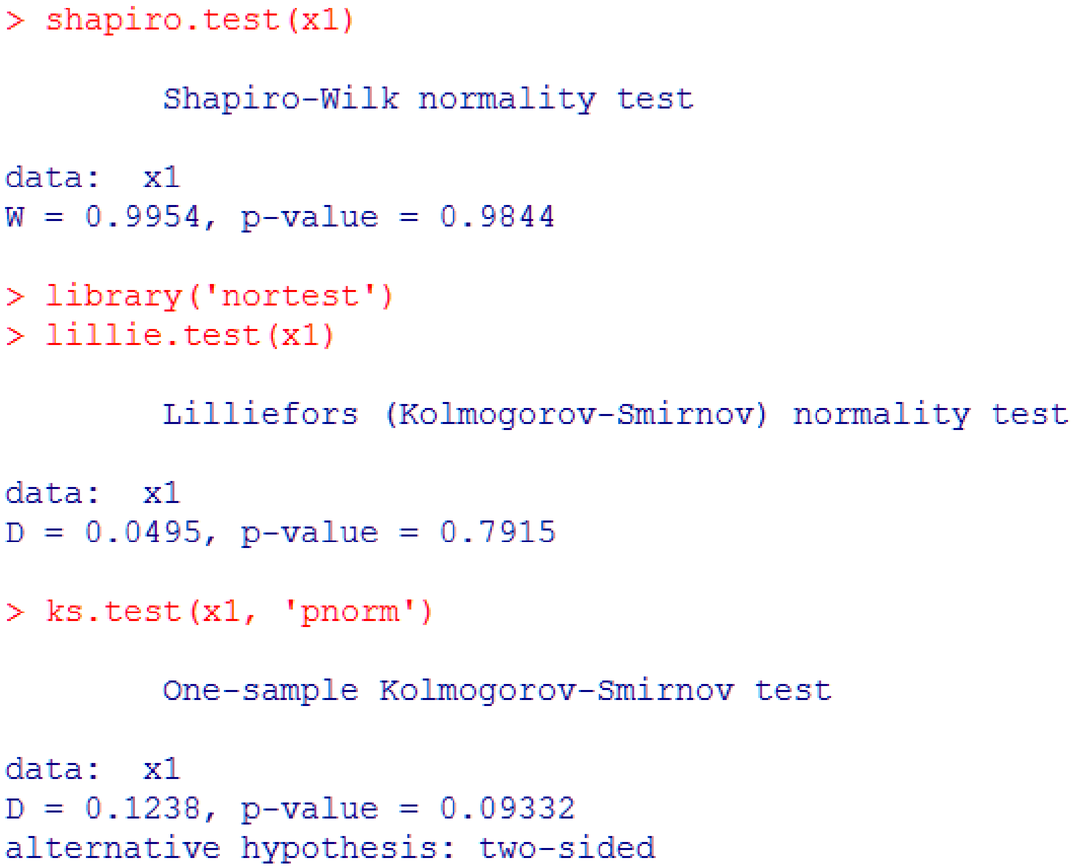
Hypothesis test for the mean



```
> x1<-rnorm(100)
> t.test(x1)

        One Sample t-test

data:  x1
t = 1.7548, df = 99, p-value = 0.08238
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -0.01969387  0.32103009
sample estimates:
mean of x
0.1506681

> temp<-t.test(x1)
> names(temp)
[1] "statistic"   "parameter"   "p.value"     "conf.int"    "estimate"
[6] "null.value"  "alternative" "method"      "data.name"
> temp$p.val
[1] 0.08238023
> temp$estim
mean of x
0.1506681
```

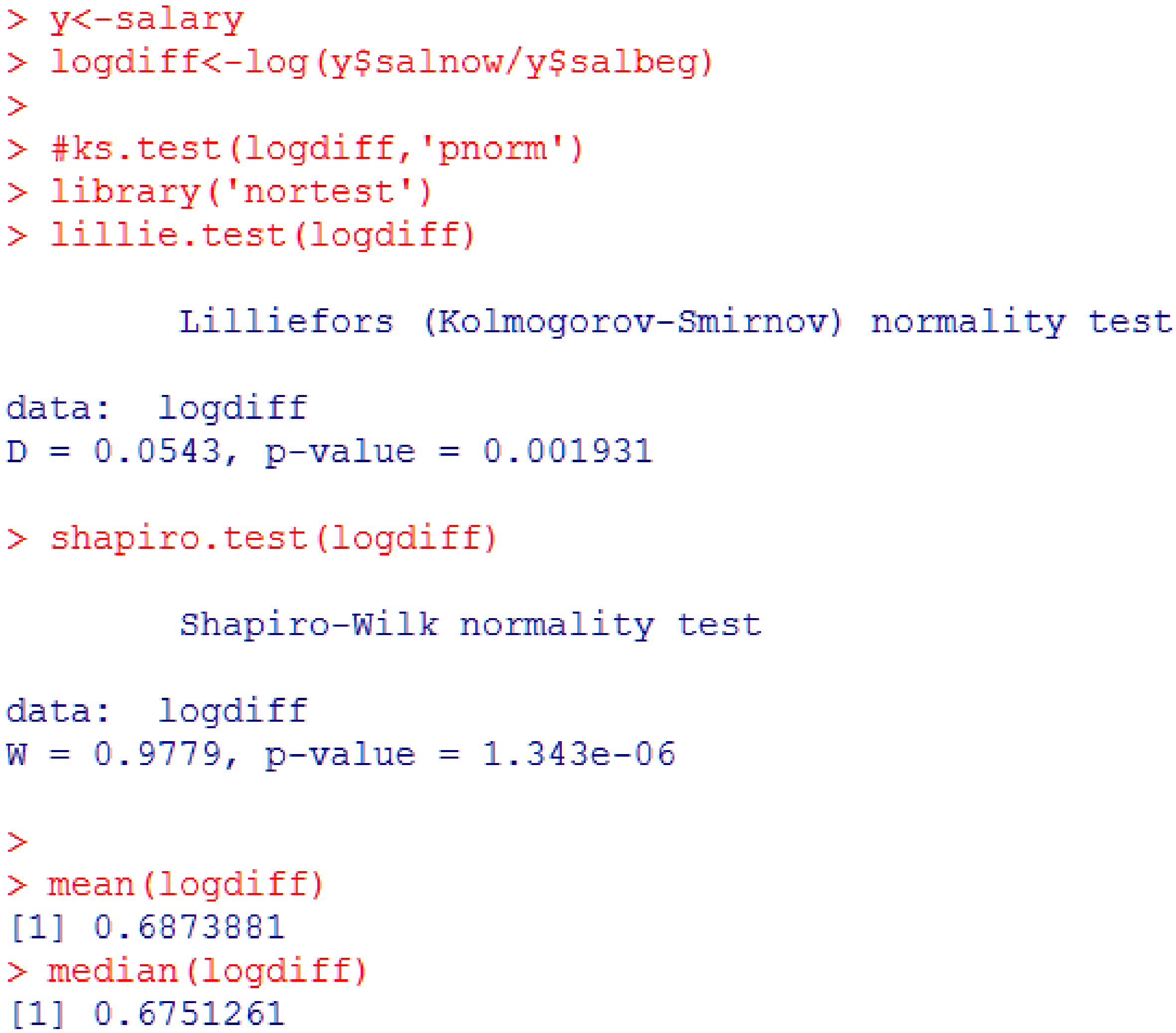
4.4. *Hypothesis tests for a single continuous variable*

Hypothesis test for the mean



```
> shapiro.test(x1)

        Shapiro-Wilk normality test

data:  x1
W = 0.9954, p-value = 0.9844

> library('nortest')
> lillie.test(x1)

        Lilliefors (Kolmogorov-Smirnov) normality test

data:  x1
D = 0.0495, p-value = 0.7915

> ks.test(x1, 'pnorm')

        One-sample Kolmogorov-Smirnov test

data:  x1
D = 0.1238, p-value = 0.09332
alternative hypothesis: two-sided
```

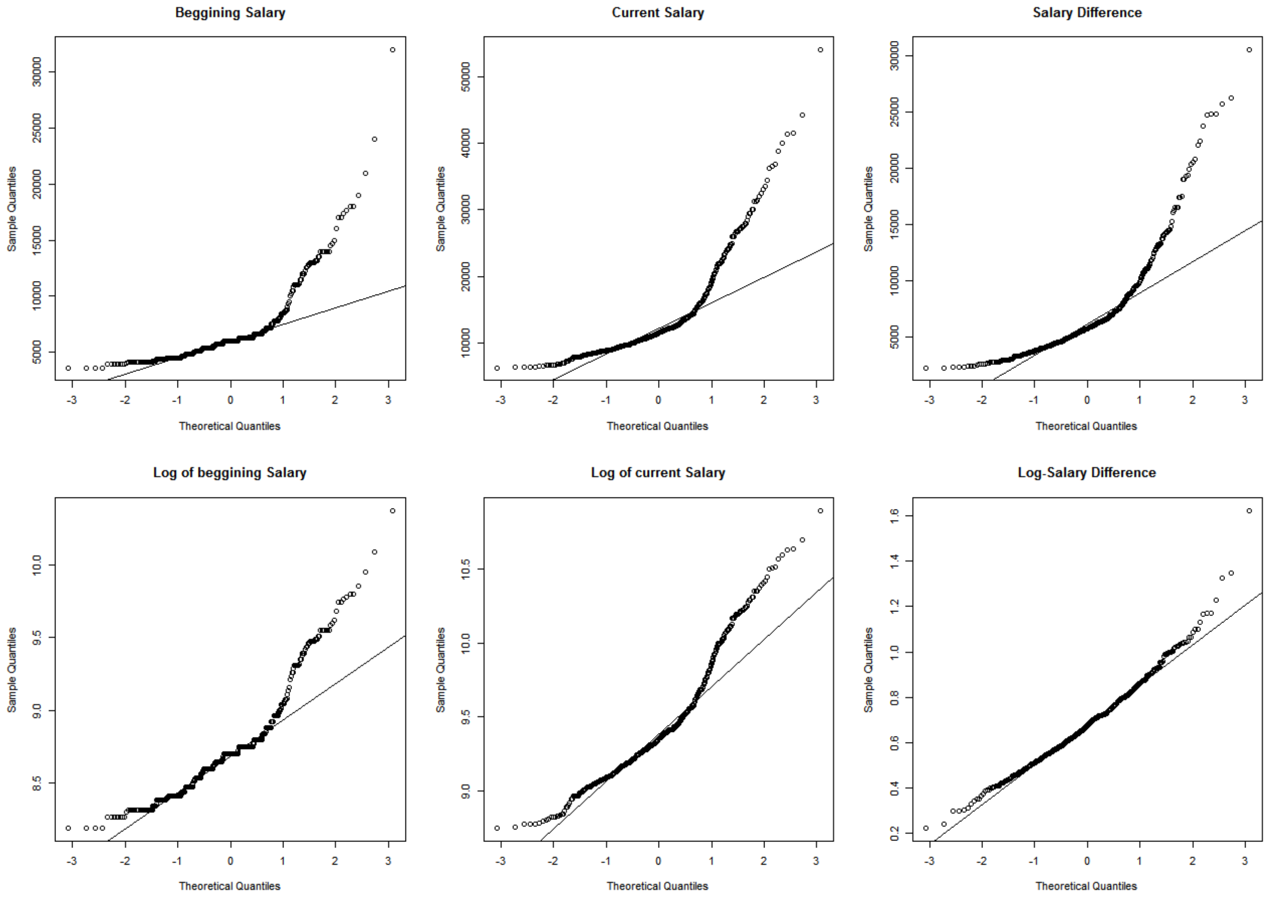
4.4. *Hypothesis tests for a single continuous variable*



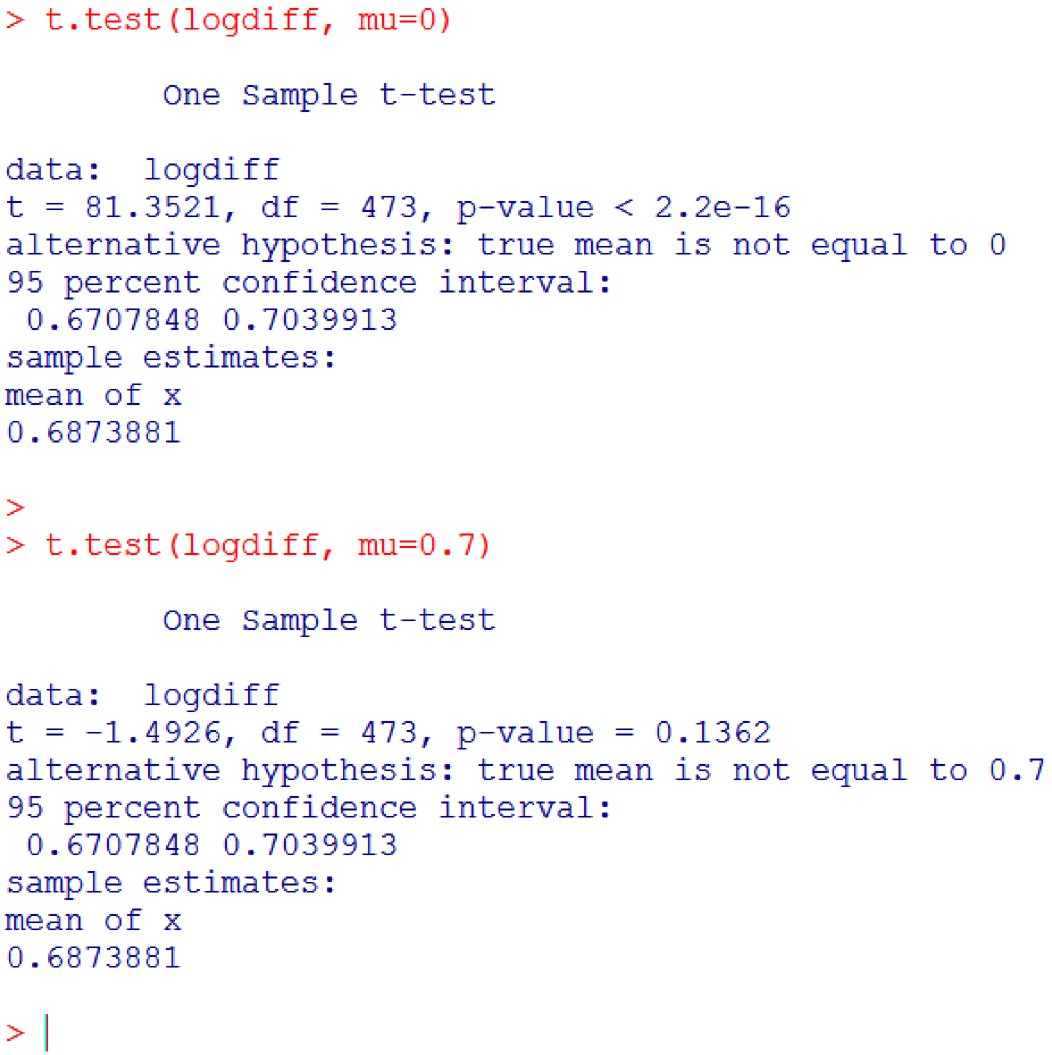
```
> y<-salary
> logdiff<-log(y$salnow/y$salbeg)
>
> #ks.test(logdiff,'pnorm')
> library('nortest')
> lillie.test(logdiff)

        Lilliefors (Kolmogorov-Smirnov) normality test

data:  logdiff
D = 0.0543, p-value = 0.001931

> shapiro.test(logdiff)

        Shapiro-Wilk normality test

data:  logdiff
W = 0.9779, p-value = 1.343e-06

>
> mean(logdiff)
[1] 0.6873881
> median(logdiff)
[1] 0.6751261
```

4.4. *Hypothesis tests for a single continuous variable*

Hypothesis test for the mean

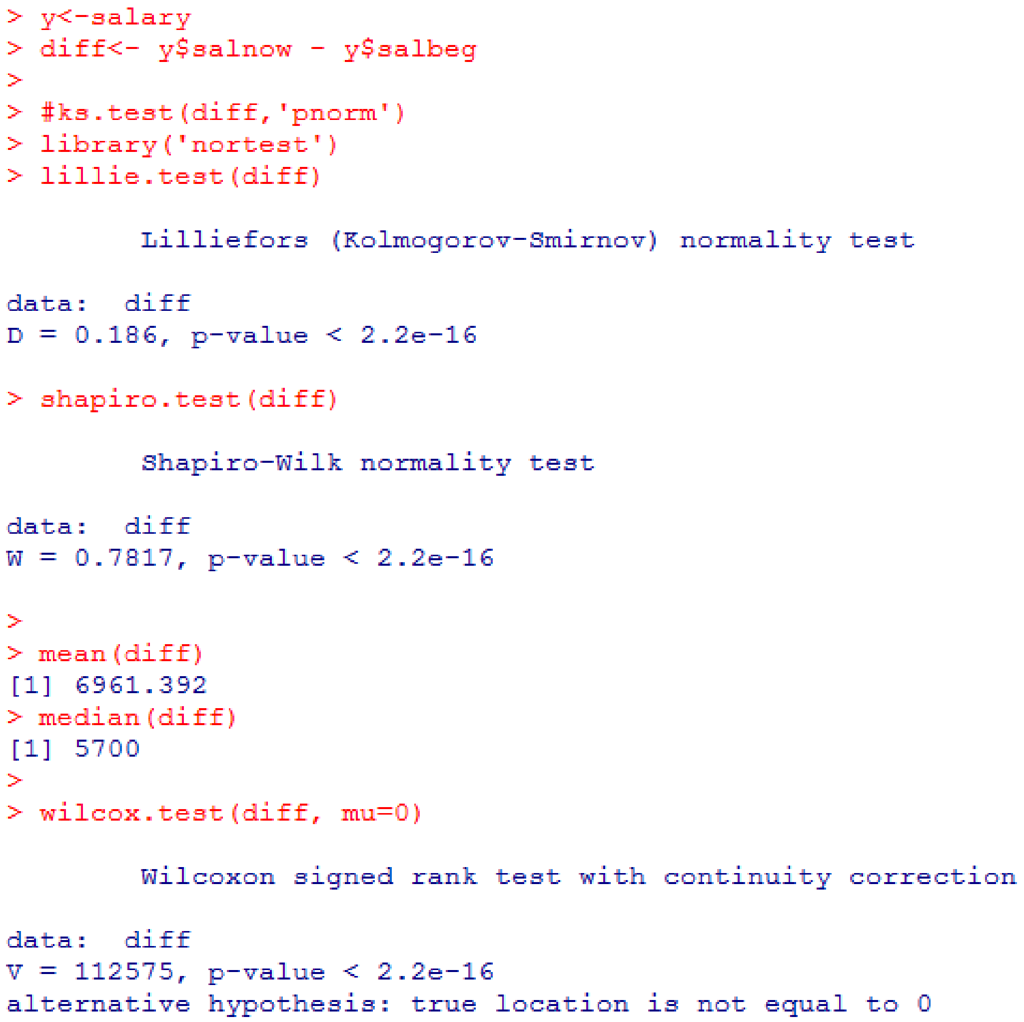


4.4. *Hypothesis tests for a single continuous variable*



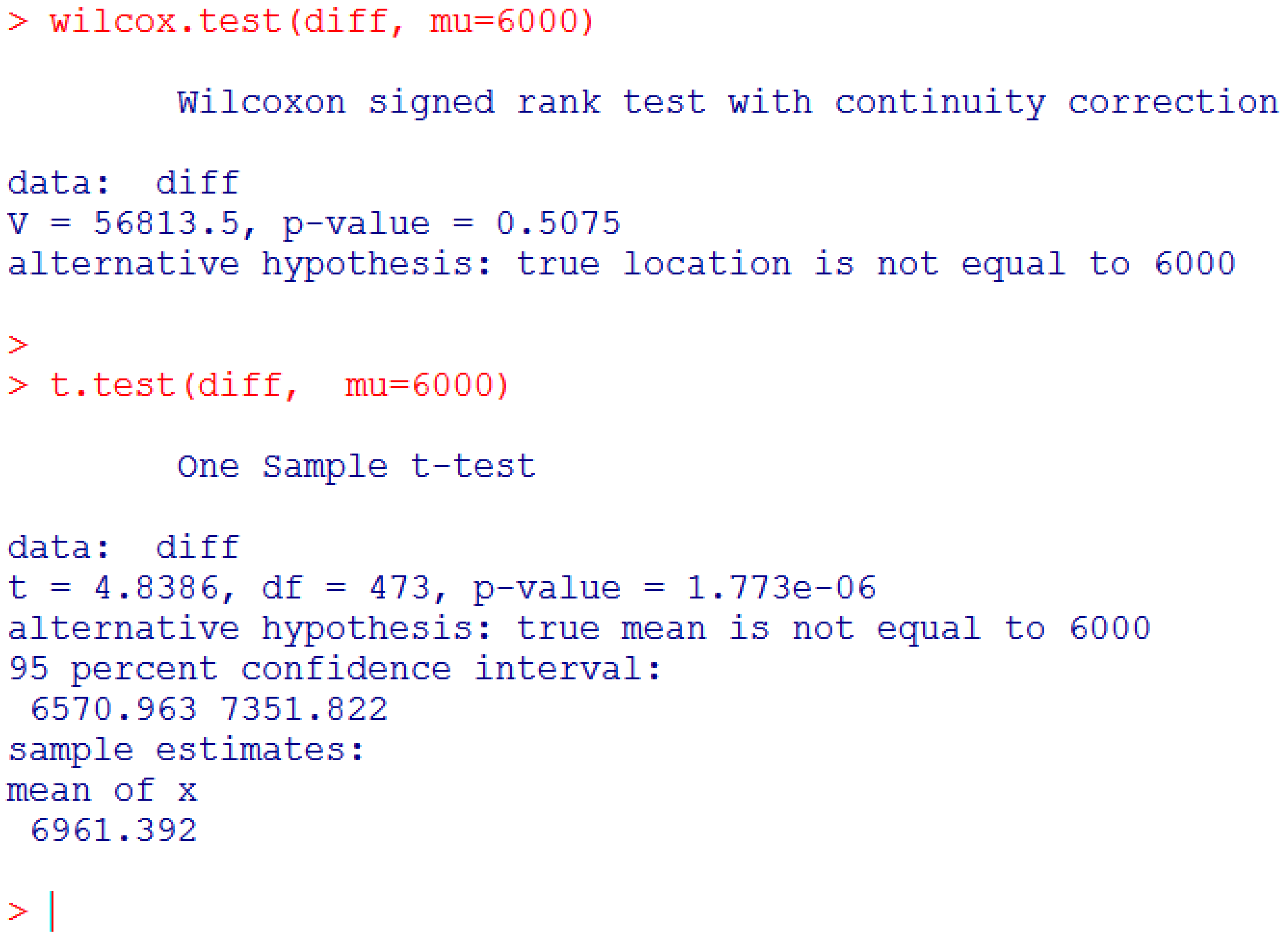
```
> t.test(logdiff, mu=0)

        One Sample t-test

data:  logdiff
t = 81.3521, df = 473, p-value < 2.2e-16
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 0.6707848 0.7039913
sample estimates:
mean of x
0.6873881

>
> t.test(logdiff, mu=0.7)

        One Sample t-test

data:  logdiff
t = -1.4926, df = 473, p-value = 0.1362
alternative hypothesis: true mean is not equal to 0.7
95 percent confidence interval:
 0.6707848 0.7039913
sample estimates:
mean of x
0.6873881

> 
```

4.4. *Hypothesis tests for a single continuous variable*



```
> y<-salary
> diff<- y$salnow - y$salbeg
>
> #ks.test(diff,'pnorm')
> library('nortest')
> lillie.test(diff)

        Lilliefors (Kolmogorov-Smirnov) normality test

data:  diff
D = 0.186, p-value < 2.2e-16

> shapiro.test(diff)

        Shapiro-Wilk normality test

data:  diff
W = 0.7817, p-value < 2.2e-16

>
> mean(diff)
[1] 6961.392
> median(diff)
[1] 5700
>
> wilcox.test(diff, mu=0)

        Wilcoxon signed rank test with continuity correction

data:  diff
V = 112575, p-value < 2.2e-16
alternative hypothesis: true location is not equal to 0
```

4.4. *Hypothesis tests for a single continuous variable*



```
> wilcox.test(diff, mu=6000)

        Wilcoxon signed rank test with continuity correction

data:  diff
V = 56813.5, p-value = 0.5075
alternative hypothesis: true location is not equal to 6000

>
> t.test(diff,  mu=6000)

        One Sample t-test

data:  diff
t = 4.8386, df = 473, p-value = 1.773e-06
alternative hypothesis: true mean is not equal to 6000
95 percent confidence interval:
 6570.963 7351.822
sample estimates:
mean of x
 6961.392

> |
```

4.5. *Hypothesis tests for two dependent samples*

- Examples
  - Grades in 2 different courses
  - Cholesterol measurements **before and after** a treatment
  - Sales before and after a marketing policy
  - Treatment effects in left and right hand of a patient
  - Behavior study of twins under different environmental conditions
- Examines the association between:
  - Grades + course
  - Cholesterol + treatment
  - Sales + marketing policy
  - Disease + treatment
  - Behavior + environmental conditions

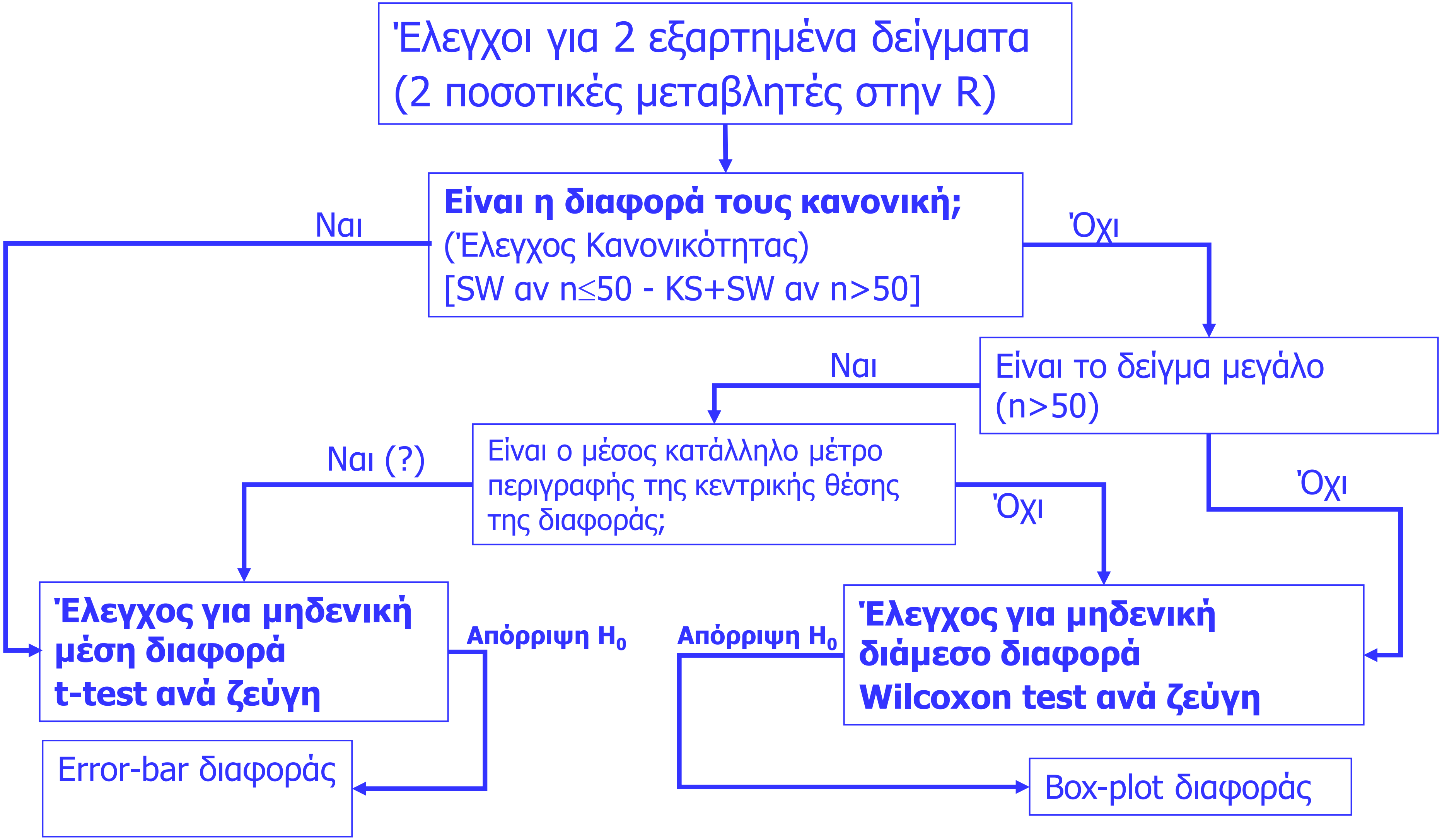
4.5. *Hypothesis tests for two dependent samples*

- We are interest to test for difference between the two dependent values – measurements
- We eliminate correlation by using the difference of each pair: $\Delta_i = X_{1i} - X_{2i}$
- We test if the mean of the difference is zero or not
- Hence we implement a one-sample t-test for the difference

  $H_0: \mu_\Delta = 0$ vs $H_1: \mu_\Delta \neq 0$.
- This is the paired t-test or the dependent samples t-test
- **ASSUMPTION**
  - The difference should follow the normal distribution or the sample size to be large (n>50)
- If normality assumption is rejected then we use the non-parametric test

  $H_0: M_\Delta = 0$ vs $H_1: M_\Delta \neq 0$

4.5. *Hypothesis tests for two dependent samples*



Έλεγχοι για 2 εξαρτημένα δείγματα (2 ποσοτικές μεταβλητές στην R)

Είναι η διαφορά τους κανονική;
(Έλεγχος Κανονικότητας)
[SW αν n≤50 - KS+SW αν n>50]

Ναι

Όχι

Είναι το δείγμα μεγάλο (n>50)

Ναι

Όχι

Είναι ο μέσος κατάλληλο μέτρο περιγραφής της κεντρικής θέσης της διαφοράς;

Ναι (?)

Όχι

Έλεγχος για μηδενική μέση διαφορά
t-test ανά ζεύγη

Απόρριψη H_0

Error-bar διαφοράς

Έλεγχος για μηδενική διάμεσο διαφορά
Wilcoxon test ανά ζεύγη

Απόρριψη H_0

Box-plot διαφοράς

4.6. *Hypothesis tests for two samples*

Testing for the association between a continuous and a categorical variable

Hypothesis test for the equality of the means between two independent samples/groups

- Two variables:
  - One continuous
  - One binary-categorical specifying the groups
- H_0: $\mu_1=\mu_2$ vs H_1: $\mu_1 \neq \mu_2$
- T-test for normal data with unknown variance
- **Assumption**: Normality or large sample for each group n>30 or >50
- Different degrees of freedom for equal and unequal variances

4.6. *Hypothesis tests for two samples*

Testing for the association between a continuous and a categorical variable

Hypothesis test for the equality of the medians between two independent samples/groups

- Two variables:
  - One continuous
  - One binary-categorical specifying the groups
- H_0: $M_1 = M_2$ vs H_1: $M_1 \neq M_2$
- Wilcoxon rank-sum test (or Mann-Whitney)
- No assumptions
- Can be used when the normality assumption is not valid or the mean cannot be used as representative of the "usual" observation

4.6. *Hypothesis tests for two samples*

Testing for the association between a continuous and a categorical variable

- Measurements of the **same variable** in **two groups** of **different** individuals/research units
- For example: 2 groups of randomly selected patients
  - 1st group receives placebo (virtual of fake treatment)
  - 2nd group receives new treatment
- Response: Cholesterol
- We study the association/relationship between a quantitative and a binary qualitative (treatment) variable
- In R data frame => 2 columns = 1 numeric (cholesterol) and a factor with two levels (treatment)

4.6. *Hypothesis tests for two samples*

Testing for the association between a continuous and a categorical variable

- Examples
    - Performance/Grades in the same course in two subsequent years
    - Cholesterol for two groups of patients receiving different treatment
    - Sales in two different groups of shops in which different business strategy was implemented
    - Study of the psychological scales of two groups of individuals tested under different conditions of stress.
- Examines the association between
    - Performance/Grades + year
    - Cholesterol + treatment
    - Sales + business strategy
    - Psychological state + stress conditions

4.6. *Hypothesis tests for two samples*

Testing for the association between a continuous and a categorical variable

- We are interested to test for differences on the values of the quantitative variable for the two groups (are the means of the medians equal?)
- Testing for the equality of means:

  $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$.
- This is the independent samples t-test
- In practice, it examines the relationship between the binary and the numeric variable since if the means on average are the same then the state of the binary does not influence the (conditional?) mean
- The hypothesis test can be re-written as

  $H_0: E(Y|X=1) = E(Y|X=2)$ vs $H_1: E(Y|X=1) \neq E(Y|X=2)$.

Y is the numeric variable, X is the factor

4.6. *Hypothesis tests for two samples*

Testing for the association between a continuous and a categorical variable

- We are interested to test for differences on the values of the quantitative variable for the two groups (are the means of the medians equal?)
- Testing for the equality of means:

  $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$.
- This is the independent samples t-test

BASIC ASSUMPTION:

  - Normality in each group or the size to be high (n_1, n_2>50)
- If the previous assumption is not valid then we use a non-parametric test (the WILCOXON SUM-RANKED TEST)

  $H_0: M_1 = M_2$ vs $H_1: M_1 \neq M_2$

4.6. *Hypothesis tests for two samples*

Testing for the association between a continuous and a categorical variable

Example 4-5 [dataset1.dat]

- The education manager of a company wants to evaluate the efficiency of two different teaching methods of the employees.
- For this reason, he selects randomly 24 employees and divides them in two groups of equal size.
- Every group is educated in a new software by a different teaching method
- 2 employees of the 2nd group did not completed the seminar
- In the end, all employees were tested in a common examination and graded with a scale from 0 to 100.

4.6. *Hypothesis tests for two samples*

Testing for the association between a continuous and a categorical variable

Example 4-5 [dataset1.dat]

Group A	70	93	82	90	77	86	79	84	98	73	81	85
Group B	89	78	94	83	88	80	91	92	87	97		

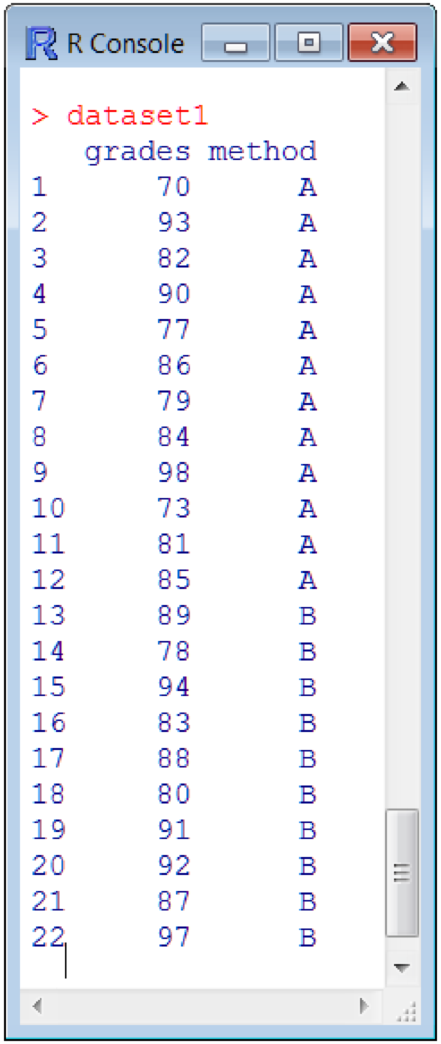
- **AIM**: To identify which method is more efficient in order to follow it in the next seminars.

4.6. *Hypothesis tests for two samples*

Testing for the association between a continuous and a categorical variable

Example 4-5 [dataset1.dat]

- Study unit:
  - employee
  - Sample size n=22 employees (rows)
- Characteristics – variables:
  - Performance – Grades (numeric) + teaching method (binary factor)
  - p=2 (columns)
- Size of each group
  - n_1=12
  - n_2=10



```
> dataset1
   grades method
1      70      A
2      93      A
3      82      A
4      90      A
5      77      A
6      86      A
7      79      A
8      84      A
9      98      A
10     73      A
11     81      A
12     85      A
13     89      B
14     78      B
15     94      B
16     83      B
17     88      B
18     80      B
19     91      B
20     92      B
21     87      B
22     97      B
```

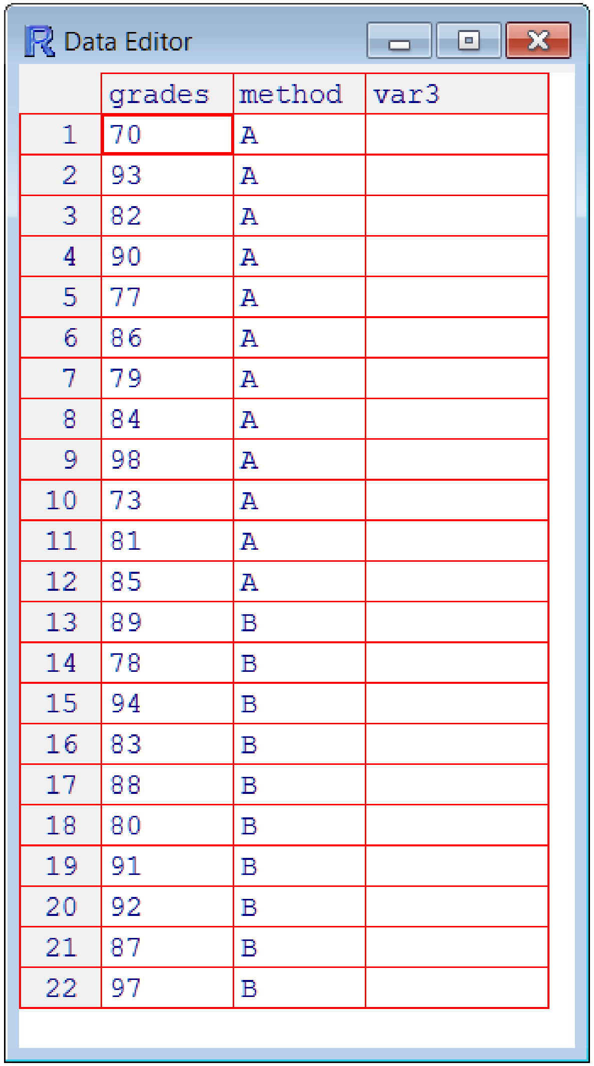
4.6. *Hypothesis tests for two samples*

Testing for the association between a continuous and a categorical variable

Example 4-5 [dataset1.dat]

- Study unit:
  - employee
  - Sample size n=22 employees (rows)
- Characteristics – variables:
  - Performance – Grades (numeric) + teaching method (binary factor)
  - p=2 (columns)
- Size of each group
  - n_1=12
  - n_2=10

```
dataset1 <- edit(dataset1)
```

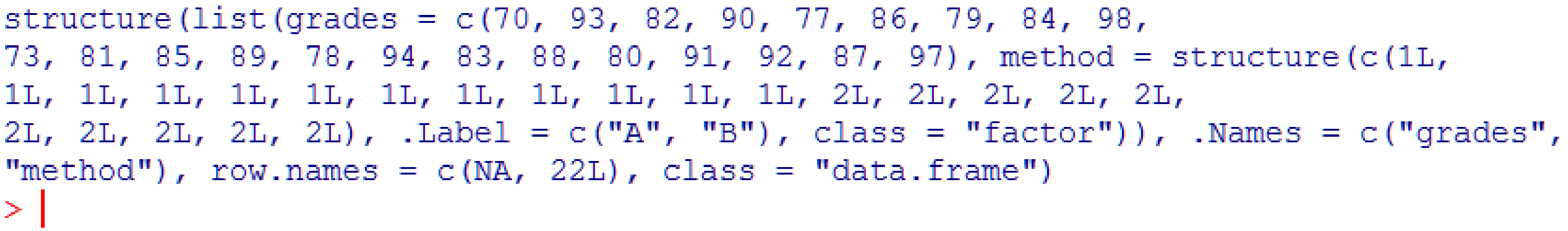


	grades	method	var3
1	70	A	
2	93	A	
3	82	A	
4	90	A	
5	77	A	
6	86	A	
7	79	A	
8	84	A	
9	98	A	
10	73	A	
11	81	A	
12	85	A	
13	89	B	
14	78	B	
15	94	B	
16	83	B	
17	88	B	
18	80	B	
19	91	B	
20	92	B	
21	87	B	
22	97	B	

4.6. *Hypothesis tests for two samples*

Testing for the association between a continuous and a categorical variable

```
groupA <- c(70 ,93 ,82 ,90 ,77 ,86 ,79 ,84 ,98 ,73 ,81 ,85)
groupB <-c(89 ,78 ,94 ,83 ,88 ,80 ,91 ,92 ,87 ,97)
n1<-length(groupA)
n2<-length(groupB)

dataset1 <- data.frame( grades=c(groupA, groupB),
          method=factor( rep(1:2, c(n1,n2)), labels=c('A','B') ) )

dataset1 <- edit(dataset1)

dput(dataset1, 'dataset1.dat')
dput(dataset1)
```



```
structure(list(grades = c(70, 93, 82, 90, 77, 86, 79, 84, 98,
73, 81, 85, 89, 78, 94, 83, 88, 80, 91, 92, 87, 97), method = structure(c(1L,
1L, 1L, 1L, 1L, 1L, 1L, 1L, 1L, 1L, 1L, 1L, 2L, 2L, 2L, 2L, 2L,
2L, 2L, 2L, 2L, 2L), .Label = c("A", "B"), class = "factor")), .Names = c("grades",
"method"), row.names = c(NA, 22L), class = "data.frame")
> |
```

4.6. *Hypothesis tests for two samples*

Testing for the association between a continuous and a categorical variable

Steps for implementing hypothesis tests in two independent samples

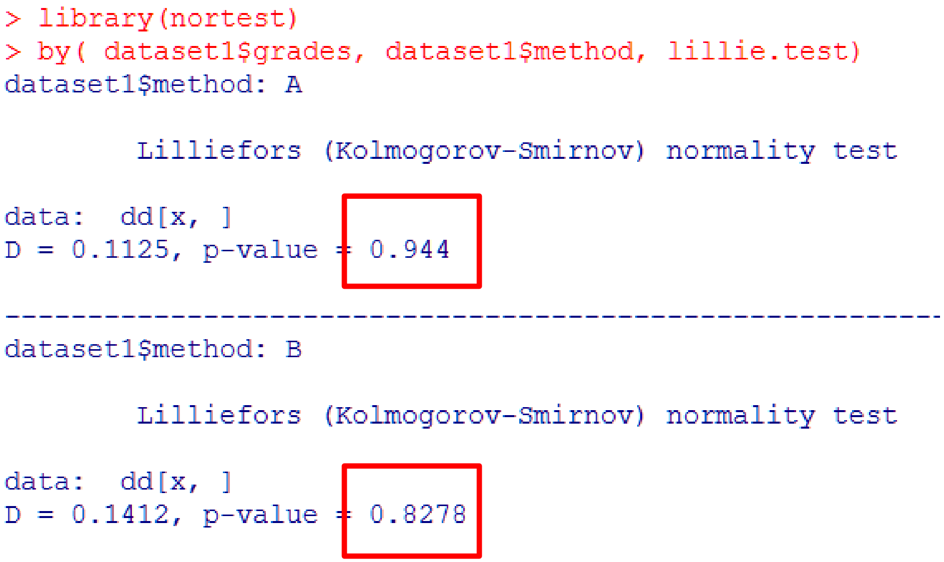
1. We test for normality in each group
2. We implement the appropriate test
   a) If normality is not rejected
      i. We test for the equality of variances (homoscedasticity)
      ii. We select the appropriate t-test (with equal or unequal variances)
      iii. If there are differences we visualize them using ERROR BARS
   b) If normality is rejected
      i. We implement the WILCOXON rank-sum test
      ii. If differences are traced then we visualize them using BOX PLOTS

4.6. *Hypothesis tests for two samples*

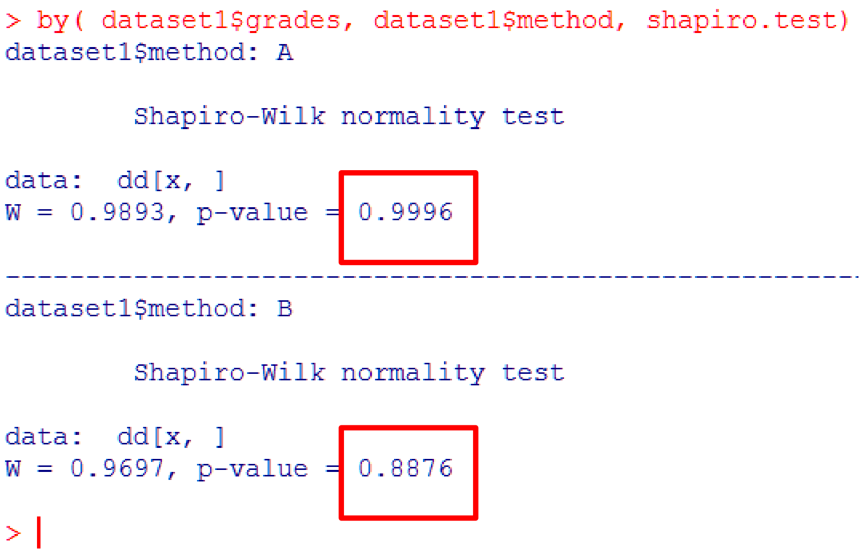
Testing for the association between a continuous and a categorical variable

Example 4-5 [dataset1.dat]

1. We test for the normality in each group



```
> library(nortest)
> by( dataset1$grades, dataset1$method, lillie.test)
dataset1$method: A

        Lilliefors (Kolmogorov-Smirnov) normality test

data:   dd[x, ]
D = 0.1125, p-value = 0.944

---------------------------------------------------------
dataset1$method: B

        Lilliefors (Kolmogorov-Smirnov) normality test

data:   dd[x, ]
D = 0.1412, p-value = 0.8278
```



```
> by( dataset1$grades, dataset1$method, shapiro.test)
dataset1$method: A

        Shapiro-Wilk normality test

data:   dd[x, ]
W = 0.9893, p-value = 0.9996

---------------------------------------------------
dataset1$method: B

        Shapiro-Wilk normality test

data:   dd[x, ]
W = 0.9697, p-value = 0.8876

> |
```

All p-values>0.5 => we do not reject normality in either of the two groups

4.6. *Hypothesis tests for two samples*

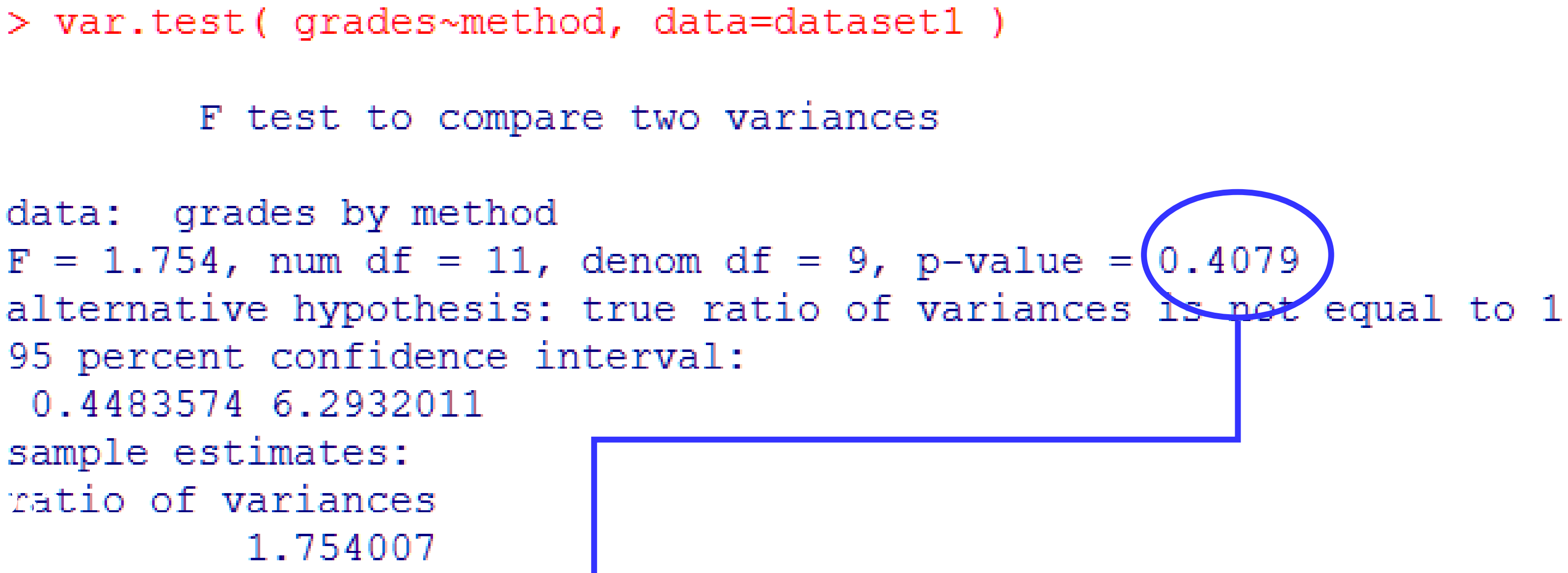
Testing for the association between a continuous and a categorical variable

Example 4-5 [dataset1.dat]

var.test(xA, xB)
var.test(x[[1]], x[[2]])
var.test(grades~method, data=dataset1)

2. α) independent samples t-test

i. Checking for the equality of variances



```
> var.test( grades~method, data=dataset1 )

        F test to compare two variances

data:  grades by method
F = 1.754, num df = 11, denom df = 9, p-value = 0.4079
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 0.4483574 6.2932011
sample estimates:
ratio of variances
          1.754007
```

P-value=0.41>0.05 => we do not reject H_0 =>
we can assume equal variances

4.6. *Hypothesis tests for two samples*

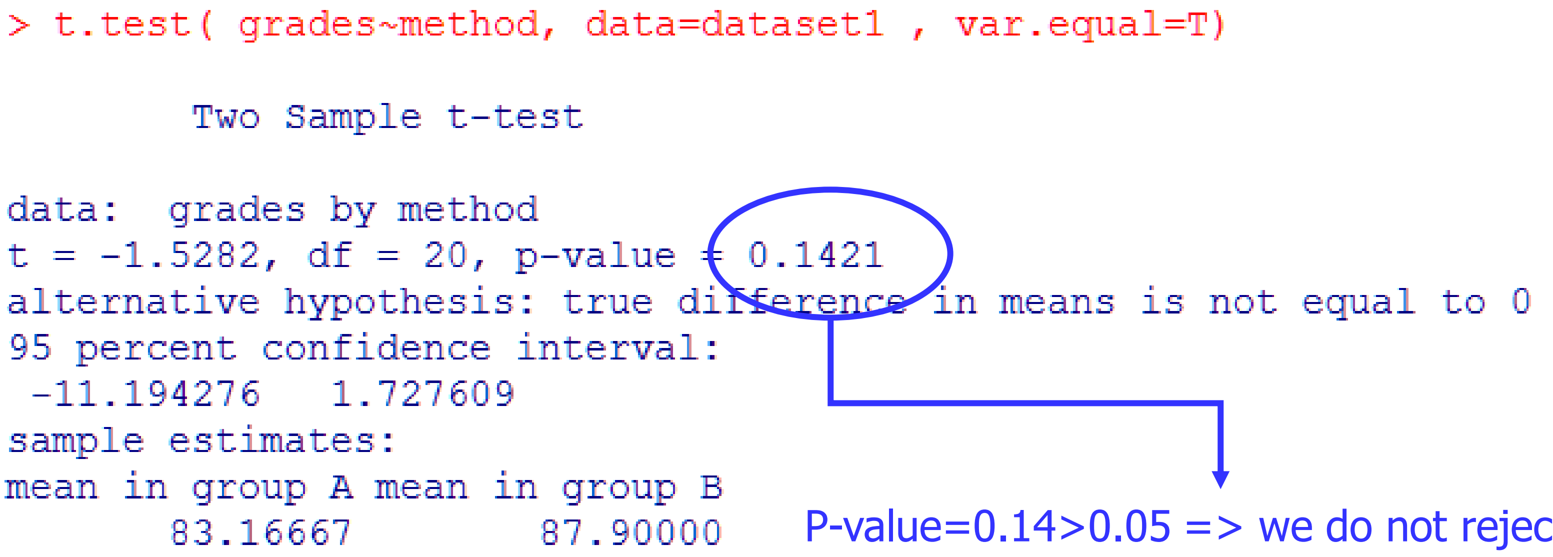
Testing for the association between a continuous and a categorical variable

```
t.test(x$A, x$B, var.equal=T)
t.test(x[[1]], x[[2]], var.equal=T)
t.test( grades~method, data=dataset1 ,
        var.equal=T)
```

Example 4-5 [dataset1.dat]

2. α) independent samples t-test

ii. Using the t-test for equal variances



```
> t.test( grades~method, data=dataset1 , var.equal=T)

        Two Sample t-test

data:  grades by method
t = -1.5282, df = 20, p-value = 0.1421
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -11.194276   1.727609
sample estimates:
mean in group A mean in group B
       83.16667        87.90000
```

P-value=0.14>0.05 => we do not reject the H_0 => no differences between the performance of the employees with different training

4.6. *Hypothesis tests for two samples*

Testing for the association between a continuous and a categorical variable

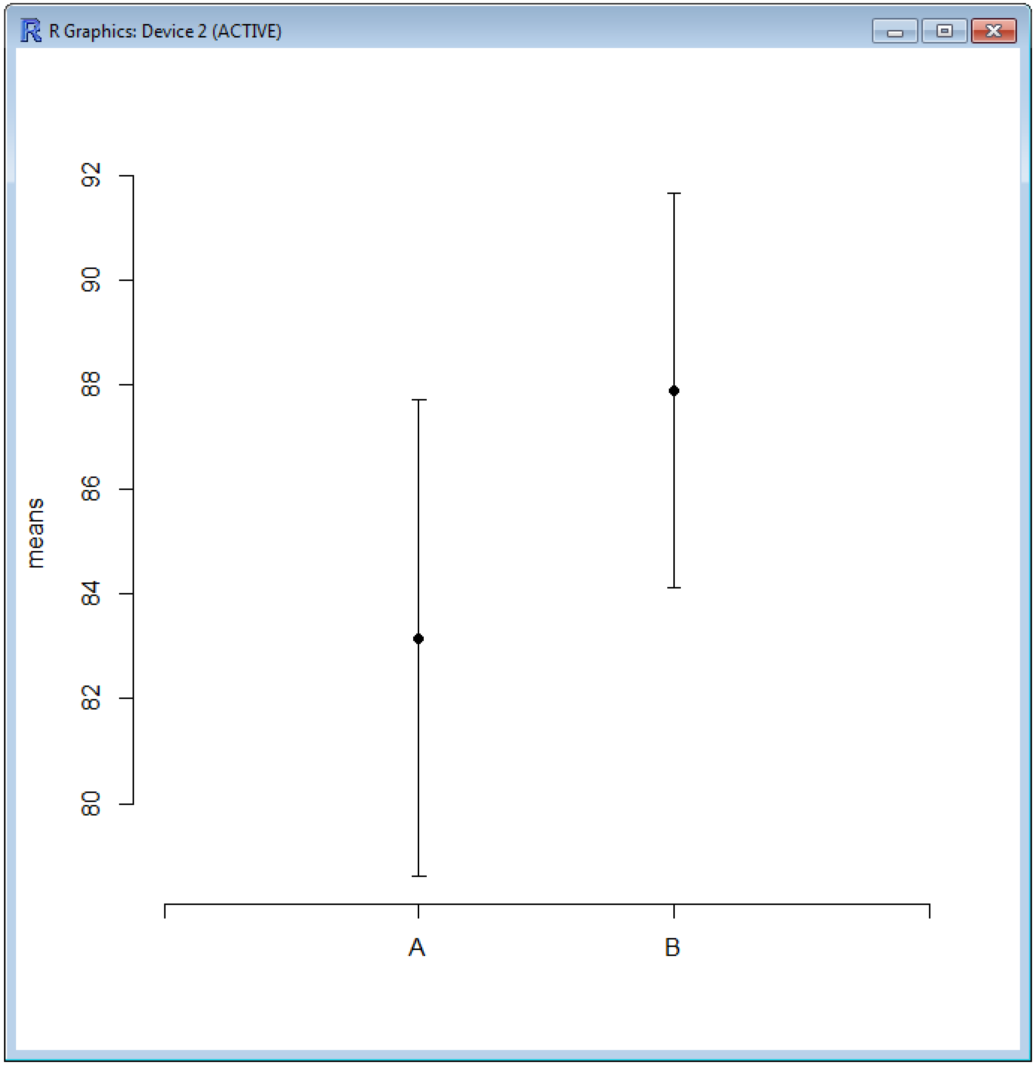
Example 4-5

[dataset1.dat]

2. α) Independent samples t-test

iii. Visualization using error bars

```
library(Hmisc)
myerrorbar( dataset1$grades,
            dataset1$method )
```



4.6. *Hypothesis tests for two samples*

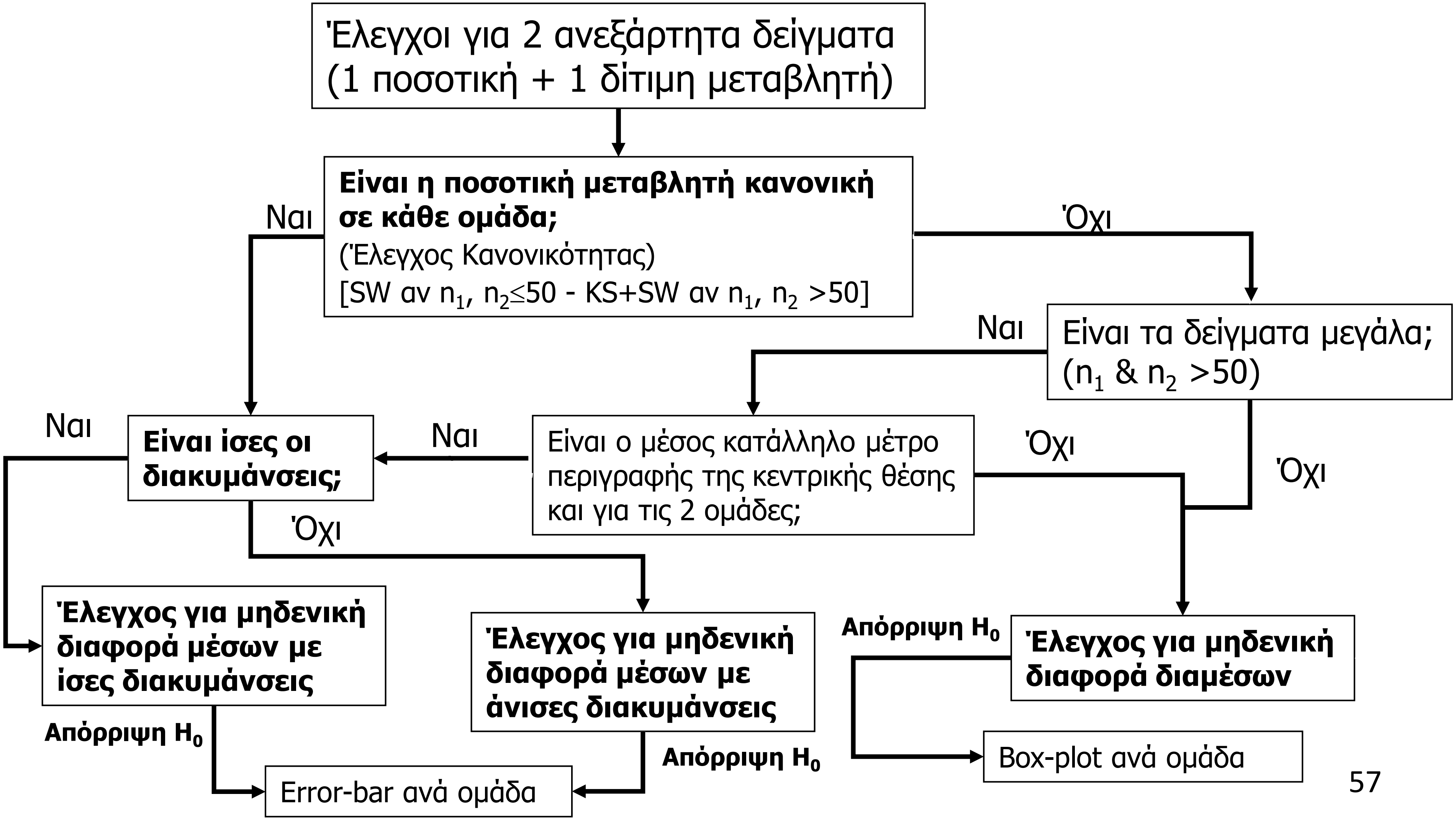
Testing for the association between a continuous and a categorical variable

```
myerrorbar<-function(x,y, horizontal=F){
        a<-0.05
        sdata <- split(x,y)
        means <- sapply( sdata,mean )
        sds <- sapply( split(x,y), sd )
        ns <- table(y)
        LB <- means + qnorm( a/2 ) * sds /sqrt(ns)
        UB <- means + qnorm( 1-a/2 ) * sds /sqrt(ns)
        nlev <- nlevels(y)
        if (horizontal) { errbar( levels(y), means, UB, LB )
                          } else {
                    errbar( 1:nlev, means, UB, LB,
                                  xlim=c(0,nlev+1), axes=F, xlab=" )
                    axis(2)
                    axis(1, at=0:(nlev+1), labels=c(",levels(y),"))
                    }
}
```

4.6. *Hypothesis tests for two samples*

Testing for the association between a continuous and a categorical variable

ΟΠΑ
AUEB



Έλεγχοι για 2 ανεξάρτητα δείγματα (1 ποσοτική + 1 δίτιμη μεταβλητή)

Είναι η ποσοτική μεταβλητή κανονική σε κάθε ομάδα;
(Έλεγχος Κανονικότητας)
[SW αν n_1, $n_2 \leq 50$ - KS+SW αν n_1, $n_2 > 50$]

Ναι

Όχι

Είναι τα δείγματα μεγάλα; (n_1 & n_2 >50)

Ναι

Όχι

Είναι ο μέσος κατάλληλο μέτρο περιγραφής της κεντρικής θέσης και για τις 2 ομάδες;

Ναι

Όχι

Είναι ίσες οι διακυμάνσεις;

Ναι

Όχι

Έλεγχος για μηδενική διαφορά μέσων με ίσες διακυμάνσεις

Απόρριψη H_0

Error-bar ανά ομάδα

Έλεγχος για μηδενική διαφορά μέσων με άνισες διακυμάνσεις

Απόρριψη H_0

Έλεγχος για μηδενική διαφορά διαμέσων

Απόρριψη H_0

Box-plot ανά ομάδα

4.7. *Hypothesis tests for multiple samples*

Testing for the association between a continuous and a categorical variable

- Introduction to analysis of variance (ANOVA)
- Assumptions
- Multiple comparisons
- Non parametric hypothesis test
- The link between t-test and ANOVA

4.7. *Hypothesis tests for multiple samples*

4.7.1. Analysis of variance

- Let us assume that we have measurement of the **same quantitative variable** in **κ(>2) groups of different individuals**
- For example 3 groups of patients are randomly selected
  - 1η group with virtual/placebo treatment
  - 2η group receives a standard treatment
  - 3η group received a new treatment
- We examine for a possible relationship between a quantitative and a categorical variable
- In R data frame => 2 columns = 1 numeric and a factor with 3 levels (or k levels in a more general setup)

4.7. *Hypothesis tests for multiple samples*

Testing for the association between a continuous and a categorical variable

- Introduction to analysis of variance (ANOVA)
- Assumptions
- Multiple comparisons
- Non parametric hypothesis test
- The link between t-test and ANOVA

4.7. *Hypothesis tests for multiple samples*

4.7.1. Analysis of variance

- Let us assume that we have measurement of the **same quantitative variable** in **κ(>2) groups of different individuals**
- For example 3 groups of patients are randomly selected
  - 1η group with virtual/placebo treatment
  - 2η group receives a standard treatment
  - 3η group received a new treatment
- We examine for a possible relationship between a quantitative and a categorical variable
- In R data frame => 2 columns = 1 numeric and a factor with 3 levels (or k levels in a more general setup)

4.7. *Hypothesis tests for multiple samples*

4.7.1. Analysis of variance

- Examples
    - Grades/student performance in 4 different academic years
    - Sales in 3 different groups of sale points with different business strategies
- Examines the association
    - Grades/performance + year
    - Sales + strategy

4.7. *Hypothesis tests for multiple samples*

4.7.1. Analysis of variance

- We are interested to test for the differences between the quantitative variable and the groups i.e. are the means or the medians equal across groups
- We test for

  H_0: $\mu_1 = \mu_2 = \ldots = \mu_\kappa$ vs
  H_1: $\mu_k \neq \mu_j$ for some $k \neq j \in \{1,2, \ldots K\}$.
- This hypothesis test is called analysis of variance (ANOVA: Analysis of Variance)
- **ASSUMPTIONS**:
  - ✓ Residuals' normality or the sample size to be large (n>50)
  - ✓ Equal variances
- If the above hypotheses are rejected then we may use a non-parametric method (KRUSKAL-WALLIS TEST)

  H_0: $M_1 = M_2 = \ldots = M_K$ vs
  H_1: $M_i \neq M_j$ for some $i \neq j = 1,2, \ldots \kappa$.

4.7. *Hypothesis tests for multiple samples*

4.7.1. Analysis of variance

- Let us assume Y quantitative variable and X categorical with κ levels.
- n_j : is the sample size of group j, j=1,2,…,κ
- $n = \sum_{j=1}^{\kappa} n_j$: total sample size
- Y_{ij} is the i-th observation of group j
- $\overline{Y}_j$ is the sample mean of Y for group j
- $e_{ij} = Y_{ij} - \overline{Y}_j$ is the residual values of i observation of group j

4.7. *Hypothesis tests for multiple samples*

4.7.1. Analysis of variance

Sum of squares

- $TSS = \sum_{j=1}^{\kappa} \sum_{\iota=1}^{n_j} (Y_{ij} - \bar{Y})^2$ is the total sum of squares (variance of Y?)
- $RSS = \sum_{j=1}^{\kappa} \sum_{\iota=1}^{n_j} (Y_{ij} - \bar{Y}_j)^2$ it the residual sum of squares and expresses the variability within groups [within groups sum of squares]
- $BSS = TSS - RSS = \sum_{j=1}^{\kappa} n_j (\bar{Y}_j - \bar{Y})^2$ is the between groups sum of squares measuring the variance of the samples means of different groups

– We use the test Statistic:

$$F^* = \frac{BSS/(\kappa-1)}{RSS/(n-\kappa)} \sim F_{\kappa-1,n-\kappa}$$

4.7. *Hypothesis tests for multiple samples*

4.7.1. Analysis of variance

- So we compare variances for testing for the equality of means. WHY???

- **BE CAREFULL**: Rejection of H_0 implies only that some mean differ. The test does not provide any information concerning which means differ.
  - We implement multiple pairwise comparisons
  - We visualize differences using error-bars

4.7. *Hypothesis tests for multiple samples*

4.7.1. Analysis of variance

Example 4-7:

- Let us reconsider example 4-6 with the different training methods.
- Now we consider 15 employees divided in 3 different groups of training
- The data are given in the following table:

Training method	**Grade/performance**				
A	86	79	81	70	84
B	90	76	88	82	89
C	82	68	73	71	81

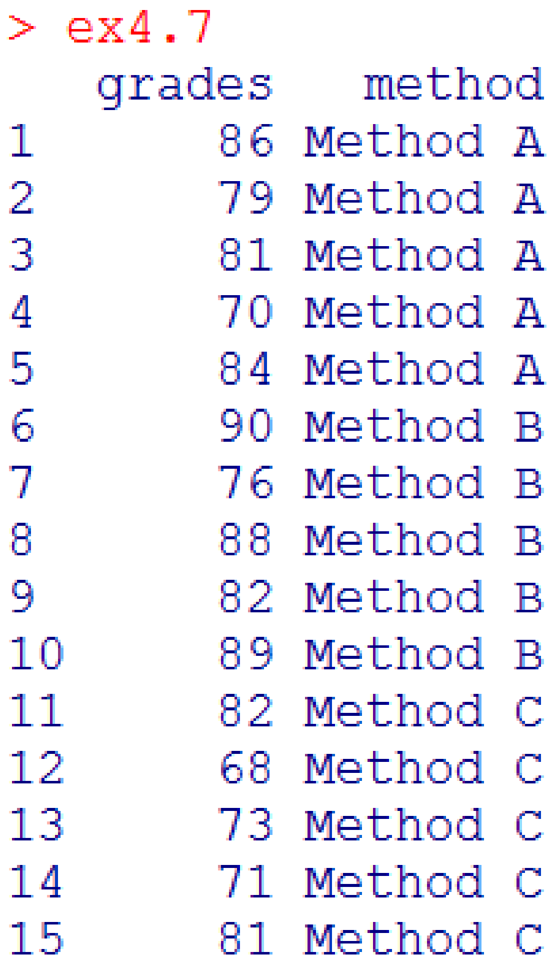
4.7. *Hypothesis tests for multiple samples*

4.7.1. Analysis of variance

Example 4-7:

- Unit of study: the employee
- Sample size: n=15
- Characteristics p=2
  - Training method
  - Grade/performance

```
grades<-c(86,79,81,70,84,90,76,88,82,89,82,
          68,73, 71,81)
method<-rep(1:3,rep(5,3))
method<-factor(method, labels=paste('Method',
                                   LETTERS[1:3]) )

ex4.7<-data.frame( grades=grades,
          method=method )
rm(grades)
rm(method)
```



```
> ex4.7
   grades   method
1      86 Method A
2      79 Method A
3      81 Method A
4      70 Method A
5      84 Method A
6      90 Method B
7      76 Method B
8      88 Method B
9      82 Method B
10     89 Method B
11     82 Method C
12     68 Method C
13     73 Method C
14     71 Method C
15     81 Method C
```

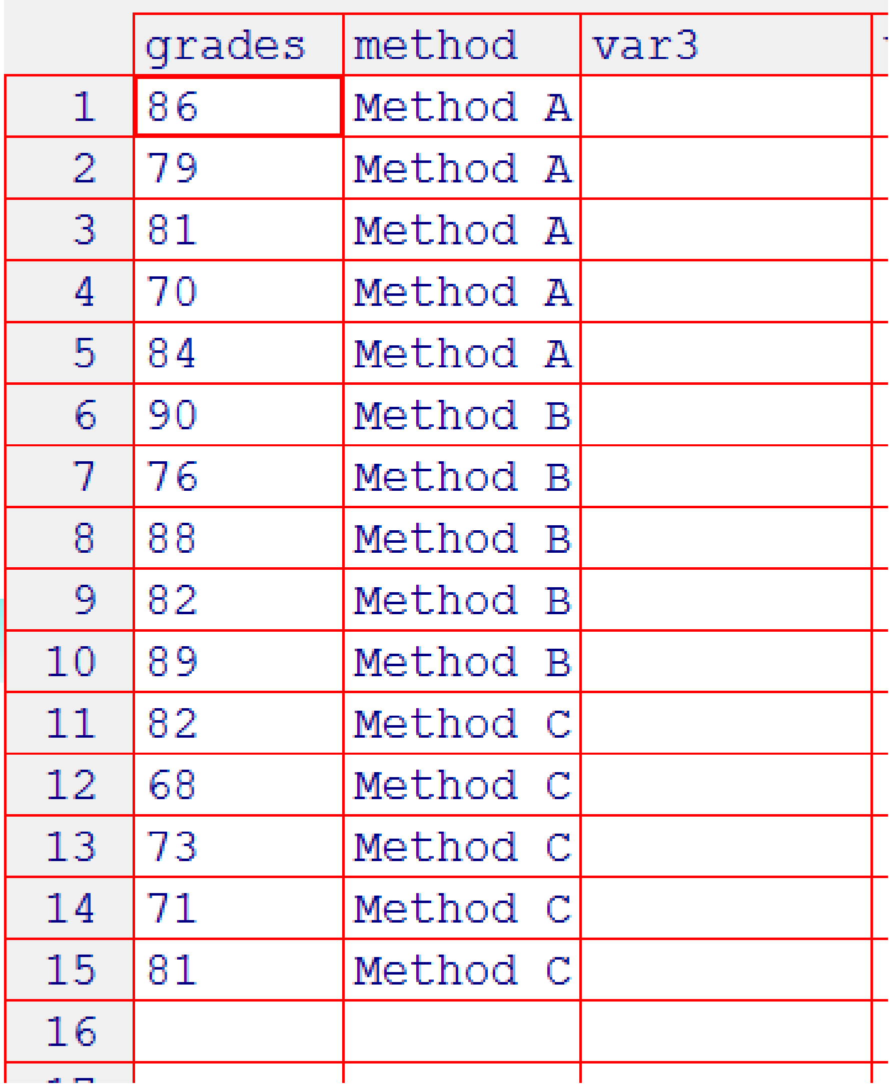
4.7. *Hypothesis tests for multiple samples*

4.7.1. Analysis of variance

Example 4-7:

- Unit of study: the employee
- Sample size: n=15
- Characteristics p=2
  - Training method
  - Grade/performance

```
ex4.7<-edit(ex4.7)
```

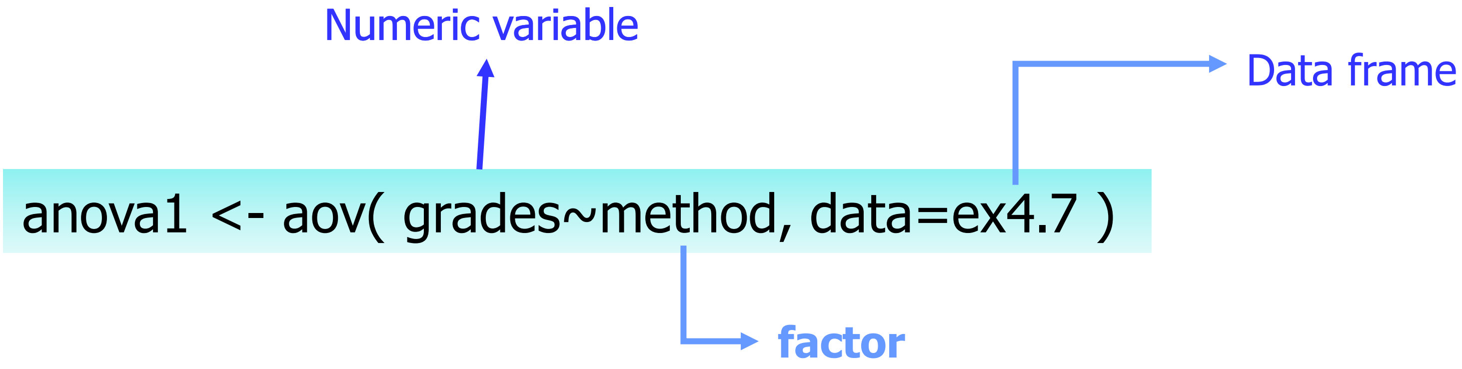


	grades	method	var3
1	86	Method A	
2	79	Method A	
3	81	Method A	
4	70	Method A	
5	84	Method A	
6	90	Method B	
7	76	Method B	
8	88	Method B	
9	82	Method B	
10	89	Method B	
11	82	Method C	
12	68	Method C	
13	73	Method C	
14	71	Method C	
15	81	Method C	
16			

4.7. *Hypothesis tests for multiple samples*

4.7.1. Analysis of variance

Example 4-7: ANOVA using aov function in R



```
anova1 <- aov( grades~method, data=ex4.7 )
```

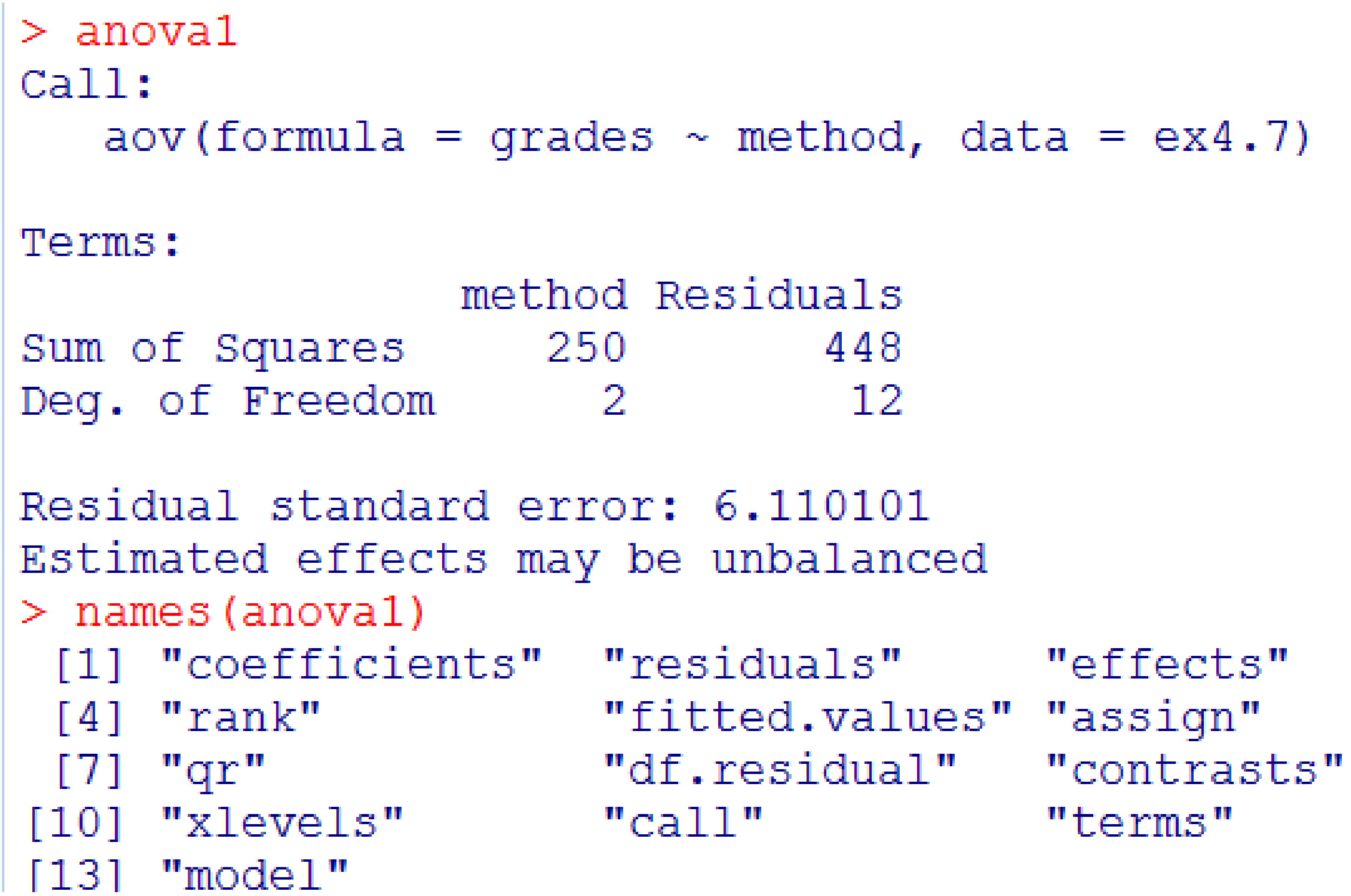
also see

```
anova2 <- oneway.test( grades~method, data=ex4.7 )
```

4.7. *Hypothesis tests for multiple samples*

4.7.1. Analysis of variance

Example 4-7: ANOVA RESULTS IN R



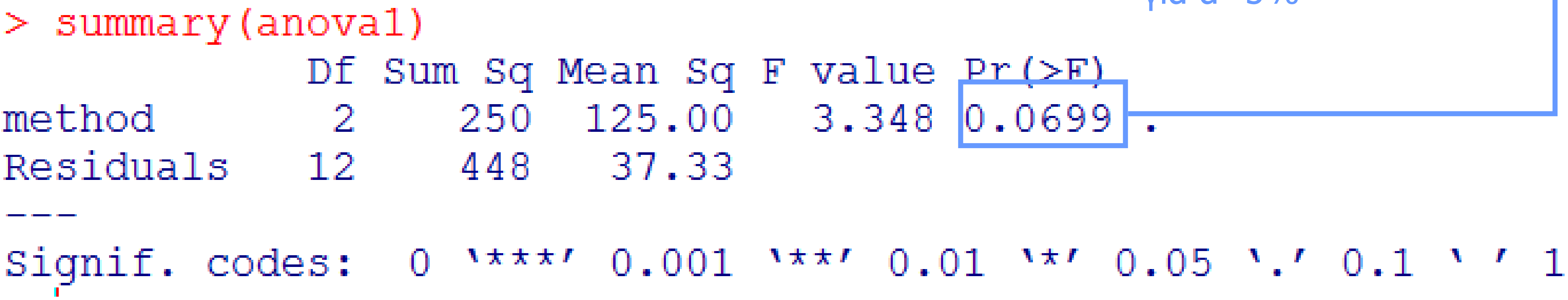
```
> anova1
Call:
   aov(formula = grades ~ method, data = ex4.7)

Terms:
                method Residuals
Sum of Squares     250       448
Deg. of Freedom      2        12

Residual standard error: 6.110101
Estimated effects may be unbalanced
> names(anova1)
 [1] "coefficients"  "residuals"     "effects"
 [4] "rank"          "fitted.values" "assign"
 [7] "qr"            "df.residual"   "contrasts"
[10] "xlevels"       "call"          "terms"
[13] "model"
```

4.7. *Hypothesis tests for multiple samples*

4.7.1. Analysis of variance

Example 4-7: Anova table using the summary function in R

Δεν απορρίπτεται η H_0
για α=5%

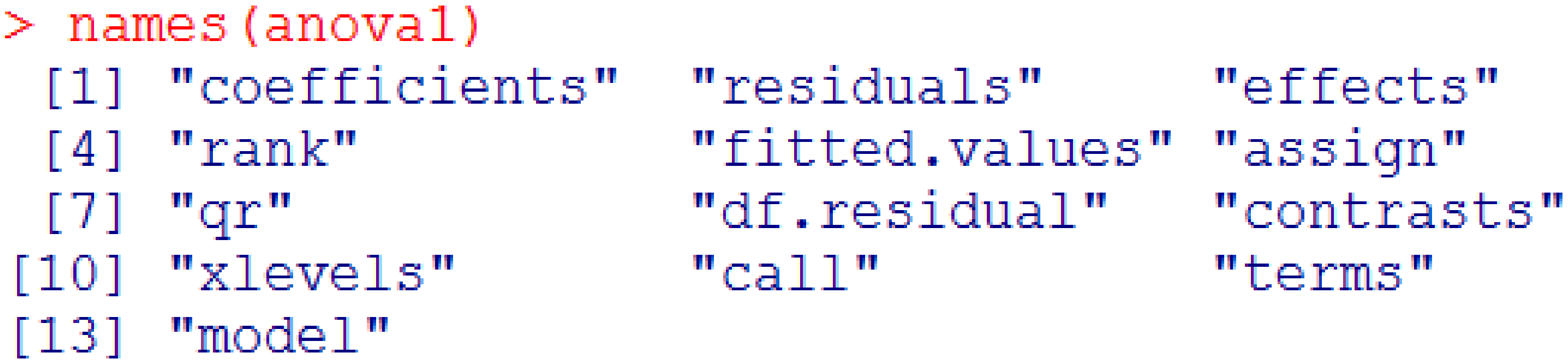


```
> summary(anova1)
            Df Sum Sq Mean Sq F value Pr(>F)
method       2    250  125.00   3.348 0.0699 .
Residuals   12    448   37.33
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

4.7. *Hypothesis tests for multiple samples*

4.7.1. Analysis of variance

Example 4-7: ANOVA RESULTS IN R



```
> names(anova1)
 [1] "coefficients"  "residuals"      "effects"
 [4] "rank"          "fitted.values"  "assign"
 [7] "qr"            "df.residual"    "contrasts"
[10] "xlevels"       "call"           "terms"
[13] "model"
```

4.7. *Hypothesis tests for multiple samples*

4.7.1. Analysis of variance

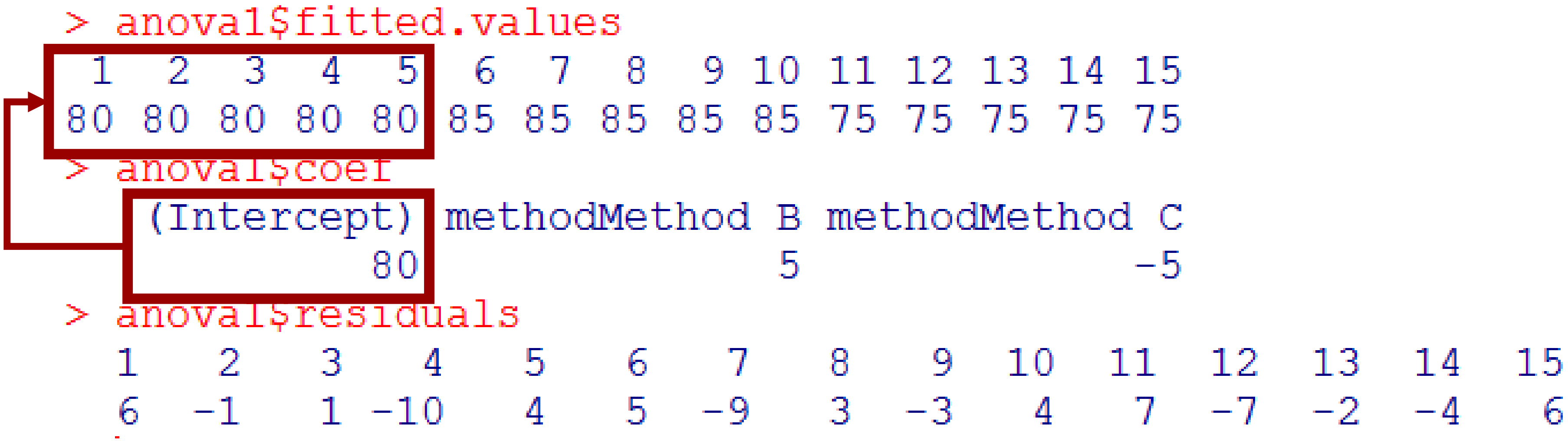
Example 4-7: ANOVA RESULTS IN R

coefficients	model coefficients
fitted.values	the fitted mean values. in one-way anova fitted values = the group mean
residuals	the residuals: observed value – fitted value
df.residual	the residual degrees of freedom.
call	the exact expression used to generate this object
contrasts	The parametrization (constraints) used
xlevels	the levels of the factors used
y	the response used (only if y=TRUE)
x	the model matrix used (only if x=TRUE)
model	the model frame used (only if model=TRUE – default value)
rank	rank of data/design matrix X => i.e. the number of parameters => in one-way anova the number of groups

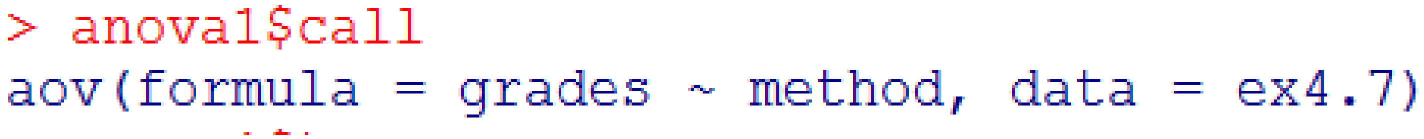
4.7. *Hypothesis tests for multiple samples*

4.7.1. Analysis of variance

Example 4-7: ANOVA RESULTS IN R



```
> anova1$fitted.values
 1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 
80 80 80 80 80 85 85 85 85 85 75 75 75 75 75 
> anova1$coef
    (Intercept) methodMethod B methodMethod C 
             80              5             -5 
> anova1$residuals
  1   2   3   4   5   6   7   8   9  10  11  12  13  14  15 
  6  -1   1 -10   4   5  -9   3  -3   4   7  -7  -2  -4   6 
```



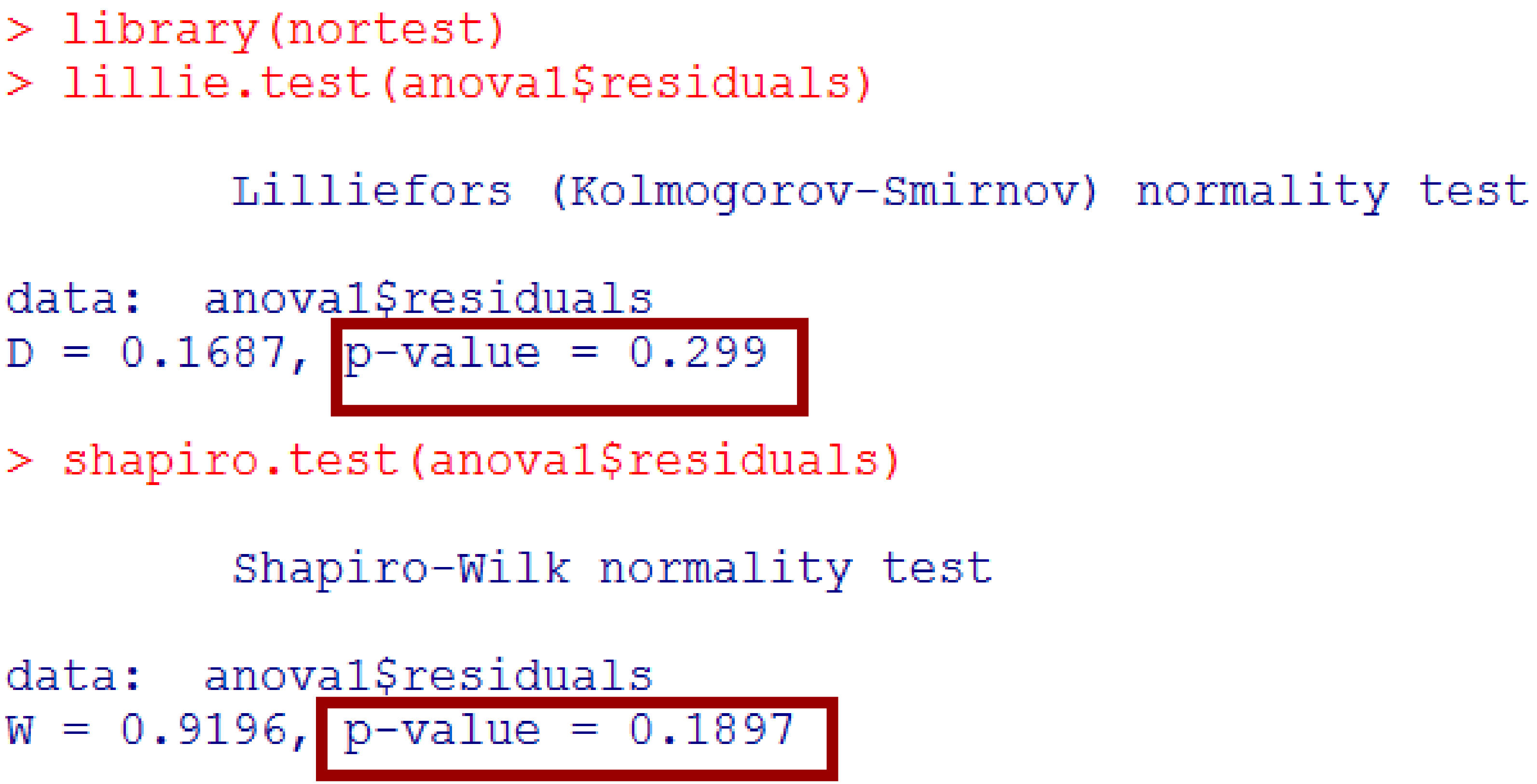
```
> anova1$call
aov(formula = grades ~ method, data = ex4.7)
```

4.7. *Hypothesis tests for multiple samples*

4.7.2. Checking for the assumptions of ANOVA

Example 4-7: Checking the assumptions of anova

1) Normality of the residuals



```
> library(nortest)
> lillie.test(anova1$residuals)

        Lilliefors (Kolmogorov-Smirnov) normality test

data:  anova1$residuals
D = 0.1687, p-value = 0.299

> shapiro.test(anova1$residuals)

        Shapiro-Wilk normality test

data:  anova1$residuals
W = 0.9196, p-value = 0.1897
```

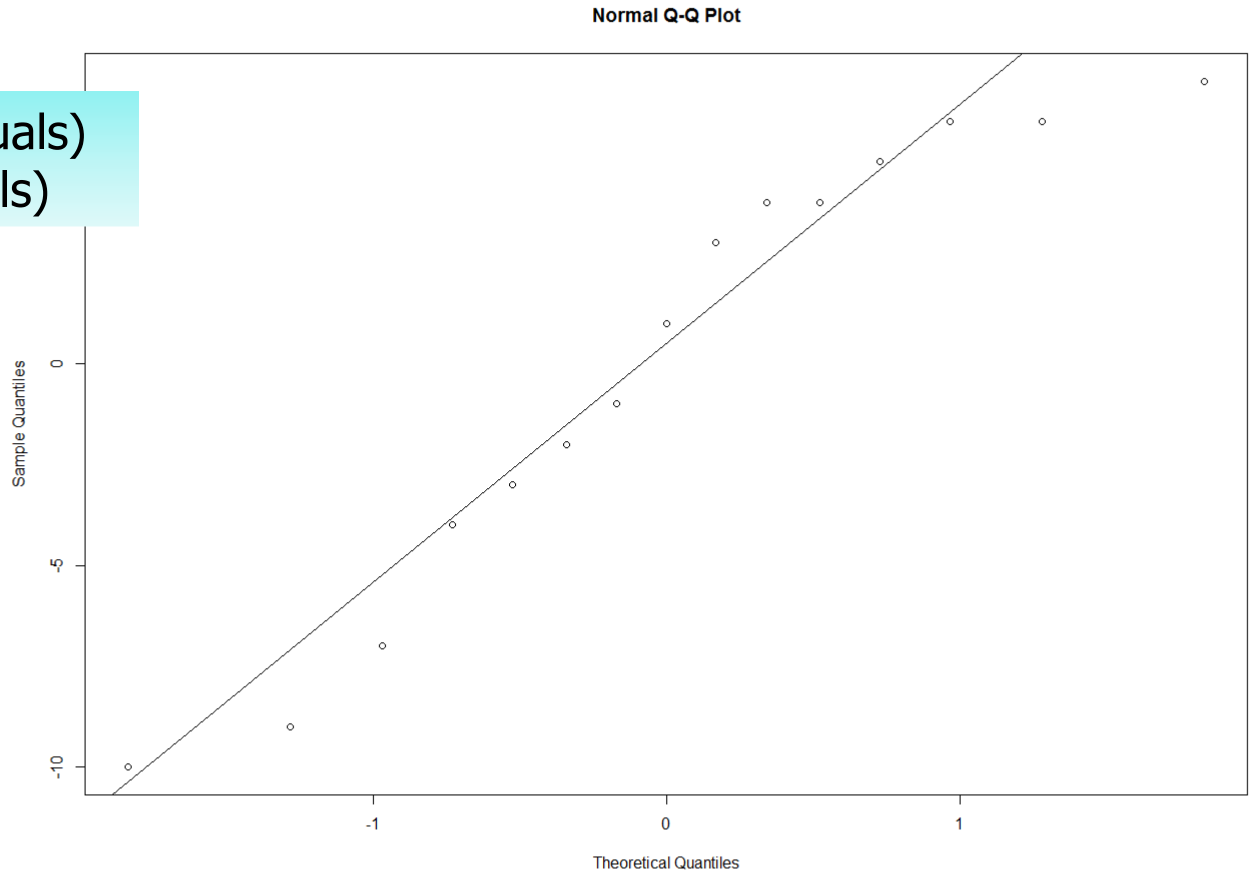
4.7. *Hypothesis tests for multiple samples*

4.7.2. Checking for the assumptions of ANOVA

Example 4-7: Checking the assumptions of anova

1) Normality of the residuals

```
qqnorm(anova1$residuals)
qqline(anova1$residuals)
```



4.7. *Hypothesis tests for multiple samples*

4.7.2. Checking for the assumptions of ANOVA

Example 4-7: Checking the assumptions of anova

2) Homogeneity of variances

H_0: $\sigma_1^2 = \sigma_2^2 = ... = \sigma_\kappa^2$ vs H_1: $\sigma_k^2 \neq \sigma_j^2$ for some $j,k \in \{1,2,\ldots,\kappa\}$

There are many ways of testing data for homogeneity of variance. Three methods are shown here.

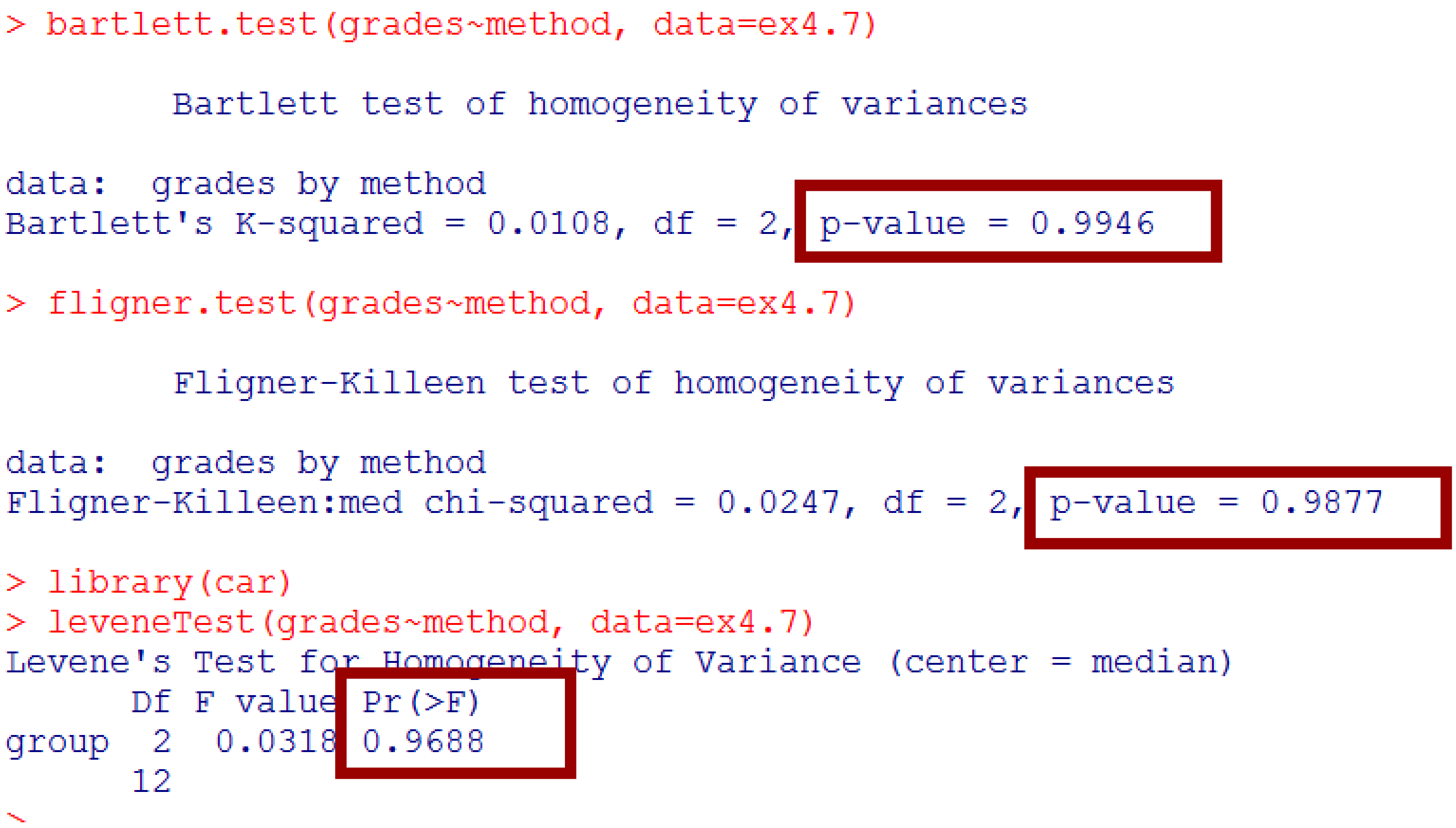
- ***Bartlett's test*** - If the data is normally distributed, this is the best test to use. It is sensitive to data which is not non-normally distribution; it is more likely to return a "false positive" when the data is non-normal.
- ***Levene's test*** - this is more robust to departures from normality than Bartlett's test. It is in the car package.
- ***Fligner-Killeen test*** - this is a non-parametric test which is very robust against departures from normality.

4.7. *Hypothesis tests for multiple samples*

4.7.2. Checking for the assumptions of ANOVA

Example 4-7: Checking the assumptions of anova

2) Homogeneity of variances



```
> bartlett.test(grades~method, data=ex4.7)

        Bartlett test of homogeneity of variances

data:  grades by method
Bartlett's K-squared = 0.0108, df = 2, p-value = 0.9946

> fligner.test(grades~method, data=ex4.7)

        Fligner-Killeen test of homogeneity of variances

data:  grades by method
Fligner-Killeen:med chi-squared = 0.0247, df = 2, p-value = 0.9877

> library(car)
> leveneTest(grades~method, data=ex4.7)
Levene's Test for Homogeneity of Variance (center = median)
      Df F value Pr(>F)
group  2  0.0318 0.9688
      12
>
```

4.8. *Two categorical variables*

- Testing for the equility of proportions/probabilities in independent groups/samples
    - ✓ Testing for the independence between binaries => comparison of success probabilities/proportions for two groups
        - t-test for the difference of proportions
        - t-test for the log-ratio of proportions (relative risk)
        - t-test for the log-odds ratio
        - Testing for independence in 2x2 contingency tables
    - ✓ Testing for independence in IxJ contingency tables
- Equality of proportions in dependent samples/paired values
    - ✓ Testing for the equality of marginal proportions using the McNemar test for 2x2 contingency tables

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey

The effect of gender on the knowledge for products

- ✓ Alcoholic drinks
- ✓ Bakery products
- ✓ Desserts and sweets
- ✓ Cosmetics

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey

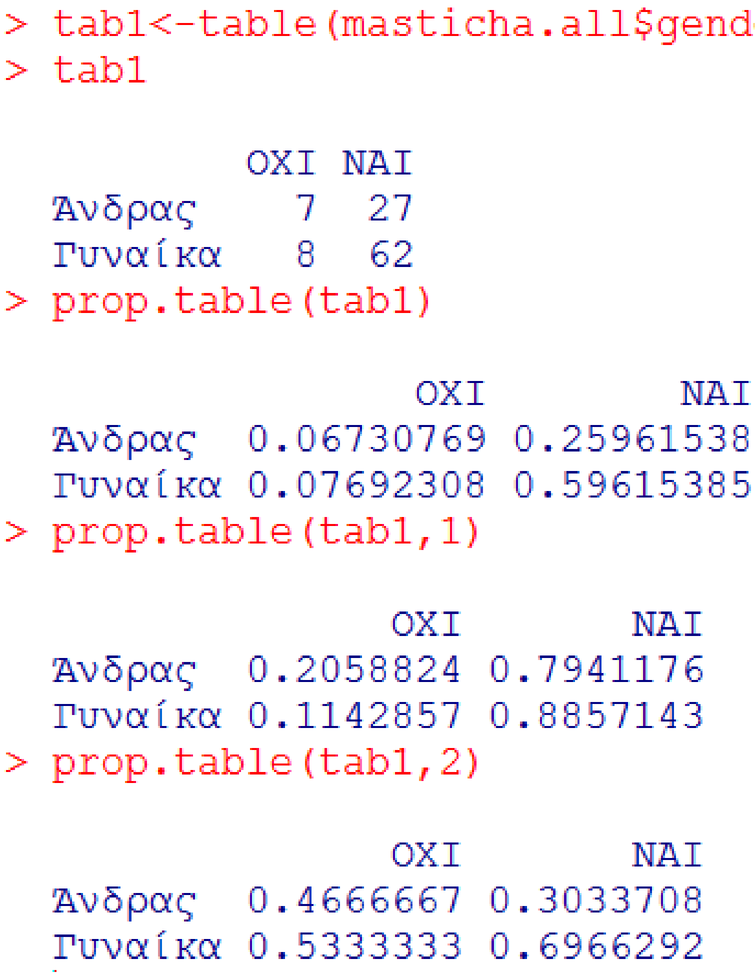
The effect of gender on the knowledge for alcoholic drinks

- We are interest to test whether there is difference between males and females concerning the knowledge of the existence of some products $\Rightarrow$
- We would like to test for the equality of proportions of males or females that were informed for the existence of a type of product (for example alcoholic drinks) $\Rightarrow$
- $H_0: \pi_{males} = \pi_{females}$ vs $H_1: \pi_{males} \neq \pi_{females} \Leftrightarrow$
- H_0: "Independence between gender and knowledge of a product" vs
- H_1: "there is association between gender and product knowledge"

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey



```
> tab1<-table(masticha.all$gender, masticha.all$a4_drinks )
> tab1

          OXI NAI
  Άνδρας    7  27
  Γυναίκα   8  62
> prop.table(tab1)

                 OXI        NAI
  Άνδρας  0.06730769 0.25961538
  Γυναίκα 0.07692308 0.59615385
> prop.table(tab1,1)

                OXI       NAI
  Άνδρας  0.2058824 0.7941176
  Γυναίκα 0.1142857 0.8857143
> prop.table(tab1,2)

                OXI       NAI
  Άνδρας  0.4666667 0.3033708
  Γυναίκα 0.5333333 0.6966292
```

Total table proportions

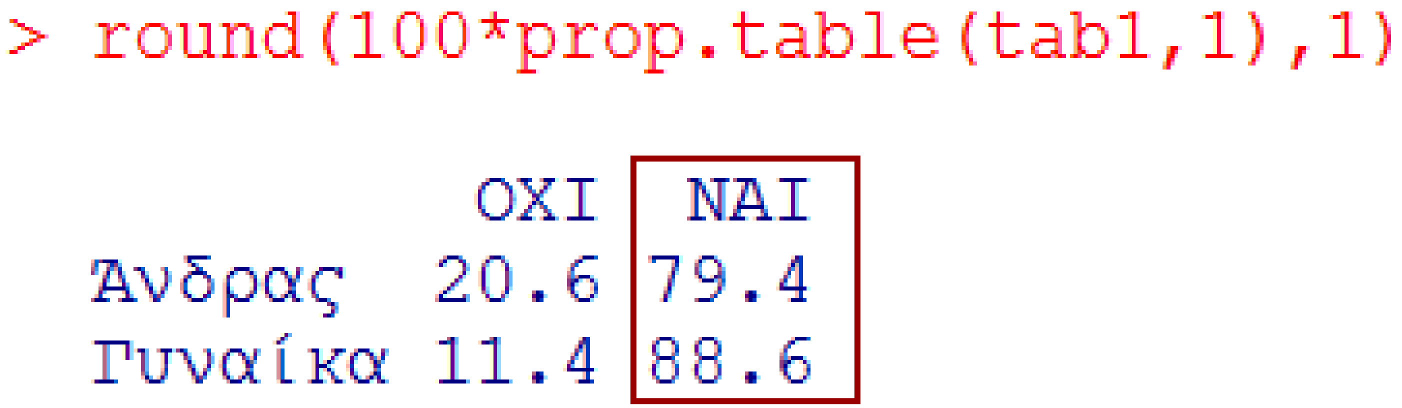
Row proportions

Column proportions

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey

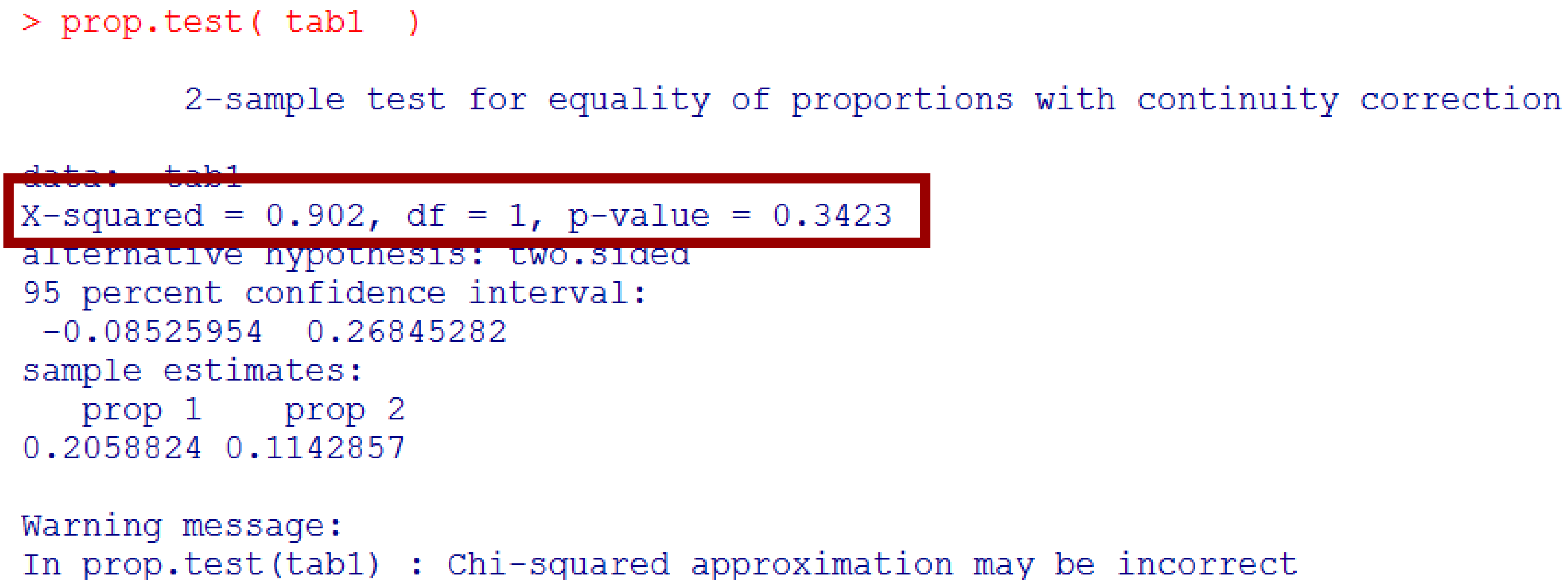


```
> round(100*prop.table(tab1,1),1)

         ΟΧΙ  ΝΑΙ
Ανδρας  20.6 79.4
Γυναίκα 11.4 88.6
```

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey



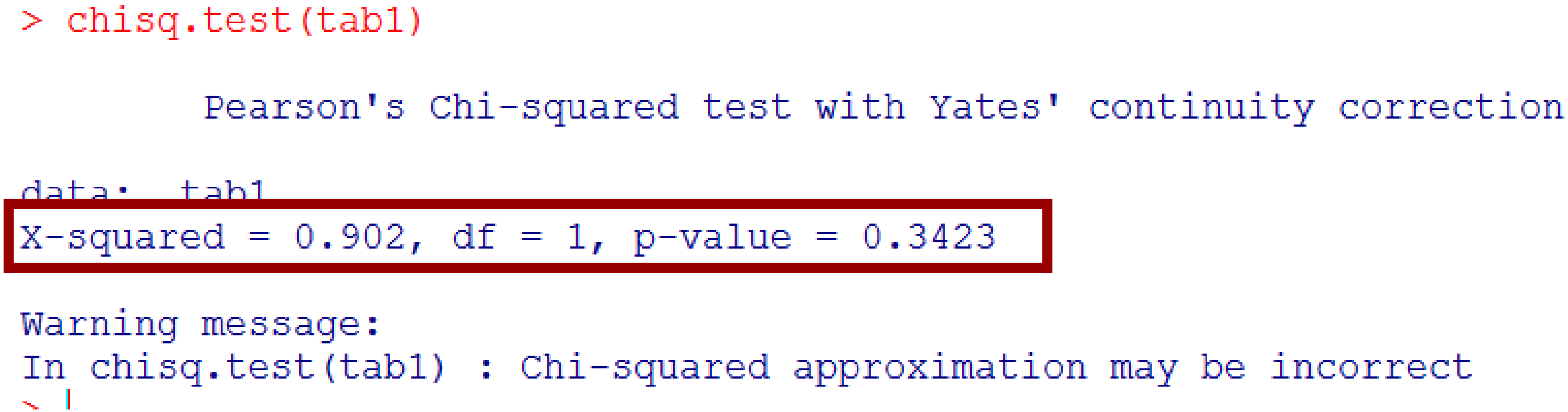
```
> prop.test( tab1  )

        2-sample test for equality of proportions with continuity correction

data:  tab1
X-squared = 0.902, df = 1, p-value = 0.3423
alternative hypothesis: two.sided
95 percent confidence interval:
 -0.08525954  0.26845282
sample estimates:
   prop 1    prop 2 
0.2058824 0.1142857 

Warning message:
In prop.test(tab1) : Chi-squared approximation may be incorrect
```

prop.test implements the Pearson's chi-square statistics for independence

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey



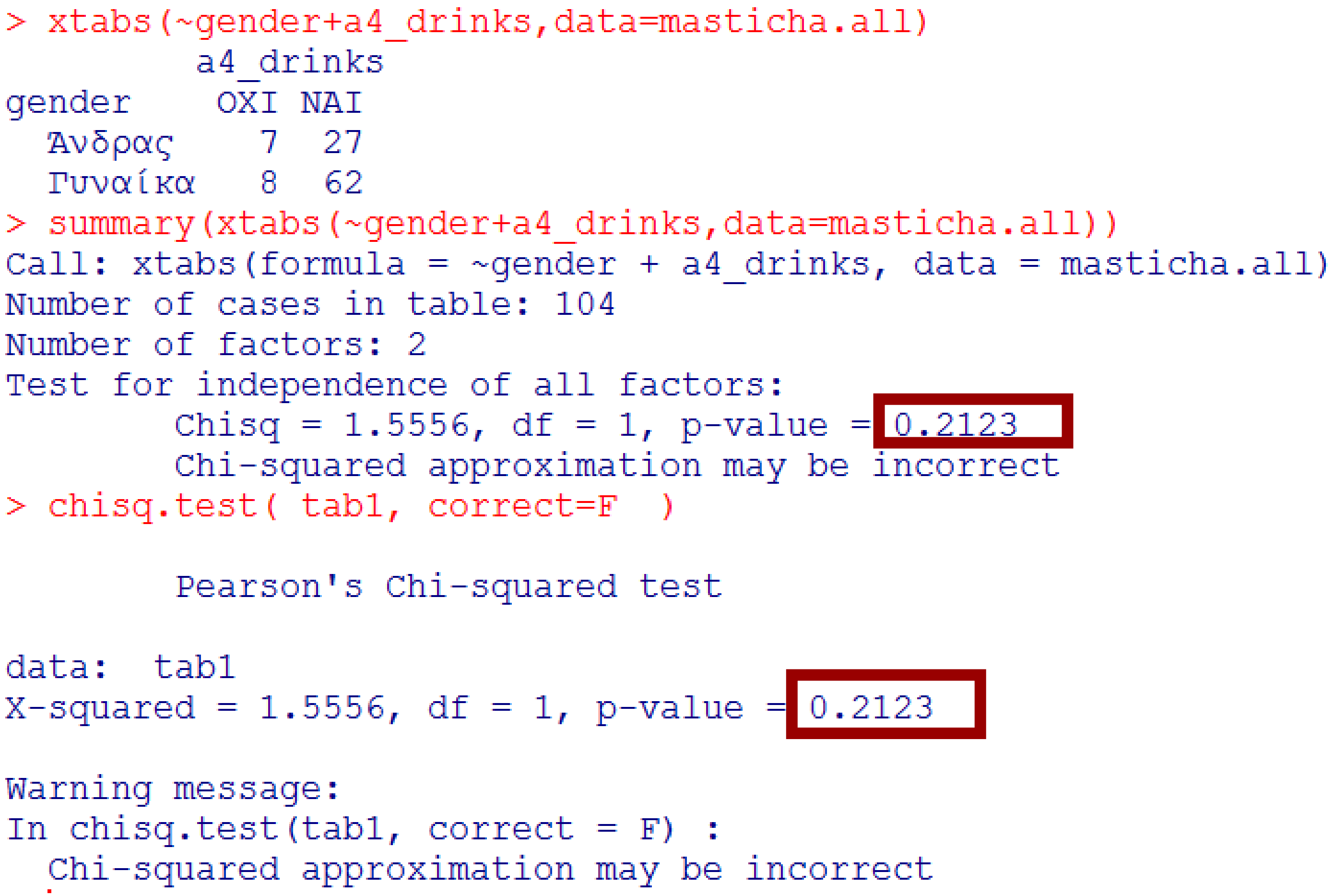
```
> chisq.test(tab1)

        Pearson's Chi-squared test with Yates' continuity correction

data:  tab1
X-squared = 0.902, df = 1, p-value = 0.3423

Warning message:
In chisq.test(tab1) : Chi-squared approximation may be incorrect
> 
```

Same also for chisq.test

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey

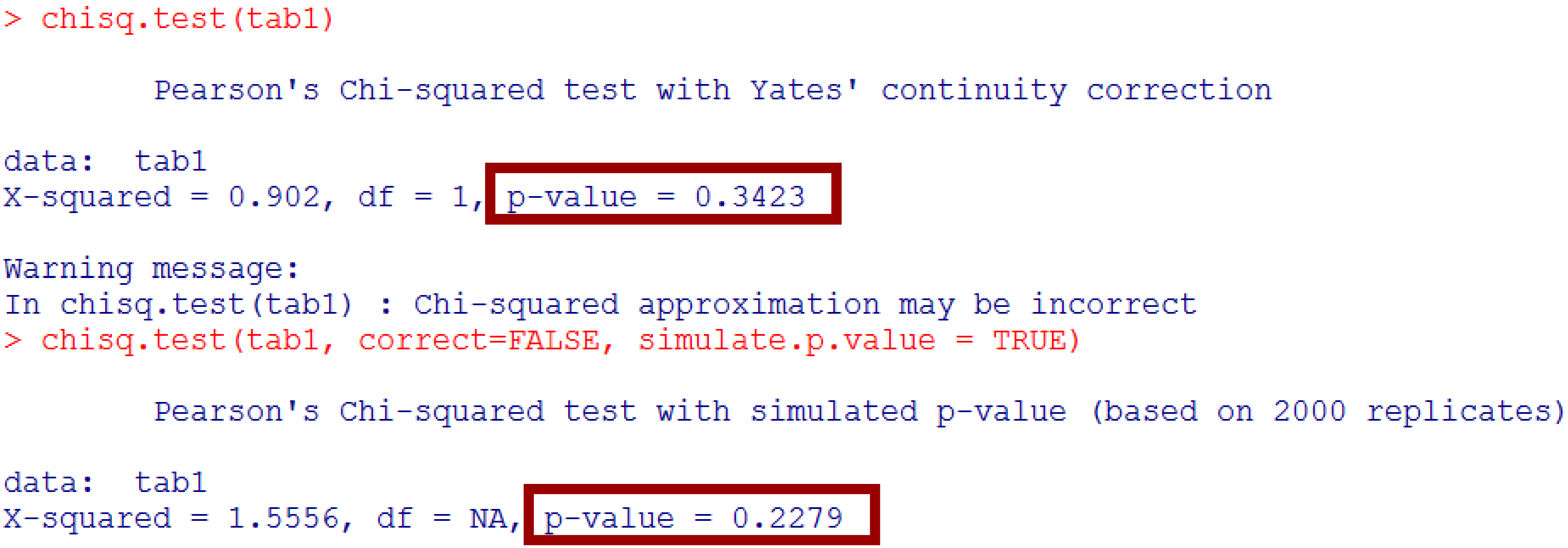


```
> xtabs(~gender+a4_drinks,data=masticha.all)
          a4_drinks
gender     OXI NAI
  Άνδρας     7  27
  Γυναίκα    8  62
> summary(xtabs(~gender+a4_drinks,data=masticha.all))
Call: xtabs(formula = ~gender + a4_drinks, data = masticha.all)
Number of cases in table: 104
Number of factors: 2
Test for independence of all factors:
        Chisq = 1.5556, df = 1, p-value = 0.2123
        Chi-squared approximation may be incorrect
> chisq.test( tab1, correct=F  )

        Pearson's Chi-squared test

data:  tab1
X-squared = 1.5556, df = 1, p-value = 0.2123

Warning message:
In chisq.test(tab1, correct = F) :
  Chi-squared approximation may be incorrect
```

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey



```
> chisq.test(tab1)

        Pearson's Chi-squared test with Yates' continuity correction

data:  tab1
X-squared = 0.902, df = 1, p-value = 0.3423

Warning message:
In chisq.test(tab1) : Chi-squared approximation may be incorrect
> chisq.test(tab1, correct=FALSE, simulate.p.value = TRUE)

        Pearson's Chi-squared test with simulated p-value (based on 2000 replicates)

data:  tab1
X-squared = 1.5556, df = NA, p-value = 0.2279
```

chisq.test is more flexible than prop.test

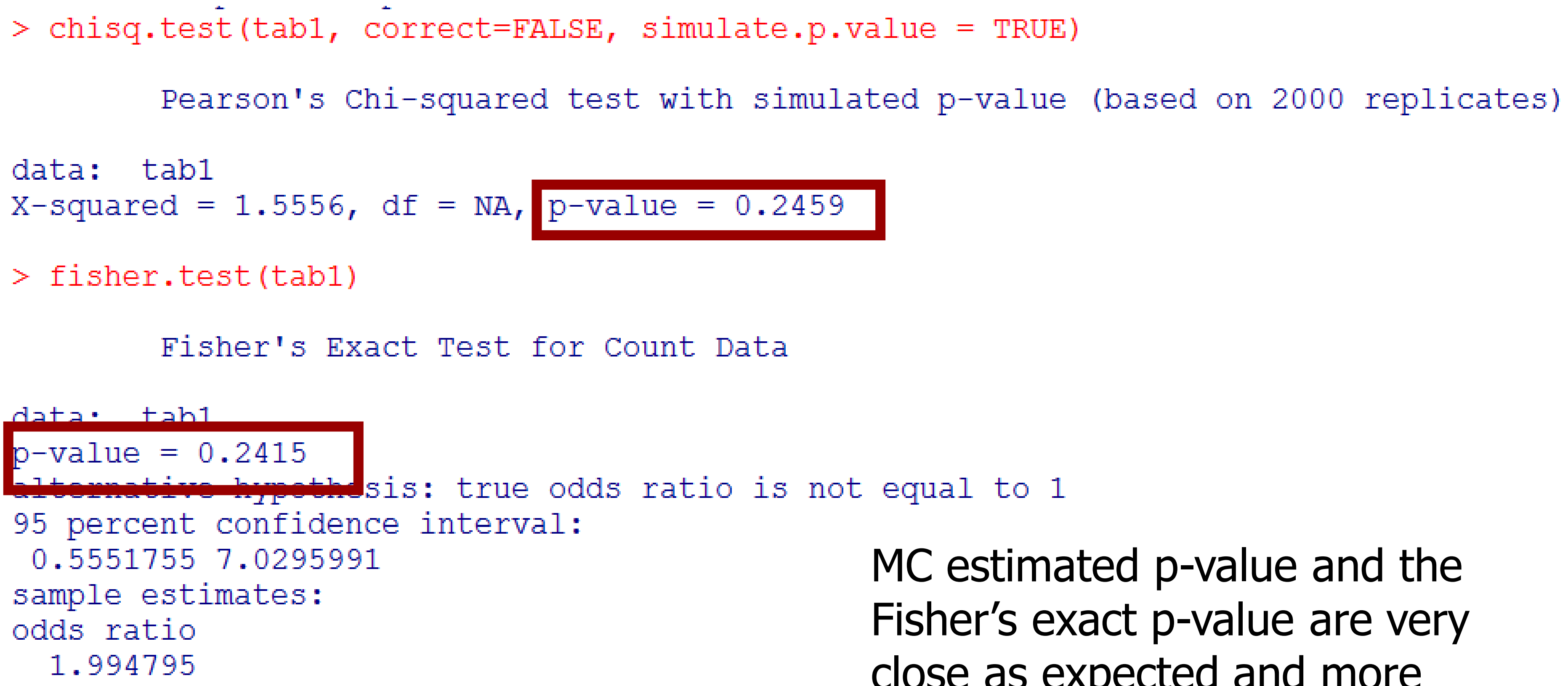
If small expected values (<5) arize then use similate.p.value to obtain a Monte Carlo estimate of the p-values which is more accurate

Otherwise use the Fisher’s exact test

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey



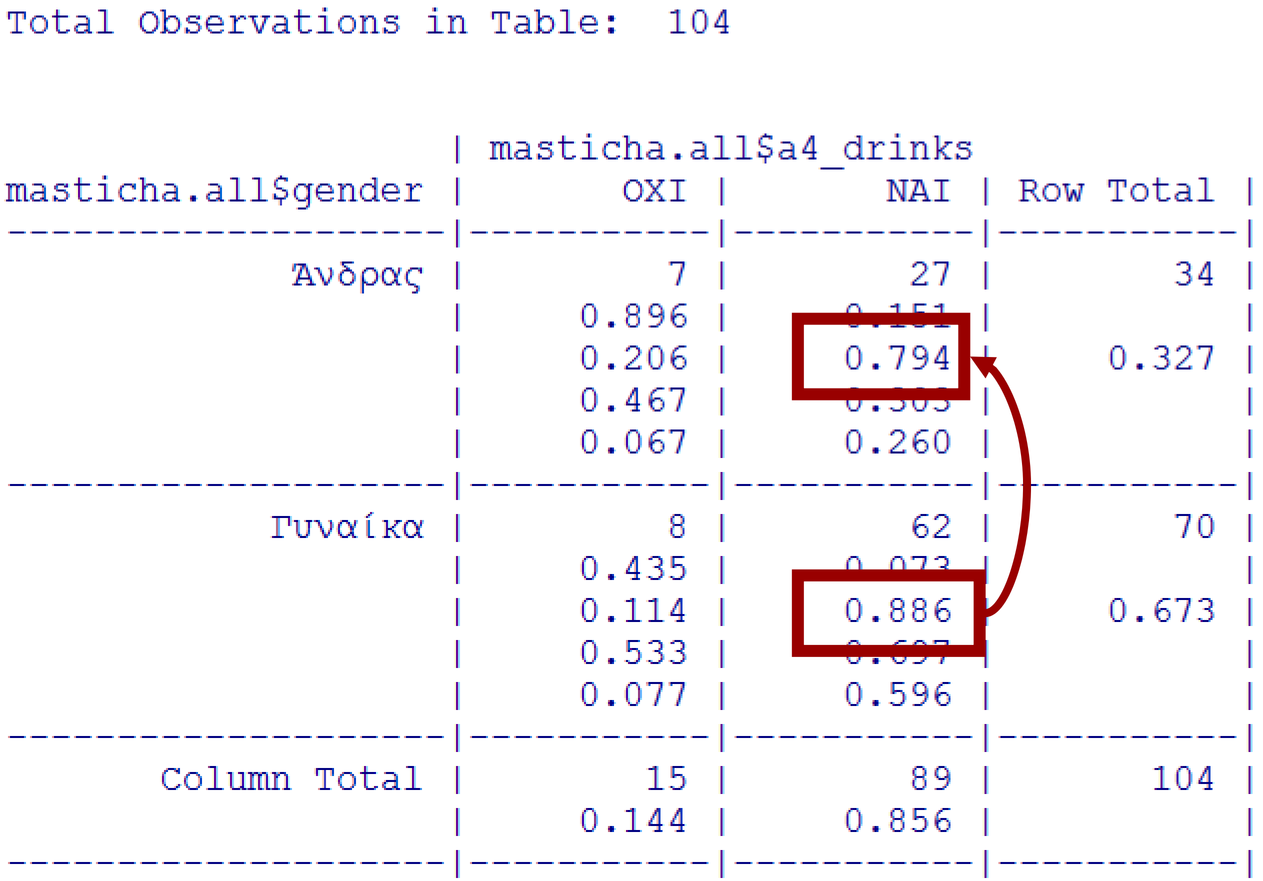
```
> chisq.test(tab1, correct=FALSE, simulate.p.value = TRUE)

        Pearson's Chi-squared test with simulated p-value (based on 2000 replicates)

data:  tab1
X-squared = 1.5556, df = NA, p-value = 0.2459

> fisher.test(tab1)

        Fisher's Exact Test for Count Data

data:  tab1
p-value = 0.2415
alternative hypothesis: true odds ratio is not equal to 1
95 percent confidence interval:
 0.5551755 7.0295991
sample estimates:
odds ratio
  1.994795
```

MC estimated p-value and the Fisher’s exact p-value are very close as expected and more accurate than approximate p-value in chi-sq test

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey



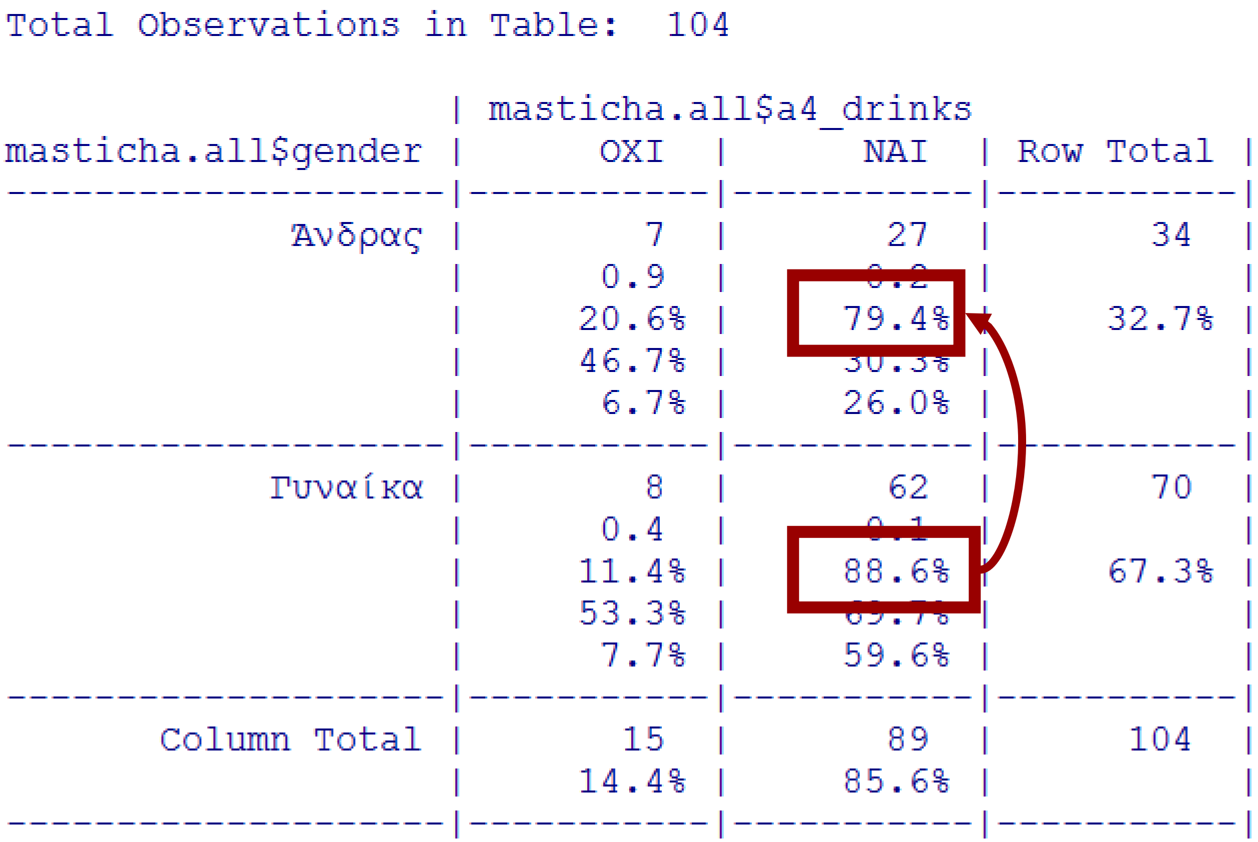
```
Total Observations in Table:  104


                    | masticha.all$a4_drinks
masticha.all$gender |       OXI |       NAI | Row Total |
--------------------|-----------|-----------|-----------|
             Άνδρας |         7 |        27 |        34 |
                    |     0.896 |     0.151 |           |
                    |     0.206 |     0.794 |     0.327 |
                    |     0.467 |     0.303 |           |
                    |     0.067 |     0.260 |           |
--------------------|-----------|-----------|-----------|
            Γυναίκα |         8 |        62 |        70 |
                    |     0.435 |     0.073 |           |
                    |     0.114 |     0.886 |     0.673 |
                    |     0.533 |     0.697 |           |
                    |     0.077 |     0.596 |           |
--------------------|-----------|-----------|-----------|
       Column Total |        15 |        89 |       104 |
                    |     0.144 |     0.856 |           |
--------------------|-----------|-----------|-----------|
```

```
CrossTable(masticha.all$gender, masticha.all$a4_drinks)
```

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey



```
Total Observations in Table:  104

                     | masticha.all$a4_drinks
masticha.all$gender |        OXI  |        NAI  | Row Total |
---------------------|------------|------------|------------|
               Ανδρας |          7 |         27 |        34 |
                     |        0.9 |        0.2 |            |
                     |      20.6% |      79.4% |     32.7% |
                     |      46.7% |      30.3% |            |
                     |       6.7% |      26.0% |            |
---------------------|------------|------------|------------|
              Γυναίκα |          8 |         62 |        70 |
                     |        0.4 |        0.1 |            |
                     |      11.4% |      88.6% |     67.3% |
                     |      53.3% |      69.7% |            |
                     |       7.7% |      59.6% |            |
---------------------|------------|------------|------------|
        Column Total |         15 |         89 |       104 |
                     |      14.4% |      85.6% |            |
---------------------|------------|------------|------------|
```

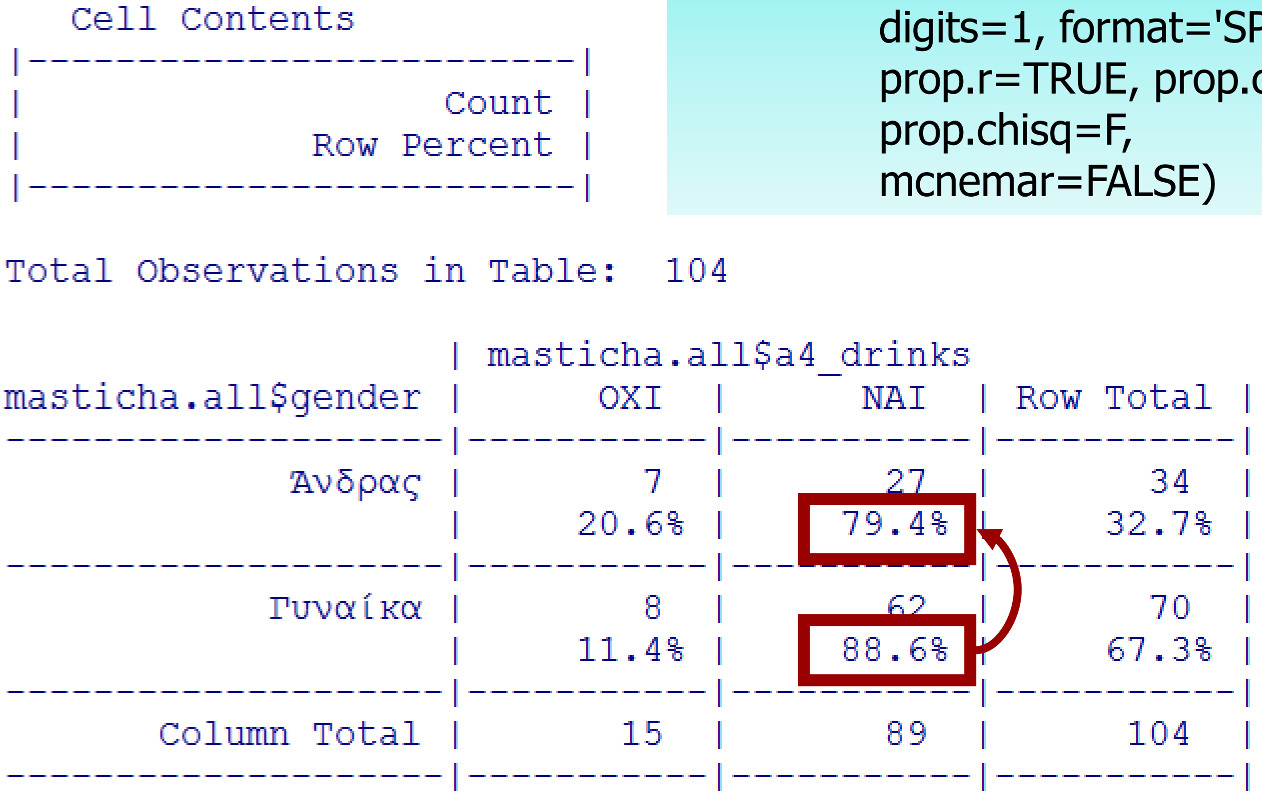
```
CrossTable(masticha.all$gender, masticha.all$a4_drinks, digits=1, format='SPSS')
```

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey

```
CrossTable(masticha.all$gender,masticha.all$a4_drinks,
           digits=1, format='SPSS', expected=FALSE,
           prop.r=TRUE, prop.c=F, prop.t=F,
           prop.chisq=F,        chisq = T, fisher=T,
           mcnemar=FALSE)
```



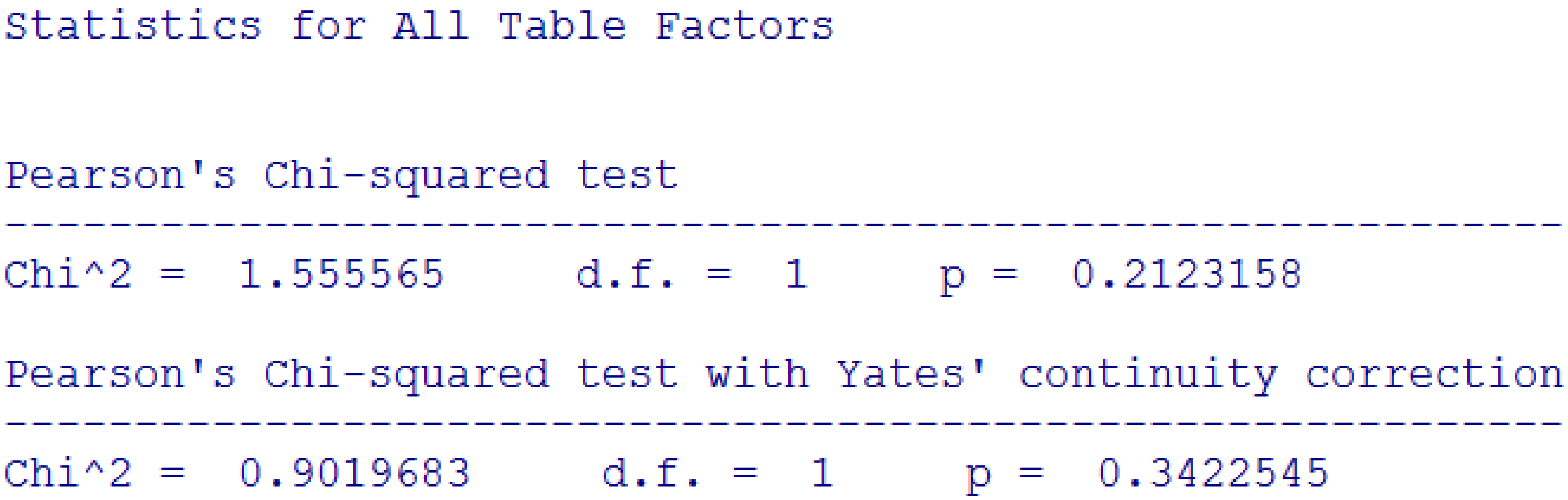
```
   Cell Contents
|-------------------------|
|                   Count |
|             Row Percent |
|-------------------------|

Total Observations in Table:  104

                    | masticha.all$a4_drinks
masticha.all$gender |       OXI  |       NAI  | Row Total |
--------------------|-----------|-----------|-----------|
             Άνδρας |         7  |        27  |        34  |
                    |     20.6%  |     79.4%  |     32.7%  |
--------------------|-----------|-----------|-----------|
            Γυναίκα |         8  |        62  |        70  |
                    |     11.4%  |     88.6%  |     67.3%  |
--------------------|-----------|-----------|-----------|
       Column Total |        15  |        89  |       104  |
--------------------|-----------|-----------|-----------|
```

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey

```
CrossTable(masticha.all$gender,masticha.all$a4_drinks,
           digits=1, format='SPSS', expected=FALSE,
           prop.r=TRUE, prop.c=F, prop.t=F,
           prop.chisq=F, chisq = T, fisher=T,
           mcnemar=FALSE)
```



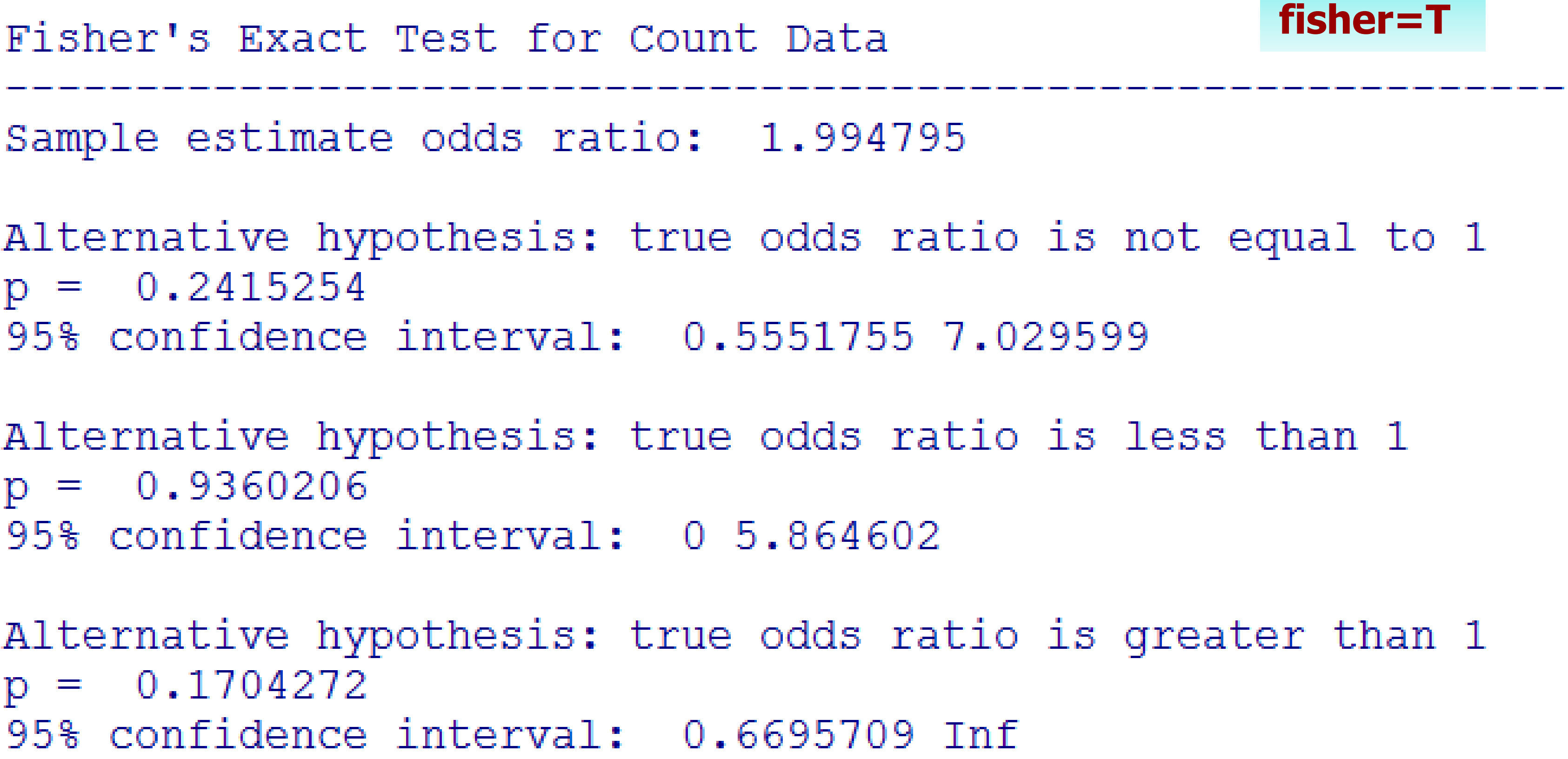
```
Statistics for All Table Factors


Pearson's Chi-squared test
------------------------------------------------------------
Chi^2 =  1.555565     d.f. =  1     p =  0.2123158

Pearson's Chi-squared test with Yates' continuity correction
------------------------------------------------------------
Chi^2 =  0.9019683     d.f. =  1     p =  0.3422545
```

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey

fisher=T



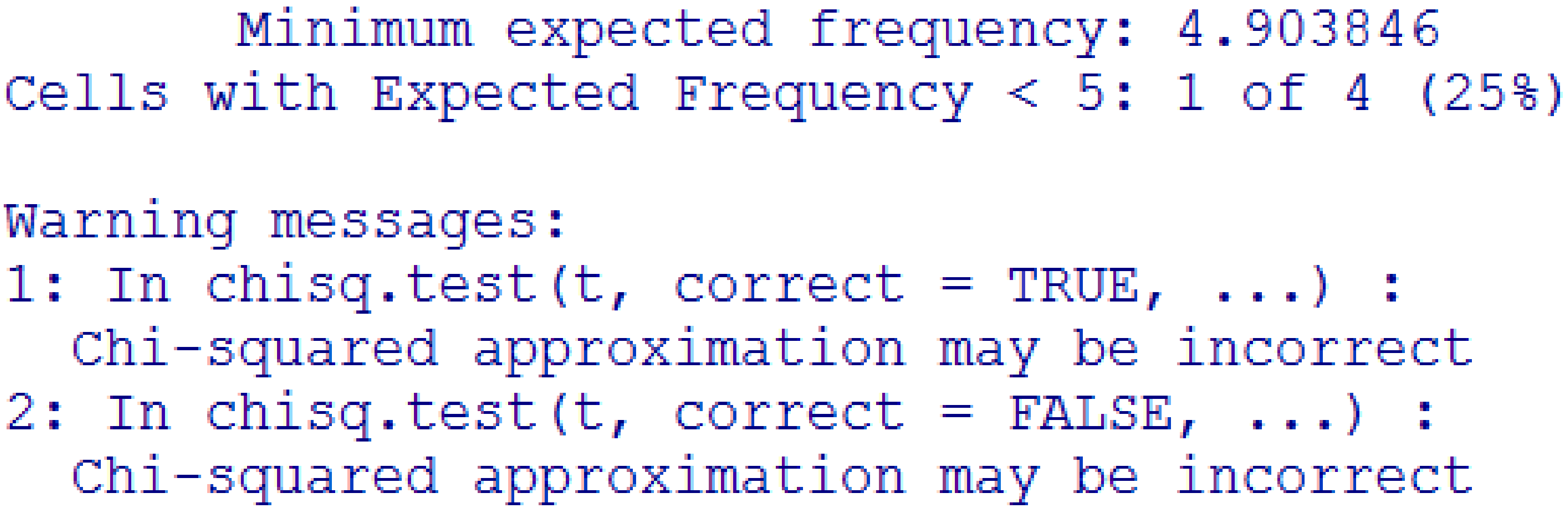
```
Fisher's Exact Test for Count Data
-----------------------------------------------------------
Sample estimate odds ratio:  1.994795

Alternative hypothesis: true odds ratio is not equal to 1
p =  0.2415254
95% confidence interval:  0.5551755 7.029599

Alternative hypothesis: true odds ratio is less than 1
p =  0.9360206
95% confidence interval:  0 5.864602

Alternative hypothesis: true odds ratio is greater than 1
p =  0.1704272
95% confidence interval:  0.6695709 Inf
```

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey

Warning for the validity of chi-square test. Expected values should be > 5 in order to accurately calculate the (approximate) p-value



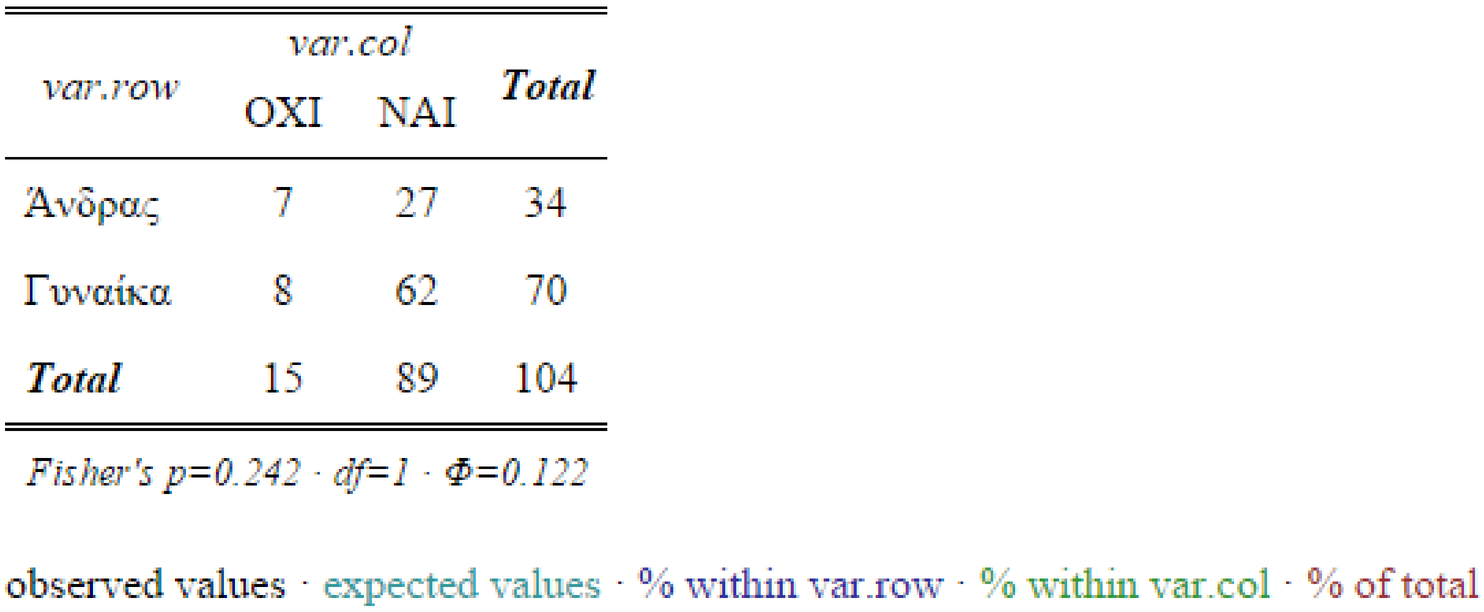
```
       Minimum expected frequency: 4.903846
Cells with Expected Frequency < 5: 1 of 4 (25%)

Warning messages:
1: In chisq.test(t, correct = TRUE, ...) :
  Chi-squared approximation may be incorrect
2: In chisq.test(t, correct = FALSE, ...) :
  Chi-squared approximation may be incorrect
```

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey

```
library(sjPlot)
sjt.xtab(masticha.all$gender, masticha.all$a4_drinks)
```



var.row	*var.col*		***Total***
	ΟΧΙ	ΝΑΙ	
Άνδρας	7	27	34
Γυναίκα	8	62	70
Total	15	89	104

Fisher's p=0.242 · df=1 · Φ=0.122

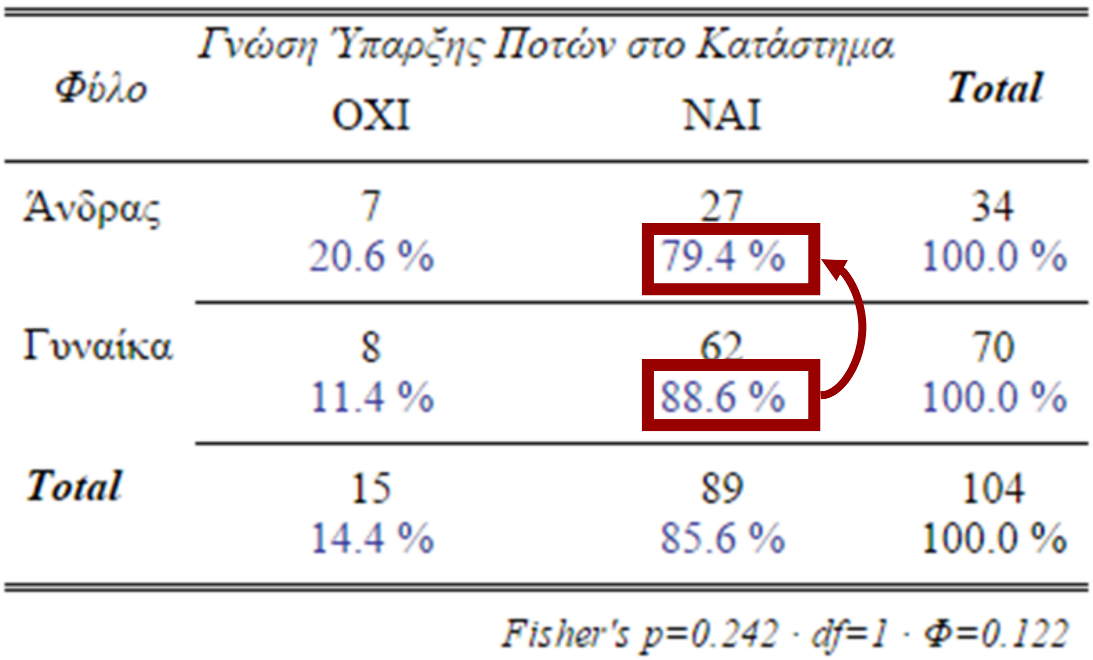
observed values · expected values · % within var.row · % within var.col · % of total

4.8. Two categorical variables

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey

```
sjt.xtab(masticha.all$gender, masticha.all$a4_drinks, show.cell.prc= FALSE,
          show.row.prc= TRUE, show.col.prc = FALSE,   show.exp= FALSE,
          var.labels = c('Φύλο', 'Γνώση 'Υπαρξης Ποτών στο Κατάστημα'),
          encoding = “UTF-8")
```



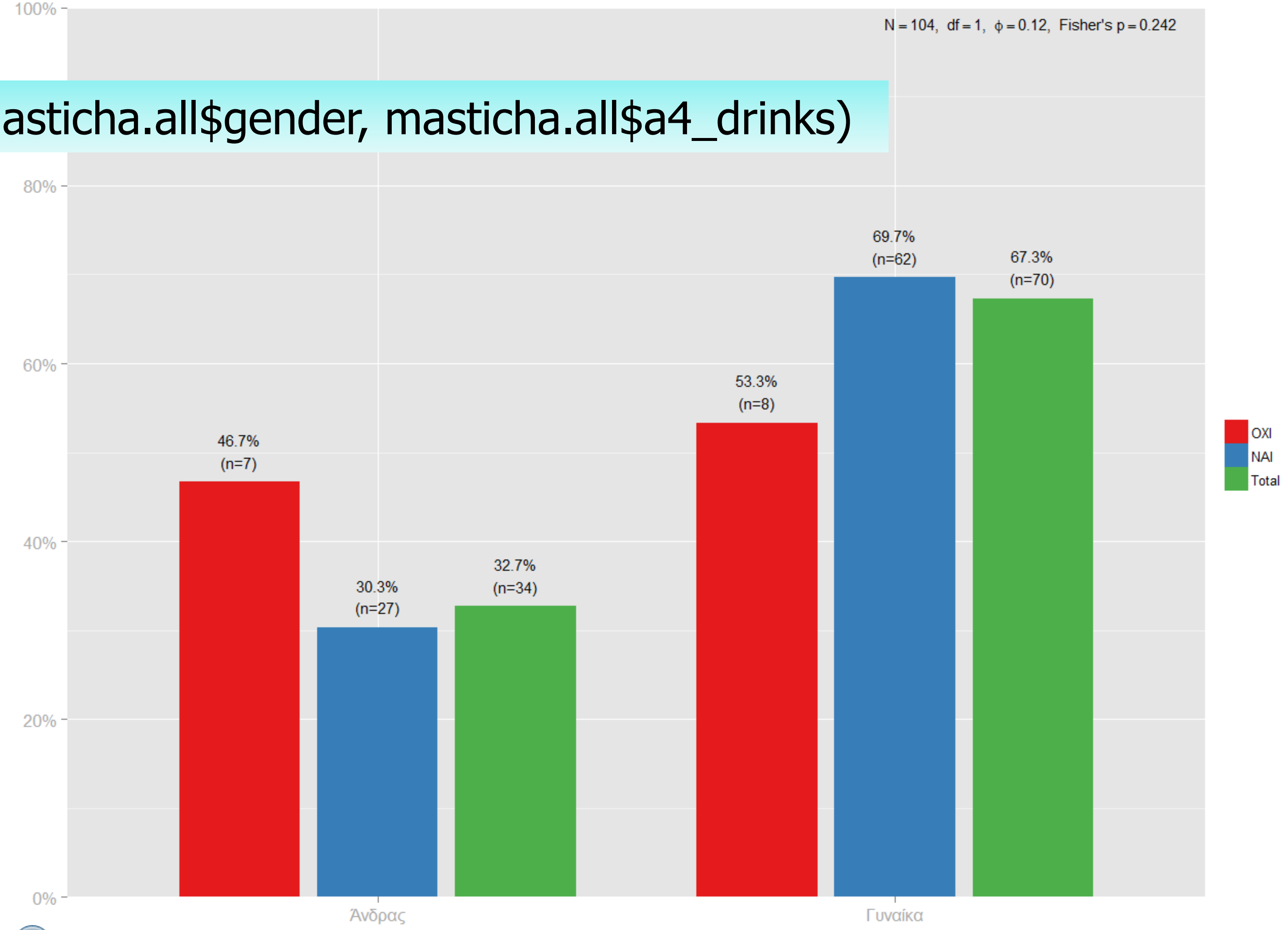
Φύλο	*Γνώση Ύπαρξης Ποτών στο Κατάστημα* ΟΧΙ	 ΝΑΙ	*Total*
Άνδρας	7 20.6 %	27 79.4 %	34 100.0 %
Γυναίκα	8 11.4 %	62 88.6 %	70 100.0 %
Total	15 14.4 %	89 85.6 %	104 100.0 %

Fisher's p=0.242 · df=1 · Φ=0.122

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

```
sjp.xtab(masticha.all$gender, masticha.all$a4_drinks)
```



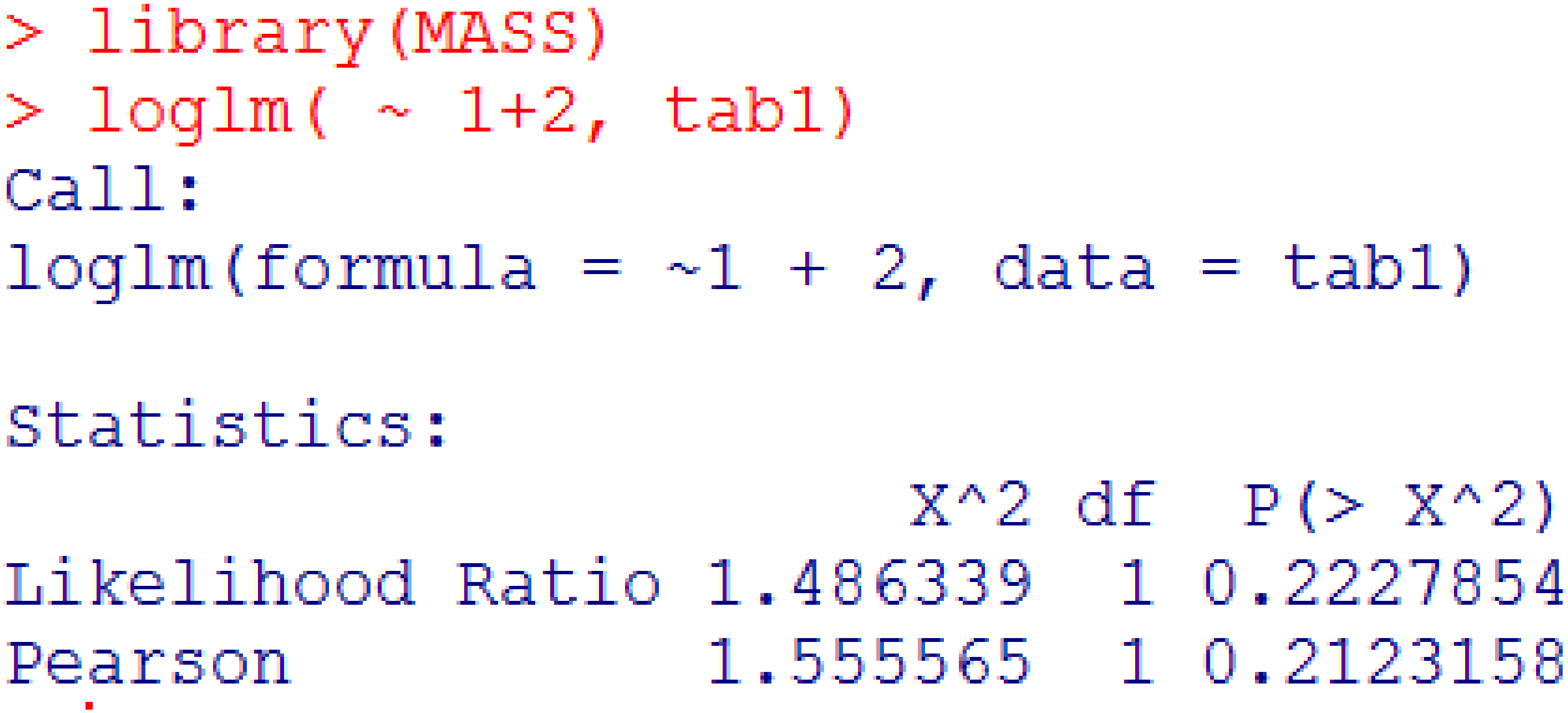
4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey

```
library(MASS)
loglm( ~ 1+2, tab1)
```

Likelihood ratio test for independence



```
> library(MASS)
> loglm( ~ 1+2, tab1)
Call:
loglm(formula = ~1 + 2, data = tab1)

Statistics:
                      X^2 df  P(> X^2)
Likelihood Ratio 1.486339  1 0.2227854
Pearson          1.555565  1 0.2123158
```

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Assumptions for tests

- Pearson's χ^2 independence test
  - Good approximation when expected values > 5
  - [Less strict assumption according to Cochran (1954, Biometrics): 80% of expected values > 5 and all of them >1]
- χ^2 test with Yates correction
  - Only for 2x2 tables [approximates better the Fisher Exact test]
- Likelihood ratio test of independence
  - Bad approximation for n/IJ < 5 (Agresti, 1990, p. 49)
- Fisher's exact test
  - It is the best since it is based on the exact distribution
- Monte Carlo estimates of p-values – it should be used when the χ^2 approximation is not valid and/or the Fisher exact test cannot be obtained

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey

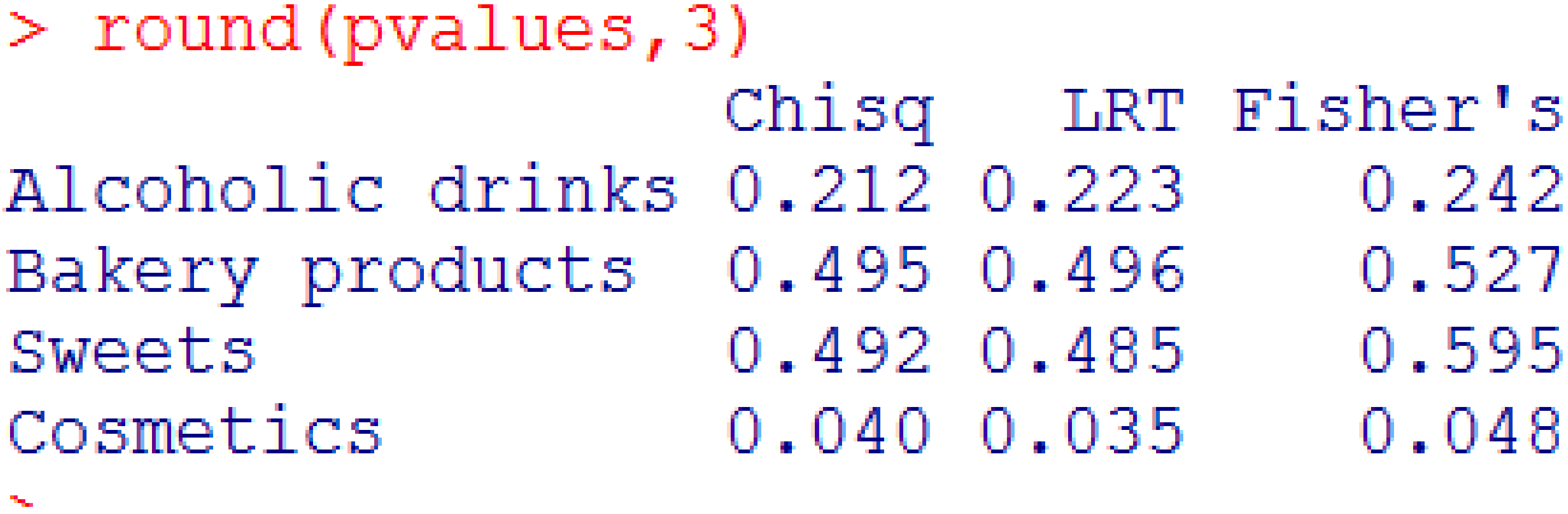
```
index<-6:9
pvalues<-matrix(nrow=4,ncol=3)
for (i in 1:4){
        var<-index[i]
        tab <- table( masticha.all$gender, masticha.all[,var] )
        pvalues[i,1]<-chisq.test(tab, correct=F)$p.value
        pvalues[i,2]<-summary(loglm( ~ 1+2, tab))$tests[1,3]
        pvalues[i,3]<-fisher.test(tab)$p.value

}
pvalues
colnames(pvalues) <- c('Chisq', 'LRT', "Fisher's")
rownames(pvalues) <- c('Alcoholic drinks', 'Bakery products', 'Sweets',
        'Cosmetics')
round(pvalues,3)
```

4.8. *Two categorical variables*

4.8.1. Testing for the association between two categorical variables (independent samples)

Example 4-11: Masticha shop customer satisfaction survey

P-values for the association of the gender with various type of products



```
> round(pvalues,3)
                 Chisq   LRT Fisher's
Alcoholic drinks 0.212 0.223    0.242
Bakery products  0.495 0.496    0.527
Sweets           0.492 0.485    0.595
Cosmetics        0.040 0.035    0.048
```

4.8. *Two categorical variables*

4.8.2. Testing for the equality of proportions in dependent samples

- Here we will consider the corresponding test of the paired t-test but for categorical variables.
- Hence, we may have the same binary or categorical measurements in two different time-point on the same study units (subjects or individuals).
- For binary variables, the resulted table is of 2x2 dimension
- We are interested to test the hypothesis:
- H_0: P(success in 1st time point)=P(success in 2nd time point) $\Rightarrow$
- $H_0: \pi_{i.}=\pi_{.i}$ vs. $H_1: \pi_{i.} \neq \pi_{.i} \Rightarrow$
- H_0: "There is no time effect in the categorical variable" vs H_1: "there is a time effect on the categorical variable"

4.8. *Two categorical variables*

4.8.2. Testing for the equality of proportions in dependent samples

Example 4-13: Prime minister's approval

- [Agresti, 2002, 2nd ed, p. 409; Table 10.1].
- Random sample of 1600 voting-age British citizens
- Two satisfaction/approval surveys with difference of 6 months
- Is the proportion of voters that approve the Prime minister's similar?

4.8. *Two categorical variables*

4.8.2. Testing for the equality of proportions in dependent samples

Example 4-13: Prime minister's approval

- H_0: "There is no difference in the approval of the Prime minister within the last 6 months" vs. H_1: "There is a difference in the approval of the Prime minister within the last 6 months" $\Rightarrow$
- H_0: P(Approval|First Survey) = P(Approval|Second Survey) vs. H_1: P(Approval|First Survey) $\neq$ P(Approval|Second Survey) $\Rightarrow$
- H_0: $\pi_{2.}=\pi_{.2}$ vs της εναλλακτικής H_1: $\pi_{2.} \neq \pi_{.2}$

4.8. *Two categorical variables*

4.8.2. Testing for the equality of proportions in dependent samples

Example 4-13: Prime minister's approval the last six months

	Second Survey		
First Survey	**Approve**	**Disapprove**	**Total**
Approve	794	150	**944**
Disapprove	86	570	**656**
Total	**880**	**720**	**1600**

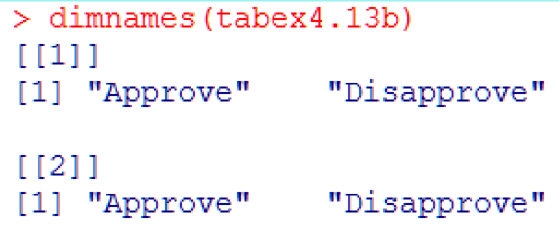
4.8. *Two categorical variables*

4.8.2. Testing for the equality of proportions in dependent samples

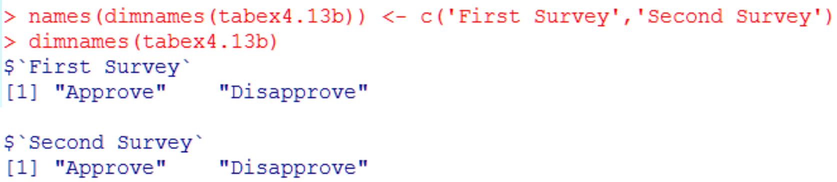
Example 4-13: Prime minister’s approval

Setting up the data directly as a table

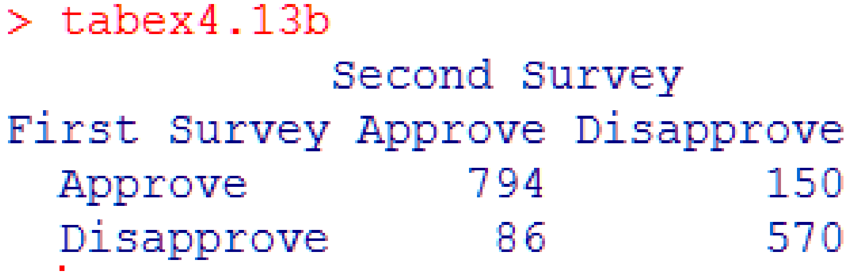
```
tabex4.13b <- as.table( matrix(c(794, 150, 86, 570),2,2, byrow=TRUE) )
tabex4.13b
rownames(tabex4.13b)<-c( 'Approve', 'Disapprove' )
colnames(tabex4.13b)<-c( 'Approve', 'Disapprove' )
tabex4.13b
dimnames(tabex4.13b)
names(dimnames(tabex4.13b)) <- c('First Survey','Second Survey')
dimnames(tabex4.13b)
tabex4.13b
```



```
> dimnames(tabex4.13b)
[[1]]
[1] "Approve"    "Disapprove"

[[2]]
[1] "Approve"    "Disapprove"
```



```
> names(dimnames(tabex4.13b)) <- c('First Survey','Second Survey')
> dimnames(tabex4.13b)
$`First Survey`
[1] "Approve"    "Disapprove"

$`Second Survey`
[1] "Approve"    "Disapprove"
```



```
> tabex4.13b
              Second Survey
First Survey Approve Disapprove
  Approve        794        150
  Disapprove      86        570
```

4.8. *Two categorical variables*

4.8.2. Testing for the equality of proportions in dependent samples

Example 4-13: Prime minister’s approval

Using the function Crosstable

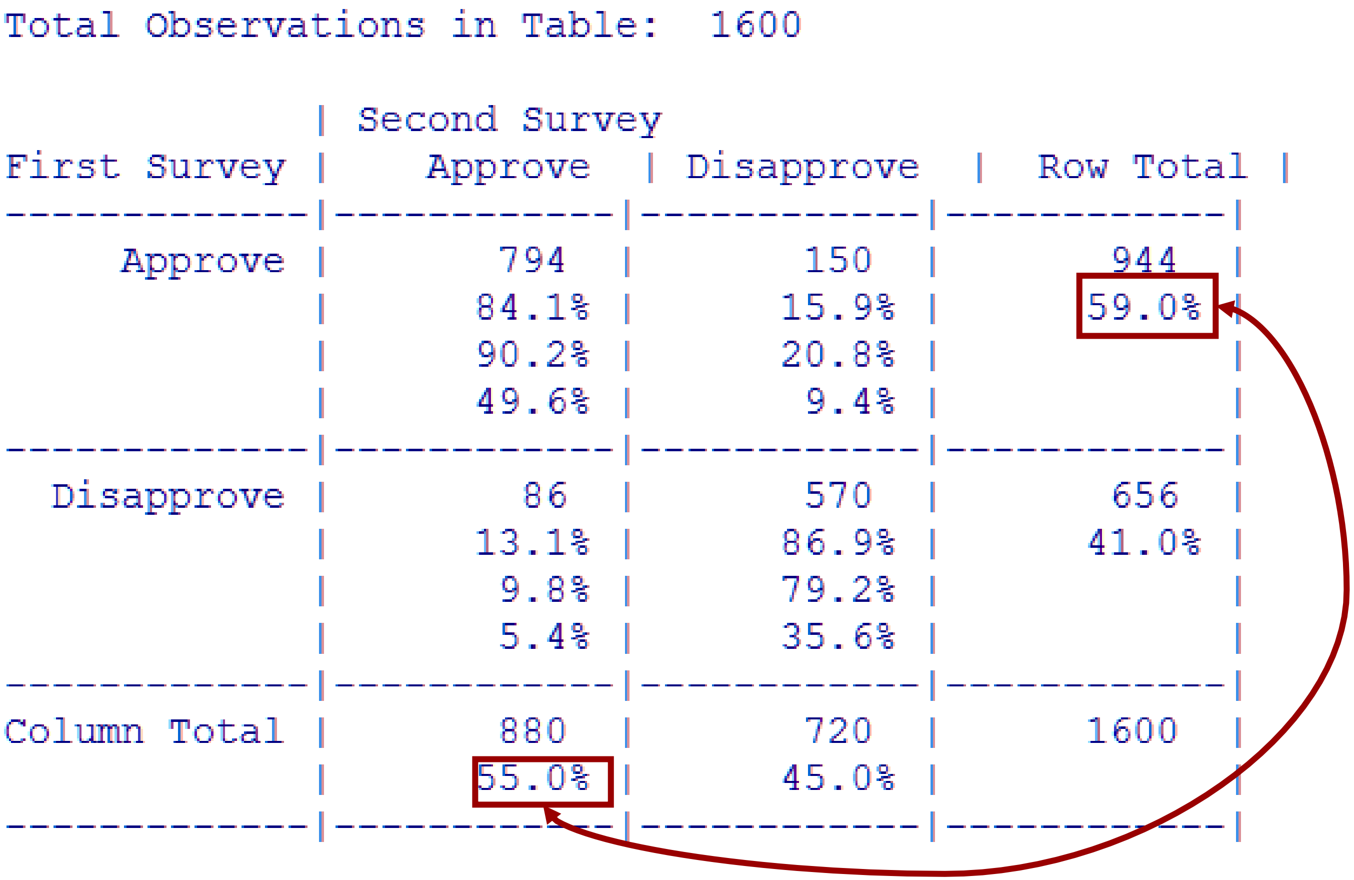
```
library(gmodels)
CrossTable(tabex4.13b, prop.r=T, prop.c=T, prop.t=T, prop.chisq=F,
          chisq = T, fisher=T,
mcnemar=TRUE, format='SPSS', digits=1)
```

4.8. *Two categorical variables*

4.8.2. Testing for the equality of proportions in dependent samples

Example 4-13: Prime minister’s approval

Using the function Crosstable



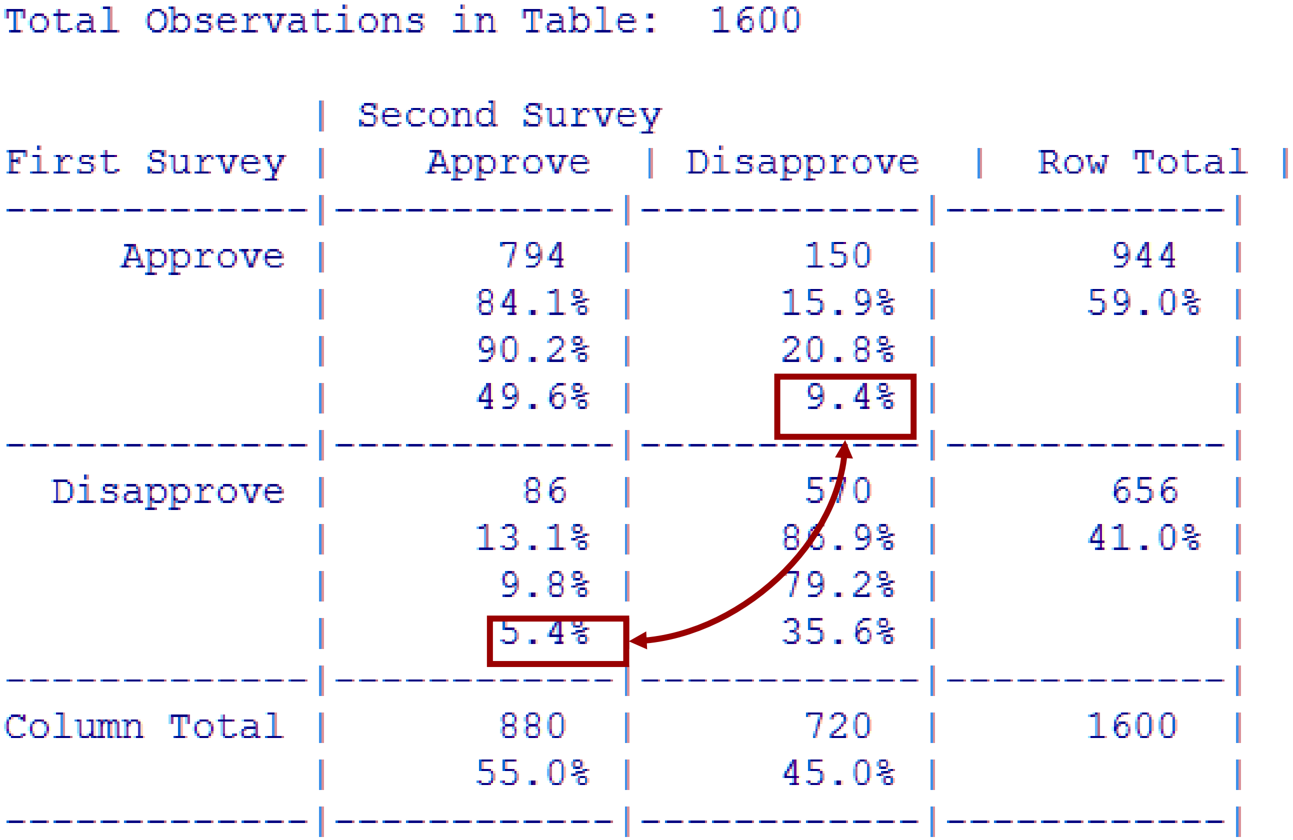
```
Total Observations in Table:  1600

              | Second Survey
 First Survey |     Approve  |  Disapprove  |   Row Total |
--------------|-------------|-------------|-------------|
      Approve |        794  |        150  |        944  |
              |      84.1% |      15.9% |      59.0% |
              |      90.2% |      20.8% |            |
              |      49.6% |       9.4% |            |
--------------|-------------|-------------|-------------|
   Disapprove |         86  |        570  |        656  |
              |      13.1% |      86.9% |      41.0% |
              |       9.8% |      79.2% |            |
              |       5.4% |      35.6% |            |
--------------|-------------|-------------|-------------|
 Column Total |        880  |        720  |       1600  |
              |      55.0% |      45.0% |            |
--------------|-------------|-------------|-------------|
```

4.8. *Two categorical variables*

4.8.2. Testing for the equality of proportions in dependent samples

Example 4-13: Prime minister's approval

Using the function Crosstable



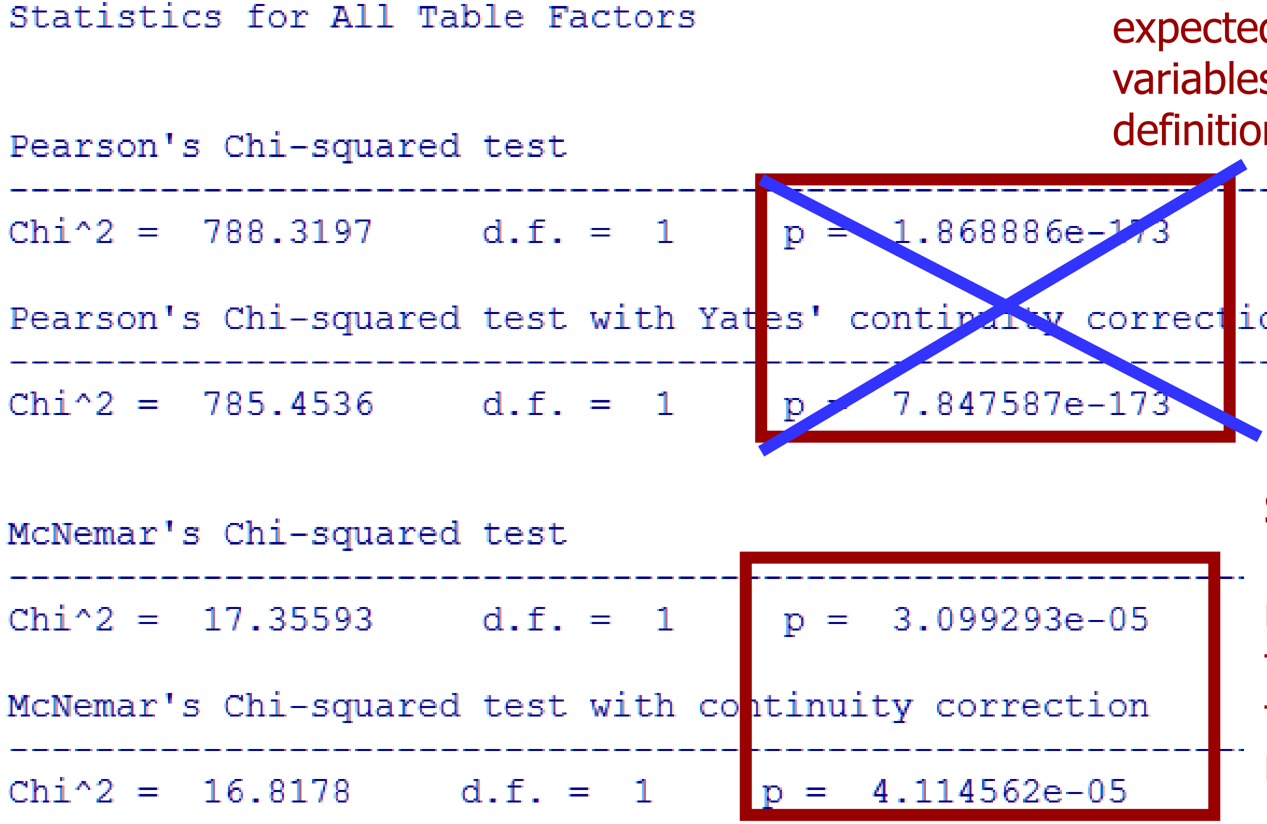
```
Total Observations in Table:  1600 

 
             | Second Survey 
First Survey |    Approve  | Disapprove  |  Row Total | 
-------------|-------------|-------------|-------------|
     Approve |       794   |       150   |       944  | 
             |     84.1%   |     15.9%   |     59.0%  | 
             |     90.2%   |     20.8%   |            | 
             |     49.6%   |      9.4%   |            | 
-------------|-------------|-------------|-------------|
  Disapprove |        86   |       570   |       656  | 
             |     13.1%   |     86.9%   |     41.0%  | 
             |      9.8%   |     79.2%   |            | 
             |      5.4%   |     35.6%   |            | 
-------------|-------------|-------------|-------------|
Column Total |       880   |       720   |      1600  | 
             |     55.0%   |     45.0%   |            | 
-------------|-------------|-------------|-------------|
```

4.8. *Two categorical variables*

4.8.2. Testing for the equality of proportions in dependent samples

Example 4-13: Prime minister's approval

Using the function Crosstable



```
Statistics for All Table Factors


Pearson's Chi-squared test
------------------------------------------------------------
Chi^2 =  788.3197     d.f. =  1     p =  1.868886e-173

Pearson's Chi-squared test with Yates' continuity correction
------------------------------------------------------------
Chi^2 =  785.4536     d.f. =  1     p =  7.847587e-173


McNemar's Chi-squared test
------------------------------------------------------------
Chi^2 =  17.35593     d.f. =  1     p =  3.099293e-05

McNemar's Chi-squared test with continuity correction
------------------------------------------------------------
Chi^2 =  16.8178     d.f. =  1     p =  4.114562e-05
```

Chi square test significant as expected since the two variables are dependent by definition

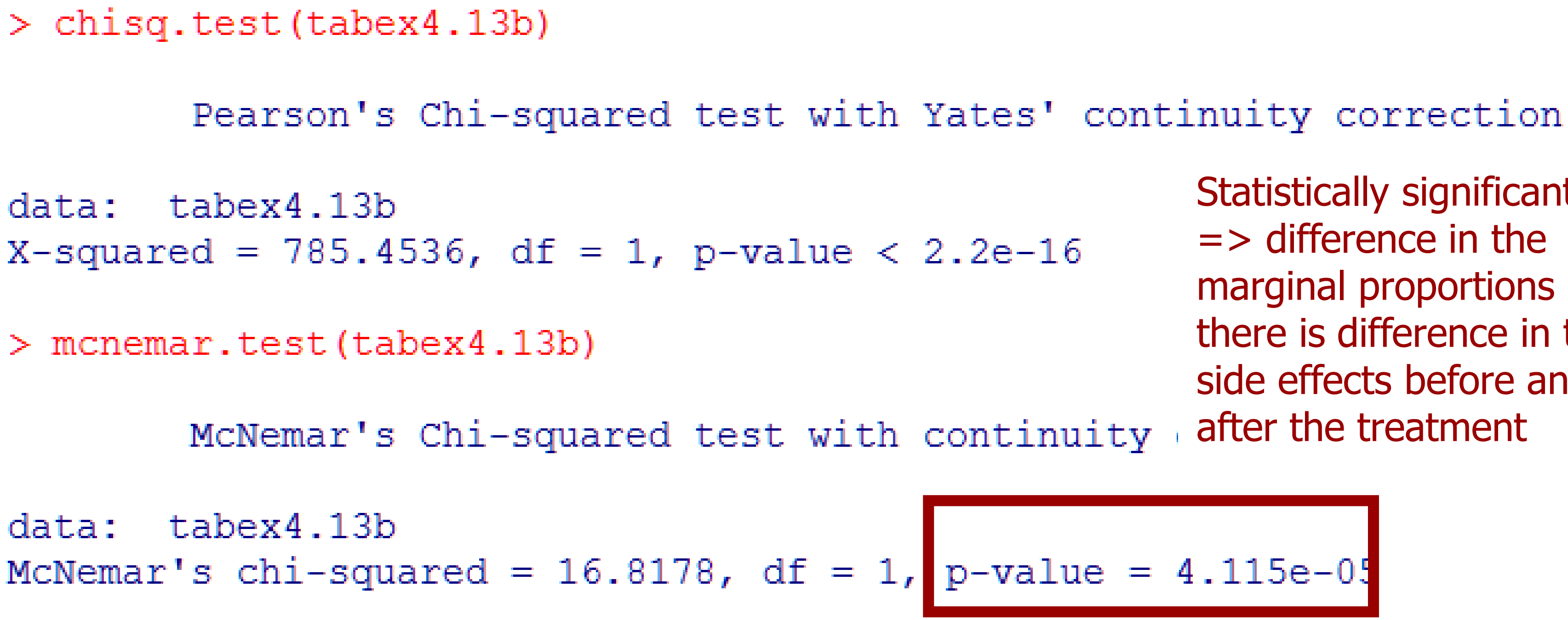
Statistically significant => difference in the marginal proportions => there is a difference in the approval of the Prime minister

4.8. *Two categorical variables*

4.8.2. Testing for the equality of proportions in dependent samples

Example 4-13: Prime minister's approval

Using the function mcnemar.test



```
> chisq.test(tabex4.13b)

        Pearson's Chi-squared test with Yates' continuity correction

data:  tabex4.13b
X-squared = 785.4536, df = 1, p-value < 2.2e-16

> mcnemar.test(tabex4.13b)

        McNemar's Chi-squared test with continuity

data:  tabex4.13b
McNemar's chi-squared = 16.8178, df = 1, p-value = 4.115e-05
```

Statistically significant => difference in the marginal proportions => there is difference in the side effects before and after the treatment

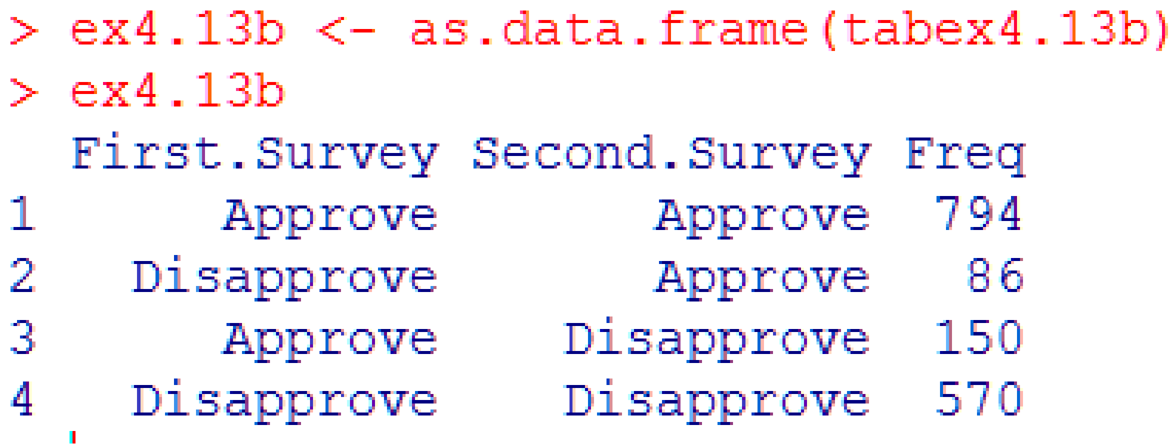
4.8. *Two categorical variables*

4.8.2. Testing for the equality of proportions in dependent samples

Example 4-13: Prime minister’s approval

Changing the table to a data.frame (with counts)

```
tabex4.13b <- as.table( matrix(c(794, 150, 86, 570),2,2, byrow=TRUE) )
rownames(tabex4.13b)<-c( 'Approve', 'Disapprove' )
colnames(tabex4.13b)<-c( 'Approve', 'Disapprove' )
dimnames(tabex4.13b)
names(dimnames(tabex4.13b)) <- c('First Survey','Second Survey')
```



```
> ex4.13b <- as.data.frame(tabex4.13b)
> ex4.13b
  First.Survey Second.Survey Freq
1      Approve       Approve  794
2   Disapprove       Approve   86
3      Approve    Disapprove  150
4   Disapprove    Disapprove  570
```

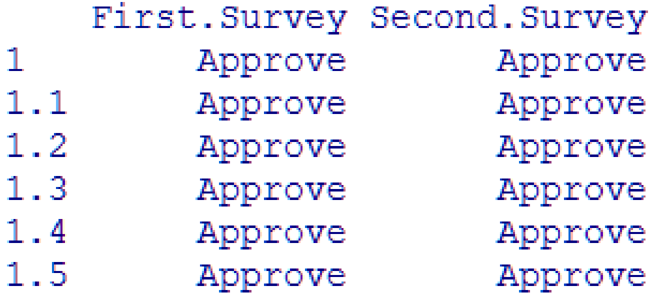
4.8. *Two categorical variables*

4.8.2. Testing for the equality of proportions in dependent samples

Example 4-13: Prime minister’s approval

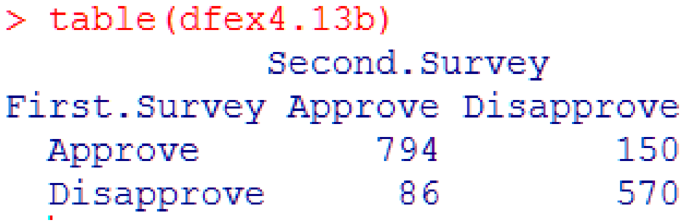
Changing the table to a data.frame (fully expanded)

```
index<-1:nrow(ex4.13b)
dfex4.13b <- ex4.13b[rep(index,ex4.13b$Freq),-3]
head(dfex4.13b)
```



```
    First.Survey Second.Survey
1        Approve       Approve
1.1      Approve       Approve
1.2      Approve       Approve
1.3      Approve       Approve
1.4      Approve       Approve
1.5      Approve       Approve
```

```
table(dfex4.13b)
```



```
> table(dfex4.13b)
            Second.Survey
First.Survey Approve Disapprove
  Approve        794        150
  Disapprove      86        570
```

4.8. *Two categorical variables*

4.8.2. Testing for the equality of proportions in dependent samples

Example 4-13: Prime minister’s approval

Changing the table to a data.frame

```
sjt.xtab(ex4.13b[,1],ex4.13b[,2], weight.by=ex4.13b[,3],
         var.labels = names(ex4.13b),
         show.cell.prc = T, show.row.prc = T,
         encoding = "UTF-8")
```

OR

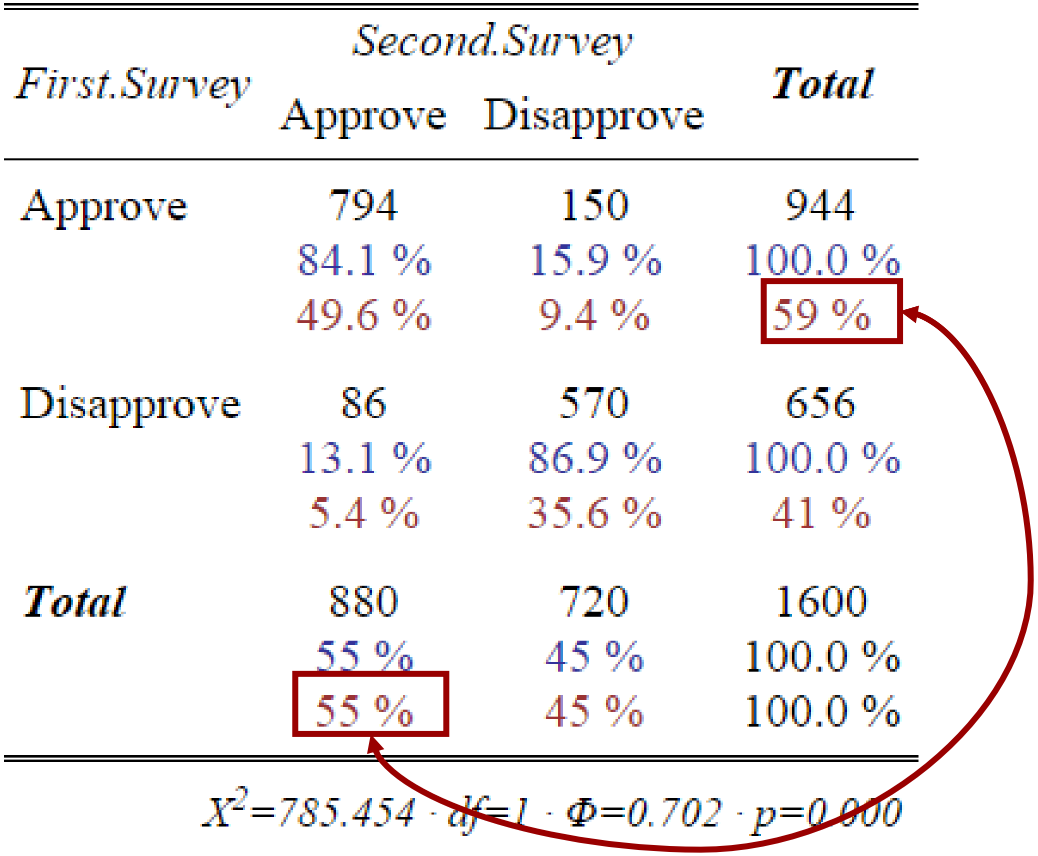
```
sjt.xtab(dfex4.13b[,1],dfex4.13b[,2], var.labels= names(ex4.13b),
         show.cell.prc= T, show.row.prc = T,
         encoding = “UTF-8")
```

4.8. *Two categorical variables*

4.8.2. Testing for the equality of proportions in dependent samples

Example 4-13: Prime minister's approval

Using the function Crosstable



First.Survey	*Second.Survey* Approve	 Disapprove	***Total***
Approve	794 84.1 % 49.6 %	150 15.9 % 9.4 %	944 100.0 % 59 %
Disapprove	86 13.1 % 5.4 %	570 86.9 % 35.6 %	656 100.0 % 41 %
Total	880 55 % 55 %	720 45 % 45 %	1600 100.0 % 100.0 %

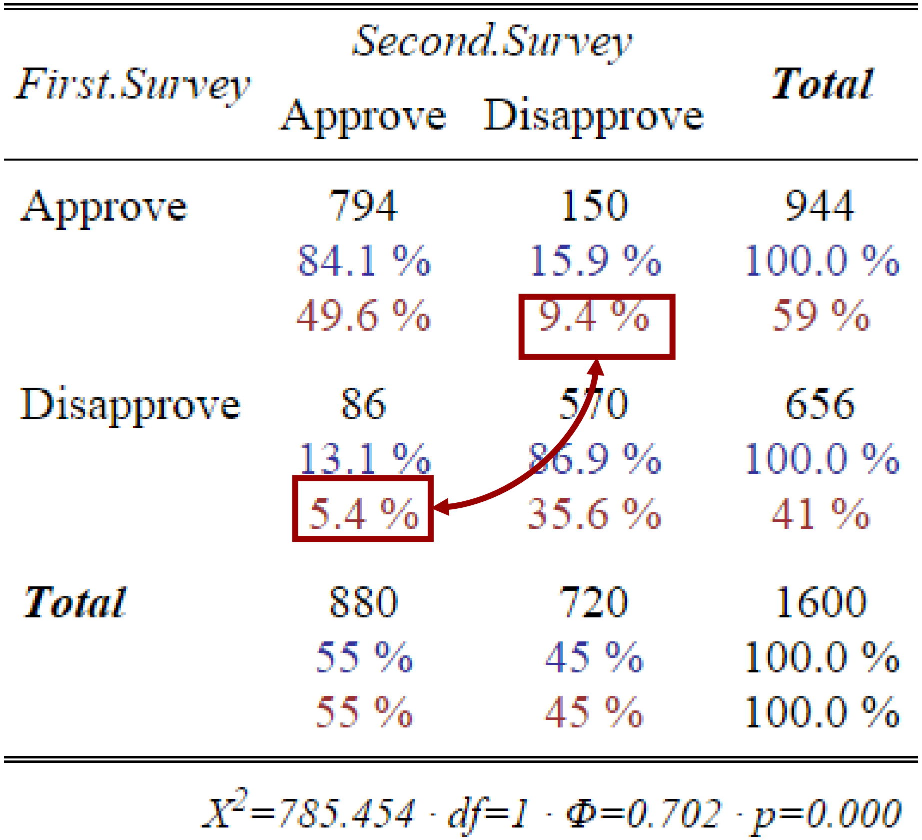
$X^2=785.454 \cdot df=1 \cdot \Phi=0.702 \cdot p=0.000$

4.8. *Two categorical variables*

4.8.2. Testing for the equality of proportions in dependent samples

Example 4-13: Prime minister’s approval

Using the function Crosstable



First.Survey	*Second.Survey* Approve	 Disapprove	***Total***
Approve	794 84.1 % 49.6 %	150 15.9 % 9.4 %	944 100.0 % 59 %
Disapprove	86 13.1 % 5.4 %	570 86.9 % 35.6 %	656 100.0 % 41 %
Total	880 55 % 55 %	720 45 % 45 %	1600 100.0 % 100.0 %

X^2=785.454 · df=1 · Φ=0.702 · p=0.000

4.8. *Two categorical variables*

4.8.2. Testing for the equality of proportions in dependent samples

The mcnemar.test for more than two levels

In the case of paired values with more than two levels, mcnemar.test implements the generalization of McNemar test which does not test the equality of the marginal distributions but the symmetry of the table which is a special case of the marginal homogeneity