

Shannon – Fano Algorithm for Data Compression

Multimedia Technology

Tutorial 1, section 3

Shannon – Fano Theory Reminder

- Shannon-Fano coding is a lossless data compression algorithm for multimedia.
 - No data is lost during compression.
 - With lossy compression, we have size reduction by removing unnecessary information.
- It assigns codes to symbols based on their probabilities of occurrence, with more frequent symbols receiving shorter codes.
 - Code length is closely related to the information content (or entropy) of each symbol.
- Why do we need data compression?
 - To optimize the representation of data for efficient transmission and storage.
- For more information refer to lecture slides.

Shannon – Fano Algorithm

Step 1:

List probabilities or frequency counts for the symbols to determine their relative frequency of occurrence.

Step 2:

Sort symbols in decreasing order of probability (most probable on the left, least probable on the right).

Step 3:

Split the list into two parts such that the total probability of both parts is as close as possible.

Step 4:

Assign 0 to the left part and **1 to the right part**.

Step 5:

Repeat the split for each part until all symbols are split into individual subgroups. (repeat steps 3 and 4)

Shannon – Fano Coding Tree Exercise

Let the alphabet {'a', 'b', 'c', 'd', 'e', 'f'}, with the following character probabilities: $P(a) = 0.25$, $P(b) = 0.15$, $P(c) = 0.30$, $P(d) = 0.10$, $P(e) = 0.15$, and $P(f) = 0.05$.

Construct a Shannon-Fano coding tree corresponding to this alphabet and calculate the average length of the resulting code.

Note: In Shannon – Fano, we only sort the nodes at the beginning of the algorithm

Shannon – Fano Algorithm

Step 1:

List probabilities or frequency counts for the symbols to determine their relative frequency of occurrence.

Symbol	a	b	c	d	e	f
Probability or Frequency	0.25	0.15	0.30	0.10	0.15	0.05

Step 2:

Sort symbols in decreasing order of probability (most probable on the left, least probable on the right).

Symbol	c	a	b	e	d	f
Probability or Frequency	0.30	0.25	0.15	0.15	0.10	0.05

Step 3:

Split the list into two parts such that the total probability of both parts is as close as possible.

Symbol	c	a	b	e	d	f
Probability or Frequency	0.30	0.25	0.15	0.15	0.10	0.05

0

1

Total Probability = 0.55

Total Probability = 0.45

Symbol	c	a
Probability or Frequency	0.30	0.25

Symbol	b	e	d	f
Probability or Frequency	0.15	0.15	0.10	0.05

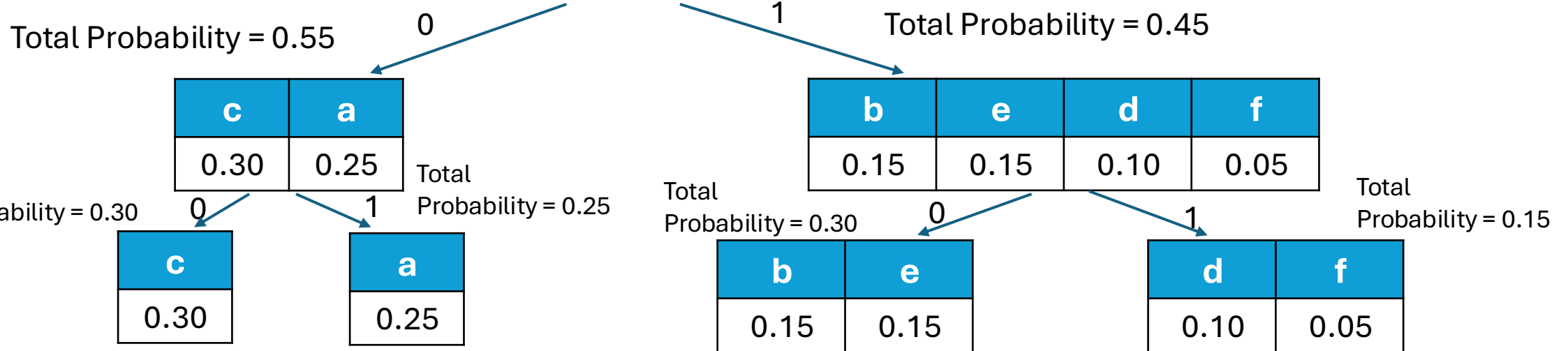
Step 4:

Assign 0 to the left part and 1 to the right part.

Step 3:

Split the list into two parts such that the total probability of both parts is as close as possible.

Symbol	c	a	b	e	d	f
Probability or Frequency	0.30	0.25	0.15	0.15	0.10	0.05

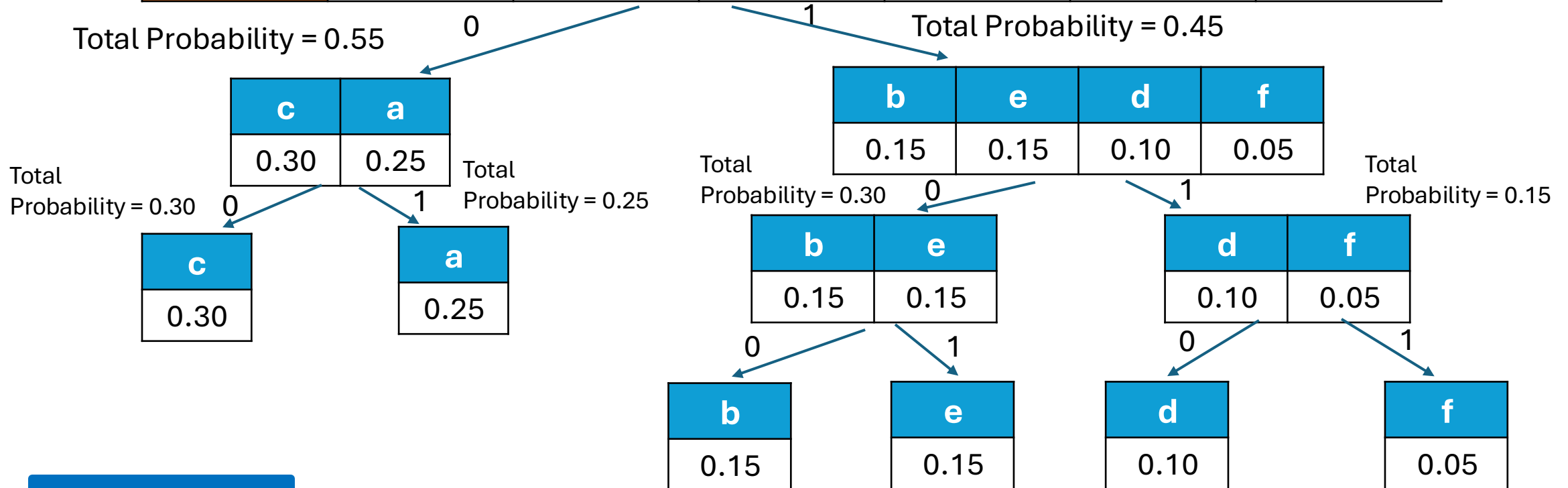
**Step 4:**

Assign 0 to the left part and 1 to the right part.

Step 3:

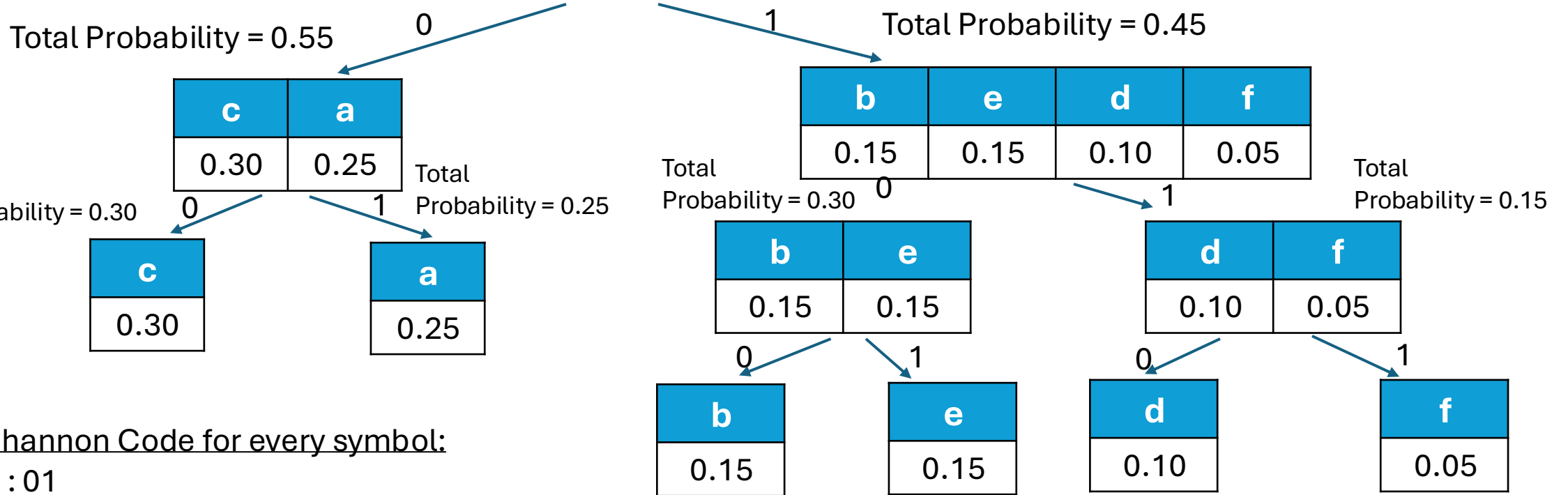
Split the list into two parts such that the total probability of both parts is as close as possible.

Symbol	c	a	b	e	d	f
Probability or Frequency	0.30	0.25	0.15	0.15	0.10	0.05

**Step 4:**

Assign 0 to the left part and 1 to the right part.

Symbol	c	a	b	e	d	f
Probability or Frequency	0.30	0.25	0.15	0.15	0.10	0.05



Shannon Code for every symbol:

- a : 01
- b : 100
- c : 00
- d : 110
- e : 101
- f : 111

Average length of the resulting code

$$\begin{aligned} &P(a) * 2\text{bit} + P(b) * 3\text{bit} + P(c) * 2\text{bit} + P(d) * 3\text{bit} + P(e) * 3\text{bit} + P(f) * 3\text{bit} = \\ &= 0.25 * 2 + 0.15 * 3 + 0.30 * 2 + 0.1 * 3 + 0.15 * 3 + 0.05 * 3 = \\ &3 * 0.45 + 2 * 0.55 = 1.35 + 1.1 = 2.45 \end{aligned}$$

Shannon Code for every symbol:

w(a) = 01

w(b) = 100

w(c) = 00

w(d) = 110

w(e) = 101

w(f) = 111

Average Length = $\sum p(s_i) * \text{number_of_bits}$,
for $i=\{0, 1, \dots, N\}$, where N the total number
of symbols