ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ — ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

ΣΧΟΛΗ ΕΠΙΣΤΗΜΩΝ & ΤΕΧΝΟΛΟΓΙΑΣ ΤΗΣ ΠΛΗΡΟΦΟΡΙΑΣ / SCHOOL OF INFORMATION SCIENCES & TECHNOLOGY — ΤΜΗΜΑ ΠΛΗΡΟΦΟΡΙΚΗΣ / DEPARTMENT OF INFORMATICS

## Ειδικά Θέματα Αλγορίθμων
# Ασκήσεις Φροντιστηρίου #6
# Algorithms for flows and matchings

**1.** Show that splitting an edge in a flow network yields an equivalent network. More formally, suppose that flow network $G$ contains edge $(u, v)$, and define a new flow network $G'$ by creating a new vertex $x$ and replacing $(u, v)$ by new edges $(u, x)$ and $(x, v)$ with $c(u, x) = c(x, v) = c(u, v)$. Show that a maximum flow in $G'$ has the same value as a maximum flow in $G$.

**2.** Extend the flow properties and definitions to the multiple-source, multiple-sink problem. Show that any flow in a multiple-source, multiple-sink flow network corresponds to a flow of identical value in the single-source, single-sink network obtained by adding a supersource and a supersink, and vice versa.

**3.** Suppose that, in addition to edge capacities, a flow network has vertex capacities. That is each vertex $v$ has a limit $l(v)$ on how much flow can pass through $v$. Show how to transform a flow network $G = (V, E)$ with vertex capacities into an equivalent flow network $G' = (V', E')$ without vertex capacities, such that a maximum flow in $G'$ has the same value as a maximum flow in G. How many vertices and edges does $G'$ have?

**4.** Suppose that each source $s_i$ in a flow network with multiple sources and sinks produces exactly pi units of flow, so that $\sum_{v \in V} f(s_i, v) = pi$. Suppose also that each sink $t_j$ consumes exactly $q_j$ units, so that $\sum_{v \in V} f(v, t_j) = q_j$, where $\sum_i p_i = \sum_j q_j$. Show how to convert the problem of finding a flow f that obeys these additional constraints into the problem of finding a maximum flow in a single-source, single-sink flow network.

**5.** Given a graph $G$ with unary weights. Show that the least amount edges that need to be deleted, in order to disconnect two vertices $s$ and $t$, is equal with the amount of maximum flow from $s$ to $t$ given that each edge has capacity equal to 1. **Hint:** What is the connection of the above quantities with the amount of edge-disjoint paths from vertex $s$ to $t$ in graph $G$.