## COMPUTER GRAPHICS COURSE

## Basic Animation

## FUNDAMENTAL CONCEPTS

## Introduction

- Computer animation: "life" given by presenting a sequence of still images (frames) in rapid succession:
- Sufficiently high rate $\rightarrow$ HVS perceives them as smooth motion or animation
- Minimum rate required for smooth motion $\approx 12 \mathrm{fps}$ :
- Below that, motion appears jerky
- In general, the frames-per-second (fps) limit is not constant; it depends on speed of movement of the objects as well as on illumination parameters


## "Tweening" and Key Frames

- Traditionally, most cartoon animation was performed by tweening, the drawing of frames in-between keyframes
- A key frame is a characteristic (or key) snapshot of the animation sequence that contains a significant stance of the moving geometry or imagery



## Computer-based Key Frame Animation

- In computer animation, key frames hold either directly the vertex data of the geometry or the parameters of higher-level entities (the simplest being transformation parameters)
- The data are interpolated using any available or suitable method (linear, spline, etc.)



## Tweening and Animation Parameters

- Typical animations require a lot of parameters if tweening is performed at a low representation level



## Animation Control Methods

- Impossible for an animator to define every animation variable for every frame; animation control methods have been developed
- Examples:
- Hierarchical mesh deformation techniques
- Procedural and representational methods for animating rigid bodies
- Physics and collision-driven approaches
- Skeletal animation for animating human-like or animal-like characters


## Animation Control Methods

- These methods use common low level techniques such as:
- Interpolation,
- Collision detection
- Motion blur
- Higher level animation control methods examined:
- Rigid body animation
- Skeletal animation
- Deformable models
- Particle systems


## Procedural Animation

- The encapsulation of the animation of an object in a procedure:
- Animation sequences can automatically be generated, often in realtime
- Particle systems: Largest subclass of procedural animation
- Rigid body and Skeletal animation: Can also be done procedurally
- Behavioral animation: Subclass of procedural animation where objects determine their own actions, taking into account their environment


## LOW-LEVEL ANIMATION TECHNIQUES

## A Lower Layer of Tools

- Interpolation techniques: Means by which computer takes over the task of tweening
- Collision detection: Essential for realism by detecting when moving objects collide so that appropriate action can be taken
- Anti-aliasing in time (Motion blur): essential to most animations
- Morphing: Allows smooth transition from one graphical object to another (in a \# of frames)


## Interpolation

- Animation uses interpolation to do the tweening work automatically
- Extreme values of the animation variables are specified by the user
- Values of animation variables are linked to frames of the animation:
- Since there is a 1-1 mapping between frames \& time, animation variables are linked to time
- Use parametric functions $f(t)$ to interpolate the animation variables between extreme values, e.g. $\mathbf{p}\left(t_{0}\right)$ and $\mathbf{p}\left(t_{1}\right)$, which become the interpolation control points


## Interpolation - Simple Example



## Choosing the Right Animation Variable

- Care must be taken in selecting the variables to be interpolated :
- Importance of animation variable selection, e.g.: Choosing (a) the endpoint and (b) the rotation angle as animation variable

(a)

(b)


## Interpolation and Control Points

- Interpolation is based on a parameter $t$ representing time
- Key frame values are control points for the animation
- They form time-value pairs, e.g.: $\left(t_{0}, v_{0}\right),\left(t_{1}, v_{1}\right)$
- Control points are not in general equidistant in the time domain:

- Interpolation functions pass through the interpolation control points, so $f\left(t_{0}\right)=v_{0}, f\left(t_{1}\right)=v_{1}$ for some $t_{0}, t_{1}$


## Linear Interpolation

- Simplest form of interpolation
- For two key points $\left(t_{0}, v_{0}\right),\left(t_{1}, v_{1}\right)$ :

$$
v=v_{0}(1-s)+v_{1} s, \quad s=\frac{t-t_{0}}{t_{1}-t_{0}}
$$

- For larger key point sequences, $t_{0}$ and $t_{1}$ are replaced with the nearest time stamps $t_{\text {prev }}$ and $t_{\text {next }}$ that $t$ falls between:

$$
t_{\text {prev }} \leq t<t_{\text {next }}
$$

## Linear Interpolation

- Good for:
- Interpolating any value in dense key frame sets
- Linear motions
- Linear state transitions of animation control variables (e.g. Euler rotation angles)
- Bad for:
- Sparse interpolation of positional data
- Interpolation of dramatic pose changes


## Bezier Interpolation

- Quadratic Bezier function interpolates between control values $v_{0}$ and $v_{2}$ using an extra value $v_{1}$ as an attractor:

$$
B^{2}(t)=(1-t)^{2} v_{0}+2 t(1-t) v_{1}+t^{2} v_{2} t \in[0,1]
$$

- The nth degree Bezier function interpolates between $v_{0}$ and $v_{n}$ using $n-1$ attractor values $v_{i}, i: 1 \ldots n-1$


## Other Parametric Interp. Functions

- Functions of parametric curves $\mathbf{X}(t)$ are good interpolation functions:
- Their tangent vector $\mathbf{X}^{\prime}(t)$ defines velocity $\rightarrow$ useful when used to describe motion
- The arc length travelled along such a curve function can be computed by integrating velocity
- Caution: In general arc length travelled is not proportional to the time parameter $t$ :
- Can not use constant differences of $t$ to get constant arc lengths of travel
- If this is desired, arc-length reparameterization $\left(^{*}\right)$ of a curve is required


## Curve Reparameterization Example



Control points
$F(t)$, equidistant $t$ samples $F(s)$, equidistant $s$ samples

## Interpolating Rotations

- Suppose we express an arbitrary rotation as a synthesis of 3 basic rotations $\mathbf{R}_{x}\left(\theta_{x}\right) \rightarrow \mathbf{R}_{y}\left(\theta_{y}\right) \rightarrow \mathbf{R}_{z}\left(\theta_{z}\right)$
- Animate this by gradually incrementing $\theta x, \theta y, \theta z \rightarrow$ problems:
- Rather difficult to estimate basic rotation angles that make up the required rotation about an arbitrary axis
- Encounter a "twisting" motion, as the rotations are applied sequentially \& the object seems to rotate alternately about the 3 axes
- Encounter a phenomenon known as gimbal lock


## Gimbal Lock Example



This particular sequence of rotations causes rotation around the x axis to be countered by rotation around the $z$ axis

## Quaternions

- Used as an alternative way to express rotation
- A quaternion consists of 4 real numbers: $\mathbf{q}=(s, x, y, z)$
$-s \rightarrow$ scalar part of quaternion $\mathbf{q}$
$-\overrightarrow{\mathbf{v}}=(x, y, z) \rightarrow$ vector part of quaternion $\mathbf{q}$
- Alternative representation: $\mathbf{q}=(s, \overrightarrow{\mathbf{v}})$
- Can be viewed as an extension of complex numbers in 4D:
- Using "imaginary units" $i, j$ and $k$ such that: $i^{2}=j^{2}=$ $k^{2}=-1$ and $i j=k, j i=-k$ and so on by cyclic permutation, quaternion q may be written as:

$$
\mathbf{q}=s+x i+y j+z k
$$

## Quaternions

- A real number $u$ corresponds to the quaternion: $\mathbf{q}=(u, \mathbf{0})$
- An ordinary vector $\overrightarrow{\mathbf{v}}$ corresponds to the quaternion: $\mathbf{q}=$ $(0, \overrightarrow{\mathbf{v}})$
- A point $\mathbf{p}$ corresponds to the quaternion: $\mathbf{q}=(0, \mathbf{p})$


## Properties of Quaternions (1)

- Addition between quaternions:

$$
q_{1}+q_{2}=\left(s_{1}, \vec{v}_{1}\right)+\left(s_{2}, \vec{v}_{2}\right)=\left(s_{1}+s_{2}, \vec{v}_{1}+\vec{v}_{2}\right)
$$

- Multiplication between quaternions:

$$
\begin{aligned}
& q_{1} \cdot q_{2}=\left(s_{1} s_{2}-\overrightarrow{\mathbf{v}}_{1} \cdot \overrightarrow{\mathbf{v}}_{2}, s_{1} \overrightarrow{\mathbf{v}}_{2}+s_{2} \overrightarrow{\mathbf{v}}_{1}+\overrightarrow{\mathbf{v}}_{1} \times \overrightarrow{\mathbf{v}}_{2}\right)= \\
& \left(s_{1} s_{2}-x_{1} x_{2}-y_{1} y_{2}-z_{1} z_{2}, s_{1} x_{2}+x_{1} s_{2}+y_{1} z_{2}-z_{1} y_{2},\right. \\
& \left.s_{1} y_{2}+y_{1} s_{2}+z_{1} x_{2}-x_{1} z_{2}, s_{1} z_{2}+z_{1} s_{2}+x_{1} y_{2}-y_{1} x_{2}\right)
\end{aligned}
$$

- Multiplication is associative
- Multiplication is not commutative


## Properties of Quaternions (2)

- The conjugate quaternion of q is defined as: $\bar{q}=(s,-\overrightarrow{\mathbf{v}})$
- It holds that: $\overline{q_{1}} \cdot q_{2}=\overline{q_{2}} \cdot \cdot q_{1}$
- The norm of $q$ is defined as:

$$
|q|^{2}=q \cdot \bar{q}=\bar{q} \cdot q=s^{2}+|\overrightarrow{\mathbf{v}}|^{2}=s^{2}+x^{2}+y^{2}+z^{2}
$$

## Properties of Quaternions (3)

- It holds that: $\left|q_{1} \cdot q_{2}\right|=\left|q_{1}\right| \times\left|q_{2}\right|$
- A unit quaternion is one whose norm: $|q|=1$
- The inverse quaternion of q is defined as: $q^{-1}=\frac{1}{|q|^{2}} \bar{q}$
- It holds that: $q \cdot q^{-1}=q^{-1} \cdot q=1$
- If $|q|=1$ then $q^{-1}=\bar{q}$


## Quaternion-based Rotations

- Quaternion rotation is more stable, requires fewer calculations \& consecutive rotations can be handled in a smooth way
- Two extreme positions of the rotation can be represented by 2 unit quaternions:
- $q_{0}=(1, \overrightarrow{\mathbf{0}})$ corresponding to the initial position and
- $q_{1}=\left(\sin \frac{\theta}{2}, \cos \frac{\theta}{2} \hat{\mathbf{n}}\right)$ corresponding to the position after rotation by $\theta$ around $\hat{\mathbf{n}}$


## Applying Quaternion Rotations

- The quaternion rotation $q=\left(\cos \frac{\theta}{2}, \sin \frac{\theta}{2} \overrightarrow{\mathbf{n}}\right)=(s, \overrightarrow{\mathbf{v}})$ can be applied to a point $\mathbf{p}$ in quaternion form as:

$$
p^{\prime}=q \cdot p \cdot q^{-1}=q \cdot p \cdot \bar{q}
$$

$$
p=(0, \mathbf{p})
$$

- Thus: $p^{\prime}=\left(0,\left(s^{2}-\overrightarrow{\mathbf{v}} \cdot \overrightarrow{\mathbf{v}}\right) \mathbf{p}+2 \overrightarrow{\mathbf{v}}(\overrightarrow{\mathbf{v}} \cdot \overrightarrow{\mathbf{p}})+2 s(\overrightarrow{\mathbf{v}} \times \overrightarrow{\mathbf{p}})\right)$

Rotated point

## Composite Rotations with Quaternions

- Expressing 2 consecutive rotations:

$$
q_{2} \cdot\left(q_{1} \cdot \mathbf{p} \cdot \overline{q_{1}}\right) \cdot \overline{q_{2}}=\left(q_{2} \cdot q_{1}\right) \cdot \mathbf{p} \cdot\left(\overline{q_{1}} \cdot \overline{q_{2}}\right)=\left(q_{2} \cdot q_{1}\right) \cdot \mathbf{p} \cdot \overline{\left(q_{2} \cdot q_{1}\right)}
$$

- The composite rotation is represented by the unit quaternion: $q=q_{2} q_{1}$
- Quaternion multiplication is simpler, requires fewer operations and is numerically more stable than rotation matrix multiplication


## Converting to and from Quaternions (1)

- Given a quaternion $q=(s, x, y, z)$, the corresponding rotation matrix is:

$$
\mathbf{R}_{q}=\left[\begin{array}{cccc}
1-2 y^{2}-2 z^{2} & 2 x y-2 s z & 2 x z+2 s y & 0 \\
2 x y+2 s z & 1-2 x^{2}-2 z^{2} & 2 y z-2 s x & 0 \\
2 x z-2 s y & 2 y z+2 s x & 1-2 x^{2}-2 y^{2} & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

## Converting to and from Quaternions (2)

- Given a rotation matrix:

$$
\mathbf{R}=\left[\begin{array}{cccc}
m_{00} & m_{01} & m_{02} & 0 \\
m_{10} & m_{11} & m_{12} & 0 \\
m_{20} & m_{21} & m_{22} & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

- The corresponding quaternion $q=(s, x, y, z)$ is:

$$
\begin{aligned}
s & =\frac{1}{2} \sqrt{m_{00}+m_{11}+m_{22}+1} \\
x & =\frac{m_{21}-m_{12}}{4 s}, \quad y=\frac{m_{02}-m_{20}}{4 s}, \quad z=\frac{m_{10}-m_{01}}{4 s}
\end{aligned}
$$

## Converting to and from Quaternions (3)

- If $s$ is near zero (or zero), a different set of values for the quaternion vector part can be used:

$$
\begin{gathered}
x=\frac{1}{2} \sqrt{m_{00}-m_{11}-m_{22}+1} \\
y=\frac{m_{01}+m_{10}}{4 x}, \quad z=\frac{m_{02}+m_{20}}{4 x}, \quad s=\frac{m_{21}-m_{12}}{4 x}
\end{gathered}
$$

## Interpolating Quaternions

- Linear interpolation between these 2 quaternions will not produce expected smooth rotation between the 2 positions:
- Instead a motion that would accelerate towards the middle
- Geometrically, unit quaternions representing rotations lie on the surface of the 4-D unit hypersphere $\rightarrow$ linear interpolation interpolates on the chord through them


## Spherical Linear Interpolation

- Smooth interpolation of the rotation can be achieved by performing spherical linear interpolation (slerp):
- Interpolation on the surface of the 4D unit hypershpere, along the great arc between $\mathbf{q}_{0}$ and $\mathbf{q}_{1}$ :

- where $\omega=\theta / 2, \theta$ the angle between the two directions


## COLLISION DETECTION

## Introduction

- At a coarse level, the objective of collision detection is to find pairs of objects which potentially intersect
- At a next level, collision detection determines:
- if a pair of objects truly intersects
- the intersection locus (point, edge, surface)


## Introduction

- When objects collide:
- We want to avoid inter-penetrations between solid/rigid objects
- Respond according to Newton's third law of motion (see physics simulation)


## Newton's third law of motion:

When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.

## Applications

- Determine collision and hit events in 2D and 3D computer games (that subsequently cause changes in the gameplay and animation sequences)
- Determine interaction points and surfaces in physics simulation



## Strategies - Continuous (a Priori)

- Consider a sweep volume as the object moves (or evolves in time) between two frames
- Accurate collision
- Very expensive to compute, especially for non-convex objects


Extruded Volumes (3D)

Sweep
Volumes (2D)

## Strategies - Discrete (a Posteriori)

- Sample the time interval and test instances of the animated object(s) at them for intersection
- Flexible and cheaper - favored in real-time applications
- Can miss intersections



## Improving Discrete Collisions

- Sampling interval can be refined based on intersection test results!
- Example:

$$
\begin{array}{c|c}
\text { Time } & \text { Test: }(\mathbf{p}-\mathbf{q}) \cdot \overrightarrow{\boldsymbol{n}} \\
\hline t_{0} & >e \\
t_{0}+\Delta t & >e \\
t_{0}+2 \Delta t & <e \text { (but not yet negative: Too close) } \\
&
\end{array}
$$

Indication that steps must be refined! $\rightarrow$ Adaptive sampling

## Collision Detection Complexity

- In general, for $N$ objects, $O\left(N^{2}\right)$ tests have to be made among them
- Each test entails $O\left(M_{i} \cdot M_{j}\right)$ primitive intersection tests, where $M_{i}, M_{j}$ the number of primitives of the two objects
- How to improve this:
- Limit $N$ (collision detection is not required between all object combinations) $\rightarrow$ collision groups
- Perform hierarchical tests
- Make Object-object or primitive-primitive tests really cheap


## Bounding Volume Hierarchies (1)

Preprocessing:

- Hierarchically group collision targets in clusters
- Find (convex) bounds for each cluster
- This bounding volume will serve as approximate collider for the cluster
- Keep grouping clusters in hierarchies

Run time, follow a branch and bound approach:

- Perform collision detection on high-level hierarchy nodes
- If successful, proceed to check lower hierarchy levels
- Otherwise, skip subtrees


## Bounding Volume Hierarchies (2)



- Simple mathematical primitives such as spheres and boxes are used as bounding volumes
- Often, the bounds are manually placed or adjusted
- Automatic bounds calculation does not know to dismiss soft or insignificant detail parts
- BVHs are also ideal for articulated models. They can follow the same structure and animation


## SKELETAL ANIMATION

## Skin and Bones

- We can think of a mesh as a "skin" that is wrapped around an animated skeleton
- The skeleton is a hierarchy of linked joints or "bones"



## Skeletal Animation Principle

- The surface is smoothly animated by:
- Assigning dependence weights of mesh vertices to bones (which joints the vertices "stick" to)
- Rigidly animating the bones
- Interpolating the motion of the skin from the bones



## A Simple Kinematic Chain (1)

- Let us define a simple chain of dependent "joint" nodes $\mathbf{p}_{j}$, each one being associated with its parent via a (rigid) transformation $\mathbf{M}_{j}=\mathbf{T}_{j} \mathbf{R}_{j}$
- Node 0 is expressed relative to the object's (or world) reference frame, so:
$\mathbf{p}_{0}=\mathbf{M}_{0} \cdot \mathbf{0}$

$\mathbf{p}_{j}=\mathbf{M}_{j} \cdot \mathbf{p}_{j-1}=\mathbf{M}_{0} \mathbf{M}_{1} \ldots \mathbf{M}_{j} \cdot \mathbf{0}$
$\mathbf{M}_{\text {Init }(j)}$


## A Simple Kinematic Chain (2)

- If we initially know the positions $\mathbf{p}_{j}$ and no other transformation is applied, then
- $\mathbf{M}_{\text {Init }(j)}=\mathbf{T}_{\mathbf{p}_{0}} \cdot \mathbf{T}_{\mathbf{p}_{1-} \mathbf{p}_{0}} \cdots \mathbf{T}_{\mathbf{p}_{j-} \mathbf{p}_{j-1}}=\mathbf{T}_{\mathbf{p}_{j}}$
- $\mathbf{M}_{\text {Init }(j)}$ expresses a point from the local reference frame of node $j$ (centered at $\mathbf{p}_{j}$ ) to the global reference frame

- Conversely, $\mathbf{M}_{\text {Init }(j)}^{-1}$ takes an arbitrary vertex from the global coordinate system and expresses it relative to $\mathbf{p}_{j}$


## A Simple Kinematic Chain (3)

- So, given an arbitrary vertex $\mathbf{v}_{i}$ on the mesh to be animated, it can be expressed relative to node $\mathbf{p}_{j}$ using $\mathbf{M}_{\text {Init }(j)}^{-1}$ :
- $\mathbf{v}_{i(j)}=\mathbf{M}_{\text {Init }(j)}^{-1} \cdot \mathbf{v}_{i}$



## Animating the Joints

- Typically an articulated structure is animated by recalculating a new local rigid motion $\mathbf{M}_{j}^{\prime}$ at each animation frame
- Then the changed position of each node and the respective local coordinate frame is defined w.r.t. the global system by the transformation:


$$
\mathbf{M}_{G l o b a l(j)}=\mathbf{M}_{0}^{\prime} \mathbf{M}_{1}^{\prime} \cdots \mathbf{M}_{j}^{\prime}
$$

## Animating the Skin Vertices

- To find the new position $\mathbf{v}_{i}^{\prime}$ of the $i$-th dependent skin vertex, we simply:
- Express it in local joint coordinates
- Apply the new global chain transformation

$$
\mathbf{v}_{j}^{\prime}=\mathbf{M}_{G l o b a l(j)} \cdot \mathbf{M}_{\text {Init }(j)}^{-1} \cdot \mathbf{v}_{j}
$$



## $\mathrm{C}_{j}$

- $\mathbf{C}_{j}$ is computed for every joint once per frame and reused for all vertices associated with the j-th joint


## Soft Skinning

- If we exclusively assign each skin vertex to a joint, the resulting animated mesh will be very rigid and abrupt folds will be formed at the joints (usually with mesh selfintersections)
- We clearly want a gradual transition of the effect of each joint across the skin to result in a smooth deformation
- Solution:
- Each vertex depends on multiple (usually adjacent) joints with a weighted contribution from each one


## Vertex Weights

- Assume we have $N$ joints in total and each vertex is allowed to be affected by max $M$ of them
- Bounding $M$ to a small value (e.g. 4) facilitates the GPU implementation of the procedure
- Each vertex then depends on the corresponding joints using a set of (convex) weights:

$$
w_{k} \geq 0 \quad k=1 \ldots M, \sum_{k=1}^{M} w_{k}=1
$$

- This in turn means that the rigid motion of a skeletal joint affects multiple vertices in a different degree of influence


## Indexing the Joints

- For each weight $w_{k}$, we also need to keep track which one of the $N$ bones is the $k$-th one used by vertex $i$
- So, each skin vertex is accompanied by:
- A table of $M$ precomputed weights $w_{k}$
- A table of (integer) associations $j(k)$ between $k$ and the ID $j$ of the referenced joint in the (global) array of joints



## Weighted Vertex Animation

- Given now:
- The skin vertices at rest pose $\mathbf{v}_{i}$
- the joint weights $w_{k}$ and indices $j(k)$,
- The final updated position $\mathbf{v}_{i}^{\prime}$ of each skin vertex is:

$$
\mathbf{v}_{i}^{\prime}=\left(\sum_{k=1}^{M} w_{k} \mathbf{C}_{j(k)}\right) \cdot \mathbf{v}_{i}
$$

## Weight Selection

- Weights are usually assigned manually or
- Assisted by automatic weight pre-calculation:
- Method A (Nearest neighbors)
- Non zero weights for closest 2 bones of each vertex
- Weighting according to distance
- Method B (Envelope)
- For each bone pair, assign a (unit) weight to each vertex it encounters within an area of effect (power envelop)
- Normalize weights for each vertex (unit sum)


## Weight Selection and Rest Pose (1)

- Bones need to be initially arranged as far apart as possible (min. interference)
- Rest pose: Crucifixion with spread



## Weight Selection and Rest Pose (2)



- Spread position: No interference across hierarchy branches

- Standing position: Arm and torso vertices share bones
- Same with legs


## Are we Done?

- The problem with using a matrix formulation for the joint transformations is that while skinning, we weight (linearly blend) the matrices to get the desired result
- This in effect has the same problems as linearly interpolating final positions rather than orientations in key frame animation (see interpolation section)
- Many undesirable artifacts arise, especially for large movements of the rig w.r.t the rest pose



## Dual Quaternion Skinning - Introduction

- To overcome the problem of linearly blending transformation matrices, we need to reshape the parameters to be interpolated into a more convenient form
- This form should have similar SLERP-like interpolation (see quaternions)
- All tuples of parameters must be compactly but fully represented ( translation + rotation, scaling is not relevant)


## Dual Quaternions

- Dual quaternions are an expansion of dual numbers to the quaternion form (or a quaternion of dual numbers, if you want)
- A dual number is defined similar to a complex number: $\hat{a}=a_{0}+\varepsilon a_{\varepsilon}, \varepsilon^{2}=0$
- A dual quaternion is similarly defined as a quaternion of dual coordinates: $\widehat{\mathbf{q}}=\widehat{w}+i \hat{x}+j \hat{y}+k \hat{z}$
- Or the concatenation of two quaternions (follows from the above): $\widehat{\mathbf{q}}=\mathbf{q}_{0}+\varepsilon \mathbf{q}_{\varepsilon}$


## Dual Quaternions and Rigid Motion

- Multiplication of a quaternion and a dual quaternion is a dual quaternion
- A unit* (i.e. unit length) dual quaternion always represents a composition of a rotation and translation
- Given:
- Rotation quat. of $\theta$ angle around $\mathbf{n}: q_{0}=\left(\sin \frac{\theta}{2}, \cos \frac{\theta}{2} \mathbf{n}\right)$
- Translation dual quat. by $\mathbf{t}: \hat{t}=1+\varepsilon \frac{1}{2} \mathbf{t}$
- A rotation followed by translation is:
$\left(\hat{\mathbf{t}}=1+\varepsilon \frac{1}{2} \mathbf{t}\right) \cdot q_{0}=q_{0}+\varepsilon \frac{1}{2} t q_{0}, \quad t=(0, \mathbf{t})$


## Revisiting Weighted Averages

- Having expressed all transformations in dual quaternion form,
- We can:
- Multiply dual quaternions instead of matrices to obtain global transformations in kinematic chains $\rightarrow$
- Always unify dual quaternions after multiplication (to ensure that they still represent a rigid motion)
- Greatly improves arithmetic stability
- Perform a weighted average of dual quaternions for skinning to solve artifacts:

$$
\operatorname{DLB}\left(\mathbf{w}, \hat{q}_{1}, \ldots, \hat{q}_{M}\right)=\frac{w_{1} \hat{q}_{1}+w_{2} \hat{q}_{2}+\cdots+w_{M} \hat{q}_{M}}{\left\|w_{1} \hat{q}_{1}+w_{2} \hat{q}_{2}+\cdots+w_{M} \hat{q}_{M}\right\|}
$$

## Dual Quaternion Results

Linear blending


Dual Quaternion Linear blending

## Motion Capture (MoCap)



It is the process of recording and transforming the motion of live subjects (humans or otherwise) in order to analyze it or transfer it to synthetic avatars

## Applications of MoCap

- Film production, games and live motion transfer (interactive applications, see for example Kinect games)
- Low latency, close to real time, results can be obtained $\rightarrow$ applicable in both offline and real-time rendering apps
- Cost reduction for keyframe-based animation sequences
- Motion complexity irrelevant
- Animator only fine-tunes results
- Many variations of a motion can be obtained quickly
- Complex movement and realistic physical interactions such as secondary motions, weight and exchange of forces can be easily recreated in a physically accurate manner


## Main MoCap Technologies

- Optical Tracking
- Marker tracking
- Markerless tracking
- Other
- Inertial Tracking
- Magnetic tracking
- Hybrid


## Optical Tracking

- Optical systems utilize data captured from image sensors to triangulate the 3D position of a point between two or more cameras calibrated to provide overlapping projections



## Optical Tracking and Interaction

- Optical tracking systems are commonly used nowadays for commodity interaction devices such as:
- Wireless position/orientation trackers (e.g. Wii mote)
- VR headset navigation tracking (head orientation and position)


SteamVR HTC Vive Lighthouse Tracking System

## Optical Tracking - Marker Tracking (1)

- Marker tracking. Markers (active or retroreflective) are placed on the subject and their position reconstructed from the multiple views
- Active markers:
- LED-powered markers enable higher marker-to-background contrast $\rightarrow$ better clarity/accuracy
- LED activation can be time-modulated to provide unique signatures (IDs) for each
 marker
- LEDs need power and control/sync circuitry


## Optical Tracking - Marker Tracking (2)

- Passive markers:
- Reflective surfaces, often beyond the visible range
- Color coding can distinguish markers or groups/configurations of them
- Easy to increase density
- Very lightweight



## Optical Tracking - Markerless Tracking

- Markerless tracking. Full-body 2D/3D geometry is captured (scanned) and salient points (corresponding to joints) are inferred by shape analysis
- No obtrusive gear or suits
- Fast setup
- Less accurate or detailed than marker tracking
- Ideal for interaction



## Optical Tracking Properties

- Pros:
- Lightweight user equipment
- Easily extensible
- Can capture groups of subjects
- Cons:
- Occlusion issues (especially in group capture). Can be mitigated with extra sensors/cameras
- Position capture only. Orientation must be inferred from neighboring detected points


## Inertial Tracking

- Inertial measurement units (IMUs), combining a gyroscope, a magnetometer, and an accelerometer, measure rotational rates. These rotations are transmitted to a base computer translated into a skeleton
- They are handy but may loose accuracy due to error propagation and need re-sync
- Good for orientation tracking and high occlusion performances
- Often complement other tracking techniques



## Magnetic Tracking

- Measure the induction current generated by the pairing of 3 perpendicular coils on both the receiver and transmitter
- Oldest tracking system
- Can be cumbersome
- Immune to any non-metallic obstacles
- Relatively small volume coverage
- Electromagnetic interference causes distortion

TEMPORAL ANTIALIASING MOTION BLURRING

## Temporal Antialiasing

- The removal of artifacts due to high-speed animation not adequately sampled by a) the image synthesis pipeline, b) the human visual system
- Shutter sync: A very distinctive artifact where objects appear either stationary or moving backwards due to inadequate frame rate
https://www.youtube.com/watch?v=y r3ngmRuGUc

- Typically addressed with super-sampling the time domain and low-pass filtering (motion blurring)


## Shutter Speed and Motion Blurring

- Motion blurring can be used to simulate the exposure time of a physical camera:
- The sensor accumulates light during the interval the shutter is open (exposure time)
- Corresponds to an integral of the input light over the open shutter time interval


Frame N

Frame N+1

## Fast-moving Objects (3)



- Motion appears jerky
- The motion is under-sampled: No information is present about the in-between positions of the object $\rightarrow$
- Aliasing in the temporal domain
- Higher velocity $\rightarrow$ greater aliasing (frame rate is fixed)


## Solutions

- Increase the frame rate
- Pre-filter the signal (filter before sampling)
- Post-filter the signal (filter after sampling)
- Similar strategies to spatial antialiasing!


## Camera Aperture



## Camera Shutter Speed

- Controls the time interval the shutter remains open



## Mathematical Formulation

- Measurement equation: Responsible for gathering the energy at a single pixel captured by the sensor. Accounts for:
- Lens aperture
- Exposure time

$$
L(\mathbf{x})=\int_{t_{1}}^{t_{2}} \int_{D(\mathbf{x})} \int_{\Omega} L(\mathbf{s}, \omega, t) W(\mathbf{s}, \omega, t) d \omega d \mathbf{s} d t
$$

- Considering only the temporal domain:

$$
L(\mathbf{x})=\int_{t_{1}}^{t_{2}} L(\mathbf{x}, t) W(\mathbf{x}, t) d t
$$

zero aperture and non-zero shutter
Reconstruction filter

## Antialiasing with Stochastic Sampling (1)

- The temporal dimension is sampled at random values of $t$, $t_{1} \leq t \leq t_{2}$ and the results are weighted according to $W(\mathbf{x}, t)$


## Antialiasing with Stochastic Sampling (2)

4 samples / pixel

## Antialiasing with Stochastic Sampling (3)

16 samples / pixel

## Antialiasing with Stochastic Sampling (4)

64 samples / pixel

## Antialiasing with Stochastic Sampling (5)

## 4x more samples to get $2 x$ better results (less variance)



## Antialiasing with Stochastic Sampling (6)



## Energy Conservation

- For a fixed exposure time, speed affects the intensity of the resulting image, as energy is "spread" to larger distances:


## Shutter Profiles (1)

instant open, instant close


Linear open, instant close


## Shutter Profiles (2)

Instant open, linear close


Linear open, linear close


## Complex profile




More motion samples towards the end of the shutter interval

## Real-time (RT) Post-filtering

- Re-use samples from previous frames
- Camera jitter + exponential averaging
- Motion vectors help recovering fragment position in the past
"Infiltrator" Unreal Engine 4 demo © Epic Games


## Motion-blur as Post-process Effect

- Typical solution for video games and real-time applications



## Temporal Pixel Reprojection and Velocity

- Locate the transformed position of the current pixel in the previous frame
- Retain transformation(s) from the previous frame(s)
- Transform and interpolate vertices
- For each pixel obtain transformed positions
- (optional) store pixel trajectories in velocity buffers



## Temporal Pixel Reprojection and Velocity



Depth buffer

Velocity buffer
2 float channels: dx, dy


## RT Post-filtering: Re-using Samples

- I found a sample from the previous frame! can I re-use it?
- Does it come from the right surface?
- Sample could be from a different object or a mix of objects (e.g. edge $\rightarrow$ background + foreground)
- Sample comes from the right object but it has drastically different properties
- e.g. don't want to re-use samples across the faces of a cube
- Did the current fragment even exist in the previous frame?
- Was partially or completely occluded?
- POV change?
- Were we even rendering it? (i.e. popped into existence in the current frame)


## RT Post-filtering: Artifacts

## Pros:

- Very fast run-time
- Easy to integrate in existing applications


## Cons:

- Visibility/occlusion is not properly resolved (can result in artifacts, "incorrect" image)



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