

Rasterization Architectures



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A High Level Rasterization Pipeline





- Geometry must be transformed in order to:
 - Be expressed in the proper coordinate system for each operation to take place
 - Get modified according to the desired arrangement of primitives / objects to form a virtual world or scene





• The vertices of the resulting primitives are then assembled into a form that can be efficiently sampled by the rasterizer (e.g. triangles):





- Redundant geometry (invisible, unimportant etc.) is culled (removed) to reduce overhead
- To further reduce/split load and avoid degenerate / problematic geometry, primitives are clipped to the boundaries of NDC regions





3D Geometry Transformations

- All coordinates have to be:
 - Transformed from their native, object space ones to a global, common reference system
 - Then expressed relative to the camera and
 - Projected on the image plane
- All of these transformations are concatenated into a single matrix, which is applied to the vertices of each triangle
- Different objects may have different transformations



Geometric Transformation Sequence





3D Geometry Setup (1)

Initial primitives (as defined/loaded by the application)





3D Geometry Setup (2)

• Transform geometry (vertices) in world coordinates to compose a 3D scene







 Coordinates as "seen" from the camera reference frame





3D Geometry Setup (5)

 Coordinates after
 perspective
 projection





3D Geometry Setup (6)

 Coordinates after perspective projection in normalized device coordinates





3D Geometry Setup (7)

 Primitives after clipping (still in normalized device coordinates)



Clipped primitives



3D Geometry Setup (8)

 Coordinates of assembled primitives after window transformation (image space – pixel units)





- With clipping we limit the extents of primitives to the viewing region
 - Avoid erroneous projection of geometry (see frustum clipping)
 - Discard invisible geometry
- In general, we clip lines and polygons in both 2D and 3D





- A hyperplane in 2D (a line) or in 3D (a plane) divides space in two halves
- The corresponding equation is positive on one side, negative on the other and zero exactly on the hyperplane:







 If a set of oriented hyperplanes f_i forms a convex region, then determining if a point p lies inside this region resolves to testing if:

 $sign(f_i(\mathbf{p})) = sign(f_j(\mathbf{p})), \forall i, j$





Point in Triangle Test

$$sign(y-s \cdot x-b)$$
$$s = \frac{y_n - y_1}{x_n - x_1} = \frac{\Delta y}{\Delta x}$$
$$b = \frac{y_1 x_n - y_n x_1}{x_n - x_1}$$

- Alternatively, we can check the barycentric coordinates of the the point w.r.t. the 3 vertices →
 - Inside: $u, v, w \ge 0$







Line Clipping on Rectangular Bounds

- 3 cases:
 - Line segment entirely outside region
 - Line segment entirely inside region
 - Line segment intersects 1 or 2 boundary segments





- Cohen-Sutherland algorithm
 - Fast segment in/out detection via binary tests
 - Recursive splitting of intersecting segments





CS Line Clipping Algorithm

```
void CS( vec3 * P1, vec3 * P2,
        float x min, float x max, float y min, float y max )
{
    unsigned char c1, c2;
    vec3 I;
     c1=Code(*P1);
     c2=Code(*P2);
     if ( ( c1|c2 == 0 ) || // both inside or
          (c1&c2 !=0)) // outside but on the same side of a
                            // clipping line (see figure)
                             // do nothing
    else
         {
             Intersect (P1, P2, &I, xmin, xmax, ymin, ymax);
             if ( IsOuside(*P1) )
                 *P1 = I;
             else
                 *P2 = I;
             CS(P1, P2, xmin, xmax, ymin, ymax);
          }
```



Polygon Clipping



- Polygon clipping cannot be regarded as multiple line clipping!
- Requires mutual edge + point containment and intersection testing



- Clips an arbitrary polygon against a convex clipping polygonal region
- Iteratively clips the input polygon against each one of the segments of the clipping region





- For each clipping line:
 - For each vertex transition of the input polygon:
 - Determine what points to generate according to the following configurations
 - Join all sequentially generated vertices to form a polygon
 - Use this polygon as input to the next iteration





• Clipped triangles against the viewing window may require re-triangulation



• Triangulation of convex shapes is trivial:





- Before rasterizing the polygons, they must be clipped against the view frustum (see projections)
- Why?
 - Coordinates behind near plane get inverted and wrap beyond the far plane → degenerate, impossible "triangles"
 - Coordinates on z=0 \rightarrow singularity in perspective division



- Frustum clipping can be done with a Sutherland-Hodgman-style method for triangles/planes
- For a 6-plane frustum (i.e. the camera frustum), this is a 6-stage triangle/plane clipping pipeline
- Clipping is performed in the post-projective space, before the perspective division. Why?
 - In all projections (perspective, too), the frustum planes are axis aligned → simplified comparisons and equations (see Chapter 5.3 in [G&V]



Frustum Clipping (3)

- Triangle/plane clipping:
 - Perform 2 line-plane clipping steps
 - Join the open edges (if any)
 - Re-triangulate if necessary





- It is possible to perform clipping at a pixel level (or pixel block level, for hierarchical implementations)
- Pixel-level clipping boils down to discarding values outside the usable range (i.e. within the 2D/3D clipping region)
 - Saves on H/W and power consumption (less circuitry)
 - Naïve implementation: Not very fast many samples to discard
 - Hierarchical / block-based implementation: efficient



Optimizations – Back-face Culling (1)



- With back-face culling (~50% fewer triangles)
- Back-face culling can dramatically reduce the rasterization load by effectively discarding all polygons facing off the eye direction
- Transparent shapes should not be BF culled



Optimizations – Back-face Culling (2)



 Back-face culling rejects polygons whose normal deviates more than 90 degrees from the viewing direction



- Conservatively discards entire objects early on, before clipping by:
 - Checking the extents (bounding box) of an object against the bounds of the frustum
- This test is very simple in post-projective space:
 - if all projected bounding box corners are outside the frustum → cull the object
 - Can be extended to non-camera frusta to cull hidden objects



http://akhanubis-eng.tumblr.com/post/24375086110/slimdx-directx-11-frustum-culling



- Rasterization is the process that generates the pixelbased samples on the stream of primitives
- Before rasterization occurs, it is convenient to transform the primitives in screen coordinates (i.e. pixel units) – see rasterization slides
- Each primitive is processed independently!



Fragments from different primitives may overlap → Ordering must be resolved (see next slides)



Line Rasterization

- Must:
 - Approximate the mathematical line as close as possible (min. error)
 - Not leave any gaps
 - Maintain a constant width
 - Be efficient





• Given a line segment in the first octant $(x_1, y_1) \rightarrow (x_2, y_2)$, the line passing through the

endpoints is defined as:




```
void Line1( float x1, float y1, float x2, float y2 )
Ł
    float s, b, y;
    float x;
   s = (y2-y1) / (x2-x1);
   b = (y1*x2 - y2*x1) / (x2-x1);
    for (x = x1; x \le x2; x = 1.0f)
    {
       y = s*x + b;
       SetPixel( floor(x+0.5f), floor(y+0.5f) );
    }
```



Result of the Line1 Algorithm

• Y values are eventually rounded to the nearest integer cell





- Y values are computed for fixed and positive X increments
- The described algorithm (Line1) is valid only for octant 1:





• The multiplication inside the loop can be simplified, since:

$$x_{i+1} = x_i + 1$$

$$y_{i+1} = sx_{i+1} + b = sx_i + b + s = y_i + s$$



```
void Line2( float x1, float y1, float x2, float y2 )
{
    float s, y;
    float x;
    s = (y2-y1) / (x2-x1);
    y = y1;
    for (x = x1; x \le x2; x = 1.0f)
    {
        SetPixel( floor(x+0.5f), floor(y+0.5f) );
     \rightarrow y = y+s;
    }
```



- If all coordinates are integer values, there are several improvements to be made to save calculations:
 - Drop the rounding, by stepping to the next Y value if the increment becomes larger than 1/2 pixel
 - Scaling all comparisons by Δx to dispense with the division





 Sampling the triangles involves traversing their interior and edges and generating a set of fragments per pixel (typically one)





• Similar to lines, triangle rasterization must not leave gaps, for thin triangles:





 Appearance must be as consistent as possible under slight sampling offsets (motion) – see antialiasing



Adapted from CG lecture notes from the Virginia University



Triangle Rasterization Issues (3)

• What is the priority of shared edges?



Adapted from CG lecture notes from the Virginia University



- Two dominant methods:
 - Edge Walking: Vertically follows edges and draws the corresponding scan line spans
 - Edge Equation: Tests the pixels for containment inside the triangle boundaries. Can be efficiently implemented in a divide and conquer manner



Edge Walking – Basic Idea

(AKA: Triangle Digital Differential Analyzer)

- Follow edges vertically
- Interpolate attributes down edges
- Fill in horizontal spans for each scanline
 - For each pixel of a scanline, interpolate edge attributes across span





Edge Walking – Procedure

Sort Vertices by Y value

Scan Convert 2 sub-triangles:

- For $y_1 \le y < y_2$:
 - Interpolate $x(x_a, x_b)$ and other values along edges
 - For $x_a \le x < x_b$: interpolate values along spans
- For $y_2 \le y < y_3$:
 - Interpolate $x(x_a, x_b)$ and other values along edges
 - For $x_a \le x < x_b$: interpolate values along spans





Edge Walking – Attribute Interpolation





- Scanline-style edge walking is reasonably good provided that you don't care about:
 - Aligned (coherent) memory access
 - Parallelism: multiple rows at a time
 - Variable sample positions
 - Ability to harness wide SIMD or build efficient hardware for it
- The above become really problematic especially in the case of thin, elongated triangles



Edge Equation Traversal – Basic Idea

 (x_{min}, y_{min})

0

- Triangle setup:
 - Find the bounding box of the triangle
 - Find the edge (line) equations of the oriented edges
 - Find triangle differentials
- For all pixels in the grid:
 - Find edge equation values $\varepsilon_1, \varepsilon_2, \varepsilon_3$
 - $\text{ If } (\varepsilon_1 > 0) \land (\varepsilon_2 > 0) \land (\varepsilon_3 > 0)$
 - Interpolate attributes
 - Issue Fragment

Embarrassingly parallel!

 (x_{max}, y_{max})



Edge Equation Values

$$y = s \cdot x + b \Longrightarrow e = sx - y + b$$

$$s = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

$$b = \frac{y_1 x_2 - y_2 x_1}{x_2 - x_1}$$





- Use barycentric coordinates!
- Can I incrementally construct the barycentric coordinates per pixel?
 - YES!
 - We can also incrementally update the edge equations per pixel

- Given two vectors \mathbf{v}_1 and \mathbf{v}_2 , the following determinant calculates the signed area of the formed parallelogram: $A_p(\mathbf{v}_1, \mathbf{v}_2) = \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix}$
- Or the signed area of the triangle formed by \mathbf{v}_1 and \mathbf{v}_2 : $A_t(\mathbf{v}_1, \mathbf{v}_2) = \frac{1}{2} \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix}$
- Remember, these quantities are signed
- The sign is determined by the order of the two vectors



- Now consider an edge $p_0 p_1$ of a triangle and an arbitrary point \boldsymbol{q}
- Using as vectors v₁ = p₀p₁ and v₂ = p₀q the determinant defines an edge function of q w.r.t. edge p₀p₁:



Source: http://fgiesen.wordpress.com/2013/02/06/the-barycentric-conspirac/



• Expanding and rearranging $F_{01}(\mathbf{q})$ we get:

$$F_{01}(\mathbf{q}) = \begin{vmatrix} x_1 - x_0 & x_q - x_0 \\ y_1 - y_0 & y_q - y_0 \end{vmatrix} \Leftrightarrow$$

$$F_{01}(\mathbf{q}) = (y_0 - y_1)x_q + (x_1 - x_0)y_q + (x_0y_1 - y_0x_1)$$

• Equivalently, for the other triangle edges:

$$F_{12}(\mathbf{q}) = (y_1 - y_2)x_q + (x_2 - x_1)y_q + (x_1y_2 - y_1x_2)$$

$$F_{20}(\mathbf{q}) = (y_2 - y_0)x_q + (x_0 - x_2)y_q + (x_2y_0 - y_2x_0)$$



 \mathbf{p}_0

 \mathbf{p}_2

 \mathbf{p}_1

- Remember that $F_{01}(\mathbf{q})$ is related to the area of the triangle $\mathbf{p}_0 \mathbf{p}_1 \mathbf{q}$
- But so is the barycentric coordinate of \mathbf{q} from \mathbf{p}_2 !
- It is easy to see that if w₀, w₁, w₂ are the 3 barycentric coordinates, then:

$$w_{0} = F_{12}(\mathbf{q})/w$$

$$w_{1} = F_{20}(\mathbf{q})/w$$

$$w_{2} = F_{01}(\mathbf{q})/w$$

$$w = F_{01}(\mathbf{q}) + F_{12}(\mathbf{q}) + F_{20}(\mathbf{q})$$



• Lets take the edge function and simplify it:

$$F_{01}(\mathbf{q}) = (y_0 - y_1)x_q + (x_1 - x_0)y_q + (x_0y_1 - y_0x_1) = A_{01}x_q + B_{01}y_q + C_{01}$$

- The terms A_{01} , B_{01} , C_{01} as well as the respective terms of the other edge functions are constant per triangle
 - Can be computed once in the triangle setup phase



Let's look now what happens for adjacent pixel coordinates:

 $F_{01}(x_q + 1, y_q) = A_{01}(x_q + 1) + B_{01}y_q + C_{01} = F_{01}(x_q, y_q) + A_{01}$ $F_{01}(x_q, y_q + 1) = A_{01}x_q + B_{01}(y_q + 1) + C_{01} = F_{01}(x_q, y_q) + B_{01}$

 So, shifting the calculation to 1 pixel ahead in either direction only involves the addition of a constant term!

Source: http://fgiesen.wordpress.com/2013/02/10/optimizing-the-basic-rasterizer/



• More importantly, for parallel (vectorized) computations:

$$F_{ij}(x_{UL} + n, y_{UL} + m) = F_{ij}(x_{UL}, y_{UL}) + nA_{ij} + mB_{ij}$$

- where (x_{UL}, y_{UL}) is the upper-left corner of the bounding box
- The barycentric coordinates (interpolation variables) are computed from $F_{ij} \rightarrow$ These are independently and cheaply computed, too!



- We can effectively reduce further the computations if we process the bounding box in blocks and discard entire blocks
 - Block discard: all block corners outside the triangle
 - Can be done hierarchically





- Is there a problem with interpolating in perspective?
 - Screen-space interpolation does not correctly interpolate perspectively projected values:



Source: Kok-Lim Low, Perspective-Correct Interpolation, Tech. Rep. 2002



• Linear in screen space \rightarrow Non-linear in eye space!





- Fortunately, we can derive functions that correctly perform this interpolation
- For the perspectively correct z:

$$z_{s} = \frac{1}{\frac{1}{z_{1}} + s\left(\frac{1}{z_{2}} - \frac{1}{z_{1}}\right)}$$

- i.e., interpolate 1/z values and invert the result
- For the derivation procedure see: Kok-Lim Low, Perspective-Correct Interpolation, Tech. Rep. 2002



• For perspectively-correct fragment attributes:

$$a_s = z_s \left(\frac{a_1}{z_1} + s \left(\frac{a_2}{z_2} - \frac{a_1}{z_1} \right) \right)$$

- i.e., divide vertex attributes by the corresponding z and multiply interpolated result by interpolated z
- For the derivation procedure see: Kok-Lim Low, Perspective-Correct Interpolation, Tech. Rep. 2002



- Aliasing in geometry boundaries due to fixed-rate sampling is a common artifact manifested as "pixelization"
 - Blocky appearance
 - Improper representation of thin structures
 - Temporal artifacts





 The problem is alleviated by mitigating the sampling issues to a higher sampling frequency by supersampling each pixel



Adapted from "Real-Time Rendering, 3rd Ed."



 Supersampling the pixel normally implies evaluating the shading at all samples taken →

– Cost: × number of samples!

 Solution: Evaluate the shading at a single location and take multiple coverage samples independently
 → MSAA (Multi-Sampled Anti-Aliasing)



Fragment shader is invoked once per pixel

Primitive coverage is evaluated independently at multiple locations



MSAA - Example



1X (no MSAA), 2X, 4X and 8X coverage samples on an NVIDIA 780Ti graphics card

Fragment shader evaluation location Coverage sample



MSAA - Deficiencies



- Shader computations may be performed for locations outside the geometry!
 - Can be fixed by moving the shading to the covered sample closest to the center



 Attributes evaluated at the pixel center my not be representative of the covered area



 Rasterized fragments overlap with previously drawn fragments from other triangles – not yet sorted






• The fragments of a primitive typically overlap fragments from other primitives



- There are many strategies to
 resolve the ordering of the
 rasterized primitives as they
 appear on screen
- Simplest:
 - Explicit order (FIFO)
- 3D: More elaborate schemes required (see 3D rasterization)





- Sorting can occur in various stages of the pipeline, depending on the type of primitives:
 - E.g., flat 2D polygons and lines can be trivially pre-sorted according to "z order" and then rasterized back to front
 - Conversely, intersecting or self-overlapping shapes may require a (post-) sorting strategy, at a fragment level (see 3D)

Can be resolved by primitive sorting



Cannot be resolved by primitive sorting – requires sorting at fragment level



Rasterization and HSE in 3D

- After projecting the primitives in NDC, we must retain only surfaces visible to the camera (HSE) →
 - Surface parts must be sorted according to depth
 - And not according to order of appearance (it is arbitrary)







- Even if polygons were depth-sorted according to some reference point on them (e.g. centroid), there is no guarantee that they do not overlap →
- Sorting must be performed per pixel





The Depth Buffer

- Separate buffer, same resolution as frame buffer
- Stores the nearest normalized depth values





- The Z-Buffer algorithm uses the depth buffer to compare each generated fragment at location (i,j) with the previous "visible" (nearest) fragment
- If the new fragment is closest to the view plane:
 - Replace the z in the depth buffer
 - Forward the fragment to the merging stage
- Else (if fragment fails the depth test)
 - Discard the fragment
- Remarks:
 - The depth test may be <, \leq or other comparison operand
 - Depth buffer is usually initialized to the "far" value



The Z-Buffer: A Simple Example





Z-Buffer – Optimization: Z Cull

- Split buffer into blocks (can use rasterization tiling)
- For each block maintain: z_{\min} , z_{\max}
- Compare the min/max z of an incoming triangle to the block's range:







- In general, the fragment (pixel) shading process defines a color and transparency value for each generated geometry fragment
 - In the simplest case of a flat-colored primitive, e.g. a 2D polygon fill, a predetermined color is assigned to the fragments
 - More elaborate shading algorithms are required for lit and textured 3D surfaces (see texturing and shading chapters)



Triangle Rasterization – HSE

• Triangle Fragments with correct order after z-buffer testing





• Triangle fragments after shading and merging





- Shaded fragments that successfully passed the depth test must contribute to the image in the frame buffer
- In general:
 - Each fragment contributes to the image pixel according to coverage
 - The color is blended with any existing one in the same pixel coordinates. This is especially true for transparent pixels
- All typical rasterization pipelines allow for a number of blending functions to be applied to the incoming fragments



- When transparency values are generated, these can control the mixing of fragments
- The value controlling this blending is the alpha value, i.e. the "opacity" (or 1-transparency)





 Extreme values (1,0), can make fragments "pass through" or opaque, to display elaborate "perforated patterns" (see texturing)



Completely transparent



Compositing: Simple Examples





- Transparency is not handled well by the Z-Buffer algorithm:
 - Result depends on the order of occurrence of the fragments: Depth test discards fragments behind transparent surfaces if the latter are already rendered





Z-Buffer and Transparency (2)

- Solution 1:
 - Render all opaque geometry first
 - Render transparent geometry next
- Still:
 - Blending of transparent surfaces is still order (and view) dependent



Image source: AMD Mecha Demo



- Is a generic antialiased fragment resolve technique, with full support for order-independent transparency
- Instead of a single (nearest) depth value, it maintains a sorted list of all fragments intersecting the pixel
- Stores per fragment transparency and coverage
- Merging:
 - Fragments are resolved front to back according to coverage (via a binary coverage mask) and their transparency



The A-Buffer (2)





The A-Buffer (3)



- Fragment token lists are updated using an atomic global counter
- The A-buffer retains a list head for each pixel



The A-Buffer (4)

- Expensive technique:
 - Must maintain a dynamic list per pixel (fragment bin)
 - Must contain additional data per fragment
 - Must sort contents in each fragment bin
 - Uses indirection (pointers) to access next datum
- H/W implementations?
 - Various optimized variants (or cut-down versions) implemented as shaders
 - Most popular variation: the k-Buffer
 - Fixed-size fragment buckets (arrays)
 - Sorting is still required



- Georgios Papaioannou
- Sources:
 - [RTR] T. Akenine-Möller, E. Haines, N. Hoffman, Read-time Rendering (3rd Ed.), AK Peters, 2008
 - [G&V] T. Theoharis, G. Papaioannou, N. Platis, N. M.
 Patrikalakis, Graphics & Visualization: Principles and Algorithms, CRC Press
 - [KV] Efficient Illumination Algorithms for Global Illumination in Interactive and Real-Time Rendering, PhD Thesis, K. Vardis, 2016
 - [OBR] <u>http://fgiesen.wordpress.com/2013/02/10/optimizing-</u> <u>the-basic-rasterizer/</u>