COMPUTER GRAPHICS COURSE

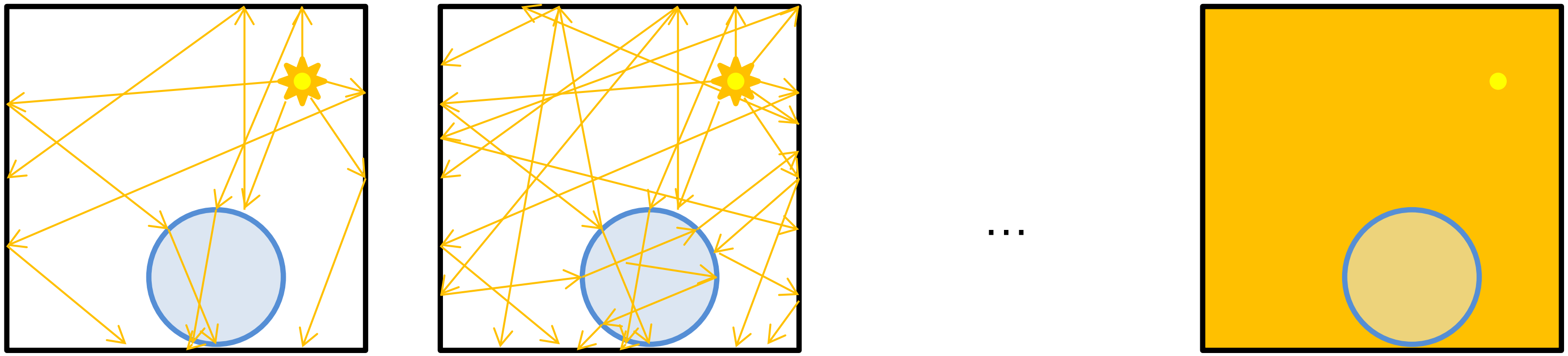
Light Transport

Foundations

Georgios Papaioannou - 2015

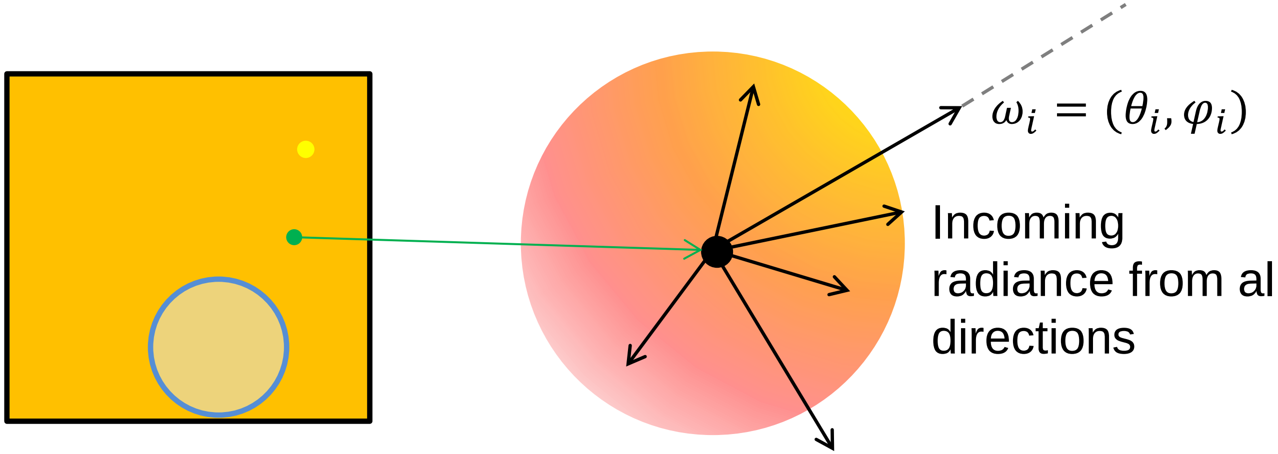
Light Transport

- Light is emitted at the light sources and scattered around a 3D environment in a practically infinite number of directions and scattering events
- This physical process, although it can be mathematically modelled, it cannot be practically solved analytically to yield the resulting illumination at each and every point in the scene



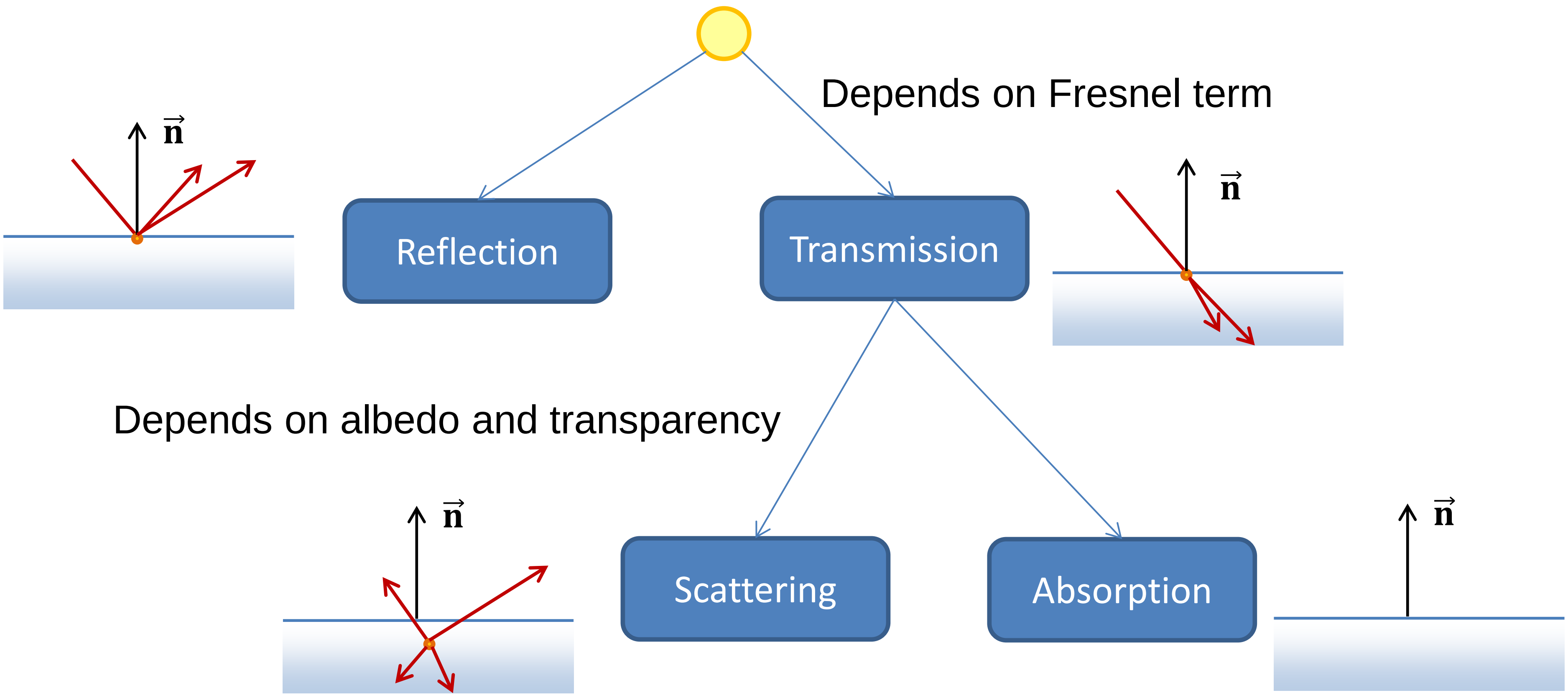
Light Transport – The Light Field

- Given:
  - The additive nature of light
  - The optical independence of the light transport directions
- We can consider the radiance at any point in space and any transport direction as a 5 DoF function, $L(\mathbf{p}, \omega)$ representing a light field:



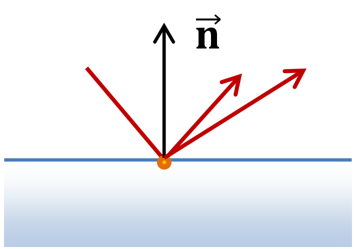
Light Transport Events (1)

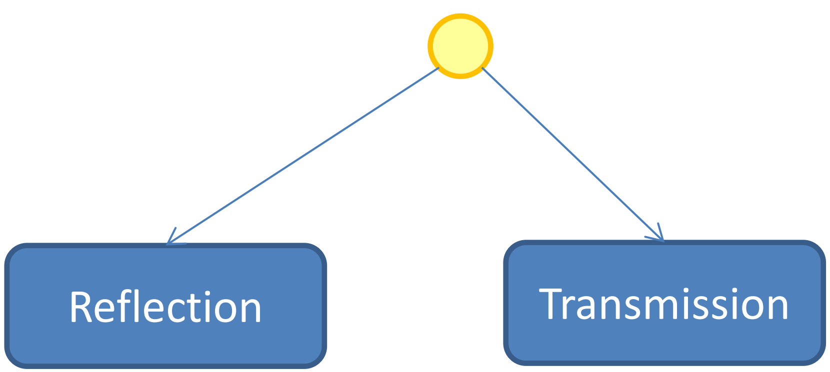
- When light hits a surface, the following events occur:



Light Transport Events (2)

- We have seen that reflected light is given by the reflectance equation using a specular BRDF
- Remember, the Fresnel term determines the splitting of energy between reflected / transmitted energy →
  - Transmitted: 1-reflected
- Reflection is a specular event*

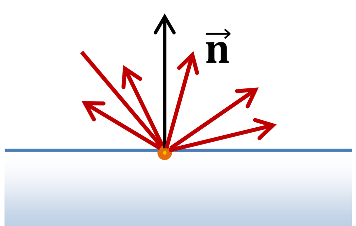


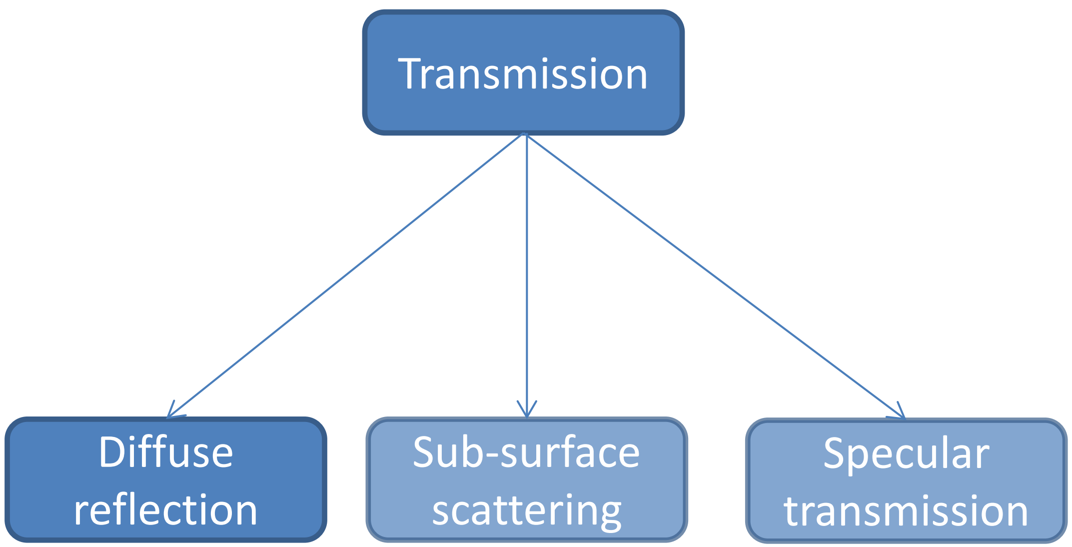


* Not to be confused with the events in path notation

Light Transport Events (3)

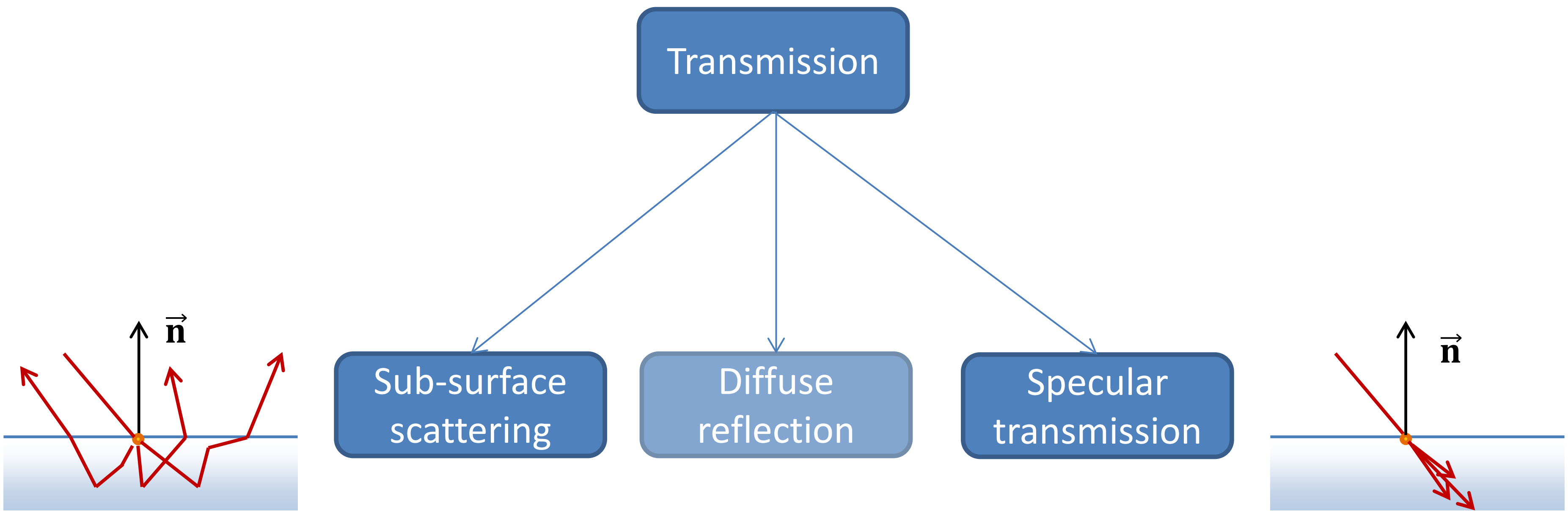
- Transmitted energy is scattered inside the body of the object
- Energy immediately scattered back towards the surface is treated as a diffuse event
  - Typically considering a uniform scattering: Lambertian surface “reflection” → Lambertian BRDF





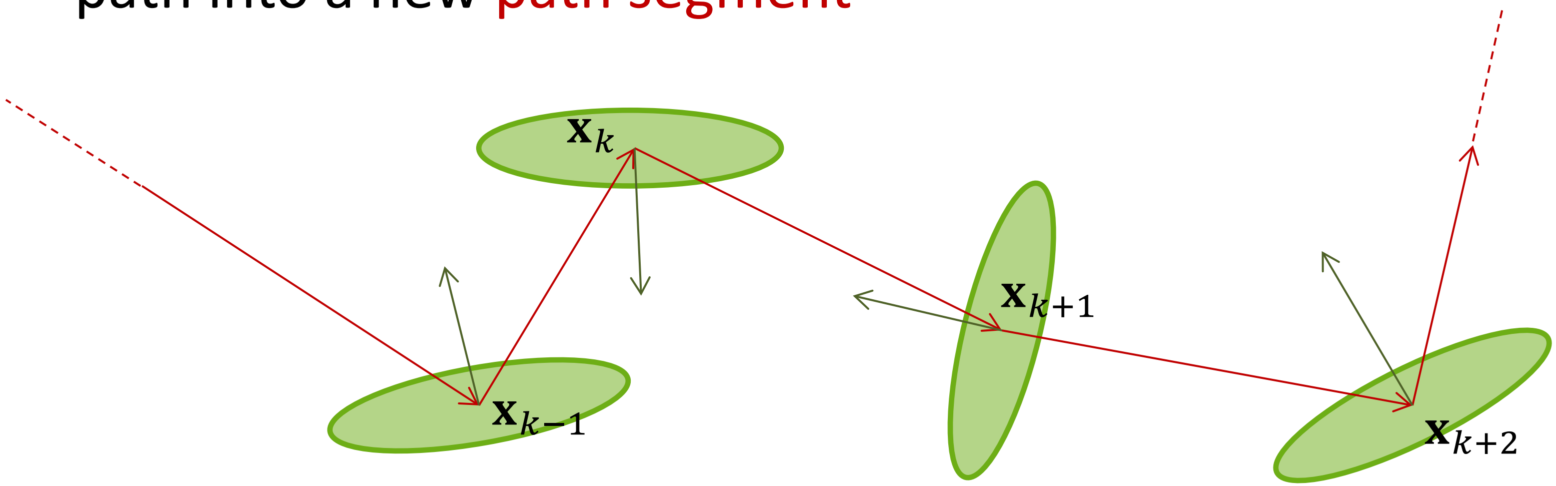
Light Transport Events (3)

- Outgoing energy after a sub-surface scattering process is also a diffuse event , but not a local one
- Highly directional transmission (e.g. in relatively clear media) is a specular event



Modeling Light Transport with Paths

- In graphics, we typically use the mechanisms of geometric optics to calculate the trajectory of transmitted light in space:
  - Radiance travels in straight paths
  - Light interacts with geometry and each event diverts its path into a new path segment

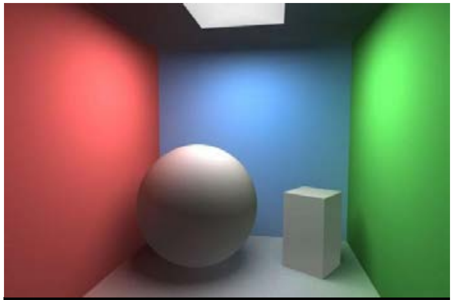


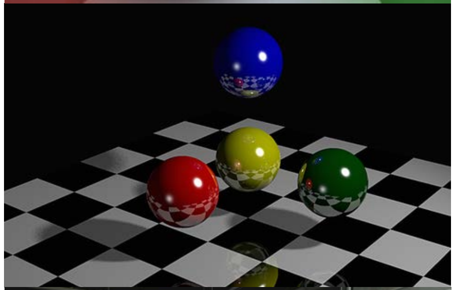
Path Notation (1)

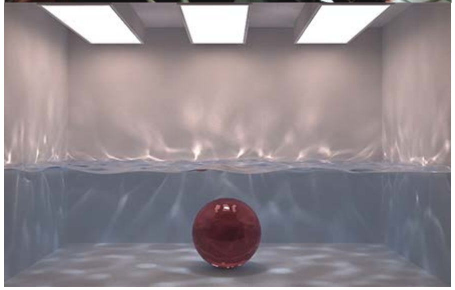
- Heckbert introduced a regular expression path notation based on the events a renderer can reproduce
- Nodes in a path represent one of the following:
  - L: light emission
  - E: “eye” sensor
  - D: diffuse scattering
  - S: ideal reflection/refraction. Regards *deterministic* paths
  - G: glossy (or non-ideal) transmission or reflection

Path Notation (2)

- Nodes are combined in regular expressions such as:
  - LD+E: Precomputed diffuse inter-reflections (radiosity algorithm)
  - ES*(D|G)L: Whitted-style recursive ray tracing
  - E(D|G)L: Local-only shading (direct rendering or ray casting)
  - L(G|S)+DS*E: Caustics







The Rendering Equation (1)

- Expresses the equilibrium of light distribution at each point in a scene
- It answers the question: "How much radiance leaves a location in a specific direction given a distribution of incident radiance values"
  - What is the total outgoing radiance (all directions)?

The Rendering Equation (2)

Taking into account the irradiance from all incident directions over the hemisphere above the surface point, the reflected radiance is:

$$L_o(\mathbf{x}, \omega_o) = \int_{\Omega_i} L(\mathbf{x}, \omega_i) f_r\left(\mathbf{x}, \varphi_o, \theta_{o,} \varphi_i, \theta_i\right) \cos\theta_i \, d\sigma(\omega_i)$$

Reflectance equation

$f_r\left(\mathbf{x}, \varphi_o, \theta_{o,} \varphi_i, \theta_i\right) = f_r\left(\mathbf{x}, \omega_o, \omega_i\right)$: BRDF

$d\sigma(\omega_i)$: Differential solid angle centered at direction ω_i

The Rendering Equation (3)

- To also account for the self-emitting surfaces (incandescence), an emission term (for most surfaces zero) is added:

$$L_o(\mathbf{x},\omega_o) = L_e(\mathbf{x},\omega_o) + \int_{\Omega_i} L(\mathbf{x},\omega_i) f_r\left(\mathbf{x},\varphi_o,\theta_o,\varphi_i,\theta_i\right)\cos\theta_i \, d\sigma(\omega_i)$$

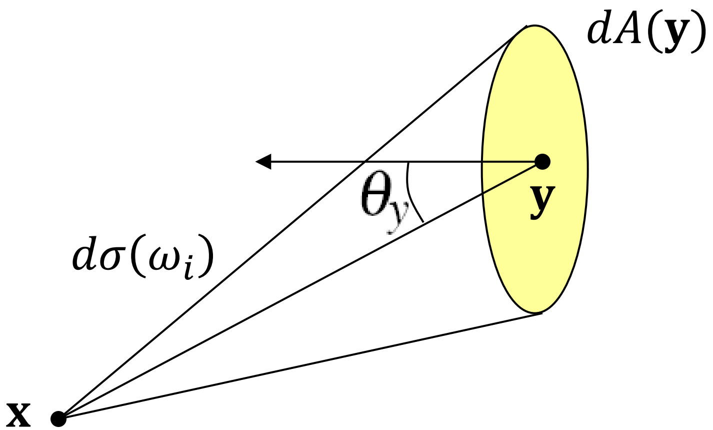
Rendering equation

- This form of the Rendering Equation is not convenient
  - Uses only quantities local to a surface

The Rendering Equation (4)

- We can replace the solid angle of incidence by the corresponding surface patch the light comes from
- If $\mathbf{x}$ is the current location, let $\mathbf{y}$ be the first visible point along the direction (ϕ_i, θ_i):

$$d\sigma(\omega_i) = \frac{\cos(\theta_y) dA(\mathbf{y})}{\|\mathbf{x} - \mathbf{y}\|^2}$$



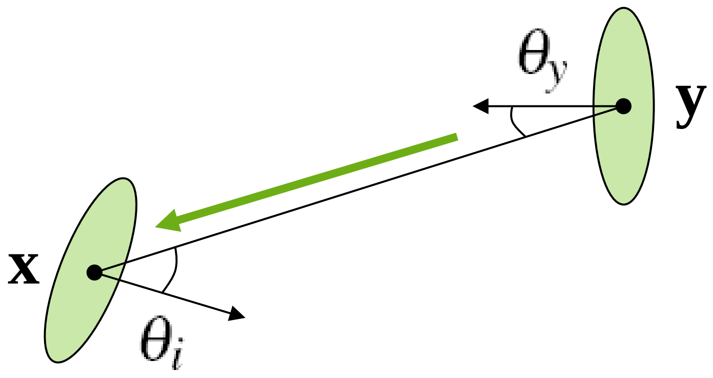
The Rendering Equation (5)

- Replacing the incident solid angles we get:

$$L(\mathbf{x},\omega_o) = L_e(\mathbf{x},\omega_o) + \int_{\mathcal{M}_{visible}} L(\mathbf{x},\omega_i) f_r\left(\mathbf{x},\varphi_o,\theta_{o,}\varphi_i,\theta_i\right)\frac{\cos\theta_i\cos\theta_y}{\|\mathbf{x}-\mathbf{y}\|^2}dA(\mathbf{y})$$

- Now as there is no attenuation (in this simple form – no participating media) as light travels on a straight line, we can assume:

$$L(\mathbf{x},\omega_i) = L(\mathbf{y},\omega_y)$$



The Rendering Equation (6)

- In the previous equation, we introduced a pure geometric term (call it $\mathrm{G}(\mathbf{x},\mathbf{y})$)
- To move from the domain of **visible** surfaces to an integration domain of **all** surfaces in the scene, we introduce a visibility function $\mathrm{V}(\mathbf{x},\mathbf{y})$:

$$L(\mathbf{x},\omega_o) = L_e(\mathbf{x},\omega_o) + \int_{\mathcal{M}} L(\mathbf{y},\omega_y) f_r(\mathbf{x},\omega_o,\omega_y) G(\mathbf{x},\mathbf{y}) V(\mathbf{x},\mathbf{y}) dA(\mathbf{y})$$

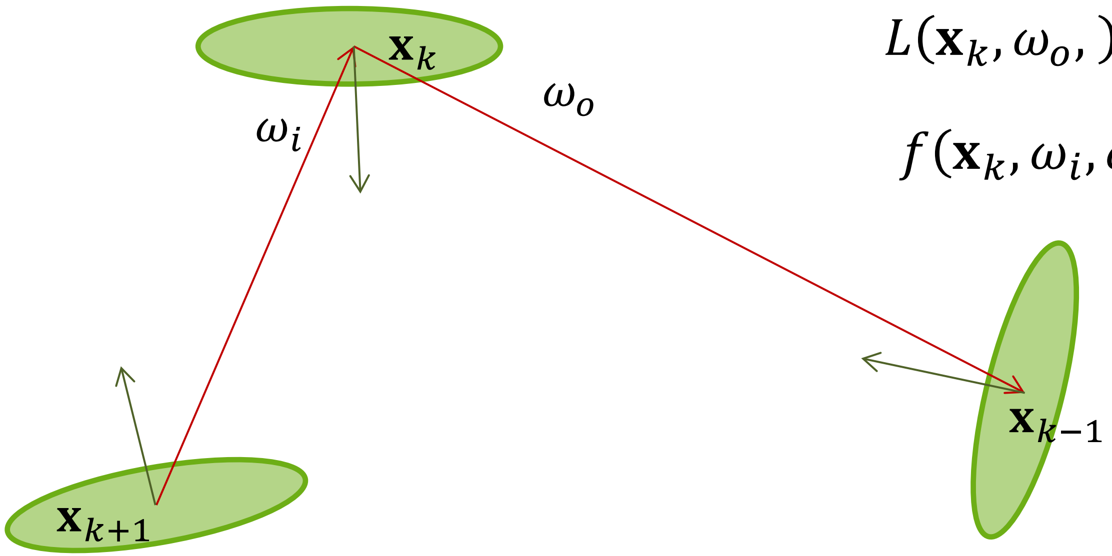
The Rendering Equation (7)

- Some times, when referring to path nodes, it is more convenient to express the rendering equation wrt a point's neighbors in a path:

$$L(\mathbf{x}_k, \omega_i,) = L(\mathbf{x}_{k+1} \rightarrow \mathbf{x}_k)$$

$$L(\mathbf{x}_k, \omega_o,) = L(\mathbf{x}_k \rightarrow \mathbf{x}_{k-1})$$

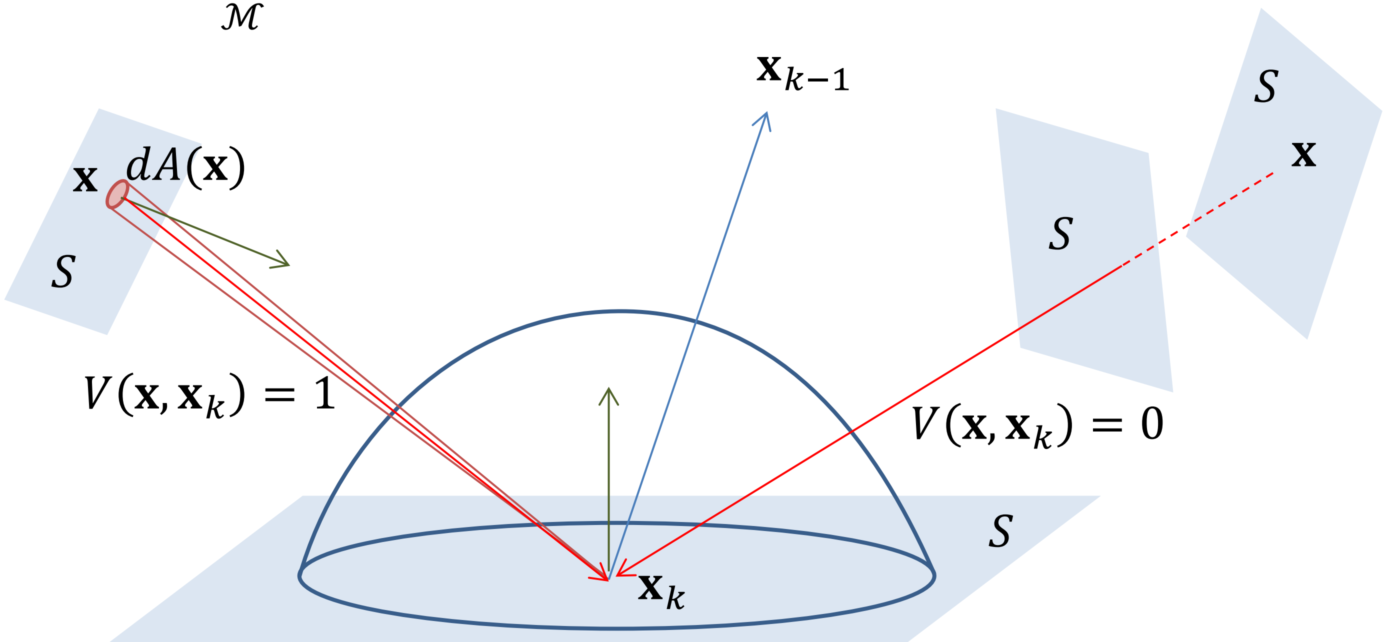
$$f(\mathbf{x}_k, \omega_i, \omega_o) = f\ (\mathbf{x}_{k+1} \rightarrow \mathbf{x}_k \rightarrow \mathbf{x}_{k-1})$$



The Rendering Equation (7)

$$L(\mathbf{x}_k \rightarrow \mathbf{x}_{k-1}) =$$

$$L_e(\mathbf{x}_k \rightarrow \mathbf{x}_{k-1}) + \int_{\mathcal{M}} L(\mathbf{x} \rightarrow \mathbf{x}_k) f_r\,(\mathbf{x} \rightarrow \mathbf{x}_k \rightarrow \mathbf{x}_{k-1}) G(\mathbf{x}, \mathbf{x}_k) V(\mathbf{x}, \mathbf{x}_k) dA(\mathbf{x})$$



Generalizing to All Scattering Events (1)

- Up to this point, our rendering equation only considered reflected light and light scattered back to the medium of incidence:

$$L_o(\mathbf{x}, \omega_o) = L_e(\mathbf{x}, \omega_o) + \int_{\Omega_{Hemisphere}} L_i(\mathbf{x}, \omega) f_r(\mathbf{x}, \omega_o, \omega) d\sigma_{\perp}(\omega)$$

$d\sigma_{\perp}(\omega) = |\cos\theta_i|\, d\sigma(\omega)$: “Projected” solid angle (on the surface)

Generalizing to All Scattering Events (2)

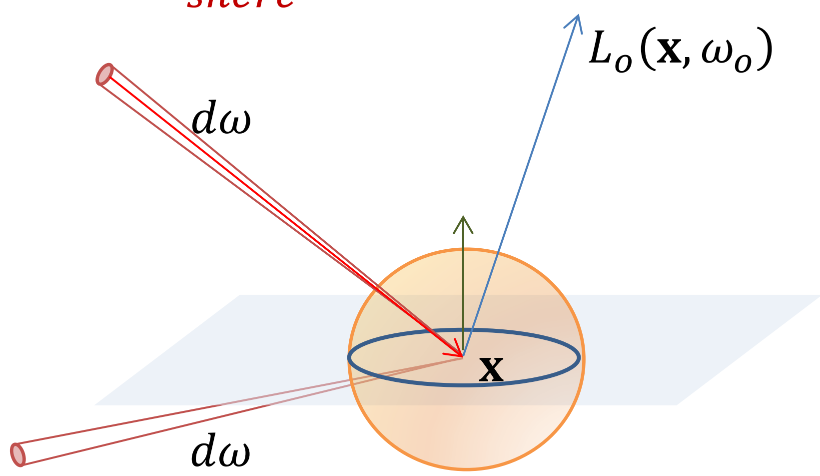
- We can extend this formulation to also include transmission of energy across an interface surface:

Scattering equation

$$L_o(\mathbf{x}, \omega_o) = L_e(\mathbf{x}, \omega_o) + \int_{\Omega_{shere}} L_i(\mathbf{x}, \omega) f_s(\mathbf{x}, \omega_o, \omega) d\sigma_\perp(\omega)$$

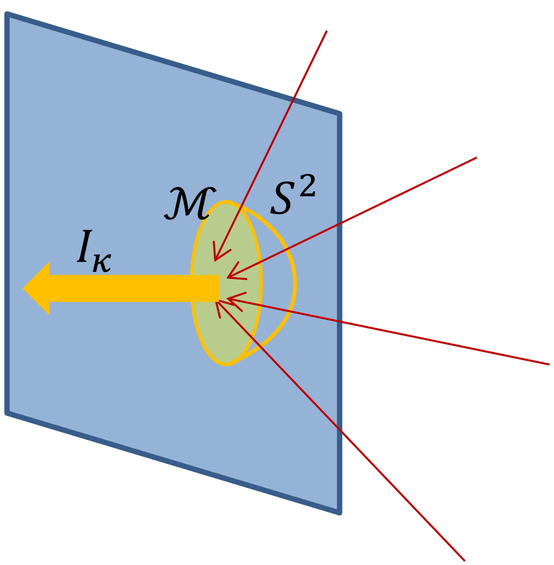
$f_s(\mathbf{x}, \omega_o, \omega)$: BSDF

Bidirectional Scattering Distribution Function



The Measurement Equation

- Light values are perceived through radiance measurements I_κ at locations on a sensor surface



- I_κ is affected by incident light in its neighborhood
- I_κ is typically affected by many incident directions (pinhole cameras don't)

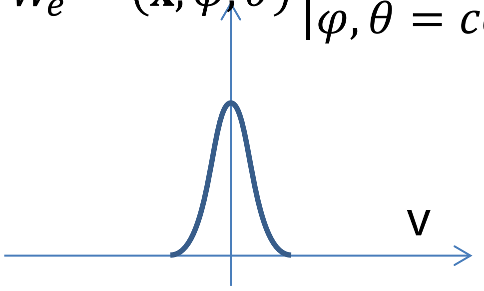
$$I_\kappa = \int_{\mathcal{M}\times S^2} W_e(\mathbf{x},\omega) L_i(\mathbf{x},\omega)\, dA(\mathbf{x}) d\sigma_\perp(\omega)$$

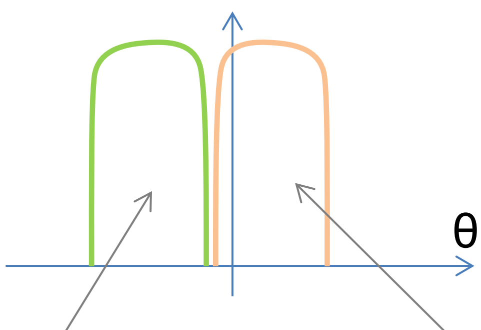
Measurement equation

- W_e: "Emitted importance"

The Measurement Equation - Example

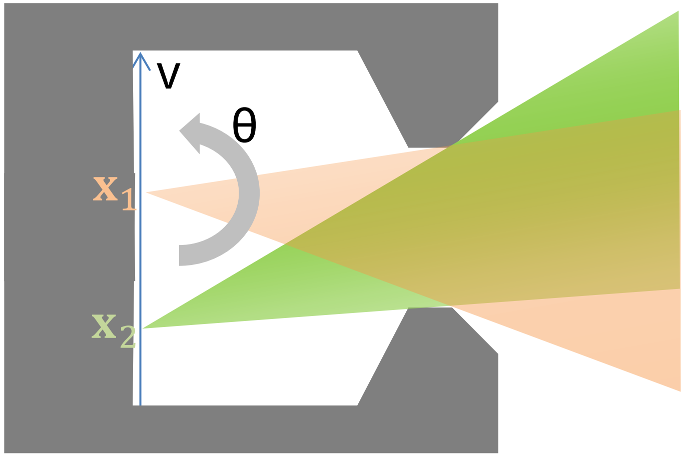
$$W_e^{(I_1)}(\mathbf{x},\varphi,\theta)\Big|_{\varphi,\theta\,=\,const}^{\mathbf{x}\,=\,(u,v)}$$





$$W_e^{(I_1)}(\mathbf{x}_1,\varphi,\theta)\Big|_{\varphi\,=\,const}$$

$$W_e^{(I_2)}(\mathbf{x}_2,\varphi,\theta)\Big|_{\varphi\,=\,const}$$



Exploring the Path Space

- The scattering equation provides the means to locally evaluate outgoing radiance at a node $\mathbf{x}_k$.
- How can we obtain the contribution of illumination at a global level?
- Two strategies:
  - Recursive evaluation
  - Path integral formulation
- Rendering algorithms are based on a mixture of the above 2 strategies

Recursive Path Evaluation (1)

- The outgoing radiance from a node $\mathbf{x}_1$ towards a reception point $\mathbf{x}_0$ (e.g. on the camera plane) is:

$$L(\mathbf{x}_1 \rightarrow \mathbf{x}_0) =$$

$$L_e(\mathbf{x}_1 \rightarrow \mathbf{x}_0) + \int_{\mathcal{M}} L(\mathbf{x}_2 \rightarrow \mathbf{x}_1) f_s\,(\mathbf{x}_2 \rightarrow \mathbf{x}_1 \rightarrow \mathbf{x}_0) G(\mathbf{x}_2, \mathbf{x}_1) V(\mathbf{x}_2, \mathbf{x}_1) dA(\mathbf{x}_2)$$

- Or more simply:

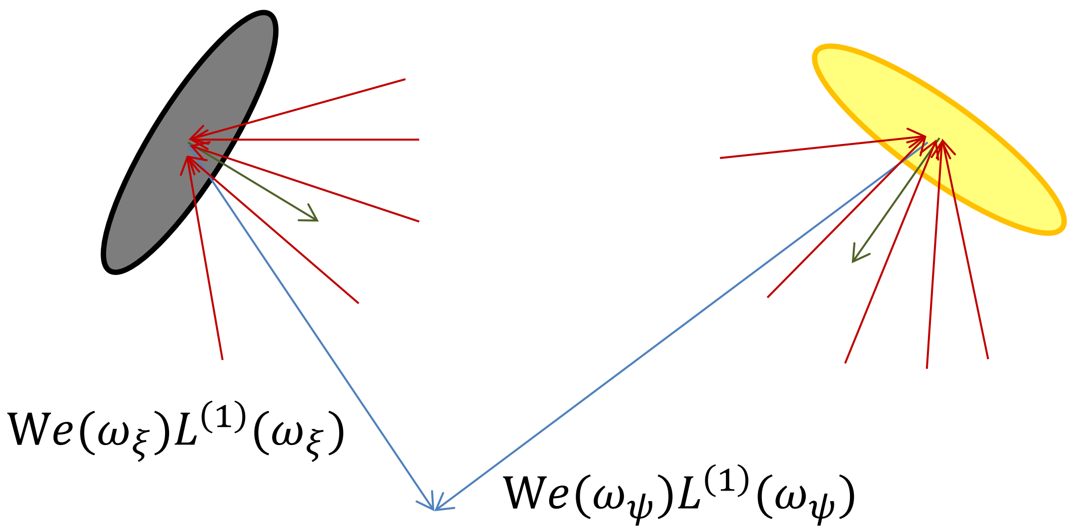
$$L^{(1)} = L_e^{(1)} + \int_{\mathcal{M}} L^{(2)} K^{(1)}\, dA(\mathbf{x}^{(2)}) \Leftrightarrow L^{(1)} = L_e^{(1)} + \boldsymbol{T} L^{(2)}$$

Light transport operator

Recursive Path Evaluation (2)

- Applying this equation recursively:

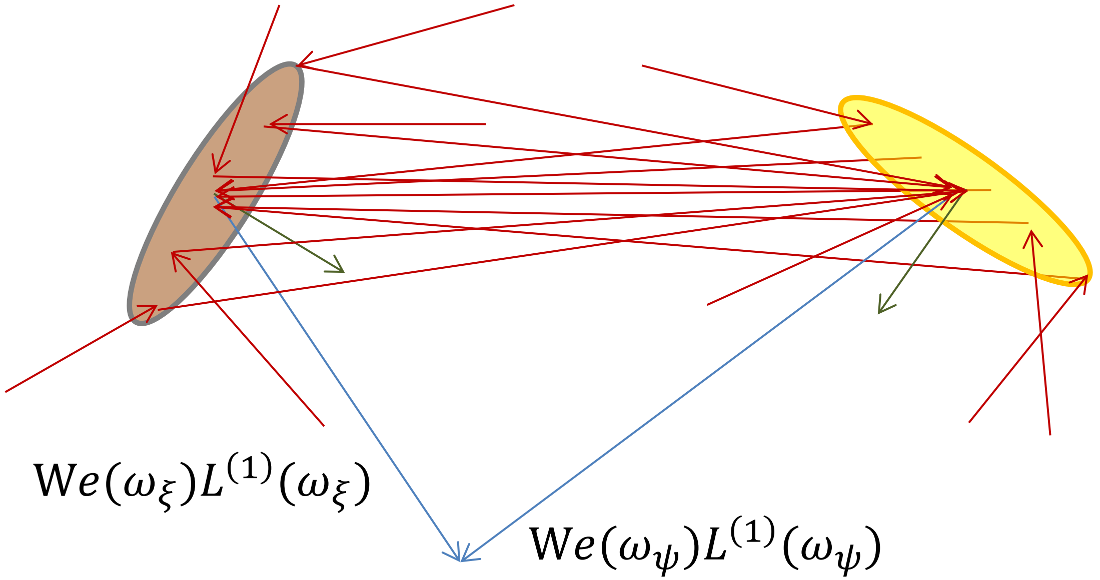
$$L^{(1)} = L_e^{(1)} + \boldsymbol{T}L^{(2)}$$



Recursive Path Evaluation (2)

- Applying this equation recursively:

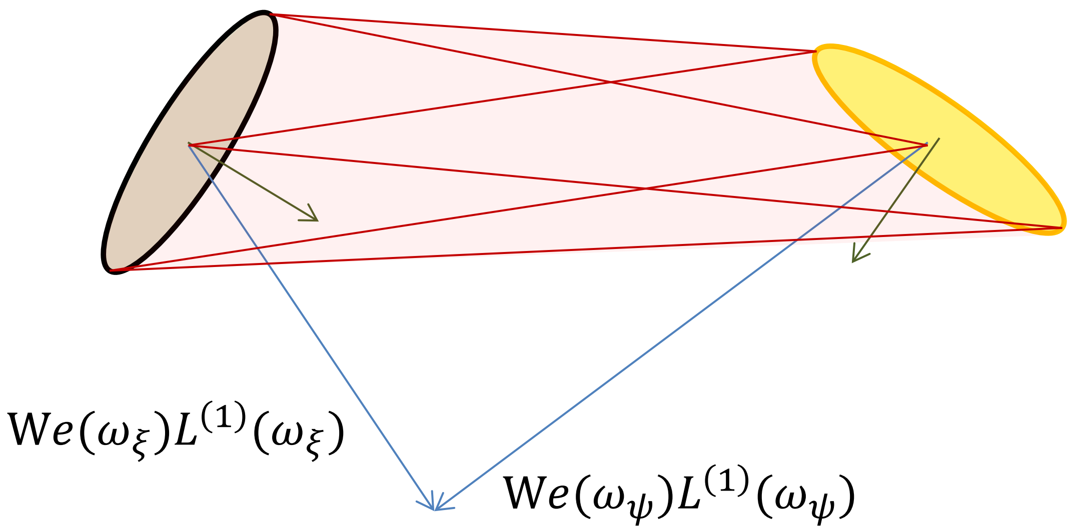
$$L^{(1)} = L_e^{(1)} + \boldsymbol{T}L^{(2)} = L_e^{(1)} + \boldsymbol{T}\left(L_e^{(2)} + \boldsymbol{T}L^{(3)}\right)$$



Recursive Path Evaluation (2)

- Applying this equation recursively:

$$L^{(1)} = L_e^{(1)} + \boldsymbol{T}L^{(2)} = L_e^{(1)} + \boldsymbol{T}\left(L_e^{(2)} + \boldsymbol{T}L^{(3)}\right) =$$
$$L_e^{(1)} + \boldsymbol{T}L_e^{(2)} + \boldsymbol{TT}L_e^{(3)} + \cdots + \prod^k \boldsymbol{T}L_e^{(k+1)},\ k \to \infty$$



Recursive Path Evaluation (2)

Notes:

- Each time the transport operator is applied, the entire surface domain is considered
- This solution explores the entire path space:
  - Takes into account the contribution of all light emitters from all possible paths → unbiased
  - Is the basis for the path tracing algorithm

The Path Integral (1)

- The path integral formulates light transport as a simple, single integral→
- Non-recursive evaluation
- In its general form it represents the aggregate light measurements from all paths of all lengths recorded on a single measurement point:

$$I_j = \int_{\Omega} f_j(\bar{x}) d\mu(\bar{x})$$

Ω : Set of paths of all lengths

μ : A measure on this space

f_j : Measurement contribution function

Source: [Vea97]

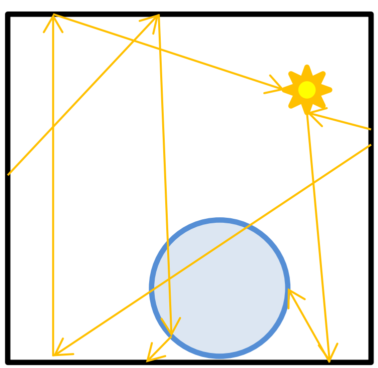
The Path Integral (2)

Why use this formulation?

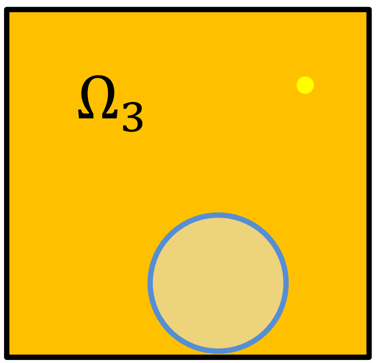
- Transforms the entire light transport into an integration problem → Can be addressed with general-purpose methods (e.g. MIS)
- Allows new techniques for sampling space:
  - The integral rendering equation represents a localized view of the light transport → only incremental path generation
  - We can now choose path nodes with other global sampling strategies → New algorithms: Bidirectional path tracing, Metropolis light transport

The Path Space (1)

- Let Ω_k represent the set of all paths $\bar{x}$ of length k: $\bar{x} = \mathbf{x}_0\mathbf{x}_1 \ldots \mathbf{x}_k, 1 < k < \infty$
- Points $\mathbf{x}_i$ are taken in the domain $\mathcal{M}$ of all surfaces of the scene



Some paths of length k=3



All paths of length k=3

The Path Space (2)

- We can now define a product measure on this space defined over a set of paths $D \subset \Omega_k$:

$$\mu_k(D) = \int_D dA(\mathbf{x}_0) \ldots dA(\mathbf{x}_k)$$

"Area" measure

- From which we can derive:

$$d\mu_k(\bar{x}) = d\mu_k(\mathbf{x}_0\mathbf{x}_1 \ldots \mathbf{x}_k) = dA(\mathbf{x}_0) \ldots dA(\mathbf{x}_k)$$

The Path Space (3)

- Now we can define the path space of all path lengths:

$$\Omega = \bigcup_{k=1}^{\infty} \Omega_k$$

- Similarly, we can extend the area measure to this space:

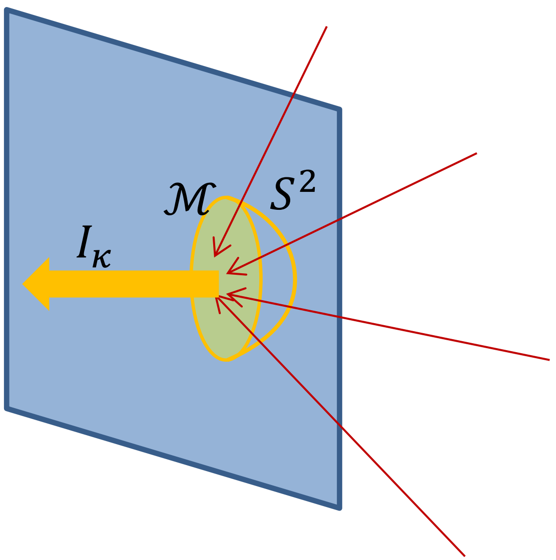
$$\mu(D) = \sum_{k=1}^{\infty} \mu_k(D \cap \Omega_k)$$

- The measure of a set of paths is the sum of the measures of the paths of each length

Rethinking the Measurement Equation (1)

- The original measurement equation regarded all incident directions and all locations around the measurement point:

$$I_\kappa = \int_{\mathcal{M}\times S^2} W_e(\mathbf{x},\omega)L_i(\mathbf{x},\omega)\,dA(\mathbf{x})d\sigma_\perp(\omega)$$



- But:

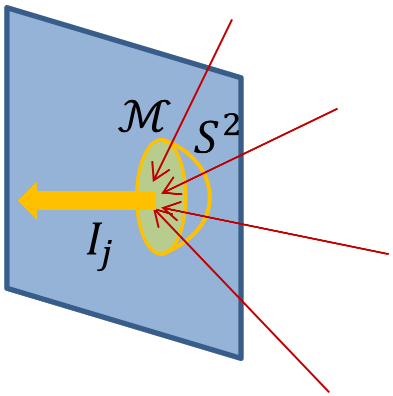
$$d\sigma_\perp(\omega) = G(\mathbf{x},\mathbf{y})dA(\mathbf{y}) = \frac{|\cos\theta_o \cos\theta_i|}{\|\mathbf{x}-\mathbf{y}\|^2}V(\mathbf{x},\mathbf{y})dA(\mathbf{y})$$

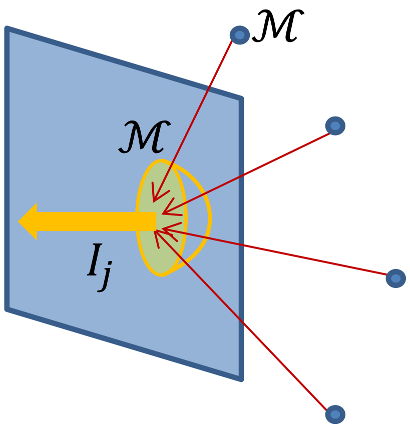
Rethinking the Measurement Equation (2)

- So the measurement equation can be mapped to an entirely surface-based domain:

$$I_j = \int_{\mathcal{M}\times S^2} W_e^{(j)}(\mathbf{x},\omega) L_i(\mathbf{x},\omega)\, dA(\mathbf{x}) d\sigma_{\perp}(\omega) =$$

$$\int_{\mathcal{M}\times\mathcal{M}} W_e^{(j)}(\mathbf{y}\to\mathbf{x}) L_i(\mathbf{y}\to\mathbf{x})\, G(\mathbf{x},\mathbf{y}) dA(\mathbf{x}) dA(\mathbf{y})$$





Source: [Vea97]

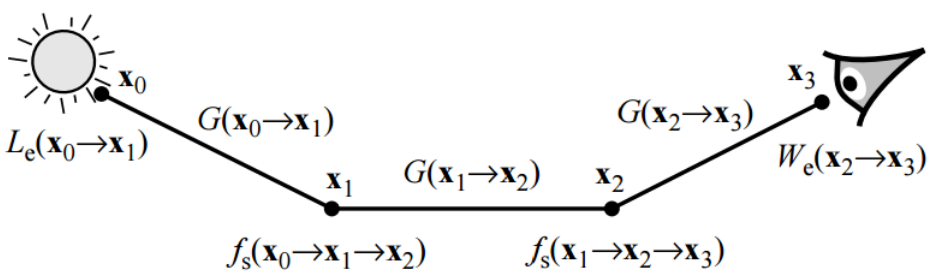
Rethinking the Measurement Equation (3)

- Expanding recursively the transport equation to replace L_i, we obtain:

$$I_j = \sum_{k=1}^{\infty} \int_{\mathcal{M}^{k+1}} W_e^{(j)}(\mathbf{x}_{k-1} \to \mathbf{x}_k) \prod_{i=1}^{k-1} [f_s(\mathbf{x}_{i-1} \to \mathbf{x}_k \to \mathbf{x}_{i+1}) G(\mathbf{x}_i, \mathbf{x}_{i+1})]$$

$$\cdot\, G(\mathbf{x}_0, \mathbf{x}_1) L_e(\mathbf{x}_0 \to \mathbf{x}_1) dA(\mathbf{x}_0) \ldots dA(\mathbf{x}_k)$$

Example:



Rethinking the Measurement Equation (4)

- So we sum the contribution of all paths of all path lengths:

$$I_j = \int_{\mathcal{M}\times\mathcal{M}} W_e^{(j)}(\mathbf{x}_0 \rightarrow \mathbf{x}_1) G(\mathbf{x}_0, \mathbf{x}_1) L_e(\mathbf{x}_0 \rightarrow \mathbf{x}_1) dA(\mathbf{x}_0) dA(\mathbf{x}_1) +$$

$$\int_{\mathcal{M}\times\mathcal{M}\times\mathcal{M}} W_e^{(j)}(\mathbf{x}_1 \rightarrow \mathbf{x}_2) G(\mathbf{x}_1, \mathbf{x}_2) G(\mathbf{x}_0, \mathbf{x}_1) f_s(\mathbf{x}_0 \rightarrow \mathbf{x}_1 \rightarrow \mathbf{x}_2)$$

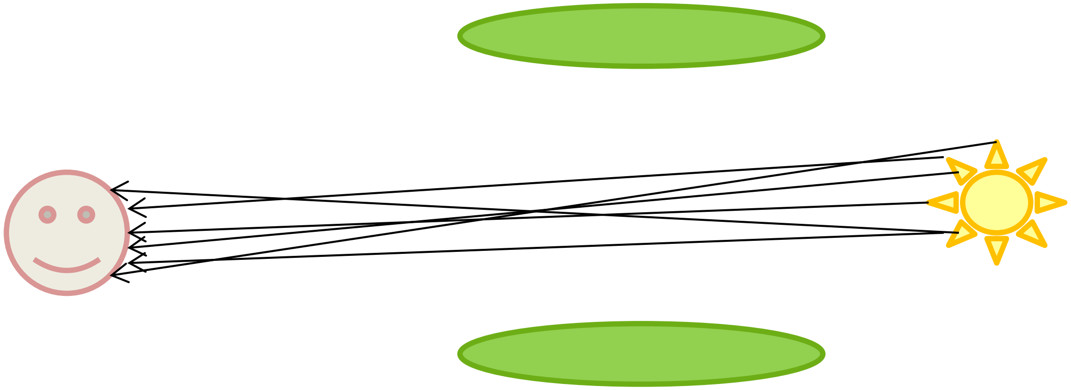
$$L_e(\mathbf{x}_0 \rightarrow \mathbf{x}_1) dA(\mathbf{x}_0) dA(\mathbf{x}_1) dA(\mathbf{x}_2) +$$

…

Source: [Vea97]

Rethinking the Measurement Equation (5)

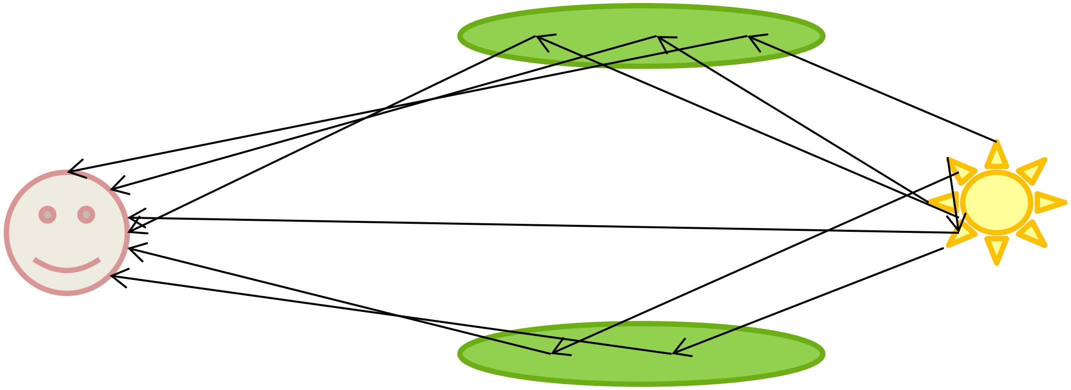
- Example:



$$\mathcal{M} \times \mathcal{M}$$

Rethinking the Measurement Equation (5)

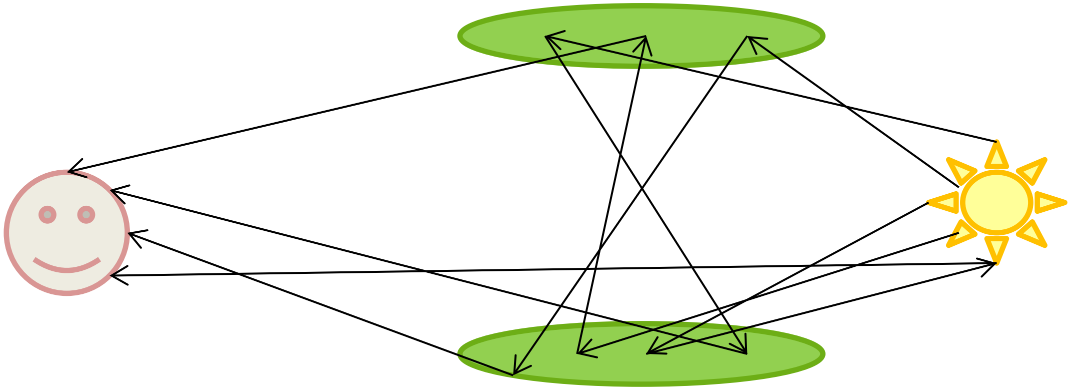
- Example:



$\mathcal{M} \times \mathcal{M} \times \mathcal{M}$

Rethinking the Measurement Equation (5)

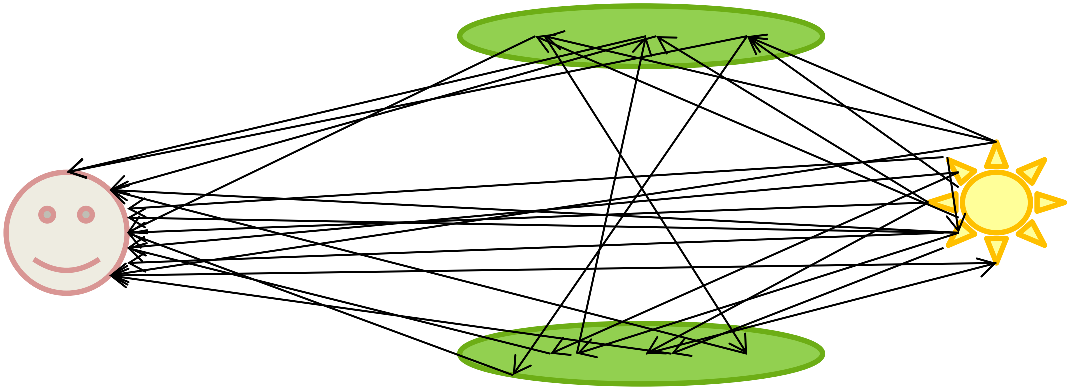
- Example:



$$\mathcal{M} \times \mathcal{M} \times \mathcal{M} \times \mathcal{M}$$

Rethinking the Measurement Equation (5)

- Example:

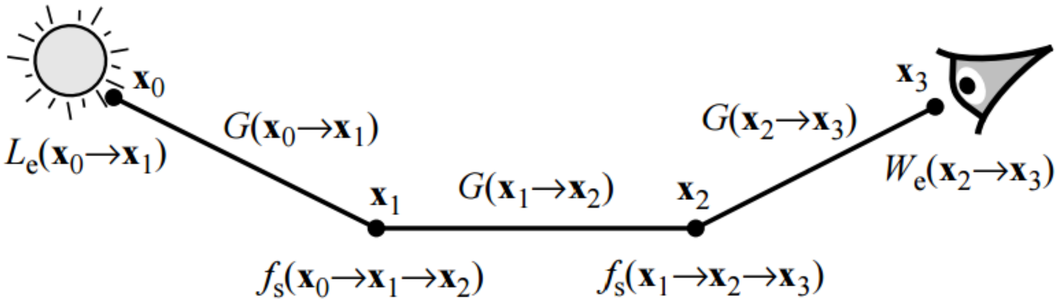


$$\mathcal{M} \times \mathcal{M} +$$
$$\mathcal{M} \times \mathcal{M} \times \mathcal{M} +$$
$$\mathcal{M} \times \mathcal{M} \times \mathcal{M} \times \mathcal{M}$$

The Measurement Contribution Function (1)

- The integrant is defined for each path length separately. This is the measurement contribution function
- For example, for $k = 3$, i.e. $\bar{x} = \mathbf{x}_0\mathbf{x}_1\mathbf{x}_2\mathbf{x}_3$ we have:

$$f_j(\bar{x}) = L_e(\mathbf{x}_0 \to \mathbf{x}_1) \cdot G(\mathbf{x}_0, \mathbf{x}_1) f_s(\mathbf{x}_0 \to \mathbf{x}_1 \to \mathbf{x}_2) \cdot G(\mathbf{x}_1, \mathbf{x}_2) f_s(\mathbf{x}_1 \to \mathbf{x}_2 \to \mathbf{x}_3) \cdot G(\mathbf{x}_2, \mathbf{x}_3) W_e^{(j)}(\mathbf{x}_2 \to \mathbf{x}_3)$$



Contributors

- Georgios Papaioannou

References:

[Vea97] Eric Veach, Robust Monte Carlo Methods for Light Transport Simulation, PhD dissertation, Stanford University, December 1997.