Lecture 2: Theoretical tools of Public Economics

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Fall 2023-24

Theoretical and Empirical Tools

Theoretical tools:

The set of tools designed to understand the mechanics behind economic decision making.

Constrained utility optimization.

Individuals maximize utility functions subject to budget constraints.

Empirical tools:

The set of tools designed to analyze data and answer questions raised by theoretical analysis.

Utility function

A utility function is a mathematical function that translates consumption into utility:

$$U = f(c)$$

where c denotes consumption:

Example log-utility: $$f(c) = \log(c)$$

Simplification with consumption bundle

- Let's assume that, c, denotes the consumption good in the economy.
- 3 units of c yield
- $f(3)$ units of utility
- We assume $f(c_2) > f(c_1)$ if $c_2 > c_1$.

Assumption: More is better (non-satiation)

- Economists assume that more of a good is always better than less.
- Mathematical representation:
- We assume if $c_2 > c_1$ then $f(c_2) > f(c_1)$.
- Since 4>3 then $f(4) > f(3)$.
- $\frac{\partial f(c)}{\partial c} > 0$ utility is an increasing function of the consumption bundle.

Example with two goods

$$U(X_1, X_2) = \sqrt{X_1 X_2}$$

- X_1 quantity of good 1, say food
- X_2 quantity of good 2, say drink

Individual utility increases with the level of consumption of each good X_1 and X_2.

Example with many goods

$$U(X_1, X_2, X_3, \dots X_N) = \sqrt{X_1, X_2, X_3, \dots X_N}$$

- $X_1, X_2, X_3, \dots X_N$ quantity of good 1,2,3… N.

For simplicity, X's can be thought as consumption bundles with goods that exhibits common features.

Preferences and Indifference curves

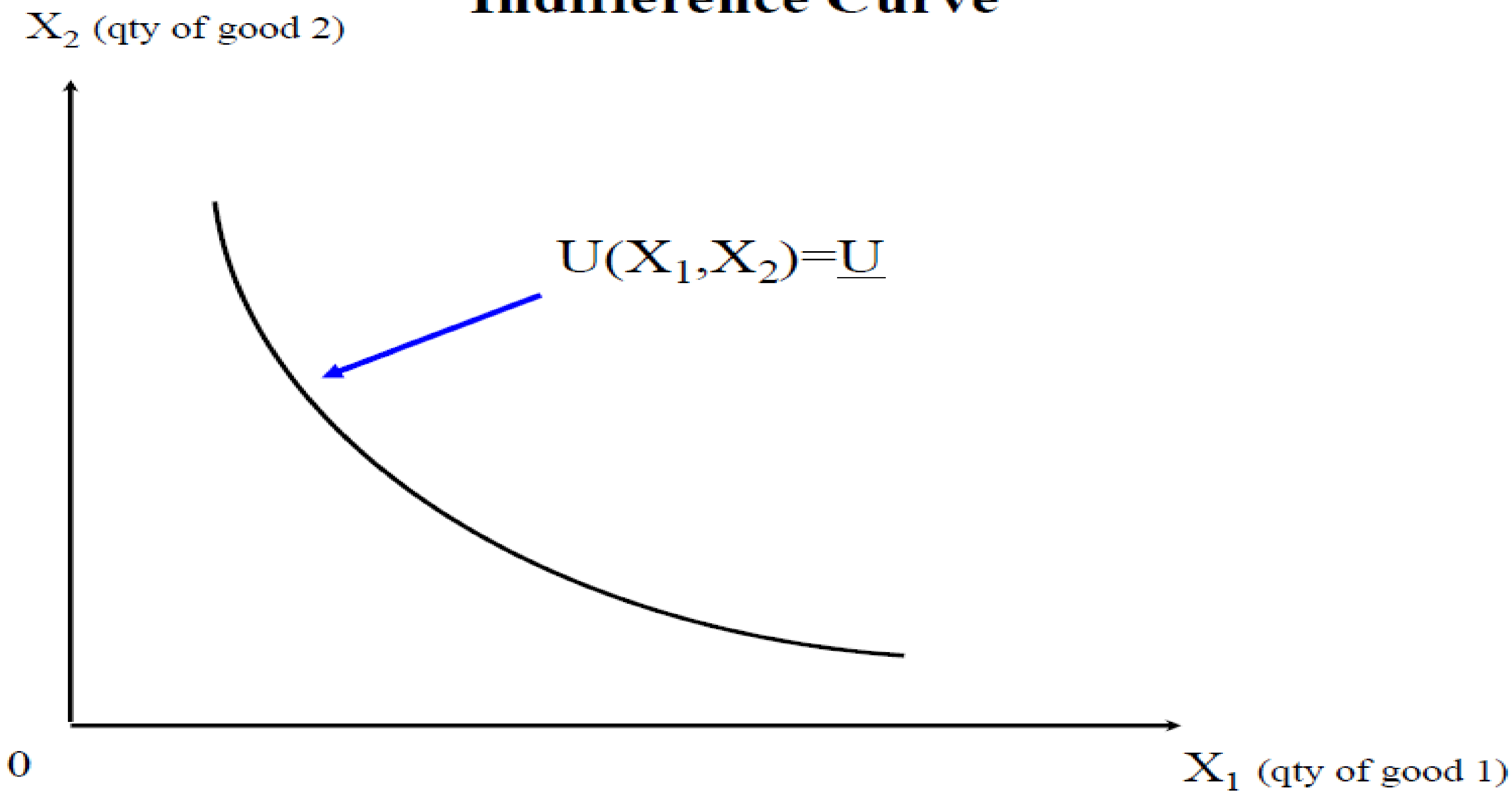
An indifference curve is a graphical representation of all bundles of goods that make an individual equally well off.

- Thus, an indifference curve yielding utility level, $\underline{U}$, by the set of bundles (x_1, x_2) such that $U(x_1, x_2) = \underline{U}$.

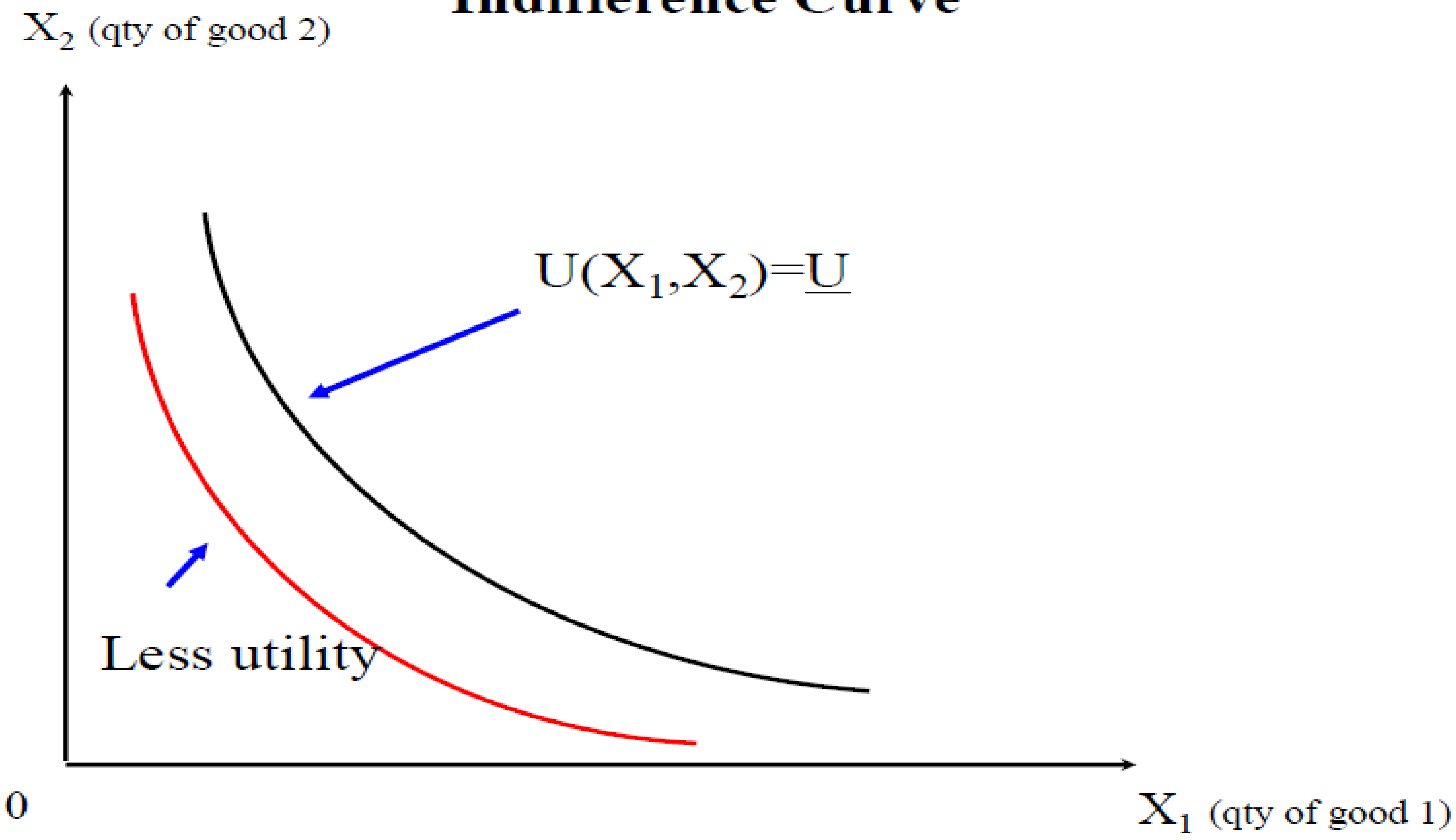
Indifference curves have two properties:

- Consumer prefer higher indifference curves.
- Indifference curves are always downward slopping.

Indifference Curve



Indifference Curve



Example

Andrea's utility for cakes (Q_c) and movies (Q_M) is:

$$U(Q_c, Q_M) = \sqrt{Q_C \times Q_M}$$

How we can construct the indifference curve:

1. Andrea is indifferent between 4 cakes and 1 movie or 1 cake and 4 movies:

- $U(Q_c, Q_M) = \sqrt{4 \times 1} = 2$
- or $U(Q_c, Q_M) = \sqrt{1 \times 4} = 2$

Example

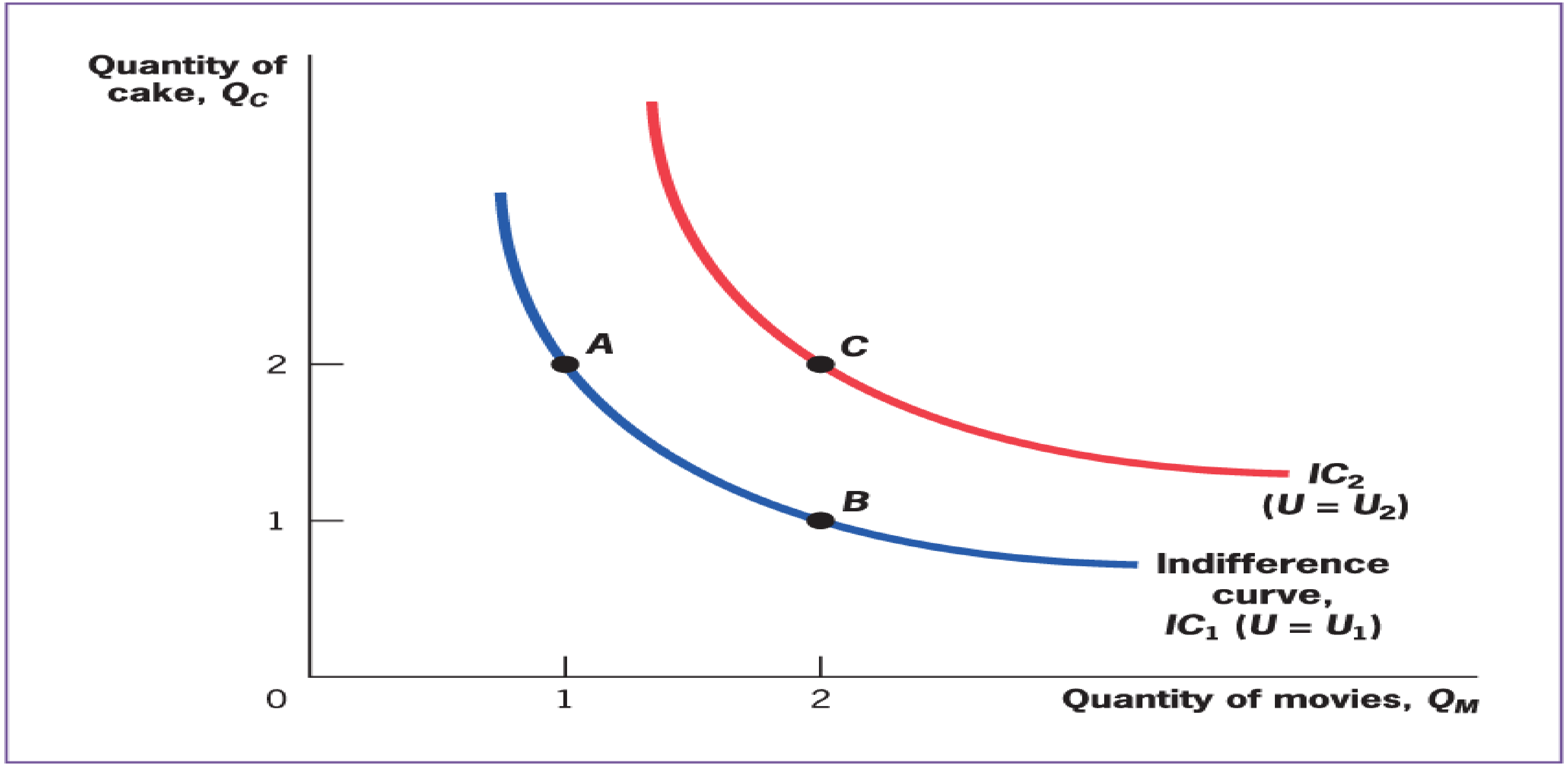
Andrea's utility for cakes (Q_c) and movies (Q_M) is:

$$U(Q_c, Q_M) = \sqrt{Q_C \times Q_M}$$

- Andrea prefers 3 cakes and 3 movies to either bundle:

$$U(Q_c, Q_M) = \sqrt{3 \times 3} = 3 > 2$$

Preferences and Indifference Curves 2



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Questions I

<u>Which of the following option is correct?</u>

1. Andrea is indifferent between bundles A and C

2. Andrea prefers bundle A from bundle B

3. Andrea prefers bundle C from bundle A

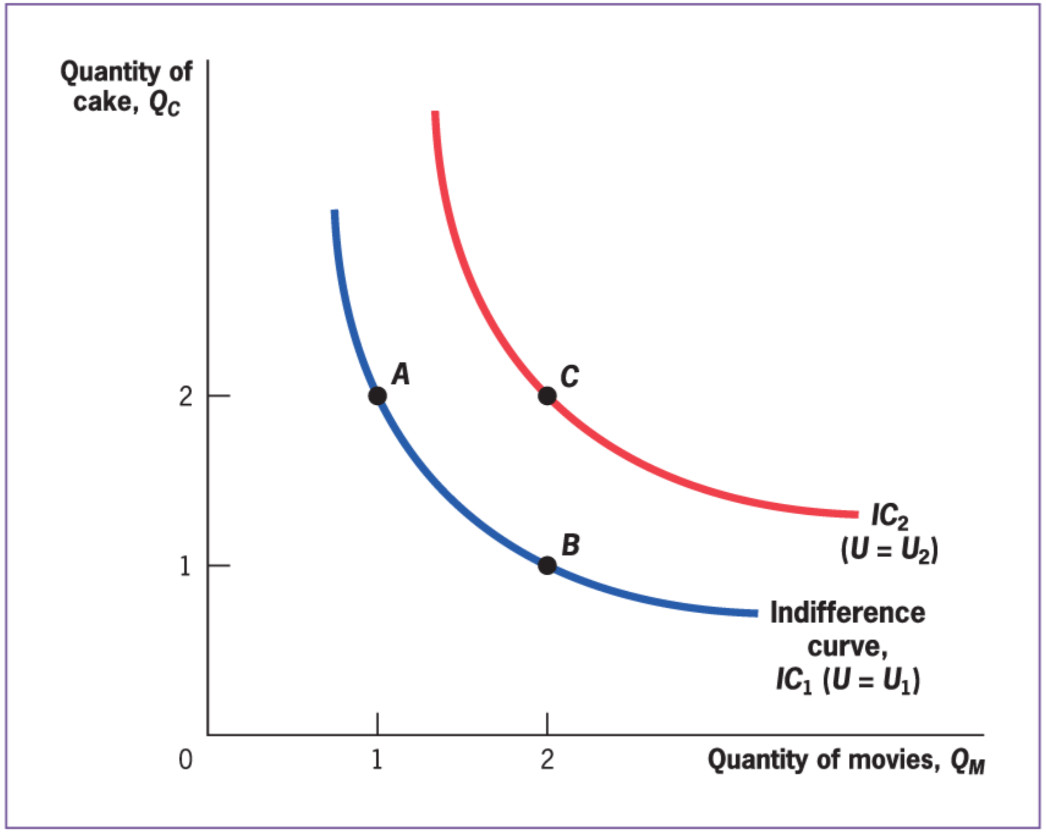
Questions II

Which of the following option is correct?

1. Andrea is indifferent between bundles A and C
2. Andrea is indifferent between bundles B and A
3. Andrea is indifferent between bundles B and C

Preferences and Indifference Curves 2

- **Consumer is indifferent between *A* and *B*.**
- ***C* is preferred to *A* or *B*.**



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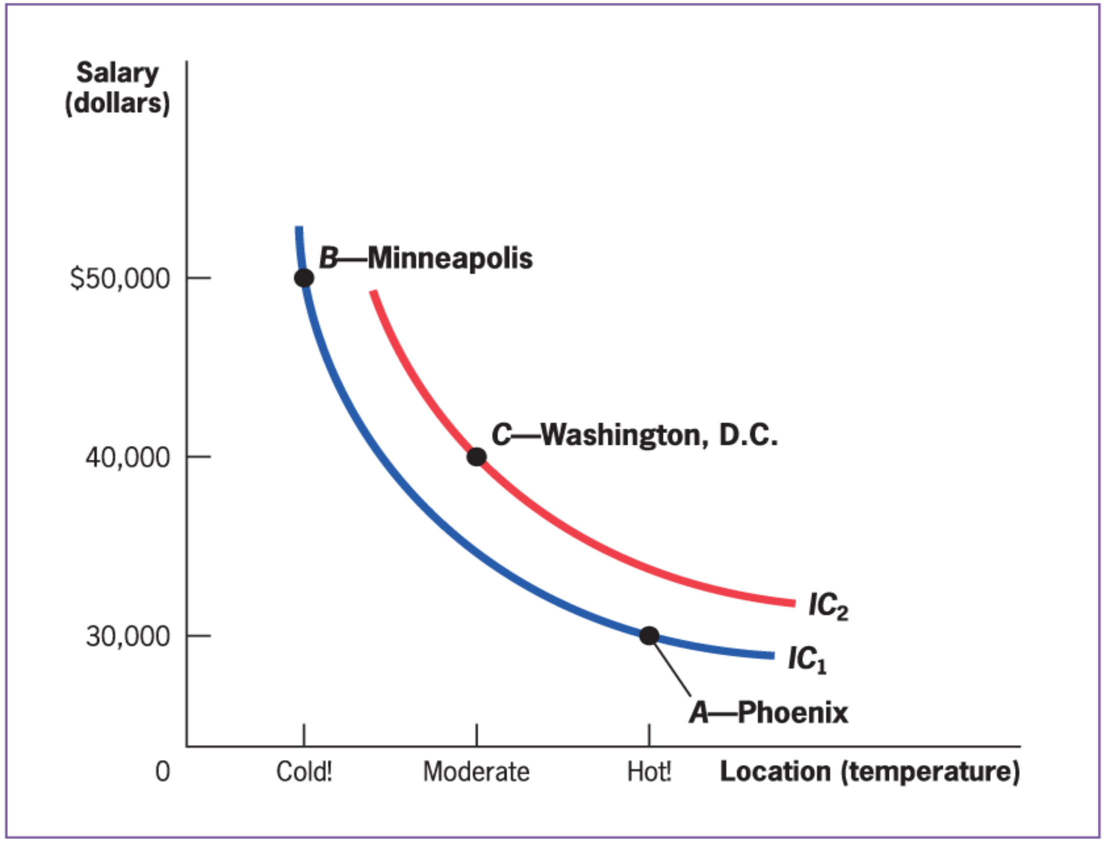
Preferences and Indifference Curves 3

- **Consumer is indifferent between *A* and *B*.**
- ***C* is preferred to *A* or *B*.**



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Marginal Utility

Marginal utility: The additional increment to utility obtained by consuming an additional unit of a good.

Marginal utility of good 1 is the derivative of utility with respect to x_1 keeping $x_2$constant:

$$U(X_1 X_2) = \sqrt{X_1 X_2}$$

$$\frac{\partial U}{\partial X_1} = \frac{\sqrt{X_2}}{2\sqrt{X_1}}$$

The utility function **exhibits the important principle of diminishing marginal utility**

Marginal utility $\frac{\partial U}{\partial x_1}$ decreases as x_1 increases

The consumption of each additional unit of a good gives less extra utility than the consumption of the previous unit.

Marginal Rate of Substitution

- The rate at which a consumer is willing to trade one good for another.
- Moving along an indifference curve keeps a consumer equally well off, so:
- The *MRS* is equal to the slope of the indifference curve, the rate at which the consumer will trade the good on the vertical axis for the good on the horizontal axis:
- $MRS = -MU_M/MU_C$

Example

$$U(Q_c, Q_M) = \sqrt{Q_C \times Q_M}$$

MRS: $MRS_{C,M} = \frac{MU_C}{MU_M}$

$$\frac{\partial U}{\partial Q_C} = \frac{\sqrt{Q_M}}{2\sqrt{Q_C}}$$

$$\frac{\partial U}{\partial Q_M} = \frac{\sqrt{Q_C}}{2\sqrt{Q_M}}$$

Example (cont'd)

$$MRS_{C,M} = \frac{MU_C}{MU_M}$$

$$\frac{\frac{\partial U}{\partial Q_C}}{\frac{\partial U}{\partial Q_M}} = \frac{\frac{\sqrt{Q_M}}{2\sqrt{Q_C}}}{\frac{\sqrt{Q_C}}{2\sqrt{Q_M}}}$$

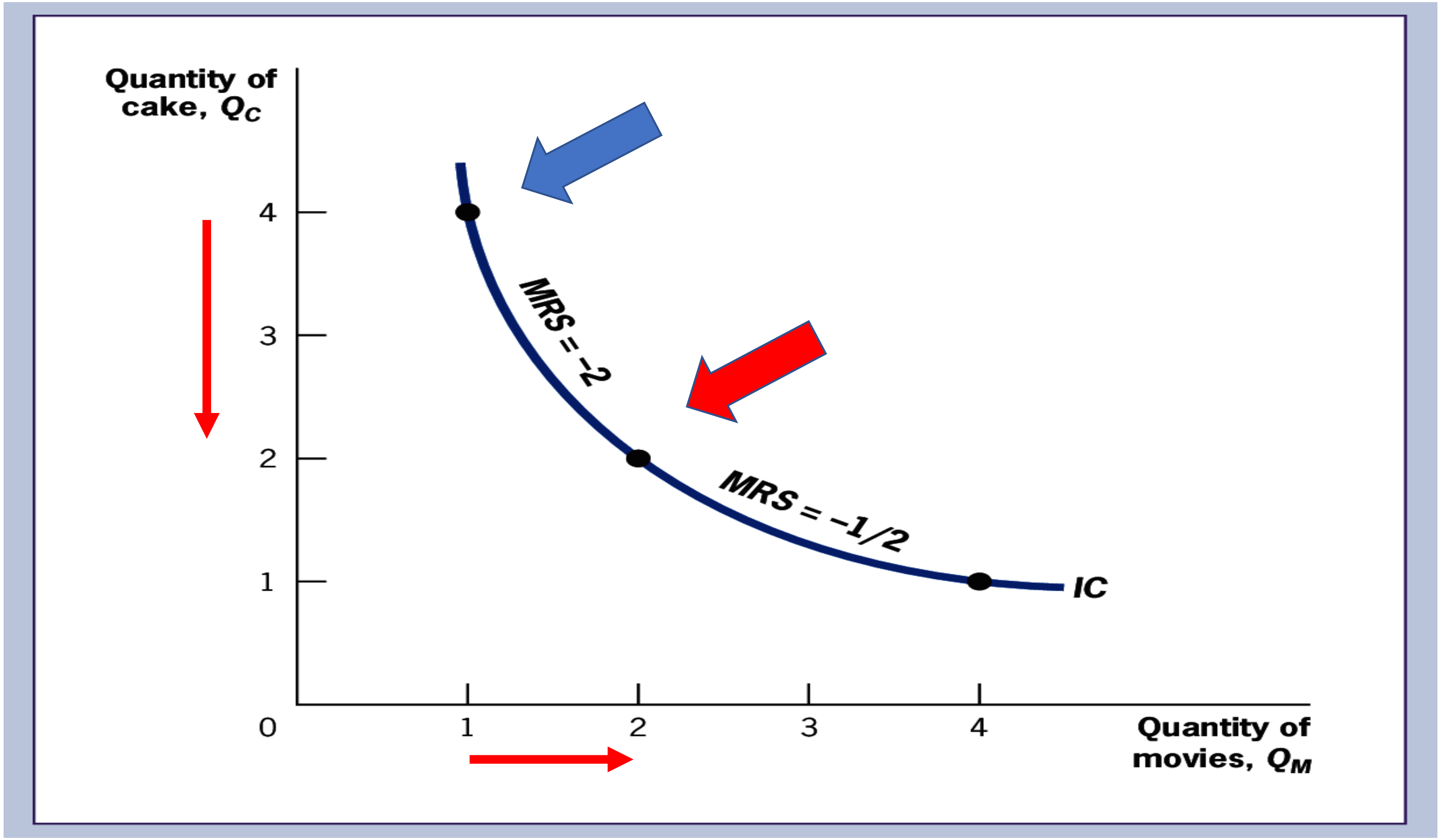
$$\frac{\frac{\partial U}{\partial Q_C}}{\frac{\partial U}{\partial Q_M}} = \frac{2\sqrt{Q_M}^2}{2\sqrt{Q_C}^2}$$

Example (cont'd)

$$U(Q_c, Q_M) = \sqrt{Q_C \times Q_M}$$

$$MRS_{C,M} = \frac{Q_M}{Q_C}$$

Intuitively: Individual is indifferent between 1 unit of good C and $MRS_{C,M}$ units of good M


Quantity of cake, Q_C
4
3
2
1
0
MRS = −2
MRS = −1/2
IC
1
2
3
4
Quantity of movies, Q_M

Budget constraint

- **Budget constraint:** A mathematical representation of all the combinations of goods an individual can afford to buy if she spends her entire income:
- $Y = p_1 X_1 + p_2 X_2$

Where p_i price of good $i = (X_1, X_2)$, and Y is disposable income.

- **Opportunity cost:** The cost of any purchase is the next best alternative use of that money or the forgone opportunity.

- *Quick hint*: When a person's budget is fixed, if he buys one thing, he is, by definition, reducing the money he has to spend on other things. Indirectly, this purchase has the same effect as a direct good-for-good trade.

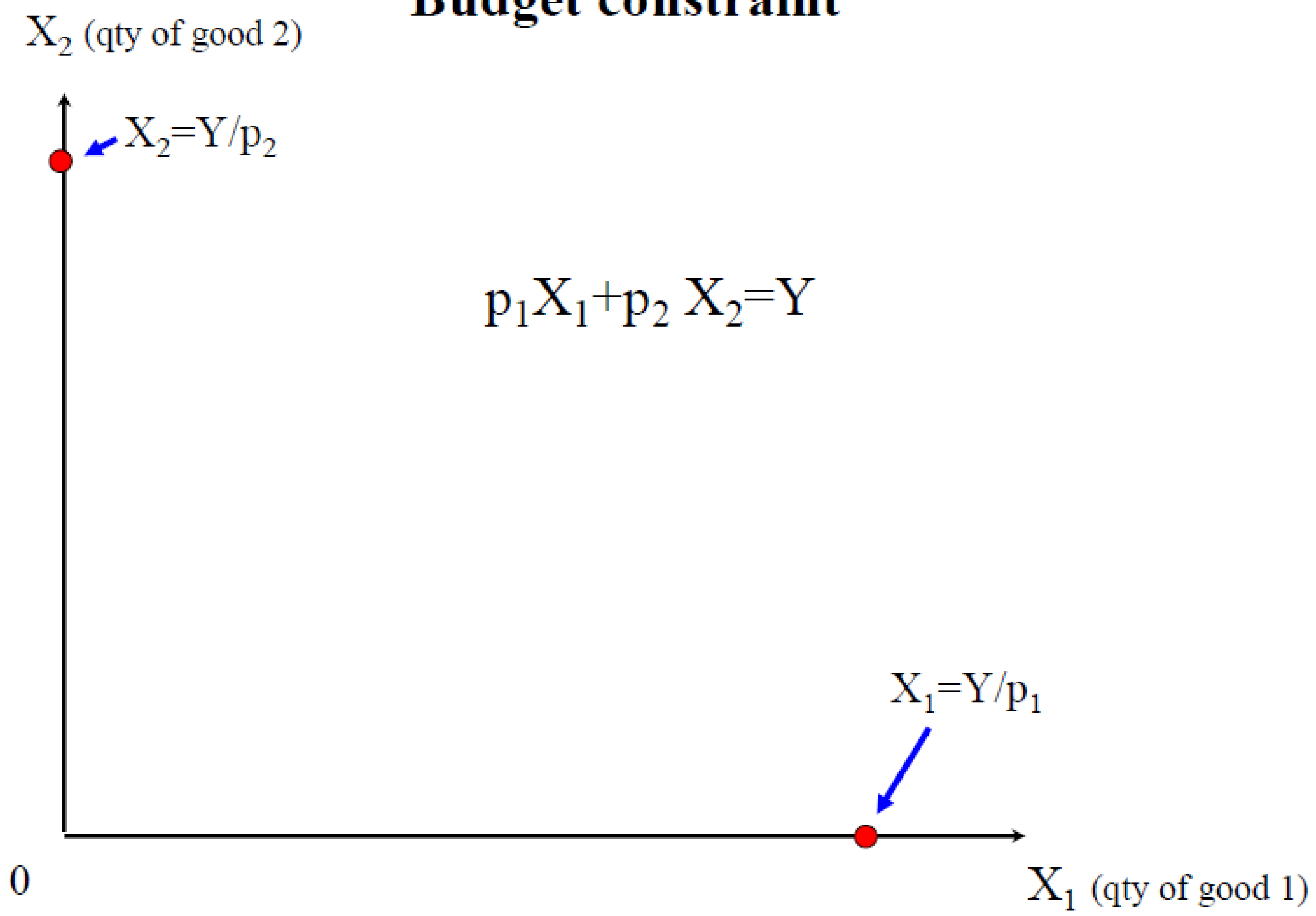
Budget constraint

- $Y = p_1 X_1 + p_2 X_2$

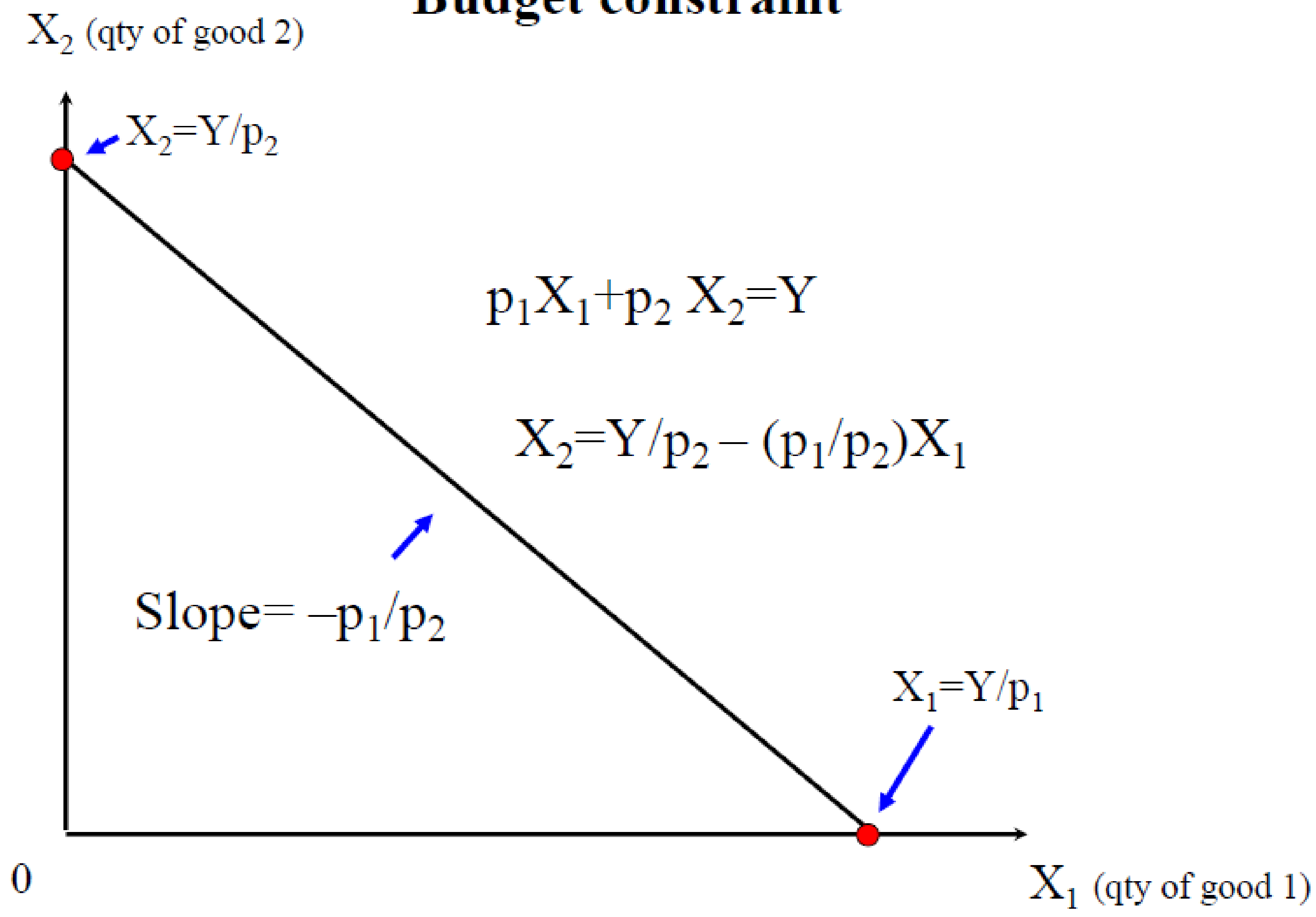
Budget constraint defines a linear set of bundles the consumer can purchase with its disposable income:

$$X_2 = \frac{Y}{p_2} - \frac{p_1}{p_2} X_1$$

Budget constraint



Budget constraint



Putting it all together

Utility and budget constraint

Constrained utility maximization

Individual maximizes utility subject to their budget constraint:

$$max_{X_1,X_2} U(X_1, X_2)$$

Subject to:

$$Y = p_1 X_1 + p_2 X_2$$

Substitute the constraint into the objective function, $X_2 = Y - \frac{p_1}{p_2} X_1$,.

Utility Maximization

Individual maximizes utility subject to budget constraint:

$$max\, U(X_1, Y - \frac{p_1}{p_2} X_1)$$

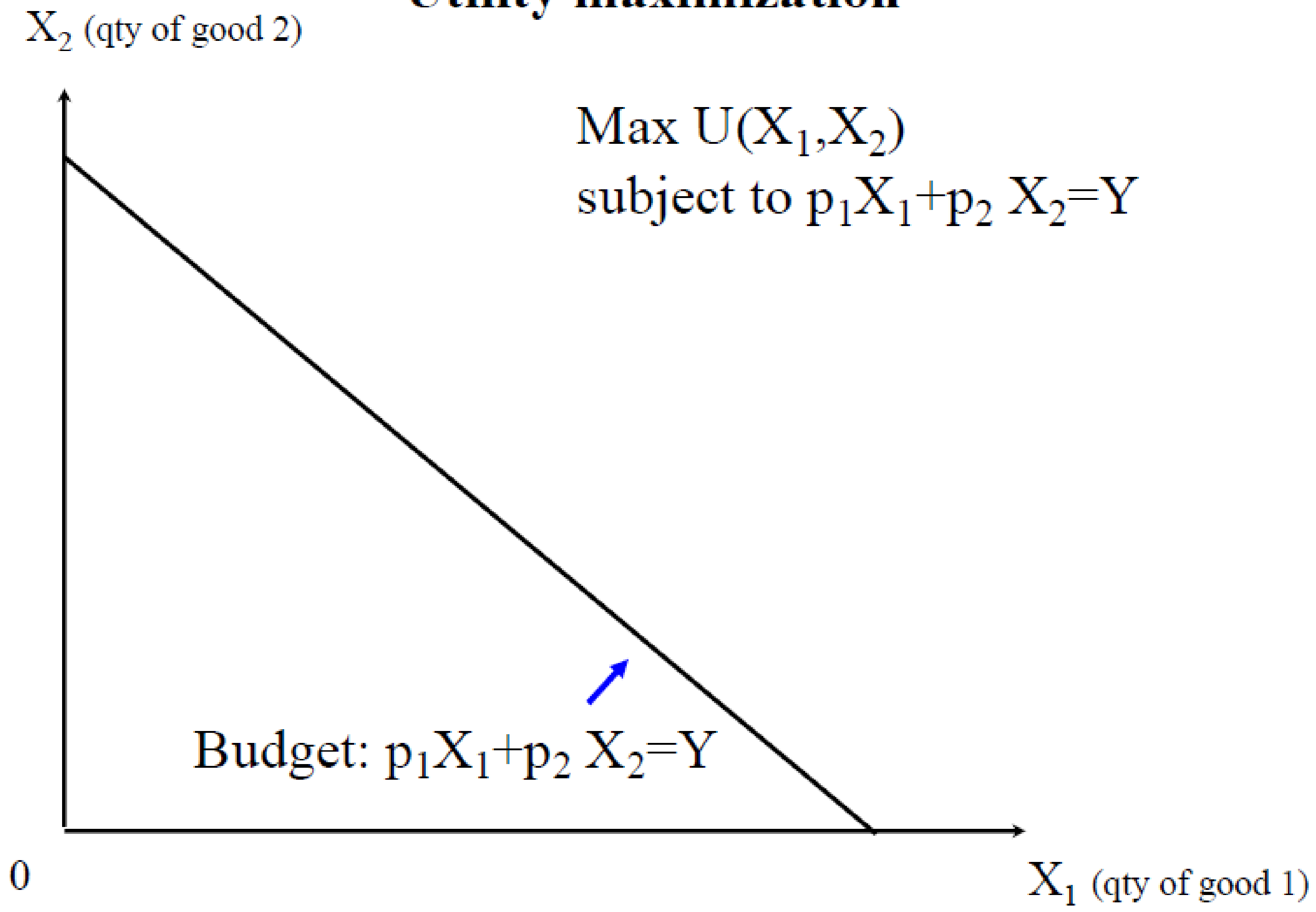
Taking the first order condition with respect to X_1:

$$\frac{\partial U(X_1, Y - \frac{p_1}{p_2} X_1)}{\partial X_1} = \frac{\partial U(.)}{\partial X_1} - \frac{p_1}{p_2} \frac{\partial U(.)}{\partial X_2}$$

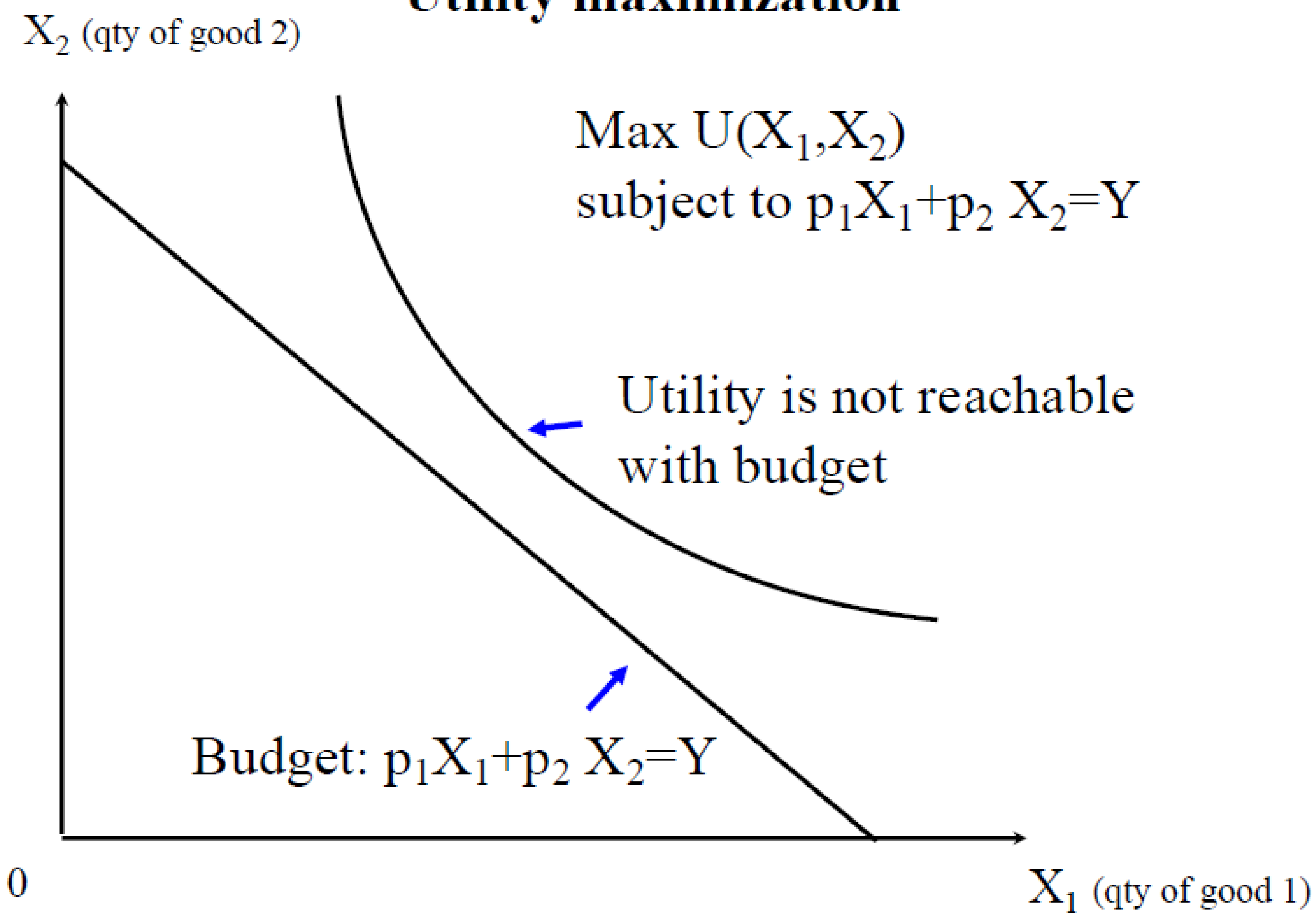
$$MRS_{1,2} = \frac{p_1}{p_2}$$

Utility maximization

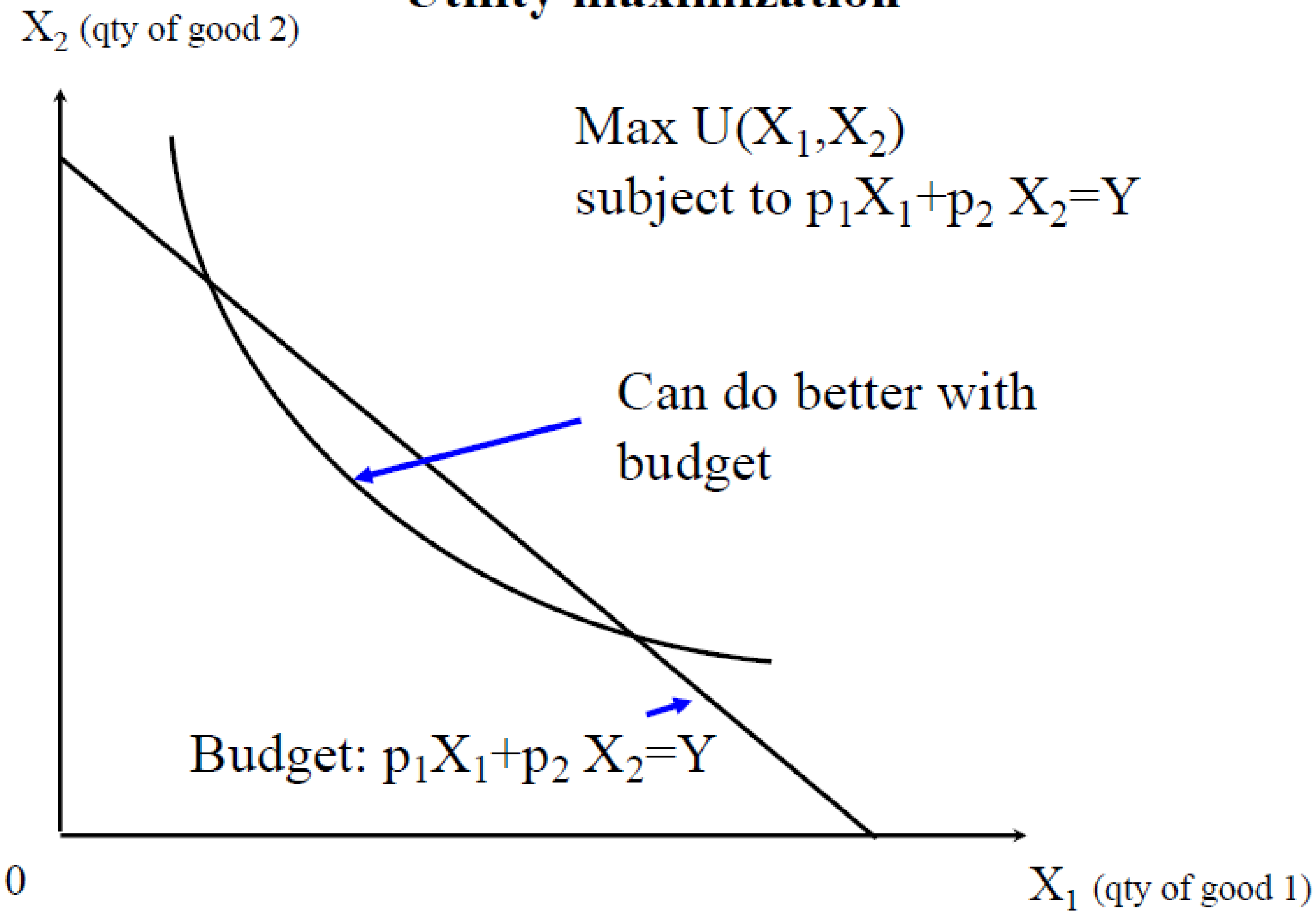
Max $U(X_1,X_2)$
subject to $p_1X_1+p_2 X_2=Y$



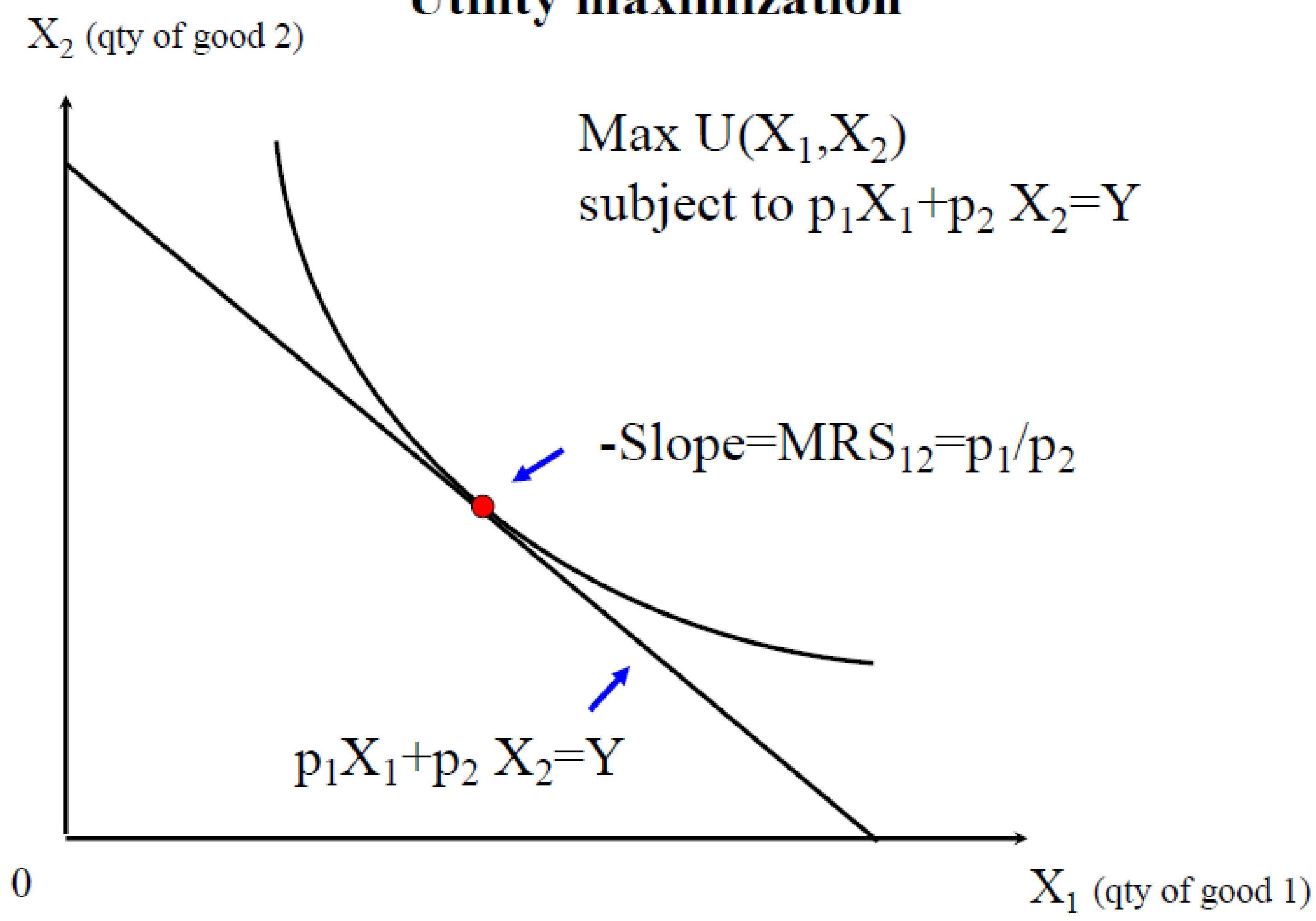
Utility maximization



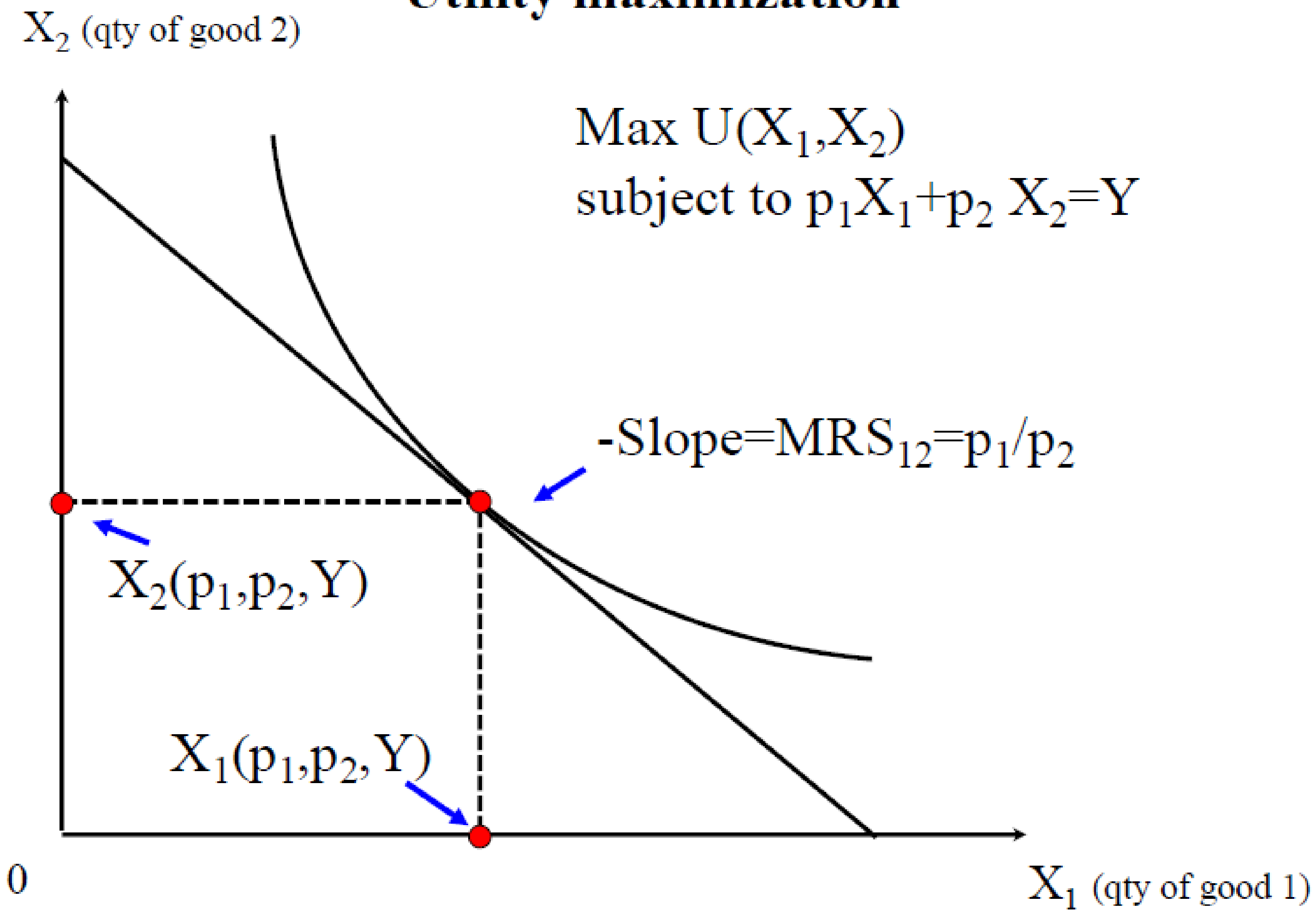
Utility maximization



Utility maximization



Utility maximization



Income and Substitution Effects

Maximization problem in slide 30 generates the following demand functions:

$$X_1(p \quad, Y)$$

$$X_2(p \quad, Y)$$

Demands for goods X_1 and X_2 are functions of price and income.

Income and Substitution Effects

Using the following functional form:

$$U(X_1, X_2) = \sqrt{X_1 \times X_2}$$

The solution of the maximization problem and the budget constraint:

$$MRS_{1,2} = \frac{X_2}{X_1} = \frac{p_1}{p_2}$$

The demand functions are:

$$X_1(p \quad , Y) = \frac{Y}{2p_1}$$

$$X_2(p \quad , Y) = \frac{Y}{2p_2}$$

Changes in Income

Suppose that the income of the individual increases by $\Delta Y > 0$:

The budget constraint changes from:

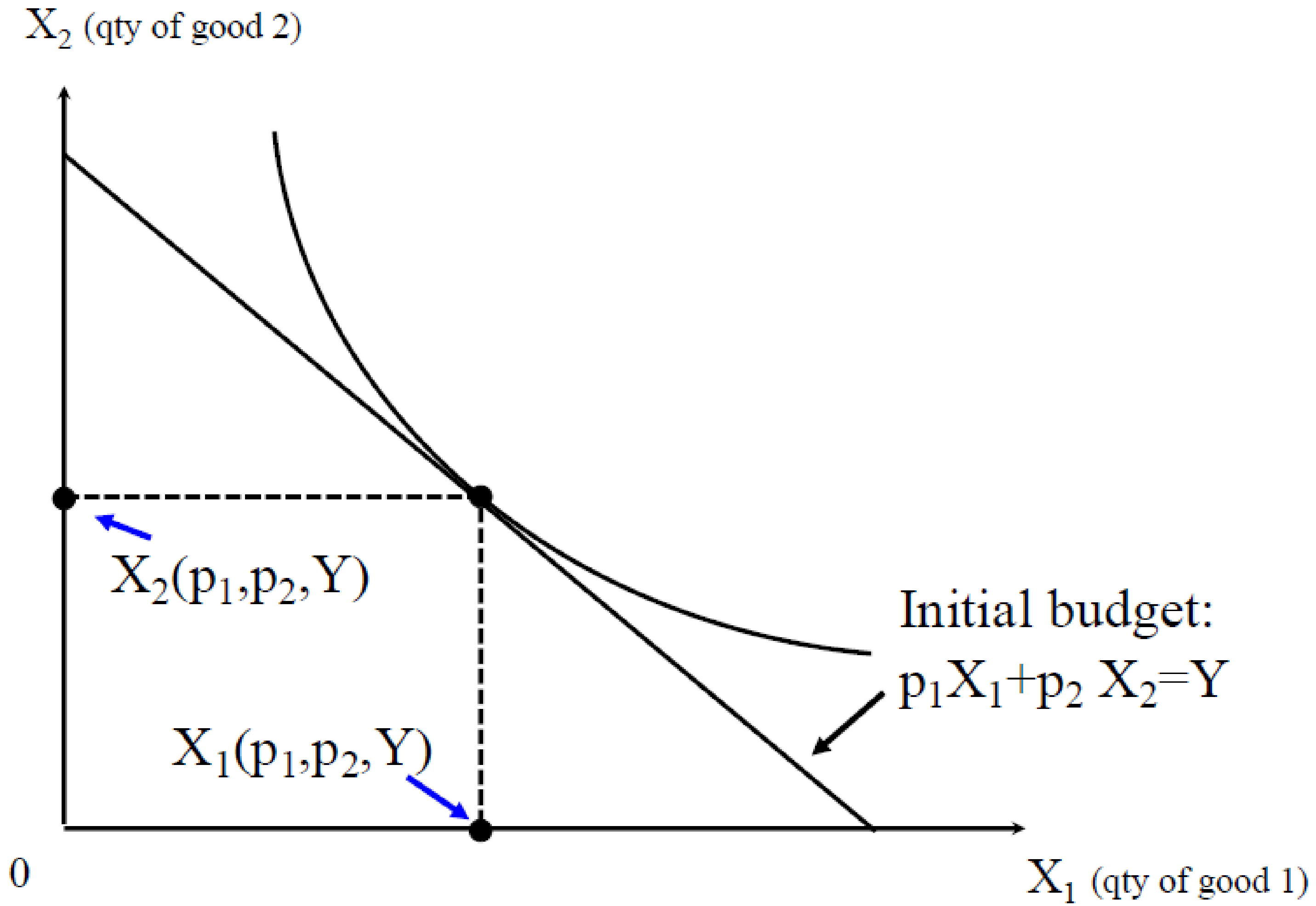
$$Y = p_1 X_1 + p_2 X_2$$

to:

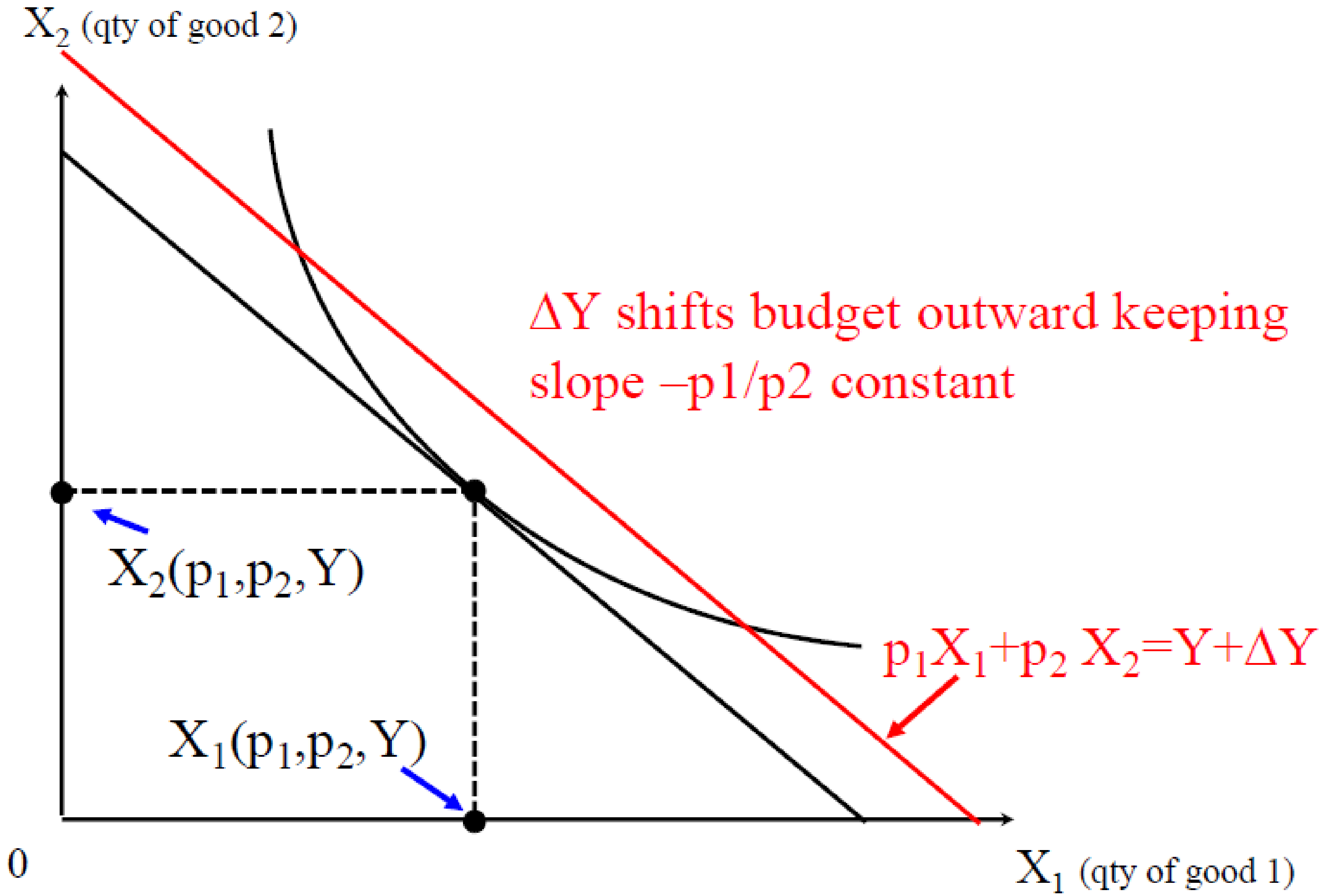
$$Y + \Delta Y = p_1 X_1 + p_2 X_2$$

How will this change the optimization problem?

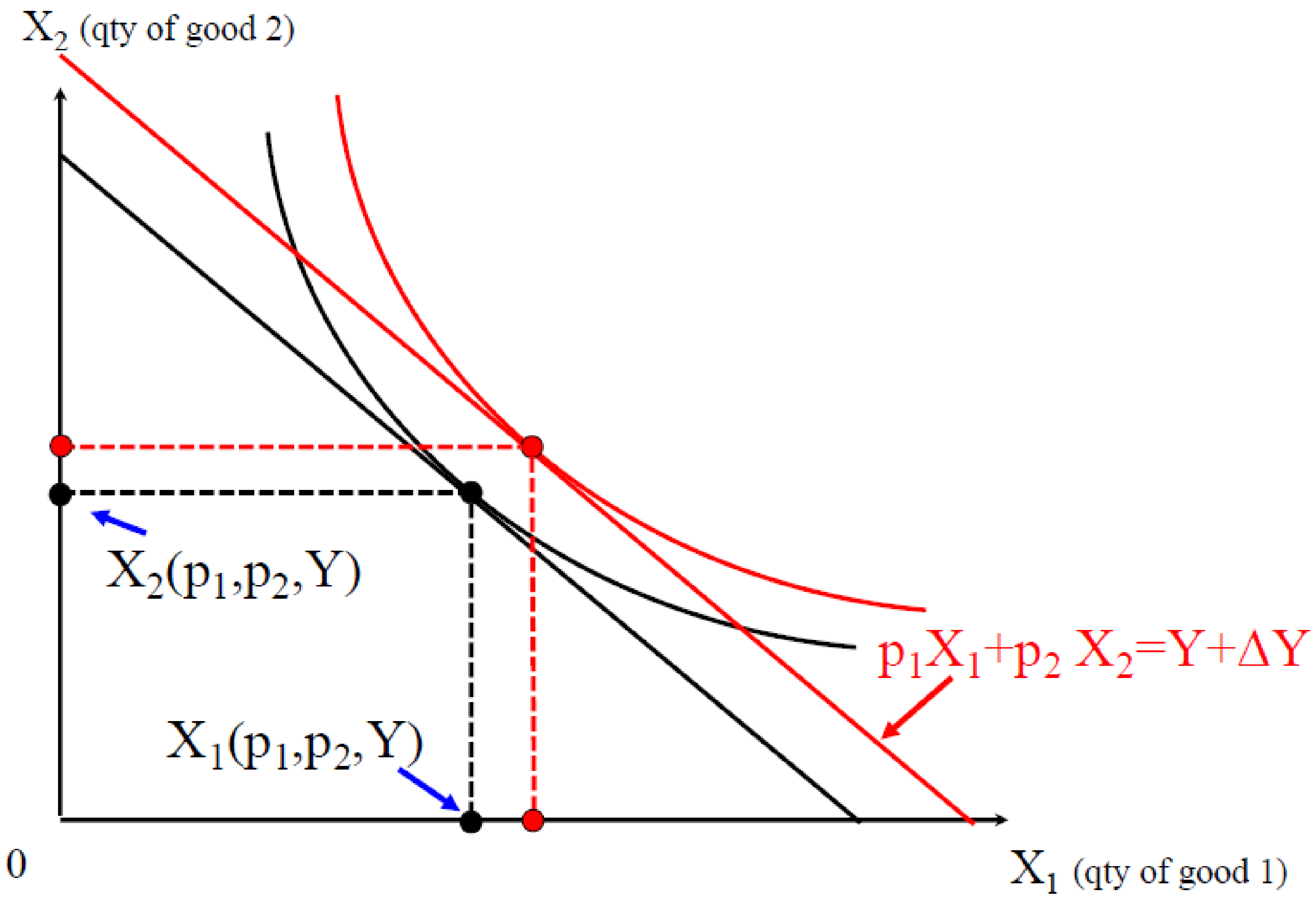
Income Effects: Y increases to Y+ΔY



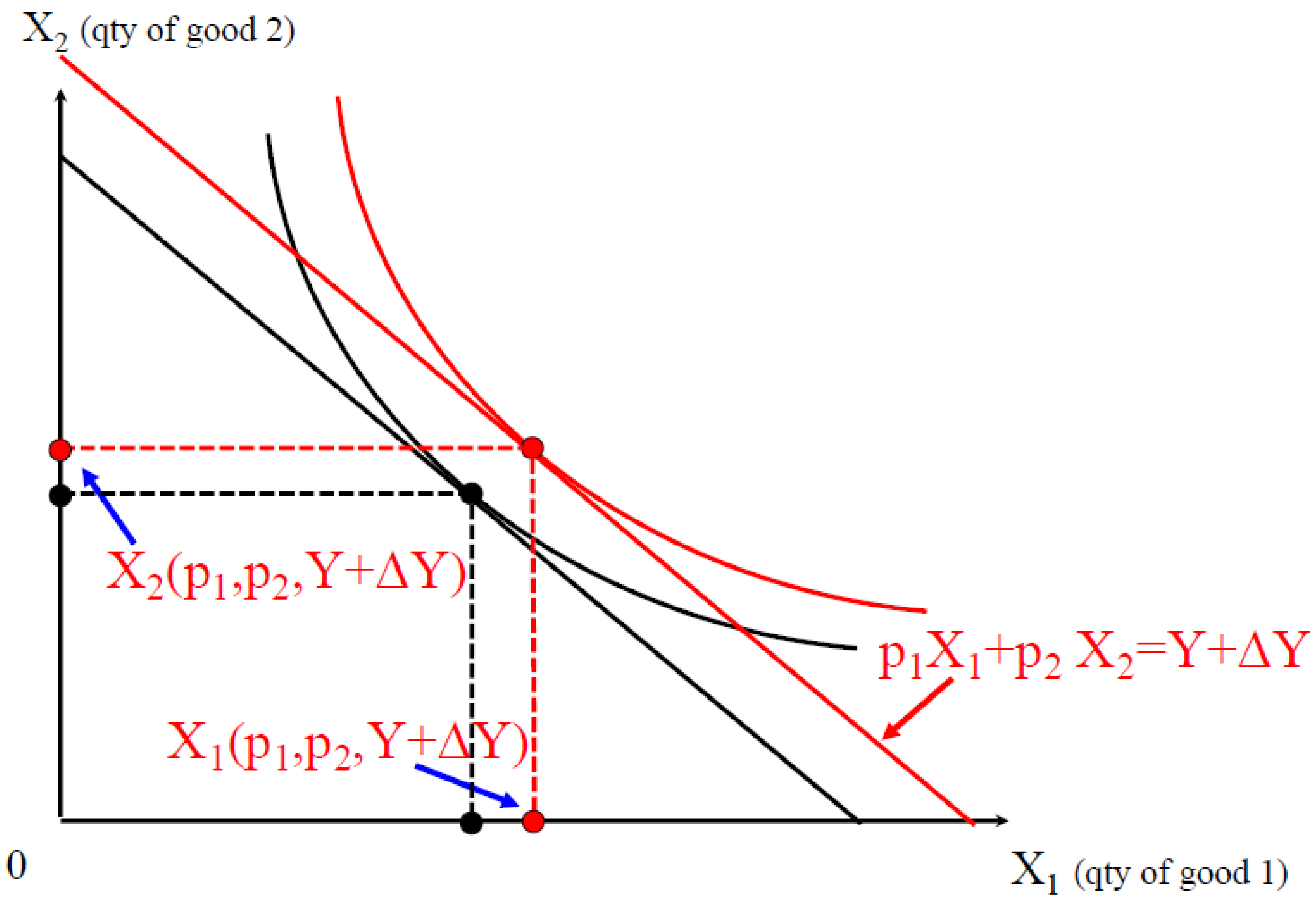
Income Effects: Y increases to Y+ΔY



Income Effects: Y increases to Y+ΔY



Income Effects: Y increases to Y+ΔY



Income effects and type of goods

How does the demand for good X_1 vary with Y?

Normal goods: Goods for which demand increases as income rises

- Y ↑
- X_1 ↑

Income effects and type of goods

How does the demand for good X_1 vary with Y?

Inferior goods: Goods for which demand falls as income rises

- Y ↑
- X_1 ↓

Change in prices

Suppose that p_1 increases by $\Delta p_1 > 0$:

The budget constraint changes from:

$$Y = p_1 X_1 + p_2 X_2$$

to:

$$Y = (p_1 + \Delta p_1) X_1 + p_2 X_2$$

How will this change the optimization problem?

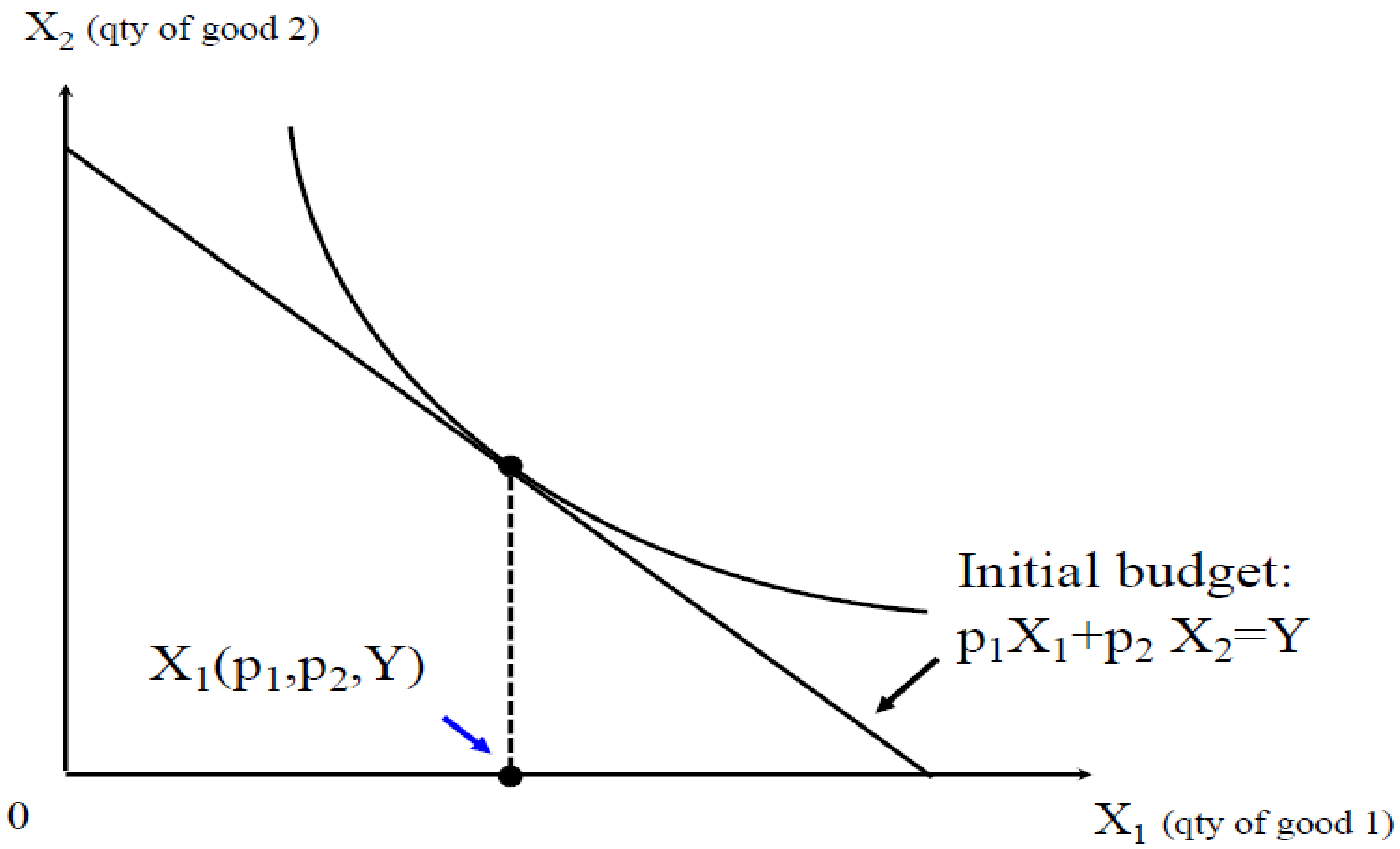
Change in prices

A change in p_1 affects the slope of the budget constraint and can be decomposed into two effects:

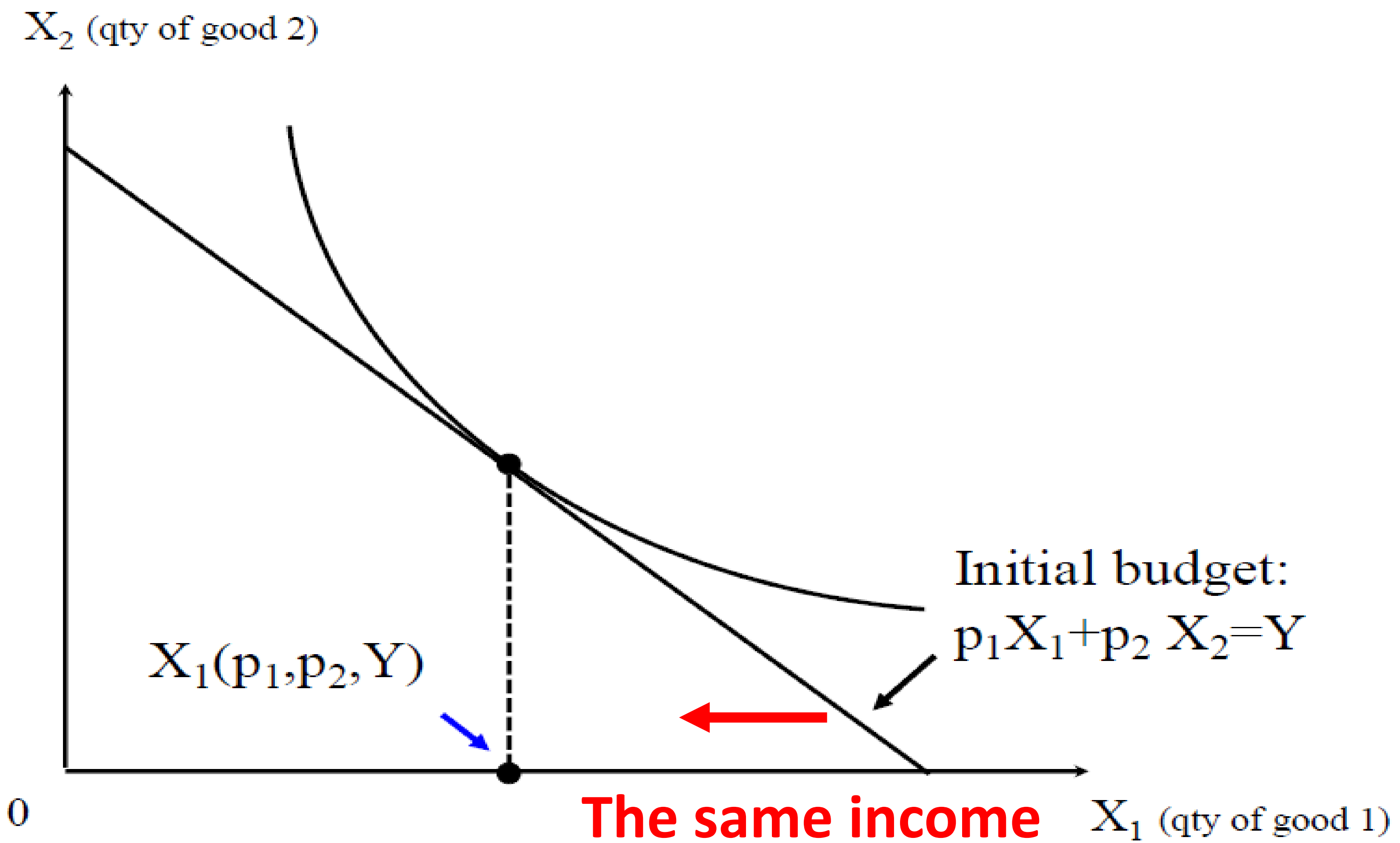
1) **Substitution effect:** **Holding utility constant**, a relative rise in the price of a good will always cause an individual to choose less of that good.

2) **Income effect:** A rise in the price of a good will typically cause an individual to choose less of all goods because her income can purchase less than before.

Price Effects: p_1 increases to $p_1+\Delta p_1$

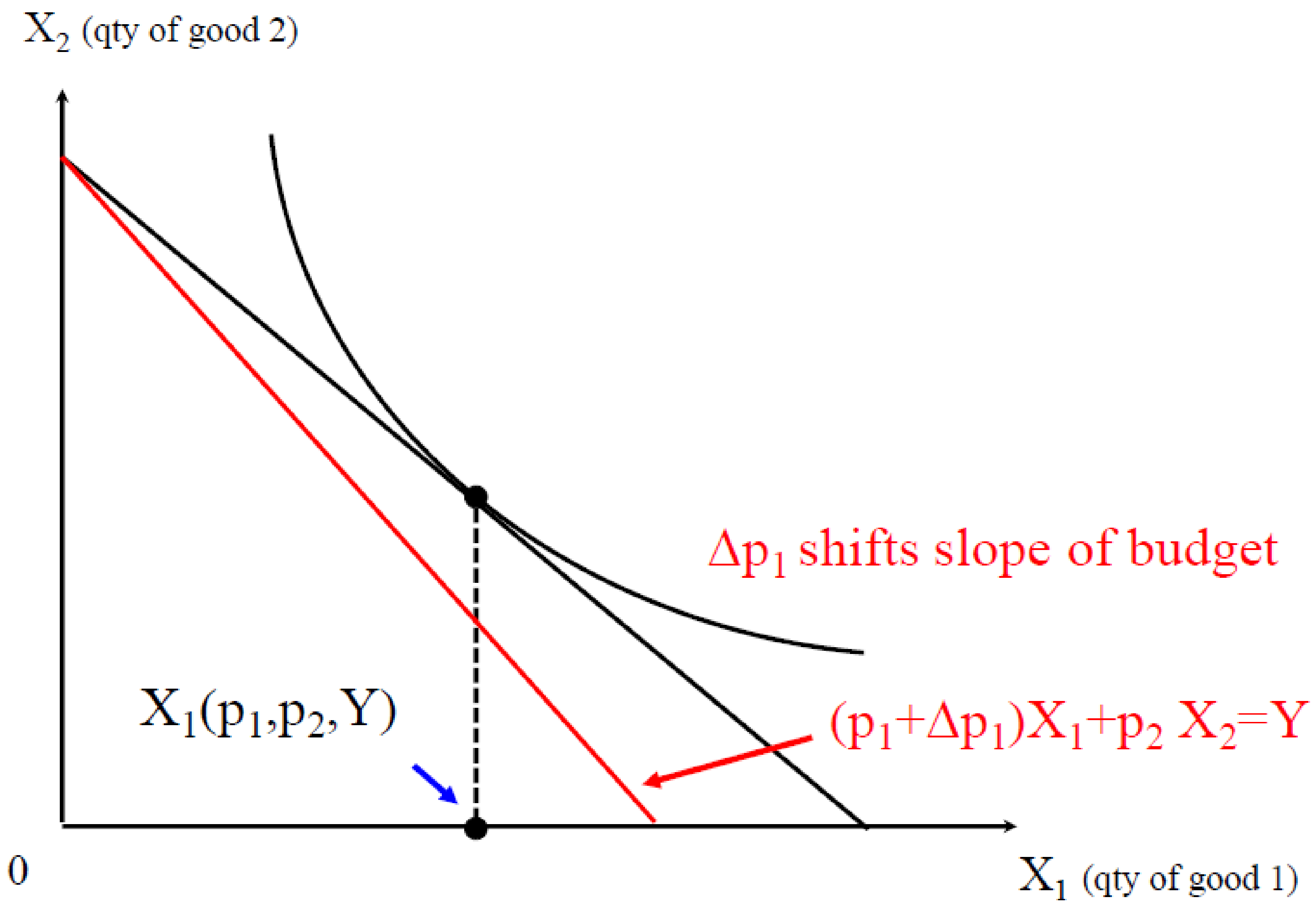


Price Effects: p_1 increases to $p_1+\Delta p_1$

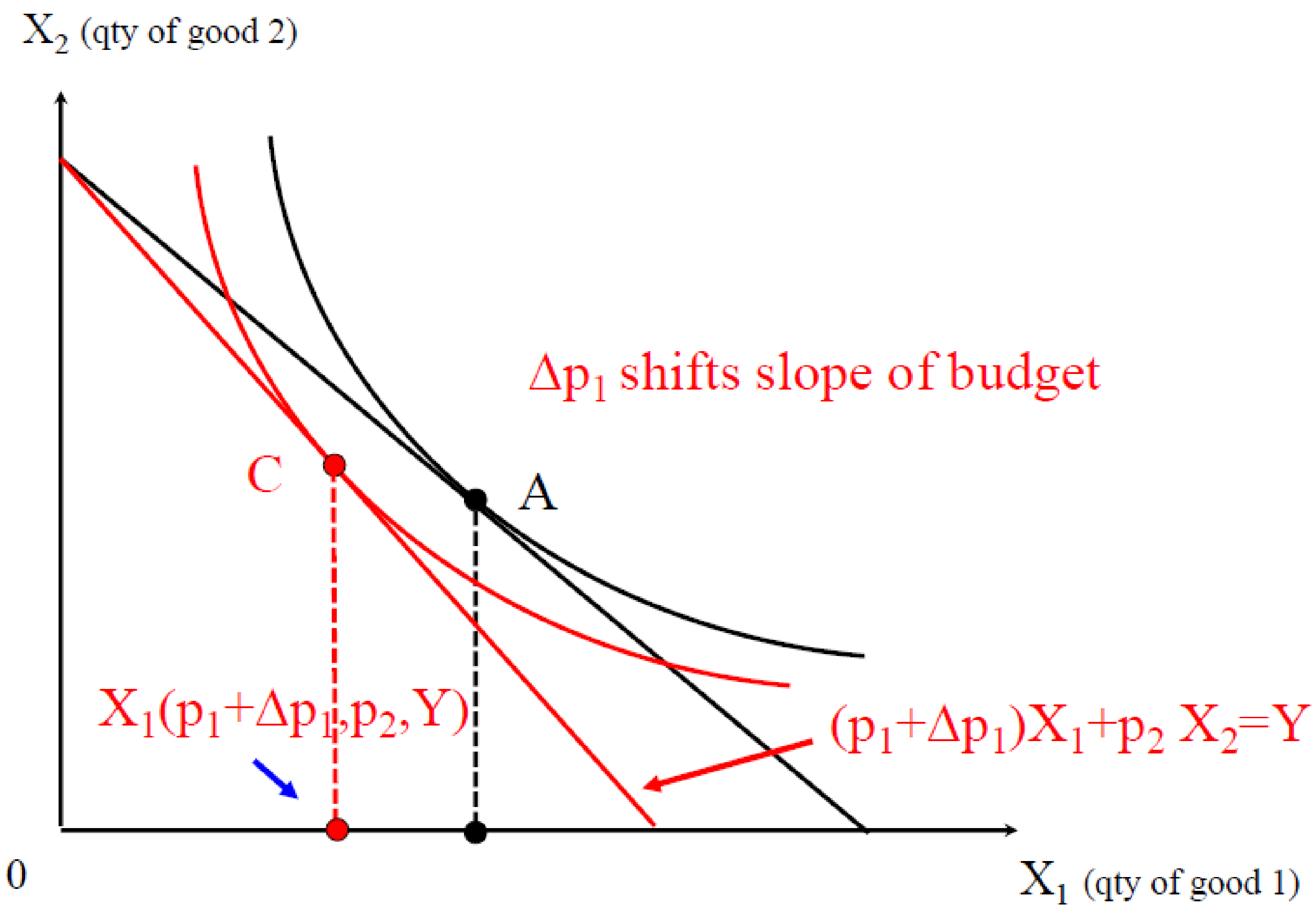


The same income buys less from good 1

Price Effects: p_1 increases to $p_1+\Delta p_1$



Price Effects: p_1 increases to $p_1+\Delta p_1$



Price Effects: p_1 increases to $p_1+\Delta p_1$